# Computing Intrinsic Link Symmetries of Hyperbolic Links

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**Abstract**

We describe a method to compute intrinsic symmetries of hyperbolic links in $S^3$ using the software package `SnapPy`.

Jeff Weeks' wonderful application `SnapPea` [Wee05] provides functionality for computing various invariants of hyperbolic manifolds. In particular, `SnapPea` can compute the mapping class group of the complement of a hyperbolic link in $S^3$, denoted $\mathrm{MCG}(S^3 \setminus L)$. My interest is in the intrinsic symmetry group of a link which can be defined as the image of the projection $\mathrm{MCG}(S^3, L) \to \mathrm{MCG}(S^3) \times \mathrm{MCG}(L)$. Thus, if we know $\mathrm{MCG}(S^3, L)$ we can compute the intrinsic symmetry group of $L$. We will describe a method for performing this calculation.

The group $\mathrm{MCG}(S^3, L)$ includes into $\mathrm{MCG}(S^3 \setminus L)$ as the maps on $S^3 \setminus L$ which extend through the missing tori. This can be made precise using the following lemma.

**Lemma 1.** *A map in* $\mathrm{Aut}(S^3 \setminus L)$ *extends to all of* $S^3$ *if and only if it sends meridians of the boundary tori to meridians. Moreover, any two such extensions are isotopic.*

Computing the image of the projection $\mathrm{MCG}(S^3, L) \to \mathrm{MCG}(S^3) \times \mathrm{MCG}(L)$ amounts to knowing the effect of the map on the knots sitting as the cores of the boundary tori. The group $\mathrm{MCG}(S^3) \times \mathrm{MCG}(L)$ is isomorphic to $\Gamma_\mu := \mathbf{Z}_2^{\mu+1} \rtimes S_\mu$ where $\mu$ is the number of components of $L$. An element $\gamma = (\epsilon_0, \epsilon_1, ..., \epsilon_\mu, p) \in \mathbf{Z}_2^{\mu+1} \rtimes S_\mu$ can be interpreted as an element of $\mathrm{MCG}(S^3) \times \mathrm{MCG}(L)$ using the following correspondence.

$$\epsilon_0 = \begin{cases} -1, & \text{if } \gamma \text{ mirrors } L \\ +1, & \text{if } \gamma \text{ does not mirror } L \end{cases}$$

and

$$\epsilon_i = \begin{cases} -1, & \text{if } \gamma \text{ reverses the orientation of the } i^{th} \text{ component of } L \\ +1, & \text{if } \gamma \text{ does not reverse the orientation of the } i^{th} \text{ component of } L \end{cases}$$

Lastly, let $p \in S_\mu$ record how the boundary tori were permuted.

Marc Culler and Nathan Dunfield have written a front-end for `SnapPea` in the scripting language `Python`. Since `SnapPea` computes $\mathrm{MCG}(S^3 \setminus L)$ we must first know which of these maps extend to $S^3$. Luckily `SnapPy` stores this informations, so this is easy to check. Given a map in $\mathrm{MCG}(S^3 \setminus L)$ we must now find its image in $\mathbf{Z}_2^{\mu+1} \rtimes S_\mu$. `SnapPy` stores maps on the boundary tori as a collection of matrices giving the images of the meridian and longitudes along with a permutation element that records how the tori were permuted. This permutation element is the one we are after, so we must now compute the effect of the map on the orientations on $S^3$ and the components of the link. This information can be extracted from the matrices representing the restriction of our map to the boundary tori. The possible matrices and their effect on the core knots and $S^3$ are summarized in Figure 1.

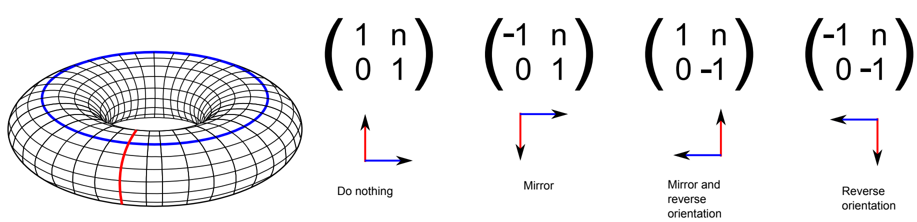


Figure 1: Torus image [Mra07]

Note that if the restriction to one of the boundary tori is orientation reversing that the map on the ambient $S^3$ is orientation reversing. There is no need to check well-definedness as these matrices came from restricting a single map on $S^3$ to the boundary tori.

Now that we can compute the projection $\mathrm{MCG}(S^3, L) \to \mathrm{MCG}(S^3) \times \mathrm{MCG}(L)$ we can simply iterate through the link complement tables provided with `SnapPy` to compute the intrinsic symmetry groups.

To carry out the computation described here I implemented the algebra of $\Gamma_\mu$ in `Python`.

I then wrote a script from within `SnapPea` which extracts the needed information about the maps on the boundary tori, converts them to a format usable by my code, and then ensures that the listed elements actually form a subgroup of $\Gamma_\mu$. The implementation of the algebra of $\Gamma_\mu$ is around 600 lines of code and the script to interface with `SnapPy` is about 75 lines.

Below you will find the output of the code which computes the symmetry subgroups for all hyperbolic 3 component links through 10 crossings.

```
Intrinsic symmetry groups as calculated by SnapPea



Out of 109 links SnapPea could not calcuate the groups for the following 8 links:
6^3_3(0,0)(0,0)(0,0)
8^3_7(0,0)(0,0)(0,0)
8^3_10(0,0)(0,0)(0,0)
9^3_21(0,0)(0,0)(0,0)
10^3_28(0,0)(0,0)(0,0)
10^3_66(0,0)(0,0)(0,0)
10^3_70(0,0)(0,0)(0,0)
10^3_71(0,0)(0,0)(0,0)
----------------------------------------



6^3_1(0,0)(0,0)(0,0)

Groups is order  12 .
(1,1,1,1,e)
(1,1,1,1,(13))
(1,1,1,-1,(12))
(1,1,1,-1,(123))
(1,1,-1,-1,(23))
(1,1,-1,-1,(132))
(1,-1,1,1,(23))
(1,-1,1,1,(132))
(1,-1,-1,1,(12))
(1,-1,-1,1,(123))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(13))
```

```
---------------------------------------
6^3_2(0,0)(0,0)(0,0)

Groups is order  48 .
(1,1,1,1,e)
(1,1,1,1,(123))
(1,1,1,1,(132))
(1,1,1,-1,(23))
(1,1,1,-1,(12))
(1,1,1,-1,(13))
(1,1,-1,1,(23))
(1,1,-1,1,(12))
(1,1,-1,1,(13))
(1,1,-1,-1,e)
(1,1,-1,-1,(123))
(1,1,-1,-1,(132))
(1,-1,1,1,(23))
(1,-1,1,1,(12))
(1,-1,1,1,(13))
(1,-1,1,-1,e)
(1,-1,1,-1,(123))
(1,-1,1,-1,(132))
(1,-1,-1,1,e)
(1,-1,-1,1,(123))
(1,-1,-1,1,(132))
(1,-1,-1,-1,(23))
(1,-1,-1,-1,(12))
(1,-1,-1,-1,(13))
(-1,1,1,1,(23))
(-1,1,1,1,(12))
(-1,1,1,1,(13))
(-1,1,1,-1,e)
(-1,1,1,-1,(123))
(-1,1,1,-1,(132))
(-1,1,-1,1,e)
(-1,1,-1,1,(123))
(-1,1,-1,1,(132))
(-1,1,-1,-1,(23))
(-1,1,-1,-1,(12))
(-1,1,-1,-1,(13))
```

```
(-1,-1,1,1,e)
(-1,-1,1,1,(123))
(-1,-1,1,1,(132))
(-1,-1,1,-1,(23))
(-1,-1,1,-1,(12))
(-1,-1,1,-1,(13))
(-1,-1,-1,1,(23))
(-1,-1,-1,1,(12))
(-1,-1,-1,1,(13))
(-1,-1,-1,-1,e)
(-1,-1,-1,-1,(123))
(-1,-1,-1,-1,(132))
----------------------------------------
6^3_3(0,0)(0,0)(0,0)

SnapPea was not able to compute the symmetry group for this link.
It is likely that  6^3_3(0,0)(0,0)(0,0)  is not hyperbolic.

----------------------------------------
7^3_1(0,0)(0,0)(0,0)

Groups is order  12 .
(1,1,1,1,e)
(1,1,1,1,(23))
(1,1,1,1,(12))
(1,1,1,1,(123))
(1,1,1,1,(132))
(1,1,1,1,(13))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(23))
(1,-1,-1,-1,(12))
(1,-1,-1,-1,(123))
(1,-1,-1,-1,(132))
(1,-1,-1,-1,(13))
----------------------------------------
8^3_1(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,-1,-1,(23))
```

```
(1,-1,1,1,(23))
(1,-1,-1,-1,e)
----------------------------------------
8^3_2(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,-1,(12))
(1,-1,-1,1,(12))
(1,-1,-1,-1,e)
----------------------------------------
8^3_3(0,0)(0,0)(0,0)

Groups is order  12 .
(1,1,1,1,e)
(1,1,1,1,(23))
(1,1,1,-1,(12))
(1,1,1,-1,(132))
(1,1,-1,1,(123))
(1,1,-1,1,(13))
(1,-1,1,-1,(123))
(1,-1,1,-1,(13))
(1,-1,-1,1,(12))
(1,-1,-1,1,(132))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(23))
----------------------------------------
8^3_4(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,-1,-1,-1,e)
(-1,1,-1,-1,(23))
(-1,-1,1,1,(23))
----------------------------------------
8^3_5(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,-1,(12))
```

```
(1,-1,-1,1,e)
(1,-1,-1,-1,(12))
----------------------------------------
8^3_6(0,0)(0,0)(0,0)

Groups is order  8 .
(1,1,1,1,e)
(1,1,1,1,(12))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(12))
(-1,1,1,-1,e)
(-1,1,1,-1,(12))
(-1,-1,-1,1,e)
(-1,-1,-1,1,(12))
----------------------------------------
8^3_7(0,0)(0,0)(0,0)

SnapPea was not able to compute the symmetry group for this link.
It is likely that  8^3_7(0,0)(0,0)(0,0)  is not hyperbolic.

----------------------------------------
8^3_8(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,1,(12))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(12))
----------------------------------------
8^3_9(0,0)(0,0)(0,0)

Groups is order  8 .
(1,1,1,1,e)
(1,1,1,1,(12))
(1,1,1,-1,e)
(1,1,1,-1,(12))
(1,-1,-1,1,e)
(1,-1,-1,1,(12))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(12))
```

```
----------------------------------------
8^3_10(0,0)(0,0)(0,0)

SnapPea was not able to compute the symmetry group for this link.
It is likely that  8^3_10(0,0)(0,0)(0,0)  is not hyperbolic.

----------------------------------------
9^3_1(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,-1,(12))
(1,-1,-1,1,(12))
(1,-1,-1,-1,e)
----------------------------------------
9^3_2(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,-1,(12))
(1,-1,-1,1,(12))
(1,-1,-1,-1,e)
----------------------------------------
9^3_3(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,1,(23))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(23))
----------------------------------------
9^3_4(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,1,(23))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(23))
----------------------------------------
9^3_5(0,0)(0,0)(0,0)
```

```
Groups is order  4 .
(1,1,1,1,e)
(1,1,1,1,(12))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(12))
----------------------------------------
9^3_6(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,-1,-1,(23))
(1,-1,1,1,(23))
(1,-1,-1,-1,e)
----------------------------------------
9^3_7(0,0)(0,0)(0,0)

Groups is order  12 .
(1,1,1,1,e)
(1,1,1,1,(23))
(1,1,1,1,(12))
(1,1,1,1,(123))
(1,1,1,1,(132))
(1,1,1,1,(13))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(23))
(1,-1,-1,-1,(12))
(1,-1,-1,-1,(123))
(1,-1,-1,-1,(132))
(1,-1,-1,-1,(13))
----------------------------------------
9^3_8(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,-1,-1,e)
(1,-1,1,1,e)
(1,-1,-1,-1,e)
----------------------------------------
9^3_9(0,0)(0,0)(0,0)
```

```
Groups is order  8 .
(1,1,1,1,e)
(1,1,1,-1,(23))
(1,1,-1,1,(23))
(1,1,-1,-1,e)
(1,-1,1,1,(23))
(1,-1,1,-1,e)
(1,-1,-1,1,e)
(1,-1,-1,-1,(23))
----------------------------------------
9^3_10(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,-1,(12))
(1,-1,-1,1,e)
(1,-1,-1,-1,(12))
----------------------------------------
9^3_11(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,1,(12))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(12))
----------------------------------------
9^3_12(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,-1,1,e)
(1,-1,1,-1,e)
(1,-1,-1,-1,e)
----------------------------------------
9^3_13(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,1,(12))
```

```
(1,-1,-1,-1,e)
(1,-1,-1,-1,(12))
----------------------------------------
9^3_14(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,1,(12))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(12))
----------------------------------------
9^3_15(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,-1,-1,(23))
(1,-1,1,1,(23))
(1,-1,-1,-1,e)
----------------------------------------
9^3_16(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,1,(23))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(23))
----------------------------------------
9^3_17(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,-1,-1,(23))
(1,-1,1,1,(23))
(1,-1,-1,-1,e)
----------------------------------------
9^3_18(0,0)(0,0)(0,0)

Groups is order  8 .
(1,1,1,1,e)
(1,1,1,-1,(12))
```

```
(1,1,-1,1,(12))
(1,1,-1,-1,e)
(1,-1,1,1,(12))
(1,-1,1,-1,e)
(1,-1,-1,1,e)
(1,-1,-1,-1,(12))
----------------------------------------
9^3_19(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,-1,-1,e)
(1,-1,1,1,e)
(1,-1,-1,-1,e)
----------------------------------------
9^3_20(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,-1,e)
(1,-1,-1,1,e)
(1,-1,-1,-1,e)
----------------------------------------
9^3_21(0,0)(0,0)(0,0)

SnapPea was not able to compute the symmetry group for this link.
It is likely that  9^3_21(0,0)(0,0)(0,0)  is not hyperbolic.

----------------------------------------
10^3_1(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,1,(23))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(23))
----------------------------------------
10^3_2(0,0)(0,0)(0,0)

Groups is order  4 .
```

```
(1,1,1,1,e)
(1,1,1,1,(23))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(23))
----------------------------------------
10^3_3(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,-1,e)
(1,-1,-1,1,e)
(1,-1,-1,-1,e)
----------------------------------------
10^3_4(0,0)(0,0)(0,0)

Groups is order  2 .
(1,1,1,1,e)
(1,-1,-1,-1,e)
----------------------------------------
10^3_5(0,0)(0,0)(0,0)

Groups is order  8 .
(1,1,1,1,e)
(1,1,-1,-1,e)
(1,-1,1,-1,e)
(1,-1,-1,1,e)
(-1,1,1,1,(13))
(-1,1,-1,-1,(13))
(-1,-1,1,-1,(13))
(-1,-1,-1,1,(13))
----------------------------------------
10^3_6(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,-1,1,e)
(1,-1,1,-1,e)
(1,-1,-1,-1,e)
----------------------------------------
10^3_7(0,0)(0,0)(0,0)
```

```
Groups is order  2 .
(1,1,1,1,e)
(1,-1,-1,-1,e)
----------------------------------------
10^3_8(0,0)(0,0)(0,0)

Groups is order  8 .
(1,1,1,1,e)
(1,1,1,1,(12))
(1,1,1,-1,e)
(1,1,1,-1,(12))
(1,-1,-1,1,e)
(1,-1,-1,1,(12))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(12))
----------------------------------------
10^3_9(0,0)(0,0)(0,0)

Groups is order  2 .
(1,1,1,1,e)
(1,-1,-1,-1,e)
----------------------------------------
10^3_10(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,1,(13))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(13))
----------------------------------------
10^3_11(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,-1,1,(13))
(1,-1,1,-1,(13))
(1,-1,-1,-1,e)
----------------------------------------
10^3_12(0,0)(0,0)(0,0)
```

```
Groups is order  4 .
(1,1,1,1,e)
(1,1,1,1,(13))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(13))
----------------------------------------
10^3_13(0,0)(0,0)(0,0)

Groups is order  2 .
(1,1,1,1,e)
(1,1,-1,1,(13))
----------------------------------------
10^3_14(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,-1,(12))
(1,-1,-1,1,(12))
(1,-1,-1,-1,e)
----------------------------------------
10^3_15(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,1,(12))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(12))
----------------------------------------
10^3_16(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,-1,(12))
(1,-1,-1,1,(12))
(1,-1,-1,-1,e)
----------------------------------------
10^3_17(0,0)(0,0)(0,0)

Groups is order  2 .
```

```
(1,1,1,1,e)
(1,-1,-1,-1,e)
----------------------------------------
10^3_18(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,-1,(12))
(1,-1,-1,1,e)
(1,-1,-1,-1,(12))
----------------------------------------
10^3_19(0,0)(0,0)(0,0)

Groups is order  2 .
(1,1,1,1,e)
(1,-1,-1,-1,e)
----------------------------------------
10^3_20(0,0)(0,0)(0,0)

Groups is order  2 .
(1,1,1,1,e)
(1,-1,-1,-1,e)
----------------------------------------
10^3_21(0,0)(0,0)(0,0)

Groups is order  2 .
(1,1,1,1,e)
(1,-1,-1,-1,e)
----------------------------------------
10^3_22(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,-1,(12))
(1,-1,-1,1,e)
(1,-1,-1,-1,(12))
----------------------------------------
10^3_23(0,0)(0,0)(0,0)

Groups is order  2 .
```

```
(1,1,1,1,e)
(1,-1,-1,-1,e)
----------------------------------------
10^3_24(0,0)(0,0)(0,0)

Groups is order  2 .
(1,1,1,1,e)
(1,-1,-1,-1,e)
----------------------------------------
10^3_25(0,0)(0,0)(0,0)

Groups is order  2 .
(1,1,1,1,e)
(1,-1,-1,-1,e)
----------------------------------------
10^3_26(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,-1,e)
(1,-1,-1,1,e)
(1,-1,-1,-1,e)
----------------------------------------
10^3_27(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,-1,1,e)
(1,-1,1,-1,e)
(1,-1,-1,-1,e)
----------------------------------------
10^3_28(0,0)(0,0)(0,0)

SnapPea was not able to compute the symmetry group for this link.
It is likely that  10^3_28(0,0)(0,0)(0,0)  is not hyperbolic.

----------------------------------------
10^3_29(0,0)(0,0)(0,0)

Groups is order  1 .
```

```
(1,1,1,1,e)
----------------------------------------
10^3_30(0,0)(0,0)(0,0)

Groups is order  8 .
(1,1,1,1,e)
(1,1,1,1,(12))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(12))
(-1,1,1,-1,e)
(-1,1,1,-1,(12))
(-1,-1,-1,1,e)
(-1,-1,-1,1,(12))
----------------------------------------
10^3_31(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,-1,(12))
(1,-1,-1,1,e)
(1,-1,-1,-1,(12))
----------------------------------------
10^3_32(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,1,(12))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(12))
----------------------------------------
10^3_33(0,0)(0,0)(0,0)

Groups is order  2 .
(1,1,1,1,e)
(1,1,-1,1,(13))
----------------------------------------
10^3_34(0,0)(0,0)(0,0)

Groups is order  2 .
(1,1,1,1,e)
```

```
(-1,1,1,1,(13))
----------------------------------------
10^3_35(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,-1,(12))
(1,-1,-1,1,(12))
(1,-1,-1,-1,e)
----------------------------------------
10^3_36(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,-1,(12))
(1,-1,-1,1,e)
(1,-1,-1,-1,(12))
----------------------------------------
10^3_37(0,0)(0,0)(0,0)

Groups is order  8 .
(1,1,1,1,e)
(1,1,1,-1,(12))
(1,1,-1,1,(12))
(1,1,-1,-1,e)
(1,-1,1,1,(12))
(1,-1,1,-1,e)
(1,-1,-1,1,e)
(1,-1,-1,-1,(12))
----------------------------------------
10^3_38(0,0)(0,0)(0,0)

Groups is order  8 .
(1,1,1,1,e)
(1,1,1,-1,(12))
(1,-1,-1,1,e)
(1,-1,-1,-1,(12))
(-1,1,1,1,(12))
(-1,1,1,-1,e)
(-1,-1,-1,1,(12))
```

```
(-1,-1,-1,-1,e)
----------------------------------------
10^3_39(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,-1,(12))
(1,-1,-1,1,(12))
(1,-1,-1,-1,e)
----------------------------------------
10^3_40(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,-1,(12))
(1,-1,-1,1,e)
(1,-1,-1,-1,(12))
----------------------------------------
10^3_41(0,0)(0,0)(0,0)

Groups is order  12 .
(1,1,1,1,e)
(1,1,1,1,(13))
(1,1,1,-1,(12))
(1,1,1,-1,(123))
(1,1,-1,-1,(23))
(1,1,-1,-1,(132))
(1,-1,1,1,(23))
(1,-1,1,1,(132))
(1,-1,-1,1,(12))
(1,-1,-1,1,(123))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(13))
----------------------------------------
10^3_42(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,1,(13))
(1,-1,-1,-1,e)
```

```
(1,-1,-1,-1,(13))
----------------------------------------
10^3_43(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,1,(13))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(13))
----------------------------------------
10^3_44(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,1,(13))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(13))
----------------------------------------
10^3_45(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,1,(23))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(23))
----------------------------------------
10^3_46(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,1,(23))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(23))
----------------------------------------
10^3_47(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,1,(12))
(1,-1,-1,-1,e)
```

```
(1,-1,-1,-1,(12))
----------------------------------------
10^3_48(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,1,(13))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(13))
----------------------------------------
10^3_49(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,-1,(12))
(1,-1,-1,1,(12))
(1,-1,-1,-1,e)
----------------------------------------
10^3_50(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,-1,(12))
(1,-1,-1,1,(12))
(1,-1,-1,-1,e)
----------------------------------------
10^3_51(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,1,(12))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(12))
----------------------------------------
10^3_52(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,-1,(12))
(1,-1,-1,1,(12))
```

```
(1,-1,-1,-1,e)
---------------------------------------
10^3_53(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,1,(12))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(12))
---------------------------------------
10^3_54(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,-1,1,(13))
(1,-1,1,-1,(13))
(1,-1,-1,-1,e)
---------------------------------------
10^3_55(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,1,(12))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(12))
---------------------------------------
10^3_56(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,-1,(12))
(1,-1,-1,1,(12))
(1,-1,-1,-1,e)
---------------------------------------
10^3_57(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,-1,1,(13))
(1,-1,1,-1,(13))
```

```
(1,-1,-1,-1,e)
----------------------------------------
10^3_58(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,1,(12))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(12))
----------------------------------------
10^3_59(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,-1,(12))
(1,-1,-1,1,(12))
(1,-1,-1,-1,e)
----------------------------------------
10^3_60(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,-1,(12))
(1,-1,-1,1,(12))
(1,-1,-1,-1,e)
----------------------------------------
10^3_61(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,1,(12))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(12))
----------------------------------------
10^3_62(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,1,(23))
(1,-1,-1,-1,e)
```

```
(1,-1,-1,-1,(23))
----------------------------------------
10^3_63(0,0)(0,0)(0,0)

Groups is order  2 .
(1,1,1,1,e)
(1,-1,-1,-1,e)
----------------------------------------
10^3_64(0,0)(0,0)(0,0)

Groups is order  2 .
(1,1,1,1,e)
(1,-1,-1,-1,e)
----------------------------------------
10^3_65(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,-1,1,(13))
(1,-1,1,-1,(13))
(1,-1,-1,-1,e)
----------------------------------------
10^3_66(0,0)(0,0)(0,0)

SnapPea was not able to compute the symmetry group for this link.
It is likely that  10^3_66(0,0)(0,0)(0,0)  is not hyperbolic.

----------------------------------------
10^3_67(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,-1,(12))
(1,-1,-1,1,(12))
(1,-1,-1,-1,e)
----------------------------------------
10^3_68(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
```

```
(1,1,1,1,(12))
(1,-1,-1,-1,e)
(1,-1,-1,-1,(12))
----------------------------------------
10^3_69(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
(1,1,1,-1,(12))
(1,-1,-1,1,(12))
(1,-1,-1,-1,e)
----------------------------------------
10^3_70(0,0)(0,0)(0,0)

SnapPea was not able to compute the symmetry group for this link.
It is likely that  10^3_70(0,0)(0,0)(0,0)  is not hyperbolic.

----------------------------------------
10^3_71(0,0)(0,0)(0,0)

SnapPea was not able to compute the symmetry group for this link.
It is likely that  10^3_71(0,0)(0,0)(0,0)  is not hyperbolic.

----------------------------------------
10^3_72(0,0)(0,0)(0,0)

Groups is order  2 .
(1,1,1,1,e)
(1,-1,-1,-1,e)
----------------------------------------
10^3_73(0,0)(0,0)(0,0)

Groups is order  2 .
(1,1,1,1,e)
(-1,1,1,1,(23))
----------------------------------------
10^3_74(0,0)(0,0)(0,0)

Groups is order  4 .
(1,1,1,1,e)
```

```
(1,1,-1,-1,(23))
(1,-1,1,1,(23))
(1,-1,-1,-1,e)
----------------------------------------
END OF OUTPUT
```

This output was generated by the following code.

```
import classdefs
import copy


for i in C:
    try:
        S.append((i.symmetry_group()).isometries())
    except ValueError:
        S.append(0)

Gens=[]

for i in S:
    if i != 0:
        gens=[]
        for j in i:
            if j.extends_to_link():
                m=1
                r=[]
                gen=[]
                for l in range(0,len(j.cusp_maps()[0].data)):
                    m=m*j.cusp_maps()[0].data[l][l]
                for k in j.cusp_maps():
                    r.append(k.data[0][0]) #this assumes the meridian corresponds to
                                                     #but this seems to be correct...


                gen.append(m)
                gen.append(r)
                gen.append(j.cusp_images())
                for k in range(0,len(gen[2])):
                    gen[2][k]=gen[2][k]+1
```

```
                if gens.count(list_to_gamma(gen))==0:
                    gens.append(list_to_gamma(gen))
        Gens.append(gens)
    else:
        Gens.append(0)

Groups=[]

for i in Gens:
    if i != 0:
        Groups.append(sub_gamma(i))
    else:
        Groups.append(0)

##the following code will print the groups to file

outfile = open(str(N)+"_comps.txt",'w')
print >>outfile, "Intrinsic symmetry groups as calculated by SnapPea"
print >>outfile, ""
print >>outfile, ""
print >>outfile, "Out of", len(C), "links SnapPea could not calcuate the groups",
print >>outfile, "for the following", S.count(0), "links:"
for i in range(0,len(C)):
    if S[i] == 0:
        print >>outfile, C[i]
print >>outfile, "----------------------------------------"
print >>outfile, ""
print >>outfile, ""

for i in range(0,len(C)):
    print >>outfile, C[i]
    print >>outfile, ""
    if Groups[i] == 0:
        print >>outfile,"SnapPea was not able to compute the symmetry group for this
        print >>outfile, "It is likely that ",C[i]," is not hyperbolic."
        print >>outfile, ""
    else:
        print >>outfile, "Group is order ",len(Groups[i]),"."
        for k in Groups[i]:
            print >>outfile, k
```

```
    print >>outfile, "------------------------------------"

print>>"END OF OUTPUT"
outfile.close()
```

## References


[Mra07] Yassine Mrabet, *Torus svg file*, Wikipedia Commons, Wikipedia Foundation, 2007.

[Wee05] J. R. Weeks, *Computations of hyperbolic structures in knot theory*, Handbook of Knot Theory, 2005.