Games and Boolean Models
Solved Exercises

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These solutions are un-official, student-made, not checked, and probably completely wrong. Use your head!
Fellow students, it's a common interest game: if you spot an error in this paper, send me an e-mail at davidecristiani@gmail.com

Exercise 5

Exercise 5.1

Consider voting quota game $v : 2^N \longrightarrow \{0,1\}$ with weights $\omega_0 = 0.4$ as well as $\omega_1 = 0.i$ for $i \in N$ and player set $N = \{1,2,3,4\}$

Compute the Banzhaf value $\phi^{Ba}(v) = (\phi_1^{Ba}(v), ..., \phi_4^{Ba}(v))$.

We can use the Banzhaf value formula wich is:

$$\phi^{Ba}(v) = \sum_{A \subseteq N \setminus i} \frac{v(A \cup i) - v(A)}{2^{n-1}}$$

Let's analyze player 1 with $\omega_1 = 0.1$
In this case the only voting coalition that loses if player 1 swings is { 3 }

because

$v(\{3\} \cup 1) \rightarrow (1)$

$(\omega_3 + \omega_1) >= \omega_0$

$(0.3 + 0.1) >= 0.4$

so:

$$\phi_1^{Ba}(v) = \frac{v(\{3\} \cup 1) - v(\{3\})}{2^{4-1}} = \frac{1-0}{8} = \frac{1}{8}$$

Exercise 5.3

Consider simple game $v : 2^N = \{1, ..., 4\}$ and minimal winning coalitions {1,4},{2,3} and {3,4}

Identify weights $\omega_0, \omega_1, ..., \omega_4$ such as that the resulting voting quota game has Banzhaf value equal to $\phi^{Ba}(v)$

Values computated by empirical methods:

$\omega_0 = 6$

$\omega_1 = 1$

$\omega_2 = 3$

$\omega_3 = 3$

$\omega_4 = 5$

Exercise 6

Let $X = \mathbb{N}_{10} = \{1, 2, ..., 9, 10\}$ be a set of money values, with utility function $u(n) = \ln n, 1 \leq n \leq 10$. Consider two lotteries $p, q \subset \Delta_X$ defined as follows: $p(n) = \frac{8-n}{28}$ if $1 \leq n \leq 7$ and $p(n) = 0$ if $7 < n \leq 10$ while $q(n) = \frac{7-n}{21}$ if $1 \leq n \leq 6$ and $q(n) = 0$ if $6 < n \leq 10$.

Compute the vN-M expected utility of the two lotteries, i.e. $Eu(p)$ **and** $Eu(q)$

$$u(p) = \sum_{1 \leq n \leq 7} \frac{8-n}{28} \ln n$$

$$u(p) = \sum_{1 \leq n \leq 6} \frac{7-n}{21} \ln n$$

Exercise 7

For player set $N = \{1, ..., 100\}$ with binary strategy sets $\mathbb{S} = \{0, 1\}$ for all $i \in N$, every strategy profile $(s_1, ..., s_n) = s(n = 100)$ is an element of $\{0, 1\}^N$. For all players $i \in N$, defines utilities

$$u_i : 0, 1^n \to \left\{ \frac{1}{2n}, \frac{1}{2(n-1)}, ..., \frac{1}{2} \right\}$$

at any strategy profile $s = (s_i, s_{-i} \in \{0, 1\}^n$, by:

$$u_i(s_i, s_{-i}) = \frac{1}{2 \sum_{j \in N^{s_j}}} \text{ if } s_i = 1$$

$$u_i(s_i, s_{-i}) = \frac{1}{2(n - \sum_{j \in N^{s_j}})} \text{ if } s_i = 0$$

Can you find a Nash equilibrium with random strategies?

Let all the players $\backslash i$ have 50% of possibilities to choose one of the two strategies:

$$\overline{\overline{\frac{1}{2}}} = \left\{ \frac{1}{2}, \cdots \frac{1}{2} \right\} \in [0, 1]^{N-1}$$

When player i choose strategy 0 so $s_i = 0$ and the other players randomize, the expected payoff is in these values:

$$Eu_i \left(1, \overline{\overline{\frac{1}{2}}} \right) \in \left\{ \frac{1}{200}, \frac{1}{198}, .., \frac{1}{2} \right\}$$

To calculate the payoff when all players but i randomize we can use this formula where k are the players that randomly choose $s_{j \in k} = 1$ with probability p

$$\binom{n-1}{k} p^k (1-p)^{n-1-k}$$

So expected utility is:

$$Eu_i\left(1, \frac{\overline{\overline{1}}}{2}\right) = \sum_{0 \leq k \leq n} \binom{n-1}{k} \frac{1}{2^{n-1}} \frac{1}{2(k+1)}$$

$$Eu_i\left(0, \frac{\overline{\overline{1}}}{2}\right) = \sum_{0 \leq k \leq n} \binom{n-1}{k} \frac{1}{2^{n-1}} \frac{1}{2(k+1)}$$

We can see that the expected payoff is the same:

$$Eu_i\left(0, \frac{\overline{\overline{1}}}{2}\right) = Eu_i\left(1, \frac{\overline{\overline{1}}}{2}\right)$$

1. Are there dominated/dominant strategies?

No, neither $s_i = 0$ dominate $s_i = 1$ neither the opposite.
As already shown above, the expected utilities of both strategies are the same when other players use mixed strategies.

$$Eu_i\left(0, \frac{\overline{\overline{1}}}{2}\right) = Eu_i\left(1, \frac{\overline{\overline{1}}}{2}\right)$$

We must avoid the error of state that $s_i = 0$ (or $s_i = 1$) is dominated if the majority of other players choose the same strategy. That's because a strategy dominates another strategy regardless of the strategy profile of other players.

1. Are there Pareto-dominated strategy profiles?

Yes, there are.
If all players choose 0 or all players choose 1.
Because in these cases, if one player i change strategy and goes from strategy

$s_i = 0$ to $s_i = 1$ improves his payoff (from $u_i(s_i, s_{-i}) = \frac{1}{200}$ to $u_i(s_i, s_{-i}) = \frac{1}{2}$)) and also the payoff of all other players (from $u_i(s_{-i}, s_i) = \frac{1}{200}$ to $u_i(s_{-i}, s_i) = \frac{1}{198}$)

2. Are there pure strategy equilibria?

Yes, there is one.

If players are equally splitted between the two strategies, their payoff is

$$u_i(s_i, s_{-i}) = \frac{1}{100}$$

No player has an incentive to unilaterally deviate and change strategy , because his payoff will be lower:

$$u_i(s_i, s_{-i}) = \frac{1}{102}$$

3. Verify whether the profile σ_1^* ... is an equilibrium or not?

TO DO

4. Is this a common interest game?

No.

Players could coordinate their strategies to be equally split to get the equilibrium: $s_i = 0$ with $0 < i \leq 50$ and $s_j = 1$ with $51 \leq j \leq 100$. In this case $u_i = u_j = \frac{1}{98}$

But this equilibrium is not pareto efficient to a profile of strategies where $s_i = 0$ with $0 < i \leq 49$ and $s_j = 1$ with $50 \leq j \leq 100$.

In this last case, the payoff of player i is $u_i = \frac{1}{98}$ so they don't have any interest to coordinate to the equilibrium because they would got a lower payoff. There is not common interest to get the equilibrium.

4. Is it a costant-sum game?

No.

There is two strategies profile where the global payoff is different.

When all players choose the same strategy $s_i = 0$ or $s_i = 1$ with $0 < i \leq 100$, the sum of global payoff is:

$$\sum_{0<i\leq 100} \frac{1}{200} = \frac{1}{2}$$

In all other cases we have that sum of players $u_i(1)$ is $\frac{1}{2}$ and and sum of players $u_j(0)$ is $\frac{1}{2}$, so the global payoff is 1.
Without any doubt, $\frac{1}{2} \neq 1$.

Altri simboli

Inizio paragrafo

corsivo

Nota[1].

Lettere greche $\alpha + \omega = \beta + \gamma + \lambda$.

- lista

[1]"nota"