

# Sottogruppo

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## 1 Definizione

A nonempty subset,  $H$ , of a group  $G$  is called a *subgroup* of  $G$  if, relative to the product in  $G$ ,  $H$  itself forms a group.

## 2 Note

### 2.1 relative to the product in $G$

We stress the phrase "relative to the product in  $G$ ". Take, for instance, the subset  $A = \{1, -1\}$  in  $Z$ , the set of integers. Under the multiplication of integers,  $A$  is a group. But  $A$  is not a subgroup of  $Z$  viewed as a group with respect to  $+$ .

### 2.2 From intro to paragraph about Subgroup From Abstract Algebra by Herstein

In order for us to find out more about the makeup of a given group  $G$ , it may be too much of a task to tackle all of  $G$  head-on. It might be desirable to focus our attention on appropriate pieces of  $G$ , which are smaller, over which we have some control, and are such that the information gathered about them can be used to get relevant information and insight about  $G$  itself. The question then becomes: What should serve as suitable pieces for this kind of dissection of  $G$ ? Clearly, whatever we choose as such pieces, we want them to reflect the fact that  $G$  is a group, not merely any old set.

A group is distinguished from an ordinary set by the fact that it is endowed with a well-behaved operation. It is thus natural to demand that such pieces above behaved reasonably with respect to the operation of  $G$ . Once this is granted, we are led almost immediately to the concept of a subgroup of a group.