XƎLATEX

Rat nephron model review

Chanoknun Sintavanuruk

April 6, 2023

Rat nephron model review

Chanoknun Sintavanuruk

April 6, 2023

Outline

Outline

1 Overview of structures

2 Model formulation

Outline

Outline

1 Overview of structures

2 Model formulation

Compartments

1. nephrons
2. medullary vasculature
3. medullary interstitium

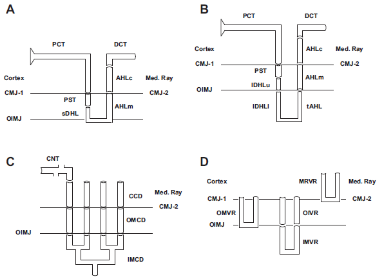
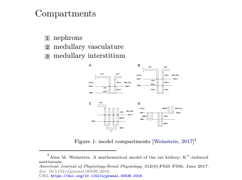


Figure 1: model compartments [Weinstein, 2017][1]

[1] Alan M. Weinstein. A mathematical model of the rat kidney: K^+-induced natriuresis.
American Journal of Physiology-Renal Physiology, 312(6):F925–F950, June 2017.
doi: 10.1152/ajprenal.00536.2016.
URL https://doi.org/10.1152/ajprenal.00536.2016



The model has 3 compartments: nephrons, medullary vasculature and medullary interstitium.

Nephrons and blood vessels are divided into multiple segments. In terms of spatial dimension, each segment has only length with the direction either going up or down the medulla.

The interstitial compartment is divided into 8 different homogeneous sub-compartments; each represents different depth and contains some segments of blood vessels and nephrons.

Tubular segments of nephrons (1/2)

1. unbranched segments
   1. Superficial (SF) nephrons
   2. Juxtamedullary (JM) nephrons
2. distal nephron ensembles: collecting ducts

Tubular segments of nephrons (1/2)

1. unbranched segments
   1. Superficial (SF) nephrons
   2. Juxtamedullary (JM) nephrons
2. distal nephron ensembles: collecting ducts

The nephron compartment is composed of many distinct tubular segments which can be grouped into unbranched segments and the distal nephron ensembles or the collecting ducts.

Tubular segments of nephrons (1/2)

1. unbranched segments
   1. Superficial (SF) nephrons
   2. Juxtamedullary (JM) nephrons
2. distal nephron ensembles: collecting ducts

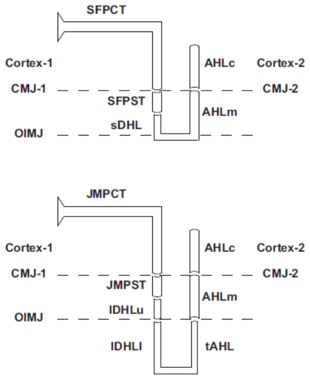
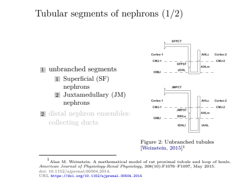


Figure 2: Unbranched tubules [Weinstein, 2015][1]

[1] Alan M. Weinstein. A mathematical model of rat proximal tubule and loop of henle. *American Journal of Physiology-Renal Physiology*, 308(10):F1076–F1097, May 2015. doi: 10.1152/ajprenal.00504.2014. URL https://doi.org/10.1152/ajprenal.00504.2014



The unbranched segments are connected in series in mainly two different ways resulting the superficial or SF and juxtamedullary or JM nephron.

These nephrons have distal segments that are identical, which are the thick ascending limbs, or AHL, in the outer medulla and the cortex. Meanwhile, the proximal convoluted tubule, or PCT, and straight tubule, or PST, of these two differ in the glomerular filtration rate, which determines the initial flow and pressure at the PCT, and ion permeabilities — which are mostly higher in the JM nephron.

For the segments in the middle, the SF nephron has only one segment of the descending Henle's limb, or DHL, at the outer medulla; while in JM nephron, DHL descends into the inner medulla in 5 different depths and comes back up via thin ascending Henle's limb, or tAHL.

Tubular structure of DHL and tAHL

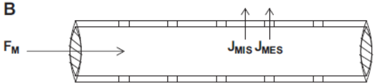
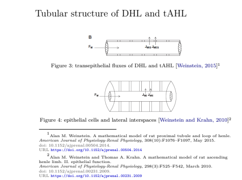


Figure 3: transepithelial fluxes of DHL and tAHL [Weinstein, 2015][1]

Figure 4: epithelial cells and lateral interspaces [Weinstein and Krahn, 2010][2]

[1] Alan M. Weinstein. A mathematical model of rat proximal tubule and loop of henle. *American Journal of Physiology-Renal Physiology*, 308(10):F1076–F1097, May 2015. doi: 10.1152/ajprenal.00504.2014. URL https://doi.org/10.1152/ajprenal.00504.2014

[2] Alan M. Weinstein and Thomas A. Krahn. A mathematical model of rat ascending henle limb. II. epithelial function. *American Journal of Physiology-Renal Physiology*, 298(3):F525–F542, March 2010. doi: 10.1152/ajprenal.00231.2009. URL https://doi.org/10.1152/ajprenal.00231.2009



These segments of DHL and tAHL in the middle, which are parts of the thin Henle's loop, do not have cellular structures and lateral interspaces. So transepithelial fluxes of ions and water are all passive.

The rest of the tubular segments including those of collecting ducts, however, have epithelial cells and lateral interspaces included which separate the luminal and the basement membranes.

Tubular segments of nephrons (2/2)

1. unbranched segments
   1. Superficial (SF) nephrons
   2. Juxtamedullary (JM) nephrons
2. distal nephron ensembles: collecting ducts

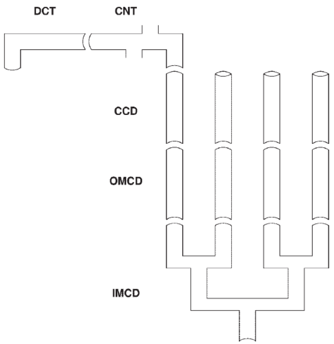
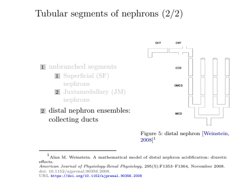


Figure 5: distal nephron [Weinstein, 2008][1]

[1] Alan M. Weinstein. A mathematical model of distal nephron acidification: diuretic effects.
American Journal of Physiology-Renal Physiology, 295(5):F1353–F1364, November 2008.
doi: 10.1152/ajprenal.90356.2008.
URL https://doi.org/10.1152/ajprenal.90356.2008



In distal nephron, the distal convoluted tubules, or DCT, come after the AHL. So there are total of 6 different profiles of DCT which receive flow from either 1 SF nephron or 5 JM nephrons. All of these flows into the connecting tubule, or CNT, in the way that each CNT are identical.

Similar branching can also be found at the inner medullary collecting ducts, or IMCD.

Vasa recta

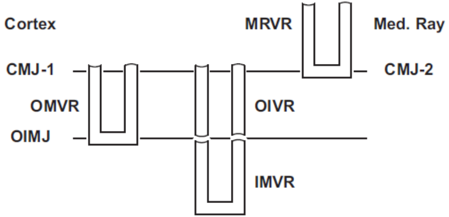
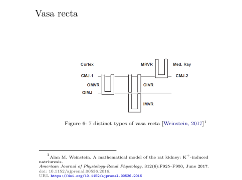


Figure 6: 7 distinct types of vasa recta [Weinstein, 2017][1]

[1] Alan M. Weinstein. A mathematical model of the rat kidney: K^+-induced natriuresis. *American Journal of Physiology-Renal Physiology*, 312(6):F925–F950, June 2017. doi: 10.1152/ajprenal.00536.2016. URL https://doi.org/10.1152/ajprenal.00536.2016



In the vascular compartments, there are 5 different kinds of vasa recta: the short one, the long ones which reaches 5 different depths, and the medullary ray vasa recta, or MRVR, which is the blood vessels that surround the cortical AHL.

Similar to the thin Henle's loop, vasa recta are assumed to not have cellular and paracellular structures. Each types of vasa recta is composed of segments with 3 different profiles of solute permeability, first at outer medulla and MRVR, which is assumed to be the same as MRVR, second at the inner medulla vasa recta, or IMVR, at the descending direction; and this is different from the third profile of ascending IMVR.

Vasa recta

AVR:DVR $= 2 : 1$.

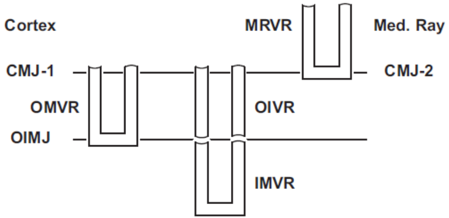
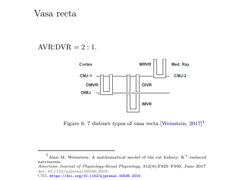


Figure 6: 7 distinct types of vasa recta [Weinstein, 2017][1]

[1] Alan M. Weinstein. A mathematical model of the rat kidney: K^+-induced natriuresis.
American Journal of Physiology-Renal Physiology, 312(6):F925–F950, June 2017.
doi: 10.1152/ajprenal.00536.2016.
URL https://doi.org/10.1152/ajprenal.00536.2016



It is also worth noting that, the branching of vasa recta is represented by the ratio between the number ascending vasa recta or AVR to the descending DVR; which was set to be $2 : 1$.

Medullary interstitium

8 sub-compartments

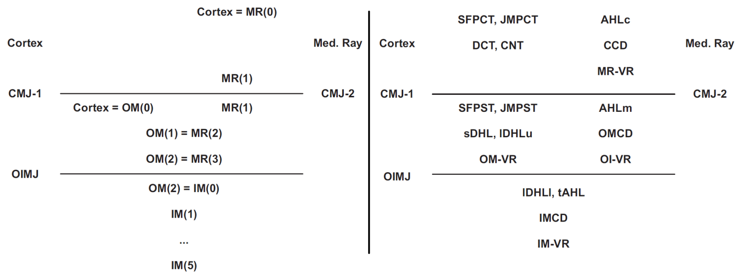
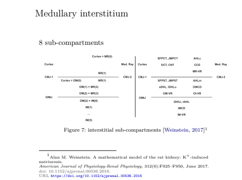


Figure 7: interstitial sub-compartments [Weinstein, 2017][1]

[1] Alan M. Weinstein. A mathematical model of the rat kidney: K^+-induced natriuresis.
American Journal of Physiology-Renal Physiology, 312(6):F925–F950, June 2017.
doi: 10.1152/ajprenal.00536.2016.
URL https://doi.org/10.1152/ajprenal.00536.2016



The third and last compartment is the medullary interstitium which is divided into 8 sub-compartments: 2 outer medulla, 5 inner medullae, and part of medullary ray that reaches the superficial layer. The cortical interstitium is not included and its solute concentrations and hydrostatic pressure are kept constant.

At the cortex, we have the PCT of SF and JM nephron, DCT, and CNT.

At the outer medulla junction, the cortex-medulla junction is separated into the part that continues from the cortex, which contains the PST, and from the medullary ray, which has the distal portion of medulla AHL. The two outer medulla sub-compartments then contain the short DHL, upper long DHL, and the outer medullary collecting duct and vasa recta.

Medullary interstitium

8 sub-compartments

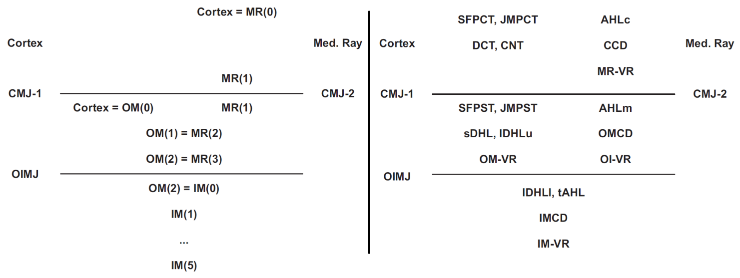
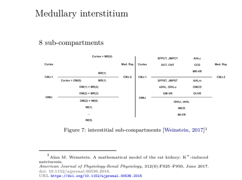


Figure 7: interstitial sub-compartments [Weinstein, 2017][1]

[1] Alan M. Weinstein. A mathematical model of the rat kidney: K^+-induced natriuresis.
American Journal of Physiology-Renal Physiology, 312(6):F925–F950, June 2017.
doi: 10.1152/ajprenal.00536.2016.
URL https://doi.org/10.1152/ajprenal.00536.2016



The inner medulla has 5 levels of depth after the outer-inner medullary junction. These contain the rest of the thin Henle's loop, and the inner medullary collecting ducts and vasa recta.

Lastly, the medullary ray. This contains the AHL, cortical collecting ducts, or CCD, and the MRVR.

Outline

Outline
1 Overview of structures
2 Model formulation

Thick tubules (with cellular structures)

Rat nephron

C. Sint

Overview of structures

Model formulation

References

1. lumen (M)
2. epithelial cell (I)
3. lateral interspace (E)

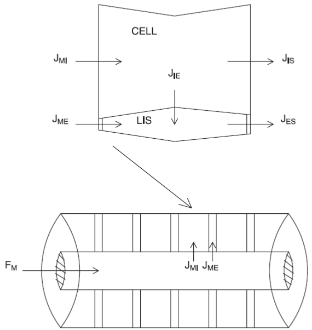
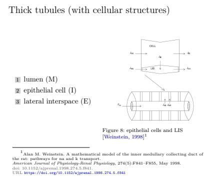


Figure 8: epithelial cells and LIS [Weinstein, 1998][1]

[1] Alan M. Weinstein. A mathematical model of the inner medullary collecting duct of the rat: pathways for na and k transport. *American Journal of Physiology-Renal Physiology*, 274(5):F841–F855, May 1998. doi: 10.1152/ajprenal.1998.274.5.f841. URL https://doi.org/10.1152/ajprenal.1998.274.5.f841



Thick tubules: lumen (M)

balance equations:

$$\frac{\partial A_\mathrm{M}}{\partial t} = -\frac{\partial F_{\mathrm{vM}}}{\partial x} - B_\mathrm{M}\left(J_{\mathrm{vME}} + J_{\mathrm{vMI}}\right), \tag{2.1}$$

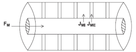
$$\frac{\partial}{\partial t}\left(A_\mathrm{M} C_{\mathrm{M},i}\right) = -\frac{\partial F_{\mathrm{M},i}}{\partial x} - B_\mathrm{M}\left(J_{\mathrm{ME},i} + J_{\mathrm{ME},i}\right) + s_{\mathrm{M},i}. \tag{2.2}$$

Thick tubules: lumen (M)

balance equations:

$$\frac{\partial A_\mathrm{M}}{\partial t} = -\frac{\partial F_{\mathrm{vM}}}{\partial x} - B_\mathrm{M}\left(J_{\mathrm{vME}} + J_{\mathrm{vMI}}\right), \tag{2.1}$$

$$\frac{\partial}{\partial t}\left(A_\mathrm{M} C_{\mathrm{M},i}\right) = -\frac{\partial F_{\mathrm{M},i}}{\partial x} - B_\mathrm{M}\left(J_{\mathrm{ME},i} + J_{\mathrm{ME},i}\right) + s_{\mathrm{M},i}. \tag{2.2}$$

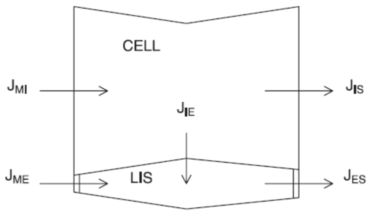


Thick tubules: cells (I) & LIS (E)

balance equations for α (α represents I or E.):

$$\frac{\partial V_\alpha}{\partial t} = J_{\mathrm{vM}\alpha} \mp J_{\mathrm{vIE}} - J_{\mathrm{v}\alpha\mathrm{S}}, \tag{2.3}$$

$$\frac{\partial}{\partial t}\left(V_\alpha C_{\alpha,i}\right) = J_{\mathrm{M}\alpha,i} \mp J_{\mathrm{IE},i} - J_{\alpha\mathrm{S},i} + s_{\alpha,i} \tag{2.4}$$



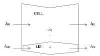
2023-04-06

Thick tubules: cells (I) & LIS (E)

balance equations for α (α represents I or E.):

$$\frac{\partial V_\alpha}{\partial t} = J_{\mathrm{vM}\alpha} \mp J_{\mathrm{vIE}} - J_{\mathrm{v}\alpha\mathrm{S}}, \tag{2.3}$$

$$\frac{\partial}{\partial t}\left(V_\alpha C_{\alpha,i}\right) = J_{\mathrm{M}\alpha,i} \mp J_{\mathrm{IE},i} - J_{\alpha\mathrm{S},i} + s_{\alpha,i} \tag{2.4}$$



Thin tubules

Only lumen:

$$\frac{\partial A_{\mathrm{M}}}{\partial t} = -\frac{\partial F_{\mathrm{vM}}}{\partial x} - B_{\mathrm{M}}\left(J_{\mathrm{vMS}} + J_{\mathrm{vMS}}\right), \tag{2.5}$$

$$\frac{\partial}{\partial t}\left(A_{\mathrm{M}} C_{\mathrm{M},i}\right) = -\frac{\partial F_{\mathrm{M},i}}{\partial x} - B_{\mathrm{M}}\left(J_{\mathrm{MS},i} + J_{\mathrm{MS},i}\right) + s_{\mathrm{M},i}. \tag{2.6}$$

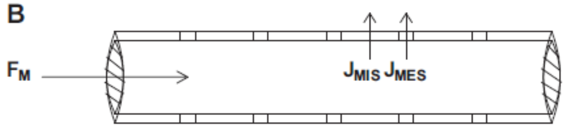


Figure 9: luminal flow & transepithelial fluxes [Weinstein, 2015][1]

[1] Alan M. Weinstein. A mathematical model of rat proximal tubule and loop of henle. *American Journal of Physiology-Renal Physiology*, 308(10):F1076–F1097, May 2015. doi: 10.1152/ajprenal.00504.2014. URL https://doi.org/10.1152/ajprenal.00504.2014

Blood vessels (C)

Only lumen [Weinstein, 2017][1]:

$$\frac{\partial F_{\mathrm{vC}}}{\partial x} = -B_{\mathrm{C}} J_{\mathrm{vCS}}, \tag{2.7}$$

$$\frac{\partial F_{\mathrm{C},i}}{\partial x} = -B_{\mathrm{C}} J_{\mathrm{CS},i} + s_{\mathrm{C},i}. \tag{2.8}$$

[1] Alan M. Weinstein. A mathematical model of the rat kidney: K^{+}-induced natriuresis.
American Journal of Physiology-Renal Physiology, 312(6):F925–F950, June 2017.
doi: 10.1152/ajprenal.00536.2016.
URL https://doi.org/10.1152/ajprenal.00536.2016

Blood vessels (C)

Only lumen [Weinstein, 2017][1]:

$$\frac{\partial F_{\mathrm{vC}}}{\partial x} = -B_{\mathrm{C}} J_{\mathrm{vCS}}, \tag{2.7}$$

$$\frac{\partial F_{\mathrm{C},i}}{\partial x} = -B_{\mathrm{C}} J_{\mathrm{CS},i} + s_{\mathrm{C},i}. \tag{2.8}$$

[1] Alan M. Weinstein. A mathematical model of the rat kidney: K^{+}-induced natriuresis.
American Journal of Physiology-Renal Physiology, 312(6):F925–F950, June 2017.
doi: 10.1152/ajprenal.00536.2016.
URL https://doi.org/10.1152/ajprenal.00536.2016

Blood vessels (C)

Only lumen [Weinstein, 2017][1]:

$$\frac{\partial F_{\mathrm{vC}}}{\partial x} = -B_{\mathrm{C}} J_{\mathrm{vCS}}, \tag{2.7}$$

$$\frac{\partial F_{\mathrm{C},i}}{\partial x} = -B_{\mathrm{C}} J_{\mathrm{CS},i} + s_{\mathrm{C},i}. \tag{2.8}$$

Relation between blood (F_{bC}) and volume flow (F_{vC}):

$$F_{\mathrm{vC}} = F_{\mathrm{bC}} \left(1 - \mathrm{Hct}\right), \tag{2.9}$$

$$0 = \frac{\partial}{\partial x} \left(F_{\mathrm{bC}} \cdot \mathrm{Hct}\right). \tag{2.10}$$

[1] Alan M. Weinstein. A mathematical model of the rat kidney: K^{+}-induced natriuresis.
American Journal of Physiology-Renal Physiology, 312(6):F925–F950, June 2017.
doi: 10.1152/ajprenal.00536.2016.
URL https://doi.org/10.1152/ajprenal.00536.2016

Blood vessels (C)

Only lumen [Weinstein, 2017][1]:

$$\frac{\partial F_{\mathrm{vC}}}{\partial x} = -B_{\mathrm{C}} J_{\mathrm{vCS}}, \tag{2.7}$$

$$\frac{\partial F_{\mathrm{C},i}}{\partial x} = -B_{\mathrm{C}} J_{\mathrm{CS},i} + s_{\mathrm{C},i}. \tag{2.8}$$

Relation between blood (F_{bC}) and volume flow (F_{vC}):

$$F_{\mathrm{vC}} = F_{\mathrm{bC}} \left(1 - \mathrm{Hct}\right), \tag{2.9}$$

$$0 = \frac{\partial}{\partial x} \left(F_{\mathrm{bC}} \cdot \mathrm{Hct}\right). \tag{2.10}$$

[1] Alan M. Weinstein. A mathematical model of the rat kidney: K^{+}-induced natriuresis.
American Journal of Physiology-Renal Physiology, 312(6):F925–F950, June 2017.
doi: 10.1152/ajprenal.00536.2016.
URL https://doi.org/10.1152/ajprenal.00536.2016

Solute ($J_{\alpha\beta,i}$) and water fluxes ($J_{v\alpha\beta}$)

Water fluxes [Weinstein, 1983][1]:

$$J_{v\alpha\beta} = L_{v\alpha\beta} A_{\alpha\beta} \Big(P_\alpha - P_\beta + \pi_\beta - \pi_\alpha + RT \sum_i \sigma_{\alpha\beta,i} (C_{\beta,i} - C_{\alpha,i}) \Big) \quad (2.11)$$

Solute fluxes:

$$J_{\alpha\beta,i} = (1 - \sigma_{\alpha\beta,i}) J_{v\alpha\beta} \overline{C}_{\alpha\beta,i} + z_i h_{\alpha\beta,i} A_{\alpha\beta} \phi_{\alpha\beta} \left(\frac{C_{\alpha,i} - C_{\beta,i} e^{-z_i \phi_{\alpha\beta}}}{1 - e^{-z_i \phi_{\alpha\beta}}} \right) + A_{\alpha\beta} \sum_j L_{\alpha\beta;i,j} \mu_{\alpha\beta,j} + J^{\text{act}}_{\alpha\beta,i} \quad (2.12)$$

[1] A.M. Weinstein. Nonequilibrium thermodynamic model of the rat proximal tubule epithelium.
Biophysical Journal, 44(2):153–170, November 1983.
doi: 10.1016/s0006-3495(83)84287-8.

2023-04-06

Rat nephron
└Model formulation
└Solute ($J_{\alpha\beta,i}$) and water fluxes ($J_{v\alpha\beta}$)

Solute ($J_{\alpha\beta,i}$) and water fluxes ($J_{v\alpha\beta}$)
Water fluxes [Weinstein, 1983][1]:
Solute fluxes:

Hydrostatic pressure (M & C)

Poiseuille's flow equations:

- tubules [Weinstein, 1998][1]:

$$\frac{\partial P_\mathrm{M}}{\partial x} + \frac{8\pi\eta}{A_\mathrm{M}^2} \cdot F_{\mathrm{vM}} = 0, \tag{2.13}$$

- vessels [Weinstein, 2017][2]:

$$\frac{\partial P_\mathrm{C}}{\partial x} + \frac{8\pi\eta_\mathrm{C}}{A_\mathrm{C}^2} \cdot F_{\mathrm{bC}} = 0. \tag{2.14}$$

[1] Alan M. Weinstein. A mathematical model of the inner medullary collecting duct of the rat: pathways for na and k transport. *American Journal of Physiology-Renal Physiology*, 274(5):F841–F855, May 1998. doi: 10.1152/ajprenal.1998.274.5.f841. URL https://doi.org/10.1152/ajprenal.1998.274.5.f841

[2] Alan M. Weinstein. A mathematical model of the rat kidney: K^+-induced natriuresis. *American Journal of Physiology-Renal Physiology*, 312(6):F925–F950, June 2017. doi: 10.1152/ajprenal.00536.2016. URL https://doi.org/10.1152/ajprenal.00536.2016

2023-04-06

Hydrostatic pressure (M & C)

Tubular compliance

Thick tubules of nephron are compliant [Weinstein, 2015, Weinstein et al., 2007][1]:

$$R_{\mathrm{M}} = R_{\mathrm{M0}} \left(1 + \frac{1}{2} \tanh\left(\nu_{\mathrm{M}}(P_{\mathrm{M}} - P_{\mathrm{S}})\right)\right) \tag{2.15}$$

Compliant lateral interspaces:

$$A_{\mathrm{ES}} = A_{\mathrm{ES0}}\left(1 + \nu_{\mathrm{AE}}(P_{\mathrm{M}} - P_{\mathrm{S}})\right), \tag{2.16}$$

$$V_{\mathrm{E}} = V_{\mathrm{E0}}\left(1 + \nu_{\mathrm{VE}}(P_{\mathrm{M}} - P_{\mathrm{S}}).\right) \tag{2.17}$$

---

[1] Alan M. Weinstein, Sheldon Weinbaum, Yi Duan, Zhaopeng Du, QingShang Yan, and Tong Wang. Flow-dependent transport in a mathematical model of rat proximal tubule. *American Journal of Physiology-Renal Physiology*, 292(4):F1164–F1181, April 2007. doi: 10.1152/ajprenal.00392.2006. URL https://doi.org/10.1152/ajprenal.00392.2006

Alan M. Weinstein. A mathematical model of rat proximal tubule and loop of henle. *American Journal of Physiology-Renal Physiology*, 308(10):F1076–F1097, May 2015. doi: 10.1152/ajprenal.00504.2014. URL https://doi.org/10.1152/ajprenal.00504.2014

Tubular compliance

Thick tubules of nephron are compliant [Weinstein, 2015, Weinstein et al., 2007][1]:

$$R_{\mathrm{M}} = R_{\mathrm{M0}} \left(1 + \frac{1}{2} \tanh\left(\nu_{\mathrm{M}}(P_{\mathrm{M}} - P_{\mathrm{S}})\right)\right) \tag{2.15}$$

Compliant lateral interspaces:

$$A_{\mathrm{ES}} = A_{\mathrm{ES0}}\left(1 + \nu_{\mathrm{AE}}(P_{\mathrm{M}} - P_{\mathrm{S}})\right), \tag{2.16}$$

$$V_{\mathrm{E}} = V_{\mathrm{E0}}\left(1 + \nu_{\mathrm{VE}}(P_{\mathrm{M}} - P_{\mathrm{S}}).\right) \tag{2.17}$$

[1] Alan M. Weinstein, Sheldon Weinbaum, Yi Duan, Zhaopeng Du, QingShang Yan, and Tong Wang. Flow-dependent transport in a mathematical model of rat proximal tubule. *American Journal of Physiology-Renal Physiology*, 292(4):F1164–F1181, April 2007. doi: 10.1152/ajprenal.00392.2006. URL https://doi.org/10.1152/ajprenal.00392.2006

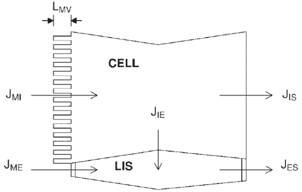
Alan M. Weinstein. A mathematical model of rat proximal tubule and loop of henle. *American Journal of Physiology-Renal Physiology*, 308(10):F1076–F1097, May 2015. doi: 10.1152/ajprenal.00504.2014. URL https://doi.org/10.1152/ajprenal.00504.2014

Flow-dependent transport of PCT

PCT transporter density parameters (Par) depend on torque of microvilous (T_M)[Weinstein et al., 2007][1]:

$$\mathrm{Par} = \mathrm{Par}_0 \left(1 + T_\mathrm{s} \left(\frac{T_\mathrm{M}}{T_\mathrm{M0}} \right) \right) \tag{2.18}$$

$$T_\mathrm{M} = \frac{8 \eta l F_\mathrm{vM}}{R_\mathrm{M}^2} \left(1 + \frac{l + \delta}{R_\mathrm{M}} + \frac{l^2}{2 R_\mathrm{M}^2} \right) \tag{2.19}$$



[1] Alan M. Weinstein, Sheldon Weinbaum, Yi Duan, Zhaopeng Du, QingShang Yan, and Tong Wang. Flow-dependent transport in a mathematical model of rat proximal tubule. *American Journal of Physiology-Renal Physiology*, 292(4):F1164–F1181, April 2007. doi: 10.1152/ajprenal.00392.2006. URL https://doi.org/10.1152/ajprenal.00392.2006

Solute generation ($s_{\alpha,i}$) in M & C

Acid-base ($H^+ + B \rightleftharpoons HB$) [Weinstein, 2017][1]:

$$s_{\alpha,\mathrm{B}} + s_{\alpha,\mathrm{HB}} = 0 \tag{2.20}$$

$$C_{\mathrm{M,H^+}} = K_1 \frac{C_{\mathrm{M,HB_1}}}{C_{\mathrm{M,B_1^-}}} = K_2 \frac{C_{\mathrm{M,HB_2}}}{C_{\mathrm{M,B_2^-}}} = \dots \tag{2.21}$$

$$\sum_i z_i s_{\mathrm{M},i} = 0, \quad \sum_i z_i C_{\mathrm{M},i} = 0 \tag{2.22}$$

[1] Alan M. Weinstein. A mathematical model of the rat kidney: K^+-induced natriuresis.
American Journal of Physiology-Renal Physiology, 312(6):F925–F950, June 2017.
doi: 10.1152/ajprenal.00536.2016.
URL https://doi.org/10.1152/ajprenal.00536.2016

Solute generation ($s_{\alpha,i}$) in M & C

Acid-base ($H^+ + B \rightleftharpoons HB$) [Weinstein, 2017][1]:

$$s_{\alpha,\mathrm{B}} + s_{\alpha,\mathrm{HB}} = 0 \tag{2.20}$$

$$C_{\mathrm{M,H^+}} = K_1 \frac{C_{\mathrm{M,HB_1}}}{C_{\mathrm{M,B_1^-}}} = K_2 \frac{C_{\mathrm{M,HB_2}}}{C_{\mathrm{M,B_2^-}}} = \dots \tag{2.21}$$

$$\sum_i z_i s_{\mathrm{M},i} = 0, \quad \sum_i z_i C_{\mathrm{M},i} = 0 \tag{2.22}$$

[1] Alan M. Weinstein. A mathematical model of the rat kidney: K^+-induced natriuresis.
American Journal of Physiology-Renal Physiology, 312(6):F925–F950, June 2017.
doi: 10.1152/ajprenal.00536.2016.
URL https://doi.org/10.1152/ajprenal.00536.2016

Solute generation ($s_{\alpha,i}$) in M & C

Acid-base ($H^+ + B \rightleftharpoons HB$) [Weinstein, 2017][1]:

$$s_{\alpha,\mathrm{B}} + s_{\alpha,\mathrm{HB}} = 0 \tag{2.20}$$

$$C_{\mathrm{M,H^+}} = K_1 \frac{C_{\mathrm{M,HB_1}}}{C_{\mathrm{M,B_1^-}}} = K_2 \frac{C_{\mathrm{M,HB_2}}}{C_{\mathrm{M,B_2^-}}} = \dots \tag{2.21}$$

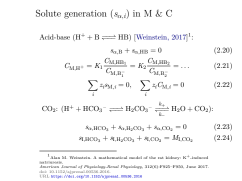
$$\sum_i z_i s_{\mathrm{M},i} = 0, \quad \sum_i z_i C_{\mathrm{M},i} = 0 \tag{2.22}$$

CO_2: ($H^+ + HCO_3{}^- \rightleftharpoons H_2CO_3{}^- \underset{k_-}{\overset{k_+}{\rightleftharpoons}} H_2O + CO_2$):

$$s_{\alpha,\mathrm{HCO_3}} + s_{\alpha,\mathrm{H_2CO_3}} + s_{\alpha,\mathrm{CO_2}} = 0 \tag{2.23}$$

$$s_{\mathrm{I,HCO_3}} + s_{\mathrm{I,H_2CO_3}} + s_{\mathrm{I,CO_2}} = M_{\mathrm{I,CO_2}} \tag{2.24}$$

[1] Alan M. Weinstein. A mathematical model of the rat kidney: K$^+$-induced natriuresis.
American Journal of Physiology-Renal Physiology, 312(6):F925–F950, June 2017.
doi: 10.1152/ajprenal.00536.2016.
URL https://doi.org/10.1152/ajprenal.00536.2016



Hemoglobin buffering species

Global variables

Global variables

Tubulo-glomerular feedback (TGF)

End-DCT pressure

Interstitial variables

Interstitial variables

References I

Alan M. Weinstein. A mathematical model of the inner medullary collecting duct of the rat: pathways for na and k transport. *American Journal of Physiology-Renal Physiology*, 274(5):F841–F855, May 1998. doi: 10.1152/ajprenal.1998.274.5.f841. URL https://doi.org/10.1152/ajprenal.1998.274.5.f841.

Alan M. Weinstein. A mathematical model of distal nephron acidification: diuretic effects. *American Journal of Physiology-Renal Physiology*, 295(5):F1353–F1364, November 2008. doi: 10.1152/ajprenal.90356.2008. URL https://doi.org/10.1152/ajprenal.90356.2008.

Alan M. Weinstein. A mathematical model of rat proximal tubule and loop of henle. *American Journal of Physiology-Renal Physiology*, 308(10):F1076–F1097, May 2015. doi: 10.1152/ajprenal.00504.2014. URL https://doi.org/10.1152/ajprenal.00504.2014.

Alan M. Weinstein. A mathematical model of the rat kidney: K^+-induced natriuresis. *American Journal of Physiology-Renal Physiology*, 312(6):F925–F950, June 2017. doi: 10.1152/ajprenal.00536.2016. URL https://doi.org/10.1152/ajprenal.00536.2016.

Alan M. Weinstein and Thomas A. Krahn. A mathematical model of rat ascending henle limb. II. epithelial function. *American Journal of Physiology-Renal Physiology*, 298(3): F525–F542, March 2010. doi: 10.1152/ajprenal.00231.2009. URL https://doi.org/10.1152/ajprenal.00231.2009.

Alan M. Weinstein, Sheldon Weinbaum, Yi Duan, Zhaopeng Du, QingShang Yan, and Tong Wang. Flow-dependent transport in a mathematical model of rat proximal tubule. *American Journal of Physiology-Renal Physiology*, 292(4):F1164–F1181, April 2007. doi: 10.1152/ajprenal.00392.2006. URL https://doi.org/10.1152/ajprenal.00392.2006.

A.M. Weinstein. Nonequilibrium thermodynamic model of the rat proximal tubule epithelium. *Biophysical Journal*, 44(2):153–170, November 1983. doi: 10.1016/s0006-3495(83)84287-8. URL https://doi.org/10.1016/s0006-3495(83)84287-8.