# A simple time-dependent model of kidney

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## 1 Model equations

## 2 Non-dimensionalization

## 3 Numerical method

Consider a multidomain type model on a (rescaled) spatial domain $\Omega = (0, 1)$, on which we have a non-dimensionalized system:

$$\frac{\partial \alpha_k}{\partial t} + \mathrm{Pe}\frac{\partial}{\partial x}\left(\alpha_k u_k\right) = -w_k, \tag{1}$$

$$\frac{\partial \alpha_0}{\partial t} + \mathrm{Pe}\frac{\partial}{\partial x}\left(\alpha_0 u_0\right) = \sum_k w_k, \tag{2}$$

$$\frac{\partial}{\partial t}\left(\alpha_k c_i^k\right) = -\frac{\partial}{\partial x} f_i^k - g_i^k, \tag{3}$$

$$\frac{\partial}{\partial t}\left(\alpha_0 c_i^0\right) = -\frac{\partial}{\partial x} f_i^0 + \sum_k g_i^k, \tag{4}$$

$$\nu_k\left(p_k - p_0\right) = \frac{\alpha_k}{\bar{\alpha}_k} - 1, \tag{5}$$

$$\alpha_0 + \sum_k \alpha_k = \alpha_*, \tag{6}$$

where

$$\frac{\rho_\cdot}{\alpha_\cdot} u_\cdot = -\frac{\partial p_\cdot}{\partial x}, \tag{7}$$

$$f_i^\cdot = -D_i^\cdot c_i^\cdot \frac{\partial \mu_i^\cdot}{\partial x} + \mathrm{Pe}\left(\alpha_\cdot u_\cdot c_i^\cdot\right), \quad \mu_i^\cdot := \ln c_i^k \tag{8}$$

$$w_k = \zeta_k\left(\psi_k - \psi_0\right), \quad \psi_\cdot := p_\cdot - \pi_\cdot, \quad \pi_\cdot := \frac{a_\cdot}{\alpha_\cdot} + \sum_i c_i^\cdot, \tag{9}$$

$$g_i^k = j_i^k + h_i^k, \quad j_i^k := \gamma_i^k(\mu_i^k - \mu_i^0). \tag{10}$$

Let $N \in \mathbb{N}$ be the number of uniformly spaced grids in $(0,1)$, and $\delta x = 1/N$ be the spatial grid size. Similarly, we denote $\delta t$ as the size of time steps. We will use the notation $\alpha_{kl}^n, c_{il}^{kn}, p_{kl}^n$ for the discretization of $\alpha_k, c_i^k$ and $p_k$ at the $l$-th spatial grid and time $t = n\delta t$.

We define difference quotient operators:

$$\Delta_x^+ y_l^n := \frac{y_{l+1}^n - y_l^n}{\delta x}, \quad \Delta_x^+ y_l^n := \frac{y_l^n - y_{l-1}^n}{\delta x}, \quad \Delta_t y_l^n = \frac{y_l^n - y_l^{n-1}}{\delta t}. \tag{11}$$

and an average operator:

$$A y_l^n := \frac{y_{l+1}^n + y_l^n}{2}. \tag{12}$$

We start with known values of $\alpha_{kl}^{n-1}, c_{il}^{k,n-1}, p_{kl}^{n-1}$, $l = 1, \ldots, N$. The first step is to update the unknowns for the next time step $n$. We have

$$\begin{gathered} \Delta_t \alpha_{kl}^n + \mathrm{Pe}(\alpha_{kl}^n u_{kl}^n) = -w_{kl}^n, \quad w_{kl}^n := \zeta_k \left(\psi_{kl}^n - \psi_{0l}^n\right) \\ \psi_{\cdot l}^n := p_{\cdot l}^n - \pi_{\cdot l}^n, \quad \pi_{\cdot l}^n := \sum_i c_{il}^{\cdot n}. \end{gathered} \tag{13}$$

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