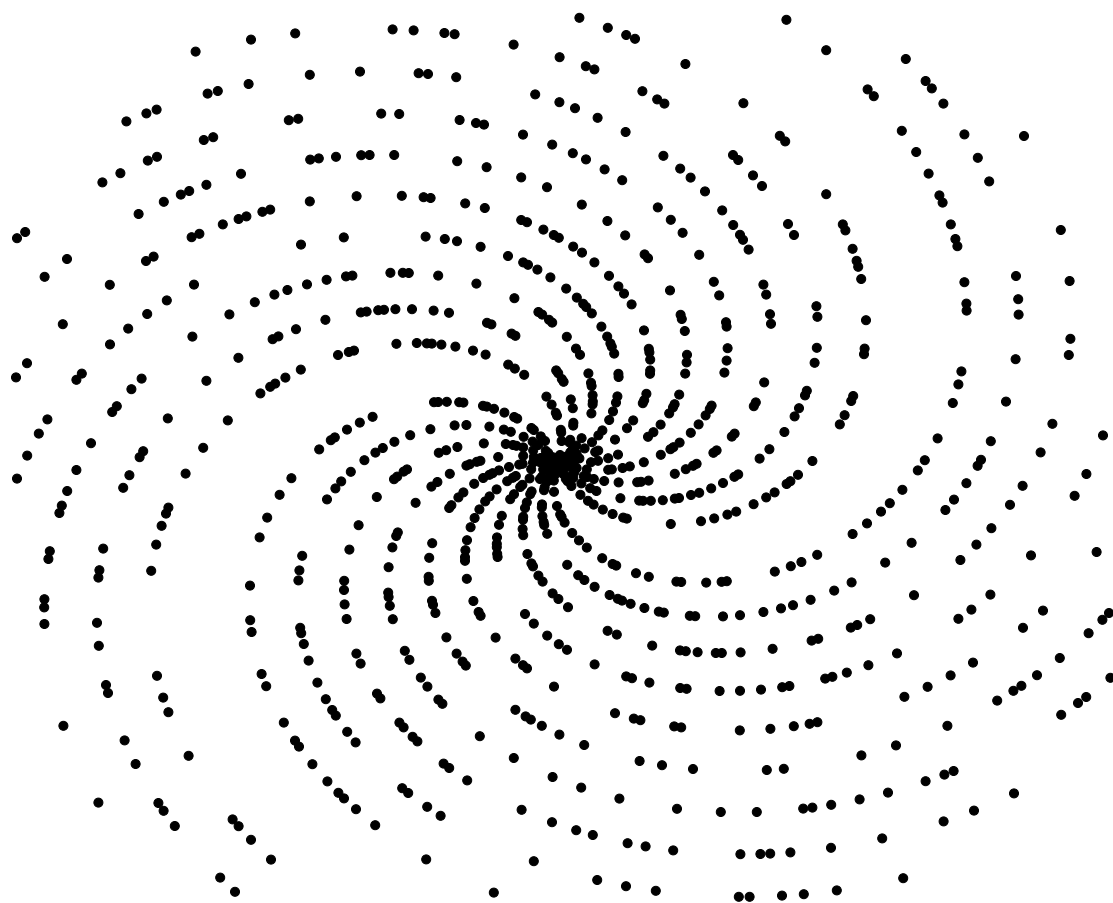


SAN DIEGO CITY COLLEGE
MATH 59 — EXPLORATIONS I
MODULES 0–7



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Module 0

Instructor Notes

Start with the following video:

<https://youtu.be/gvQNYbQQ06E4>

A New Way of Learning Mathematics

For many students, mathematics does not bring happy memories or feelings. Some even dread learning mathematics. One important question is why do so many students dislike a subject that Gauss called the “Queen of all Sciences.”

Many say how important mathematics is, and how it is used to understand the world around us through many different fields. Some say mathematics is a universal language and that humans are hardwired to do basic mathematics.

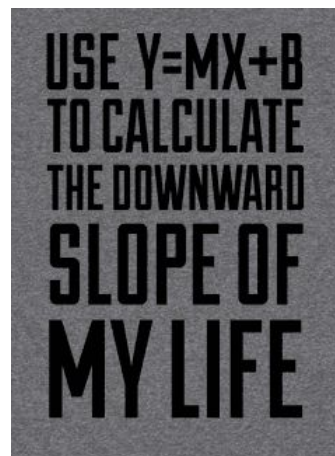
Yet many struggle and come to associate this subject with negative feelings. Most never attain an appreciation for how mathematics is used in their day-to-day lives. We are asking every student who has decided to take this course to let go of their past experiences, their erroneous belief that they are “bad at Math” or that they do not have a “Math brain,” and definitely let go of their ego and pride.

It does not matter what level you are at, Mathematics will always have challenges for the mind. There is an inherent challenging nature in this discipline, and it is in this struggle that you are indeed actually learning. In such struggles, you are developing different analytical skills and are increasing your neuroplasticity, that is literally making your brain grow. Such a realization is a very important step in dealing with this subject.

Our curriculum is intended to generate thought-provoking questions, analytical reasoning, critical thinking, and most of all, an appreciation and realization for the need and even desire to improve such skills. You will be developing into a more effective thinker and a more informed member of our 21st century society. Throughout this course, the way you will be exposed to mathematics will be very different than in a traditional algebra curriculum. The purpose of this approach is for the student to identify with the subject matter and to gain a sense of intellectual curiosity, and therefore motivate the learning process.

We will introduce ways of doing problems that at times may look very different from how you may have learned them in the past. At times this could even seem more difficult or unconventional, but the intention is for you to focus on your thought process, rather than on just trying to get a “correct” answer to a problem. Because students prefer taking courses where content is relevant and engaging, this approach to learning mathematics is intended for the student to deeply explore the subject matter. We expect to receive your feedback throughout the course. So, please communicate with your instructor anything that you would like to discuss about the structure and content presented.

Teachers act like this is how the world will be



Module 0: Appreciating Mathematical Beauty

There are many things that affect people’s views about their ability and their likes or dislikes of any subject. For some, an aversion to mathematics begins in elementary school when they are first exposed to fractions. Early failures in a math class, or in-class punishments or embarrassing moments in front of peers, or having to learn what seems like meaningless procedures that detract from creative thinking, are all experiences that can stay with some students for years. In some cultures, parents and peers maintain norms that math is hard or even unnecessary. In other cultures, girls more than boys, learn to question their mathematical ability.

It is difficult to identify a single cause for a dislike, anxiety, or negative self-view that developed over many years. One thing is for sure: ABSOLUTELY and without exception — even to the most evolved and gifted minds — mathematics will eventually challenge you to the point where you end up feeling that you hit a massive wall and thus continuing to improve in learning it seems just impossible. There are mathematical problems so incredibly complicated, that to this day remain unsolved. So, do not judge yourself by your ability to actually get a correct answer quickly, but by your resilience in continuing to systematically devote meaningful time, energy and struggle in your studies. *If you are truly struggling, then you are definitely learning.*

It is at this point that a growth mindset, a gritty attitude, and a persevering disposition are absolute necessities. For without this, that “wall-hitting” experience that you feel in the subject will grow even deeper and more unrelenting. However, with a self-advocate and positive attitude, it is very possible to overcome such feelings and modify these thoughts. Without exception, EVERY mathematician has at some point felt this feeling of doubt and struggle in this subject. We all have our strengths and weaknesses. However, as human beings we can also develop new skills and abilities through diligence, practice, tenacity, positivity, grit and becoming conscious of our thought processes.

Goals::

- Promote student self-reflection about the subject matter
- Promote a clear and specific plan for success in the math course as well as for the student’s academic and professional goals
- To maintain this plan of action in the forefront with every student, thereby making them responsible for their own learning and academic experiences.

Write a two to three-page typed, double-spaced essay about your personal math history.

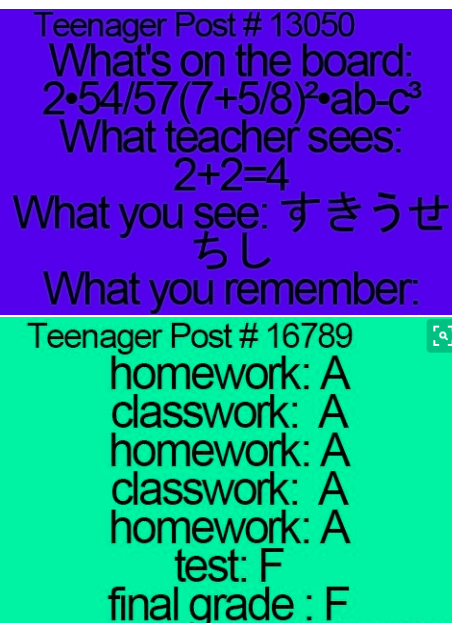
Instructor Notes

This is to be turned in by the end of the first week of the semester. Have the students begin coming up with ideas for this first written assignment. Please specify your presentation requirements for the essays.

Address the following:

- What is your mindset about mathematics?
- What math courses and teachers do you particularly remember most and why?
- Describe a positive and a negative math experiences you have had, and what did you learn about yourself through these experiences.
- What are your academic and professional goals, and explain how your life has brought you to have such goals and what role do you think math would play in achieving these goals?
- What do you think you currently need to be able to reach these goals and how do you intend to make these goals your reality?
- Describe in ample detail and being very specific what will you do to succeed in this course.

Make a pact with yourself on doing all that you have stated in your essay and perform weekly or bi-weekly evaluations on your progress. If you do feel anxiety while taking math tests or doing math problems, pay careful attention to your biography and strategize on how you will overcome challenging experiences. In your history, there may be some anxiety-provoking events that may surface. Once you finish looking at any negative math-related events from your past, put them away. Put away any self-views such as “I am just bad at math.” We don’t believe in a victim-complex in



mathematics, but we do believe that we are the creators of our future. Start with a fresh attitude and a new approach. You will see how even mathematics can be rewarding and even fun to explore, like for our blue jay friend in the right!

Instructor Notes

Show these videos on the first day of class. Each video is between 6–9 minutes. Pause to have a discussion between these two videos.

- Angela Duckworth — Grit:
<https://youtu.be/H14bBuluwB8>
- Mathematigal — Fear of dealing with Math:
<https://youtu.be/Xs9aGVUZ3YA>

Instructor Notes

Here is where you need to set a new perspective for students. The video links below can help you bring about this new perspective. The perspective here is to value and encourage critical and analytical thinking. This approach to teaching Math hinges on exploring math topics where the process is more important than some correct answer. In this class, we encourage that a significant part of the course grade be geared to the student's effort in critical thinking, and the explanations from the students to justify their reasoning on a problem. Students put an over-emphasis on grades and GPA, especially the many students that are on a transfer track. Here is where you as the instructor need to both balance the process taken AND the student's explanation, with the correct answer to a problem. Here is where you need to show them how valuable it is to struggle while doing mathematics, because their ability to learn future topics grows significantly. After discussing this first activity with the students and having them write down some ideas on how they will address the questions posed by this writing assignment, show them the following video:
How you can be good at Math By Jo Boaler (TED Talk Stanford):

<https://youtu.be/3icoSeGqQtY>

You should go thru the entire video. It has quite a bit of information for both students and instructors. This video exemplifies what SDCC Math department is looking to create in this way of doing mathematics.

Instructor Notes

For the following three links below you may want to consider some of the following activities or assignments. Assign these next 2 videos for the next class session. At the beginning of the second class you will have the students work in groups of 3. If there is one person left over, make a final group of four, and if there are two students left over, let them be a group on their own. Have the students discuss what new perspective they gained with regards to doing math by watching these videos. Have the students come up with 6 – 8 tangible things that they both learned about what it takes to learn math and how they will incorporate these strategies into this math course and future ones.

<https://youtu.be/smHZNr5q0b0> (Angela Duckworth – University of Pennsylvania Talk)

https://youtu.be/mHQ_0ehnavI (What it's like to study Mathematics in College)

For this video, among the main points is that a student HAS to be self-advocate AND they network to help themselves learn math. All students build into their study time, working in groups and working in a Tutorial Center.

Instructor Notes

Or for this video, have the students write a one page essay on how this video can relate to learning mathematics, and what does each student think is the most important factors for them in learning mathematics based on the premises of this video and have the students read their essays in class.

<https://youtu.be/LNHBMFCzznE> (Dr. Lara Boyd – Your Brain Will Never Be The Same When You Learn Anything – Neuroplasticity)

Homework due next class:

Name: _____

Watch the following video. Write down 3 things you liked or learned from it. Be ready to discuss it in class.

How You Can Be Good at Math by Jo Boaler.

URL: <https://youtu.be/3icoSeGqQtY>

QR Code:



1.

2.

3.

People Bingo

- Find someone who matches each characteristic and ask them to sign in the box.
- You can't have the same person sign more than one box on your paper.

Is Married	Has Children	Lives more than 10 miles from campus	Speaks a language other than English
Lived outside of the US	Plays a musical instrument	Wears glasses or contacts	Is under 21 years of age
Is left-handed	Is vegetarian or vegan	Works full- or part-time	Plays video games
Exercises Regularly	Does Not Have a Facebook Account	Has a January Birthday	Other: _____

Module 1

Module 1: Overview and Expectations

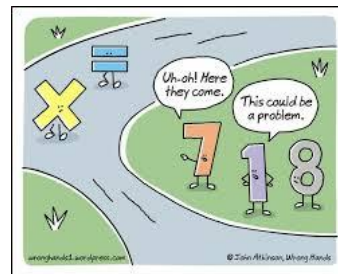
Our first module introduces you to a seemingly unrelated set of activities that are intended to challenge your thinking, as well as the way you process information. This module intends to amplify your perspective on how to analyze and dissect multiple types of problems. You will be engaging in activities that begin to develop in your mathematical thinking. These activities range from mathematically historic forms of computations, to developing a sense and understanding of numbers, their magnitudes, how to represent them and how the differences in number representations and organization will assist in effectively analyzing and solving problems.

You will get an introduction of the differences between the various types of numbers and you will begin to develop an appreciation and a sense of the structure of the system of numbers.

You will be using this newly discovered structure of the number system to attempt at recognizing patterns and properties of these patterns to assist you in solving problems. You will see how by struggling on mathematical puzzles and brainteasers, you can learn more effectively how to think about and analyze problems.

Finally, you will see how when we represent certain properties of a puzzle algebraically and articulate into written mathematical statements the rules and hypotheses we have discovered throughout our analysis, the level of thinking on a problem rises to higher and deeper level. And when you continually put this into practice, you end up realizing you are becoming a clearer and more accurate thinker.

Because there is literally no limit to how deep or how sophisticated you can think on any one problem, learning how to consistently put into practice the skills of analyzing and mentally dissecting all problems will inevitably help you learn, retain and appreciate much more efficiently mathematics with all its rules, its language its processes, and its theories.

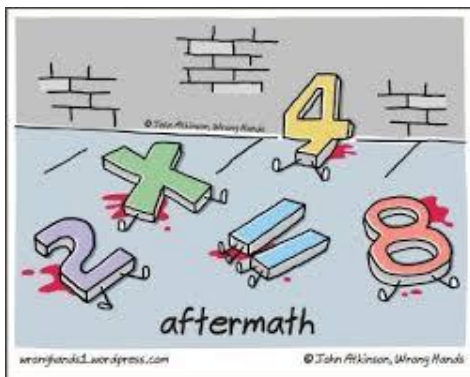


Reflective Assignment

Begin by thinking on how you initially go about analyzing and breaking apart a problem.

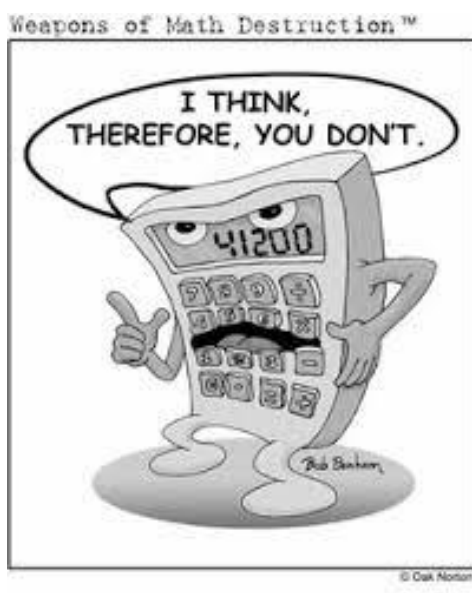
Working in groups of 3 to 4 students per group, discuss with your classmates the following questions:

- Are you systematic in analyzing a problem?
- What do you think, as it pertains to mathematics, does it mean to be systematic?
- Do you have a strategic pattern of thought when you are confronted with a conceptual problem?
- What things do you look for when analyzing a complicated problem that can give you clues on how to analyze and gain better understanding of the problem?
- What would you consider to be a reasonable amount of time to analyze/dissect/understand a mathematics problem?



Now write a 400–450 word essay addressing the following: (to be turned in the next class session)

- What is it that you currently do in analyzing a complicated mathematics problem?
- What elements do you look for in trying to gain a better understanding of a problem?
- What strategies would you recommend to someone to implement that is stuck for some time on a mathematics problem, irrespective of the type of math problem it is?
- Develop your own unique list of steps and strategies that you are going to follow in this module when dealing with the many complicated math problems that you will confront.
- Finding and receiving appropriate assistance to gain a better handle on how to tackle complicated mathematics problems is always necessary in order to improve our own problem solving abilities. Describe how you define to be an appropriate and educationally sound use of assistance in learning how to improve your own analytical skills and problem solving strategies.



Note that getting too much or inappropriate assistance is counterproductive to developing and improving your own skills to think about and to solve problems. It is in the productive struggle that you learn and progress the most. Do not cheat yourself of the opportunity to grow analytically by just trying to find solutions to problems that you yourself do not do spend the majority of the time thinking and struggling.

It is perfectly natural to feel tired, mentally drained, frustrated, even anxious about getting the answer.

The real litmus test to your overall progress is not whether you get the “correct” answer; nor is it your overall course grade or gpa. The real measure of your success is how much you have improved in your thinking skills and how much you have learned from where you started.

So finally, include in your essay VERY specific and detailed things, that you are going to do to maximize this opportunity to learn how to be a better, more accurate and clearer thinker.

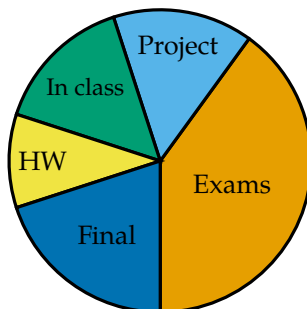
Homework Exercises

1. List specific things you will do to be a better math student.

2. List things you will avoid doing

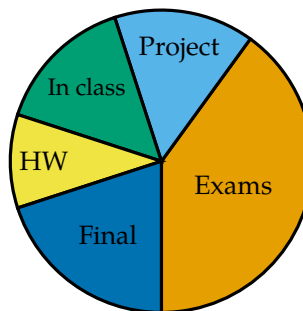
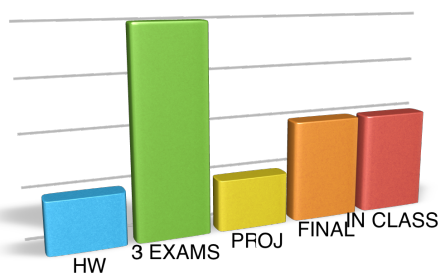
Answer the following questions.

1. Your course grade is based on several different categories. Here is a pie chart showing graphically the weights of each category. Based on the chart below. Answer the following questions.

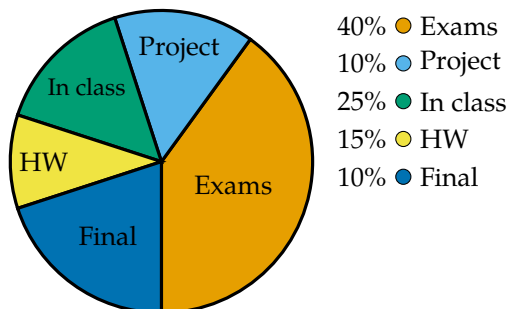
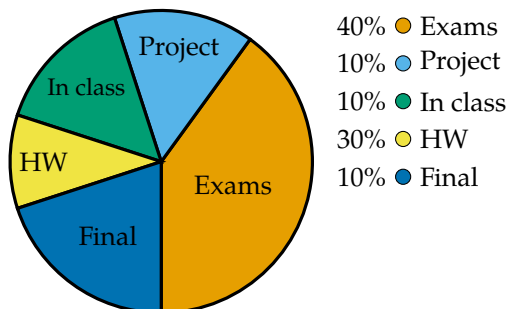
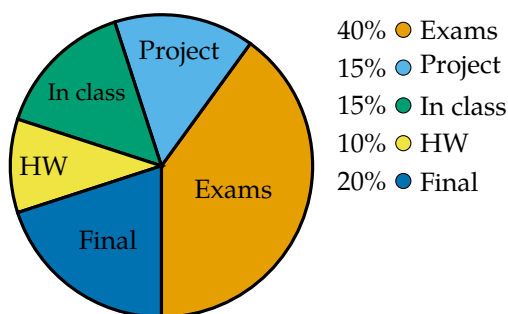
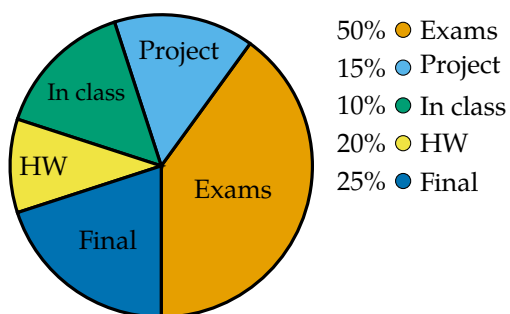


- (a) What category will affect your grade the most? Which will affect it the least? Explain your answer? Explain your answer.
- (b) Approximate the fraction of your grade that is computed from homework.
- (c) Find two categories that together make up about one-half of your grade?
- (d) If a student decided not to do any homework at all, how do you think that would affect their grade? How about if the decided to not do any in class work?

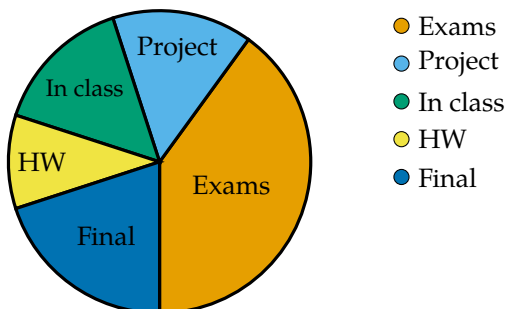
2. How do you think information in the following diagrams differs? Explain your answer.



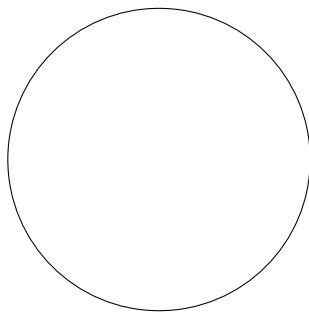
3. Put a star next to the chart whose legend looks accurate. For each of the other charts, write down at least one reason why the legend does not make sense.



4. Using the chart that you marked accurate from exercise 3, fill in the weight of each category using fractions. Write fractions in lowest terms.



5. If you were teaching this class, how would your pie chart look on your syllabus? Don't worry about attaching numbers, just represent your idea graphically.



6. Will an understanding of the pie chart change how you manage your study time for this class? Explain your answer.

Introduction

As we begin to explore math this semester, it makes sense that we start with numbers. The first written form of numbers was invented by the Egyptians, then followed by the Greeks and Romans with their own system. Today, the most widely used modern numeral system is the Hindu-Arabic numbers 0, 1, 2, 3, ... which was invented by Indian mathematicians and then adopted by Arabic mathematicians which then spread to Europe and beyond.

The first module will focus on understanding real numbers. Within the real number system there are sub-groups of numbers that are important for us to identify and understand. But first, here is a quick video to pique our interest.

Watch the following video on Interesting Numbers 1 to 50:

<https://youtu.be/Je4rK9fMGKs>

1. What are some types (or groups) of numbers that you know?

Terms and Definitions

A *set* is a collection of objects. Do you or anyone you know have an interesting or unusual collection? How many objects (or items) are in that collection?

We use *set notation* to symbolically represent sets with braces $\{ \}$. For example, the set of all schools in the San Diego Community College District consist of City College, Mesa College, and Miramar College. Using set notation, we can write this as the set of all schools in SDCCD = $\{\text{City, Mesa, Miramar}\}$. There are 3 objects (or elements) in this set. To indicate that City College is an element (or member) of the school in our district, you can write $\text{City} \in \{\text{City, Mesa, Miramar}\}$. The symbol " \in " means "is an element of" or "is a member of".

Some sets are subgroups of other larger sets and are completely contained inside another set. We call these subgroups subsets, using the symbol " \subseteq ". For example, $\{2, 3\} \subseteq \{2, 3, 7\}$ which reads as the set $\{2, 3\}$ is a subset of the set $\{2, 3, 7\}$.

2. Illustrate or explain how the following terms relate to each other:

San Diego City College

Schools in California

Schools in San Diego County

Schools in the city of San Diego

Schools in the United States

3. Can you find any two sets, A and B, where A is a subset of B? Be creative.

It is now time to take notes on some common number sets: Here are some optional videos as a resource:

- What are Real Numbers?

<https://youtu.be/3YwrcJxEbZw>

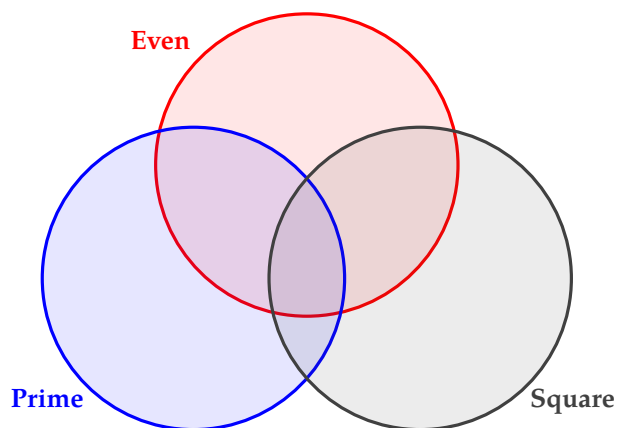
- Real Numbers-Categories

<https://youtu.be/IueVrMlmQ2I>

Write your notes here.

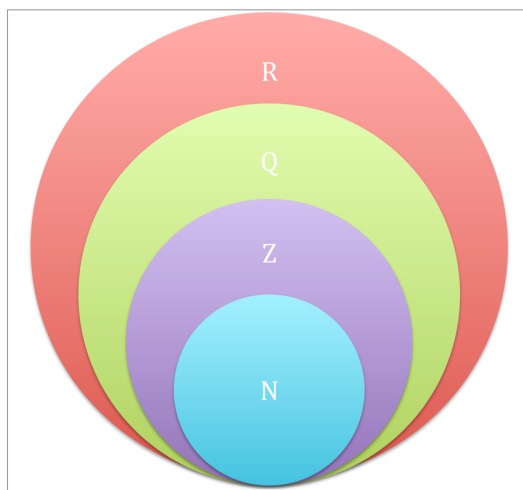
Practice

4. Place the integers $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ in the following diagram.



5. In your groups, think of three categories of students at City College that don't overlap. Draw them on your paper and label the sets. Then write your names in the appropriate set. (ex. Taking 1 class, taking 2 classes, taking 3 or more classes)
6. Think of three categories of students at City College that may overlap. (ex. Female, taking math this semester, takes trolley to school). Draw the sets and insert your groupmates' names.

7. Explain your interpretation of the figure below:



Place the following numbers in the appropriate location above:

$$\left\{ 5.3, \frac{1}{2}, -8, 0, 14, \pi \right\}$$

8. Determine if the following statements are true or false. If they are false, correct the statement using proper notation. Box your answer.

- $\{1, 2\} \subseteq \mathbb{Z}$
- $\frac{1}{2} \in \mathbb{Z}$
- $\mathbb{R} \subseteq \mathbb{Z}$
- $\frac{0}{5} \in \mathbb{Q}$
- $\frac{6}{0} \in \mathbb{Q}$
- $\frac{7}{0} \in \mathbb{R}$
- $\frac{1}{2} \in \mathbb{Q}$
- $\mathbb{Z} \subseteq \mathbb{Q}$

9. What is the difference between the symbols \in and \subseteq ?

Homework exercises

Decide whether the following statements are true or false.

1. $3 \in \{7, 4, -10, 17, 13, 3, 9, 67\}$
2. $4 \in \{14, 44, 43, 24\}$
3. $\frac{1}{3} \in \mathbb{Z}$
4. $-5 \in \mathbb{N}$
5. $-\frac{271}{113} \in \mathbb{Q}$
6. $-37 \in \mathbb{Z}$
7. $5 \in \mathbb{R}$
8. $\{2, 4, 7\} \subseteq \{-3, 2, 5, 4, 7\}$
9. $\{2, 3, 5\} \subseteq \{2, 5\}$
10. $\{2, 5, 9\} \subseteq \{2, 4, 9\}$
11. $\{-15, \frac{3}{4}, \pi\} \subseteq \mathbb{R}$
12. $\{-15, \frac{3}{4}, \pi\} \subseteq \mathbb{Q}$
13. $\{-2, 3, 0\} \subseteq \mathbb{N}$
14. $\{-2, 3, 0\} \subseteq \mathbb{Z}$
15. $\{\sqrt{2}, 271\} \subseteq \mathbb{R}$
16. $\{\sqrt{2}, 271\} \subseteq \mathbb{Q}$

Dollar Bills

Instructor Notes

Instructor: give students one or two simple examples computing volume of a rectangular box. Note that they do not need to know or understand the volume formula yet. The goal is to give them background information to do the activity.

Give each group of students a one-dollar bill.

1. Give a rough estimate for the volume of the bill. Describe your thought process.

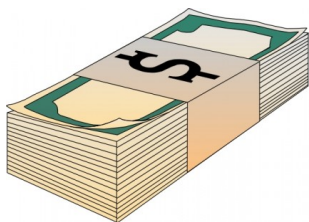
Instructor Notes

Answer is varied. One suggestion: fold the bill several times to create a thickness. Estimate the dimensions of the “box”.

Instructor Notes

Background: one dollar bills have width of 2.61 inches, length of 6.14 inches, and thickness of 0.0043 inch. Note to instructor: do not give students these measures until they finish the first activity above.

2. Find the height of a stack of 500 brand new one-dollar bills. Mark the height of the stack on the ruler.



Instructor Notes

Answer $500 \times 0.0043 = 2.15$ inches. If students give reasonable and logical estimations, that is OK, too.

3. How about a stack of 5000 brand new bills.

4. Which would you choose: a 2 in stack of one dollar bills or 5 one-hundred dollar bills? Why?

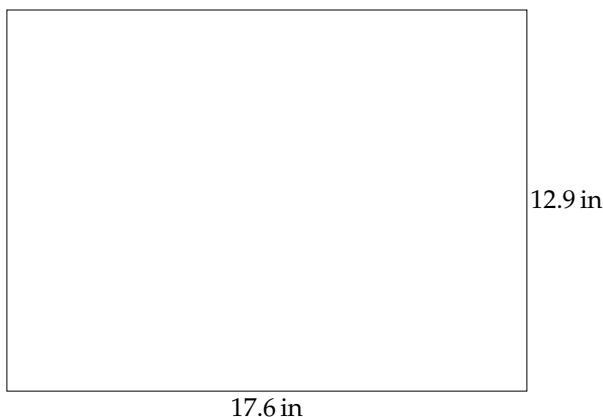
5. Give an estimate of the number of bills needed to be as long as the whiteboard when laid out lengthwise end-to-end.

6. Measure the length of the whiteboard. Use this to compute the number of bills needed. How close was your estimate?

Name: _____

Homework

The length of a briefcase is 17.6 inches, the width is 12.9 inches and the height is 5.9 inches. Feel free to use your calculators in this activity. The bottom of the briefcase has the shape of the following rectangle:



1. How many dollar bills would you need to cover as much of the bottom of the briefcase as possible? The dollar bills can't overlap. Recall that the dimensions of a dollar bill are 2.61 inches by 6.14 inches by 0.0043 inches. Draw the dollar bills in the rectangle above as best you can to scale. Show any computations you used here.
2. Can you find a way to use more dollar bills to cover the bottom if the bills don't overlap? (Hint: Is there another way to arrange the bills on the bottom?)

-
-
3. You decide to stack dollar bills on each of your dollar bills you drew in the bottom of the briefcases? How many dollar bills will fit in each of your stacks?
4. How much money fits in the briefcase? Use your drawing, and assume you stack the bills as high as possible to fit in the briefcase.
5. If instead of one \$1 bills you had \$100 bills, how much money would fit in the briefcase?

Powers of 2

How fast can rumors spread by word of mouth?

If you tell 2 friends, and they tell 2 friends, and so on, how fast can a rumor spread just by word of mouth?

1. Watch the following two commercials and compare the two videos in terms of powers of 2.

<https://youtu.be/DKUZX1Nk4yY> Faberge shampoo commercial



<https://youtu.be/JA7CKvoKEmE> Wayne's World parody



2. Let's first calculate some powers of 2 to warm up.

$$\begin{aligned}2^1 &= \underline{\hspace{2cm}} \\2^2 &= 2 \cdot 2 = \underline{\hspace{2cm}} \\2^3 &= 2 \cdot 2 \cdot 2 = \underline{\hspace{2cm}} \\2^4 &= \underline{\hspace{2cm}} = \underline{\hspace{2cm}} \\2^5 &= \underline{\hspace{2cm}} = \underline{\hspace{2cm}}\end{aligned}$$

3. City College has about 16000 students*. How many iterations of “telling” need to occur before the entire student body hears the rumor?

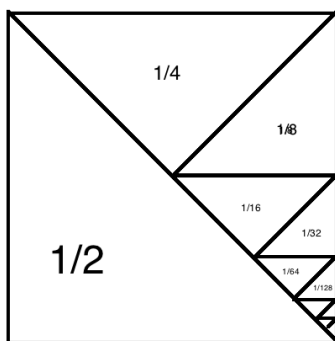
*

http://research.sdccd.edu/docs/Student%20Profiles/Demographics/20171/Student%20Profile%20CR-20171_CD.pdf

4. If each cycle took one hour, then in _____ hours, the whole school would know. What if each cycle took 15 minutes? Is this surprising?
5. Discuss with a neighbor how social media would affect the spread of rumors. Any personal experiences you can share?

Powers of $1/2$

We have just seen how quickly powers of 2 can grow. Now we will do a hands-on activity that goes in the opposite direction — powers of $1/2$. Your instructor will give your group a square piece of paper with scissors and tape. Try to construct the following image using the materials given. Label each piece with the correct power of 2. For instance, $1/2$ is $(1/2)^1$ and $1/4$ is $(1/2)^2$



6. How many powers of $1/2$ were you able to physically create?
7. Where would you encounter powers of $1/2$ in real life?

Powers of 10

We will now watch two powers of 10 videos:

<http://www.eamesoffice.com/education/powers-of-ten-2/> †

†



Name: _____

Homework Exercises

1. Find the following powers of 10 or missing exponent.

10^1	10
10^2	$10 \cdot 10 = 100$
10^3	$10 \cdot 10 \cdot 10 = 1000$
10^4	
10^5	
10^6	
$10^{\boxed{}}$	1,000,000,000
$10^{\boxed{}}$	10,000,000,000,000

2. Do you notice any pattern or relationship between the exponent and the number of zeroes?
3. How many seconds are in one minute?
4. If you could say two numbers in one second, how many numbers could you say in one minute? (Show the setup of your calculation.)
5. How many minutes are in one hour?
6. If you could say two numbers in one second, how many numbers could you say in one hour? (Show the setup of your calculation.)

7. How many hours would you need to count to one billion? (Show the setup of your calculation.)
8. Why would it take 4 to 5 days to count to one billion? (Show the setup of your calculation.)
9. How long would it take to count to one billion if you said 2 numbers every second? Can you realistically say 2 large numbers in one second?

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