Section 6.2 — Standard Normal Distribution

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Outline

Uniform Distribution

Normal Distributions

Uniform Distribution

Graphs of probability distributions

1. The area under the graph of a probability distribution is 1.
2. There is a correspondence between area and probability.

Uniform Distributions

Definition (Uniform Distribution)

A continuous random variable has a uniform distribution if its values are spread evenly over the range of possibilities. The graph of a uniform distribution results in a rectangular shape.

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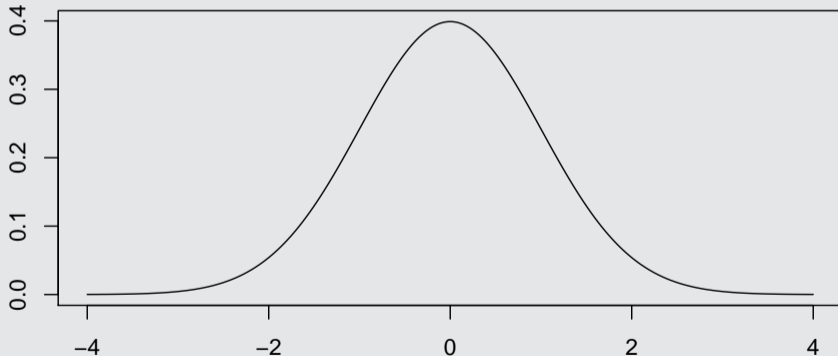- What is the probability they wait exactly 2.5 minutes?

Normal Distributions

What does it mean for a histogram to be a normal distribution?
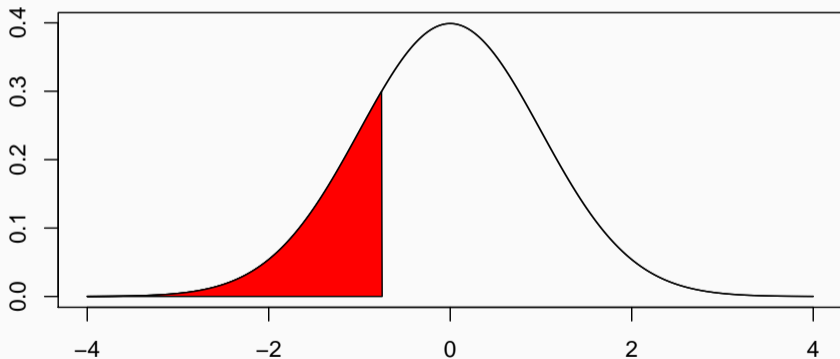
Standard Normal Distribution

Definition (Standard Normal Distribution)

The standard normal distribution is a normal distribution with the parameters $\mu = 0$ and $\sigma = 1$. The total area under its curve is 1.

Probabilities and areas

When we look up probabilities, we are talking about the cumulative area from the left.

Finding Probabilities from a *z* score

- $P(Z < a)$ is the probability that the *z*-score is less than *a*.
- $P(Z > a) = 1 - P(Z < a)$ is the probability that the *z*-score is greater than *a*.
- $P(a < Z < b) = P(Z < b) - P(Z < a)$ is the probability that the *z*-score is between *a* and *b*.

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- What is the probability that z is less than -1.55?
- What is the probability that z is greater than -0.82?
- What is the probability that z is greater than 0.91?
- What is the probability that z is between 0.25 and 1.25?
- What is the probability that z is between -2.11 and 1.15?

The Values Gone Critical!

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Percentiles and Critical Values

- P_α cuts off the bottom α percent from the top. It's the *z* score with an area of $\frac{\alpha}{100}$ to the left.
- z_α is the *z* score with an area of α to the right.

- What is $z_{0.05}$?

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- What is P_{30}?