Section 5.1 — Discrete Random Variables

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Outline

Introduction

Definition (Random Variable)

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Definition (Probability Distribution)

A probability distribution is a description that gives the probability for each value of the random variable. It is often expressed in the format of a table, formula, or graph.

Definition (Discrete Random Variable)

A discrete random variable has a collection of values that is finite or determined by a counting process.

Requirements of a Distribution

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3. $\sum P(X = x) = 1$

Examples

Marijuana Legalization

Table 1: Responses to the question "Should marijuana use be legal?"

Response	$P(X = x)$
Yes	0.409
No	0.520
Don't Know	0.070

Is this a probability distribution?

Table 2: Responses to the question "On which interview should a candidate begin salary negotiations?"

Number of Interviews x	$P(X = x)$
1	0.30
2	0.26
3	0.10

Is this a probability distribution?

$$P(X = x) = \frac{x}{10} \text{ for } x = 0, 1, 2, 3, 4$$

Is this a probability distribution?

Parameters

The expected value for a discrete random variable *X* is equal to the mean of the probability distribution. It is given by

$$E(X) = \mu = \sum \left(x_i \cdot P\left(X = x_i\right) \right)$$

Definition (Variance)

The variance of a probability distribution is either

$$\sigma^2 = \sum \left((x_i - \mu)^2 \cdot P\left(X = x_i\right) \right)$$

or

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Definition (Standard Deviation)

The standard deviation of a probability distribution is

$$\sigma = \sqrt{\sum \left(x_i^2 \cdot P(X = x_i) \right) - \mu^2}$$

MOAR EXAMPLES!!!

Genetic Disorders

Four males with an X-linked genetic disorder have one child each. The random variable *x* is the number of children among the four who inherit the genetic disorder.

Table 3: Number of children among with disorder

x	$P(X = x)$
0	0.0625
1	0.2500
2	0.3750
3	0.2500
4	0.0625

In the Texas Pick 3 lottery, you can bet $1 by selecting three digits, each between 0 and 9 inclusive. If the same three numbers are drawn in the same order, you win $500.

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- What is the probability of winning?
- What is the expected value of your winnings?