Section 4.4 — Combinations and Permutations

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Introduction

Movie tickets

Example

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What if I have tickets for everyone?

Definitions

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Combinations

Combinations of items are arrangements in which different sequences of the same items are **not** counted separately. In other words, order does **not** matter.

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- Number of ways to arrange people in class?

Rules

Factorial Rule

Theorem (Factorial Rule)

The number of different permutations of n items when all n of them selected without replacement is n!.

Permutation Rule (Different Items)

Theorem (Permutation Rule)

The number of different permutations of n items when only r of them are selected without replacement is

$$_nP_r = \frac{n!}{(n-r)!}$$

Permutation Rule (Some Identical Items)

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The number of different permutations of n items when and all of them are selected without replacement, but some of them are identical is

$$\frac{n!}{n_1! n_2! \cdots n_k!}$$

Where n_1, n_2, … n_k are the different number of alike items.

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Example

How many ways are there to rearrange the letters in "banana"?

Combinations Rule

Theorem (Combinations)

The number of different combinations (order doesn't matter) of n items when only r of them are selected without replacement is

$$_nC_r = \binom{n}{r} = \frac{n!}{(n-r)!r!}$$

Examples

Coin flips

If I flip a coin 8 times, how many possible outcomes are there? What's the probability of getting no heads?

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What is the probability of getting exactly 5 heads?

An ATM PIN is made up of four digits. How many possible PINs are there? What's the probability of guessing it right on the first try?

Mix CD

You are making a mix CD of Iggy Azalea songs for your parents. How many different combinations of 12 of her 81 songs can you make?

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What if instead the three people are put on a committee that is in charge of all three tasks?