

Section 4.3 — Multiplication

Chris Godbout

Outline

Independent Events

Introduction

Sampling

Conditional Probability

Fundamental Counting Rule

Independent Events

Definition (Independent)

Two events are **independent** if the occurrence of one does not affect the *probability* of the occurrence of the other. If they are not independent, they are **dependent**.

Example

We will draw two cards from a standard deck of playing cards. Let A be the event that the first card is a queen and B be the event that the second card is an ace.

Example

We will draw two cards from a standard deck of playing cards. Let A be the event that the first card is a queen and B be the event that the second card is an ace.

1. What if we put the first card back after drawing?

Example

We will draw two cards from a standard deck of playing cards. Let A be the event that the first card is a queen and B be the event that the second card is an ace.

1. What if we put the first card back after drawing?
2. What if we don't?

Multiplication Rule for Independent Events

Independent Events

For two independent events A and B , the probability that A and B both occur is given by

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Introduction

Notation

$P(A|B)$ is the probability of event A occurring after it is assumed that event B has already occurred.

Notation

$P(A|B)$ is the probability of event A occurring after it is assumed that event B has already occurred.

Notation

If events A and B are independent, then $P(A|B) = P(A)$

Theorem (Multiplication Rule)

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

Theorem (Multiplication Rule)

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

Theorem (Independent Events Rule)

If events A and B are independent, then
$$P(A \text{ and } B) = P(A) \cdot P(B).$$

Sampling

Replacement

Definition

Sampling with replacement means a member of the population can be selected more than once. **Sampling without replacement** means a member of the population can be selected only once.

Replacement

Definition

Sampling with replacement means a member of the population can be selected more than once. **Sampling without replacement** means a member of the population can be selected only once.

- With replacement selections are independent events.

Replacement

Definition

Sampling with replacement means a member of the population can be selected more than once. **Sampling without replacement** means a member of the population can be selected only once.

- With replacement selections are independent events.
- Without replacement selections are dependent events.

Table 1: Results of a certain drug screening

	Positive Result	Negative Result
Used drugs	44	6

Drug Screening

Table 1: Results of a certain drug screening

	Positive Result	Negative Result
Used drugs	44	6

1. If we select two people *with replacement* who used drugs, what is the probability that the first person had a positive result and the second had a negative result?

Drug Screening

Table 1: Results of a certain drug screening

	Positive Result	Negative Result
Used drugs	44	6

1. If we select two people *with replacement* who used drugs, what is the probability that the first person had a positive result and the second had a negative result?
2. What if there was no replacement?

Stop and Frisk

In 2013, New Yorkers were stopped by the police 191558 times.
169252 of them were innocent.

Stop and Frisk

In 2013, New Yorkers were stopped by the police 191558 times. 169252 of them were innocent.

Question

What is the probability that 5 people selected at random without replacement are all innocent?

Birthdays

Question

What's the probability that at least two people in this class are born on the same day of the year?

Birthdays

Question

What's the probability that at least two people in this class are born on the same day of the year?

Question

What's the probability that two people selected at random are born on a Monday?

Conditional Probability

Definition (Conditional Probability)

A **conditional probability** of an event is a probability obtained with the additional information that some other event has already occurred. $P(B|A)$ denotes the probability of event B occurring, given that A has already occurred.

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

Let's make a deal!

Civil Rights Act of 1964

	Yes	No
Democrats	152	96
Republicans	138	34

Civil Rights Act of 1964

	Yes	No
Democrats	152	96
Republicans	138	34

1. What is the probability a member of congress is a democrat given that they voted yes?

Civil Rights Act of 1964

	Yes	No
Democrats	152	96
Republicans	138	34

1. What is the probability a member of congress is a democrat given that they voted yes?
2. What is the probability that a member of congress voted yes given that they're a democrat?

Civil Rights Act of 1964

	Yes	No
Democrats	152	96
Republicans	138	34

1. What is the probability a member of congress is a democrat given that they voted yes?
2. What is the probability that a member of congress voted yes given that they're a democrat?
3. Which party was more likely to vote yes?

Civil Rights Act of 1964

Delegation	Democrats		Republicans	
	Yes	No	Yes	No
Northern	145	9	138	24
Southern	7	87	0	10

Civil Rights Act of 1964

Delegation	Democrats		Republicans	
	Yes	No	Yes	No
Northern	145	9	138	24
Southern	7	87	0	10

- What's the probability that a member of congress voted yes given that they're from the north?

Civil Rights Act of 1964

Delegation	Democrats		Republicans	
	Yes	No	Yes	No
Northern	145	9	138	24
Southern	7	87	0	10

- What's the probability that a member of congress voted yes given that they're from the north?
- What's the probability that a member of congress is from the north given that they voted yes?

0.5% of a population have a certain disease. There is a test to screen for it that is 95% accurate. If you are screened and the test comes back positive, what is the probability that you have the disease?

Fundamental Counting Rule

Fundamental Counting Rule

If two events can occur m and n ways, respectively, then the number of ways the two events can occur is $m \cdot n$.

Example

Number of two-character blocks where the first character is an upper-case letter and the second is a digit?

Example

Number of potential California license plates if the format is 2GAT123?