Section 9.2 — Comparing Two Population Means (σ unknown)

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Outline

Introduction

Notation

1. n_1 is the size of sample 1.
2. $\bar{x}_1$ is the mean for sample 1.
3. s_1 is the standard deviation for sample 1.

1. n_1 is the size of sample 2.
2. $\bar{x}_2$ is the mean for sample 2.
3. s_2 is the standard deviation for sample 2.

Requirements

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2. The samples are independent.
3. Both population standard deviations, σ_1 and σ_2, are unknown.
4. Both sample sizes are at least 30 or both population distributions are approximately normal.

Basic idea

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2. Find the margin of error.
3. Use the margin of error to construct a confidence interval.

Unequal Variances

Margin of Error

When both population standard deviations are unknown and assumed to be unequal *and* the samples taken are independent, then the margin of error for $\mu_1 - \mu_2$ is

$$E = t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Degrees of Freedom

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Simple

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Less simple

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(s_1^2/n_1\right)^2}{n_1-1} + \frac{\left(s_2^2/n_2\right)^2}{n_2-1}}$$

Do men talk less than women?

The following table gives results from a study of the words spoken in a day by men and women. Construct a 99% confidence interval for the true difference in means.

Men	Women
$n_1 = 186$	$n_2 = 210$
$\bar{x}_1 = 15668.5$	$\bar{x}_2 = 16215.0$
$s_1 = 8632.5$	$s_2 = 7301.2$

Equal Variances

Margin of Error

When both population standard deviations are unknown and assumed to be equal *and* the samples taken are independent, then the margin of error for $\mu_1 - \mu_2$ is

$$E = t_{\alpha/2}\sqrt{\frac{(n_1 - 1)\, s_1^2 + (n_2 - 1)\, s_2^2}{n_1 + n_2 - 2}}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

where the degrees of freedom is

$$df = n_1 + n_2 - 2$$

Cell Phones

Steven believes that his wife's cell phone battery does not last as long as his cell phone battery. On eight different occasions, he measured the length of time his cell phone battery lasted, and calculated that the mean was 24.3 hours with a standard deviation of 6.1 hours. He measured his wife's cell phone battery on nine different occasions and got a mean of 22.8 hours with a standard deviation of 8.3 hours. Construct and interpret a 95% confidence interval for the true difference of battery life between Steve's cell phone battery and his wife's. Assume the populations are normal distributions with equal variances.