Sections 8.1, 8.2, 8.3 — Estimating Population Means

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Outline

 σ is known

Student's t-distribution

 σ is Unknown

σ is known

Margin of error

If the population standard deviation is known, the margin of error is:

$$E = \left(z_{\alpha/2}\right) \left(\frac{\sigma}{\sqrt{n}}\right)$$

if the following requirements are met:

- 1. The sample is a simple random sample.
- 2. The sample size is at least 30 or the population distribution is normally distributed.

Home Maintenance

A survey of 85 randomly selected homeowners finds that they spend a mean of \$67 per month on home maintenance. Construct and interpret a 99% confidence interval for the mean amount of money spent per month on home maintenance by all home owners. Assume that the population standard deviation is \$14 per month.

Minimum Sample Size

The minimum sample size required for estimating a population mean at a given level of confidence with a particular margin of error is

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2$$

Credit Cards

A bank would like to estimate the mean number of credit cards college students have in their wallets. They would like to create a 98% confidence interval with a maximum error of 1 card. Assuming a standard deviation of 3.25 cards, what is the minimum number of college students they must include in their sample?

Student's t-distribution

Properties

- 1. A *t*-distribution is symmetric and bell-shaped, centered around 0.
- 2. A *t*-distribution is completely defined by its number of degrees of freedom, df.
- 3. The total area under its graph is 1.
- 4. The *x*-axis is a horizontal asymptote for a *t*-distribution.

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- 4. Find the value of *t* so the *t*-distribution with 11 degrees of freedom has an area to the left of 0.010.
- 5. Find the value of t so that the area between t and -t is 0.95

σ is Unknown

Margin of Error

If the population standard deviation is not known, the margin of error is:

$$E = \left(t_{\alpha/2}\right) \left(\frac{s}{\sqrt{n}}\right)$$

with

$$df = n - 1$$

if the following requirements are met

- 1. The sample is a simple random sample.
- 2. The sample size is at least 30 or the population is normally distributed.

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- Construct a 90% confidence interval estimate of the mean weight loss for all such subjects.
- 2. What would it mean (for \bar{x}) for the Atkins diet to work?
- 3. Based solely off of this data, do you think the Atkins diet works? Is it worth it? What other information may we want?