General Multinomial Example.

- Three Categories (R, G, B)

- Let x_i be the true color of the i^{th} marble.
- Let y_i be the observed color of the i^{th} marble.

$$P(X_i = x_i) = \begin{Bmatrix} \gamma_R, & x_i \equiv R \\ \gamma_G, & x_i \equiv G \\ \gamma_B, & x_i \equiv B \end{Bmatrix}$$

- We want to learn the abundance or 'count' of each category.

$$T_k = \sum_{i=1}^{N} \mathbb{I}(x_i = k), \qquad k \in \{`R', `B', `G'\}$$

- Instead we observe

$$Y_i = y_i \in \{`RB', `G'\}$$

$$P(Y_i = y_i) = \begin{cases} \gamma_R + \gamma_B, & y_i = `RB' \\ \gamma_G, & y_i = `G' \end{cases}$$

$$O_L = \sum_{i=1}^{N} \mathbb{I}(y_i = g), \qquad g \in \{`RB', `G'\}$$

- The probability distributions of T_K and O_L in the vectors $\vec{T}$ and $\vec{O}$ are.

$$P(\vec{T} = \vec{\mathrm{t}}) = \frac{\left(\sum_{i=1}^{K} T_i\right)!}{\prod_{i=1}^{K} (T_i)!} \cdot \gamma_R^{T_R} \cdot \gamma_B^{T_B} \cdot \gamma_G^{T_G}$$

$$P(\vec{O} = \vec{o}) = \frac{\left(\sum_{i=1}^{L} O_i\right)!}{\prod_{i=1}^{L} (O_i)!} \cdot (\gamma_R + \gamma_B)^{O_{RB}} \cdot \gamma_G^{O_G}$$

- So for the iteration step at stage p we have:

$$E[T_R \,|O_{RB}] = \frac{\gamma_R}{\gamma_R + \gamma_B} \cdot O_{RB}$$

$$E[T_B|O_{RB}] = \frac{\gamma_B}{\gamma_R + \gamma_B} \cdot O_{RB}$$

- Finally the iterative step for determining the p^{th} stage estimates of γ_R and γ_B can be written as:

$$\hat{\gamma}_R^{(p)} = \frac{\left(\dfrac{\hat{\gamma}_R^{(p-1)}}{\hat{\gamma}_R^{(p-1)} + \hat{\gamma}_B^{(p-1)}}\right) \cdot (O_{RB})}{N}$$

$$\hat{\gamma}_B^{(p)} = \frac{\left(\dfrac{\hat{\gamma}_B^{(p-1)}}{\hat{\gamma}_R^{(p-1)} + \hat{\gamma}_B^{(p-1)}}\right) \cdot (O_{RB})}{N}$$

$$\hat{\gamma}_G^{(p)} = \frac{\hat{\gamma}_G^{(p-1)}(O_G)}{N}$$