Causal Impact of Campaign Strategies Analysis

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1 Average Causal Effect (ATE)

Assumptions:
1) Consistency: $D_i = d \rightarrow Y_i(d) = Y_i$
2) Unfoundness: given X, if $\{Y_i(1), Y_i(0)\} \perp D_i|X_i$, D_i is conditionally ignorable.

In our question, $D_i = 1$: have the specific VSA; $D_i = 0$: don't have the specific VSA

$$ATE = E[E[Y_i|D_i = 1, X_i]] - E[E[Y_i|D_i = 0, X_i]]$$

$$\hat{ATE} = \frac{1}{n}\sum_{i=1}^{n}[\hat{E}[Y_i|D_i = 1, X_i] - \hat{E}[Y_i|D_i = 0, X_i]]$$

Consider $E[Y_i|D_i, X_i] = \tau D_i + X_i^\top \beta$, then $ATE = \tau \rightarrow \tau$ has a causal interpretation.

Now we need to do linear regression to fit $E[Y_i|D_i = 1, X_i]$

In this setting, Assumption 2 is relatively difficult to fulfill, because Regression is more dependent on having a comprehensive enough set of covariates to satisfy "conditional ignorability".

Randomized Controlled Trails (RCT)
RCT is also known as A/B test, which is a well-knwon method to calculate causal effect. In our settings, we cannot randomize D_i, and it's also hard to determine which variable should we include in regression so that conditional ignorability holds; what will happen when X_i is assumed to be linear in model but the CEF is not linear on X_i?

However, because our data collected before we change the campaign strategy in different age groups (16-23; other) are almost parallel, we consider to use difference in differences to analysis the causal effect.

2 Difference-in Differences

Assumptions:
1) SUTVA: Stable Unit Treatment Value Assumption $\rightarrow$ the potential outcomes of unit i are not relevant to the treatment of other units $\rightarrow$ we should choose the units of analysis to minimize interfernce across units.
2) No-Anticipation Assumption: $Y_{i,t}(g) = Y_{i,t}(\infty)$ for all $t < g \rightarrow$ Before the start of treatment, potential outcomes for units were consistent with those that did not receive treatment
(Selection Bias might be introduced here)
3) Parallel Trends Assumption: $E[Y_{i,t=2}(\infty)|G_i = 2] - E[Y_{i,t=1}(\infty)|G_i = 2] = E[Y_{i,t=2}(\infty)|G_i = \infty] - E[Y_{i,t=1}(\infty)|G_i = \infty] \rightarrow$ In the absence of treatment, the evolution of the outcome among treated units is the same as the evolution among the untreated units on average.

Under 1) 2) 3) assumptions,

$$ATT = (E[Y_{i,t=2}|G_i = 2] - E[Y_{i,t=1}|G_i = 2]) - (E[Y_{i,t=2}|G_i = \infty] - E[Y_{i,t=1}|G_i = \infty])$$

ATT represents the average treatment effect on the treated group at time t=2.

We have two methods to estimate ATT in population,
1) plug-in estimator: $\hat{\theta}^{DID} = (\overline{Y}_{g=2,t=2} - \overline{Y}_{g=2,t=1}) - (\overline{Y}_{g=\infty,t=2} - \overline{Y}_{g=\infty,t=1})$
2) two-way fixed-effects (TWFE) regression: $Y_{i,t} = \alpha_0 + \gamma_0 \mathbf{1}\{G_i = 2\} + \lambda_0 \mathbf{1}\{T_i = 2\} + \beta_0^{twfe}(\mathbf{1}\{G_i = 2\}\mathbf{1}\{T_i = 2\}) + \epsilon_{i,t}$, where β_0^{twfe} is the estimate of $\hat{\theta}^{DID}$

In our setting, we use Repeated Cross-section data $\{Y_i, G_i, T_i\}_{i=1}^n$ i.i.d. draw from the mixture distribution:

$$P(Y \leq y, G = g, T = t) = \mathbf{1}\{t = 2\}\lambda P(Y_{t=2} \leq y, G = g|T = 2) + \mathbf{1}\{t = 1\}(1 - \lambda)P(Y_{t=1} \leq y, G = g|T = 1)$$

which means select the pre-intervention time period ($T = 1$) with probability $(1 - \lambda)$ Select the post-intervention time period ($T = 2$) with probability λ. Collect data for $G = 1, G = 2$, respectively.