Aggregate Implications of Heterogeneous Inflation Expectations: The Role of Individual Experience *

Mathieu Pedemonte [1] Hiroshi Toma [2] Esteban Verdugo [2]

[1] Federal Reserve Bank of Cleveland

[2] University of Michigan

February, 2023

Abstract

We show that inflation expectations are heterogeneous and depend on past individual experiences. We propose a diagnostic expectations-augmented Kalman filter to represent consumers' heterogeneous inflation expectations, where heterogeneity comes from an anchoring-to-the-past mechanism. Using survey data, we estimate the diagnosticity parameter and show that the model can replicate consumers' inflation expectations and its heterogeneity across cohorts in the US. We introduce this mechanism into a New-Keynesian model and find that heterogeneous expectations anchor aggregate responses to the agents' memory, producing sluggish dynamics in expectations. Central banks should be active to prevent agents from remembering current shocks far into the future.

Keywords: Belief formation, heterogeneous expectations, survey data, overextrapolation

JEL codes: D84, E31, E58, E71

* We thank Andrés Blanco, Olivier Coibion, Javier Cravino, Ina Hajdini, Juan Herreño, Christopher House, Daniel John, Ed Knotek, John Leahy, Jean-Paul L'Huillier, Matthew Shapiro, Michael Weber, Donghoon Yoo, Basit Zafar, and the seminar participants at the BSE Workshop on Expectations in Macroeconomics, the KIEP-CEPR Return of Inflation Conference, the Federal Reserve Bank of Cleveland, and the University of Michigan for their useful comments. The views expressed herein are those of the individual authors and do not necessarily reflect the official positions of the Federal Reserve Bank of Cleveland or the Federal Reserve System. E-mails: mathieu.pedemontelavis@clev.frb.org, htoma@umich.edu, everdugo@umich.edu.

I Introduction

Inflation expectations matter for decisions at both the firm and the household levels (Coibion et al., 2019, 2020; Hajdini et al., 2022b). Given their importance, there is an increased interest in measuring them and exploring what determines their formation process. The recent literature shows that individuals form inflation expectations, for instance, based on their recent buying experience (D'Acunto et al., 2021) and historical experiences regarding aggregate inflation (Malmendier and Nagel, 2016). While most studies on this topic provide empirical evidence regarding how differences in inflation expectations arise at the individual level and their effects on different micro-level decisions, there is less understanding of the aggregate implications associated with the inflation expectations' heterogeneity observed in the data. There is a noticeable gap in the literature between the empirical micro-level findings and macroeconomic models.

This paper aims to fill this gap. First, we show that individuals' inflation expectations depend on their history of inflation, confirming the main empirical finding of Malmendier and Nagel (2016) but using new US and international evidence.[1] Using detailed micro-level data, we find that (i) inflation expectations are heterogeneous across cohorts, (ii) inflation experiences are clustered by age and (iii) positively correlated with individual inflation history, and (iv) there are no differences between cohorts in updating to current information.

These facts suggest that consumers use heuristic representativeness, as in Kahneman and Tversky (1972), by positively weighing their past experiences when forming expectations.[2] We propose a diagnostic expectations-augmented Kalman filter (Bordalo et al., 2018, 2019, 2020) to model the inflation expectation-formation process. We depart from rational expectations by presenting a formulation where individuals use their past inflation histories and compare them to current information that is rationally gathered from signals.

Under the proposed framework, inflation expectations have two components: a current forecast made with current shared information between agents and a reference term that depends on

[1] While Malmendier and Nagel (2016) use the University of Michigan's Survey of Consumers (MSC), in Sections III and IV of this paper we provide external validity to their result by using a different data source with panel characteristics: the Survey of Consumer Expectations (SCE) from the Federal Reserve Bank of New York. In addition, in Appendix E we show that the finding is consistent at the international level even after controlling for common cohort characteristics.

[2] Meaning that to form expectations, they use their history of inflation experienced as a way to evaluate and forecast current and future events

individuals' past experiences. We structurally estimate the diagnostic parameter that governs the expectations-formation process. As all respondents share current information, we can control for the current common forecast using time-fixed effects. The resulting estimation for the diagnostic coefficient is -0.317, implying that consumers positively weigh their inflation history to the detriment of current unexpected news.[3]

This result implies that people compare new information with a reference from their inflation history, weighing the current, unbiased distribution with their history and referring back to their experiences. Our framework generates patterns that replicate relevant features of the data since agents who experienced episodes of high inflation in the past systematically forecast higher inflation values compared to those who experienced episodes of low inflation. This experience-based bias can explain part of the inflation expectations heterogeneity across cohorts observed in surveys. Moreover, our framework predicts patterns that match the data in line with sluggish expectations as in Coibion and Gorodnichenko (2015).

We model the shared component following a standard signal-extraction procedure. Based on works exploring reference prices through the shopping experiences of consumers (D'Acunto et al., 2021), we use the lagged inflation rate of the food component of the CPI as a signal of the non-observed aggregate inflation variable. Then, we construct a cohort-specific inflation expectations measure using (i) the already computed shared component, (ii) an idiosyncratic element related to the memory of the cohort, and (iii) the diagnostic coefficient value estimated in the empirical section of the paper.

With our cohort-specific inflation expectations measure, we obtain model-based forecasts that closely match the inflation expectations observed in the data across cohorts and time. A regression between our diagnostic model-based cohort-specific inflation expectations and individual expectations observed in survey data gives a coefficient of 0.888, statistically different from zero. Although the model-based inflation forecasts do not use micro-level information on inflation expectations, they remarkably predict consumers' survey data. This paper shows that using these two simple consumer references, shopping experience, and memory, we can effectively model survey household expectations. Thus, we show that consumers' expectations surveys contain relevant and

[3]This result does not imply that agents cannot overreact to current news. Our empirical exercise shows that the proposed expectation modeling framework explains the heterogeneity observed across cohorts. However, the presence of a common component in the modeling allows for a common overreaction to some current news.

meaningful information.

We then introduce this heterogeneous inflation expectations mechanism into a New Keynesian model to explore the macro implications arising from this micro-level heterogeneity. We allow households to form expectations according to the proposed diagnostic Kalman filter. In our model, while old generations have their expectations shaped mainly by their past, new generations are highly influenced by recent developments. We find that heterogeneous expectations anchor the aggregate response of the inflation and output gap to agents' memory. At the same time, they also increase the duration of the effects of the shocks.[4] After an inflationary shock, the model produces hump-shaped expectations. This reaction is consistent with overextrapolation as in Angeletos et al. (2021). In the first few periods, consumers underreact to the inflationary shock, as they are tied to their reference from the steady state. But after witnessing higher inflation (and also after new cohorts enter the economy in a high inflation environment), consumers get tied to that reference and over-extrapolate the shock, overreacting in their forecast.

We perform an optimal Taylor rule exercise where the central bank seeks to minimize the expected volatility of the economy by optimally choosing the parameters of a Taylor rule. When we allow for heterogeneous expectations in the model, agents have long memories and remember current shocks far into the future. After a negative supply shock or a positive demand shock, the optimal response of the central bank is to be more active with respect to the full information rational expectations case. This way, the monetary authority prevents inflation from rising and also prevents agents from incorporating a high-inflation episode into their memories.

This paper has important implications for explaining past inflation dynamics and learning about the future consequences of recent economic developments. Since 2021, a new cohort of consumers worldwide has been experiencing relatively high inflation rates for the first time. According to our findings, this high-inflation episode could have consequences in the medium run since consumers incorporate this episode into their history of inflation, adjusting future expectations. Our framework shows that accommodating high inflation produces higher and more persistent inflation expectations, which in turn generate a higher and more persistent inflation rate in the future. Our findings help us understand why inflation has persisted in the past, why consumers' inflation ex-

[4]While we could incorporate other forms of heterogeneity and biases related to experience-based mechanisms on the firm or the government side of the economy, in this paper, we introduce non-rational heterogeneous expectations only at the household level. We opt for this option mainly because of data availability motives.

pectations are persistent today, what to expect from episodes of unusually high inflation, and how central banks should react to such episodes.

Recent macroeconomic models show the relevance of heterogeneity in explaining aggregate fluctuations. However, the focus has been mostly on households' financial constraints (Kaplan et al., 2018). There are few studies on the aggregate role of expectations heterogeneity across individuals. Although surveys show significant heterogeneity across firms (Coibion et al., 2018) and households inflation expectations (Hajdini et al., 2022c), few works study its macroeconomic implications. A notable exception is Afrouzi (2020), which shows that heterogeneity in firm-level inflation expectations, coming from different levels of attention due to endogenous information acquisition on competitors' beliefs, amplifies monetary non-neutrality. Our paper focuses on the heterogeneity of expectations at the household level. In our framework, households' heterogeneous inflation expectations anchor the response of aggregate variables to agents' memories, increasing the persistence of the effects of shocks. Therefore, an energetic reaction from monetary authorities prevents current high inflation and prevents agents from incorporating high-inflation episodes into their memories, thus preventing higher future inflation expectations.

Other works focus on exploring sources of heterogeneity across firms and consumers (Afrouzi, 2020; Hajdini et al., 2022c). While Malmendier and Nagel (2016) find that personal experience on aggregate inflation affects individual expectations, D'Acunto et al. (2021) show that buying experiences matter. These idiosyncratic and shared experiences create systematic differences in expectations across the population. Similarly, Coibion et al. (2019), Roth and Wohlfart (2020), and Hajdini et al. (2022b) show that consumer expectations are relevant for their decision-making process.

The rest of the paper is organized as follows. Section II discusses recent works on the topic. Section III provides empirical results regarding consumers' heterogeneity in inflation expectations. We empirically model inflation expectations depending on the history of inflation experienced by cohorts in Section IV. Section V discusses the aggregate implications arising from heterogeneous inflation expectations. Section VI shows results obtained for an optimal Taylor rule exercise. We analyze the high-inflation episode of 2021 through the lens of our theoretical model in Section VII. Finally, Section VIII concludes.

II Literature review

A vast empirical literature shows that households' inflation expectations depart from full information rational expectations and are heterogeneous. Relevant to our paper, Malmendier and Nagel (2016) document that households present learning from past inflation mechanisms when forming inflation expectations. Thus, people who have experienced higher inflation rates in the past have higher inflation expectations in the future. Therefore, heterogeneity of inflation expectations naturally arises due to different experiences with past inflation rates. Similar results are discussed in Malmendier et al. (2021).

However, households' inflation expectations depend on more than past experiences. Evidence shows they also respond to other variables such as professional forecasts (Carroll, 2003), prices exposure (D'Acunto et al., 2021), and socioeconomic characteristics (D'Acunto et al., 2022), among others.[5] This departure from full information rational expectations and the presence of heterogeneity is relevant since households' inflation expectations affect a broad set of households' decisions. For instance, Malmendier and Nagel (2016) show that inflation expectations influence individuals' financial decisions, while Coibion et al. (2019) confirm that inflation expectations partly determine households' spending on durable goods.

The model proposed in this paper features monetary policy, a learning from the past mechanism, and overlapping generations, connecting the paper to several strands of the theoretical literature. First, our model closely follows the literature enclosing behavioral New Keynesian models such as the ones proposed by Branch and McGough (2010), Gabaix (2020), and Gáti (2020). By considering overlapping generations in a New Keynesian context, we relate, for instance, to Gali (2021). By stating that cohorts show heterogeneity in expectations because of different experiences, we connect to papers unrelated to monetary policy but where different cohorts have different beliefs about the future, such as Collin-Dufresne et al. (2017) and Kuchler and Zafar (2019).

We base our model of non-rational and heterogeneous expectations on the diagnostic expectations literature, as introduced in Bordalo et al. (2018), Bordalo et al. (2019), and Bordalo et al. (2020).[6] Bianchi et al. (2021) and L'Huillier et al. (2021) provide recent applications of this frame-

[5]While we focus on the household side of the economy, there is evidence showing that professional forecasters depart from rational expectations too (Coibion and Gorodnichenko, 2015; Bordalo et al., 2020; Gáti, 2020). Because of data availability, evidence on firms' expectations is notably scarce (Candia et al., 2022).

[6]Our approach is also related to broader signal extraction/noisy information approaches such as the ones proposed

work to macro monetary settings.

Besides diagnostic expectations, a large body of literature focuses on analyzing the implications of departing from the full information rational expectations assumption. Among the main examples, we include the set of papers related to the imperfect information approach (Mankiw and Reis, 2011), the complex systems/animal spirits/heuristic approach (De Grauwe, 2011; Jump et al., 2019), the sticky information approach (Mankiw and Reis, 2002), and the adaptive learning approach (Marcet and Sargent, 1989; Evans and Honkapohja, 2001). However, only some of these papers have studied how heterogeneity in expectations arises and its macroeconomic consequences.

Our findings are closely related to those of Malmendier and Nagel (2016), who use adaptive learning to approximate cohorts' heterogeneous inflation expectations. Instead, we opt for a diagnostic Kalman filter. While we also rely on constant gain, the selection of parameters in our framework is primarily data-driven. Our approach allows us to incorporate a current shared forecast component using a standard Kalman filter and a structure that incorporates past inflation experiences.

Our paper shows that cohorts do not adjust their expectations differently in response to current inflation news. Our results suggest that while younger cohorts implicitly put more weight on current information, they adjust to the news data similarly to older cohorts. In other words, younger cohorts do not react more strongly to inflation news than older cohorts. In that sense, our modeling follows Malmendier and Nagel (2016) by incorporating a method consistent with the heuristic of representativeness of past experiences and allowing agents to use the new information to form expectations. Following recent evidence showing that agents form expectations based on their shopping experiences (D'Acunto et al., 2021), we use a Kalman filter approach where agents use signals from current food prices to model the expectation formation process. In addition, our approach is flexible enough to be incorporated into a general equilibrium framework as some other recent studies (Bianchi et al., 2021; L'Huillier et al., 2021).

in Woodford (2001) and Blanchard et al. (2013), among others.

III Empirical facts

This section reviews some empirical facts related to heterogeneous inflation expectations at the household level. We show how these expectations are correlated with past experiences regarding the aggregate inflation variable. These empirical facts motivate and guide the theoretical model of the paper.

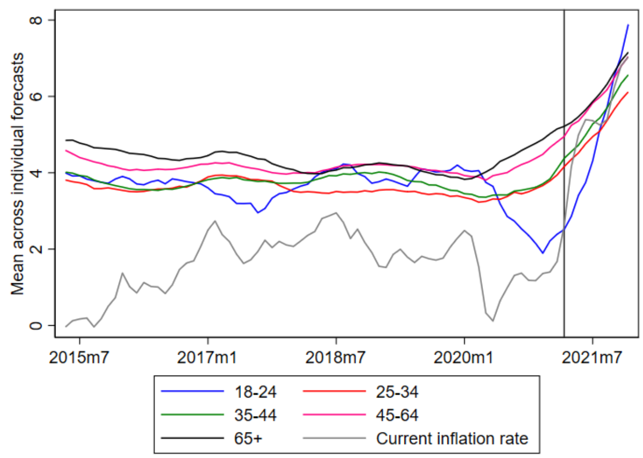
Malmendier (2021) and D'Acunto et al. (2022) document that consumers' experiences influence their inflation expectations. Thus, individual experiences are a source of expectations' heterogeneity. This paper focuses on how aggregate inflation experiences influence idiosyncratic inflation expectations, as in Malmendier and Nagel (2016). In the US economy, this heterogeneity turned out to be significant after the high-inflation episode of 2021, when inflation surged after 30 years of low and stable rates. This event could highly influence consumers' younger cohorts.

For this section, we use data from the Survey of Consumer Expectations (SCE) of the Federal Reserve Bank of New York. This data set is a US-wide rotating panel with information between March 2013 and December 2021, where each respondent is surveyed for a maximum of 12 contiguous months. This data set is handy for our purposes because it provides high-frequency data on American households' inflation expectations in two different periods of the US economy. In particular, we focus our analysis on respondents' 12-months-ahead point forecast. The 12-months-ahead inflation rate is computed as the inflation rate existing between the current month and 12 months after the current month.

Fact 1: Inflation expectations are heterogeneous across cohorts.

Figure 1 shows the mean 12-months-ahead inflation forecast by cohort. The heterogeneity across cohorts is evident. The oldest (65+) and the second oldest cohort (45-64) have higher mean inflation expectations throughout most of the sample. Those cohorts experienced a period of high inflation in the 60s, 70s, and early 80s. Regarding inflation forecast value, these cohorts are followed by intermediate cohorts (25-34 and 35-44), who experienced the stable and low inflation rates of the 90s, 00s, and 10s. Finally, the youngest cohort (18-24) shows the most volatile mean, following the current inflation rate most of the time. The mean value of this cohort notably increased after the high-inflation episode of 2021, surpassing older cohorts' expectations.

Figure 1: Average 12-months-ahead inflation expectations.



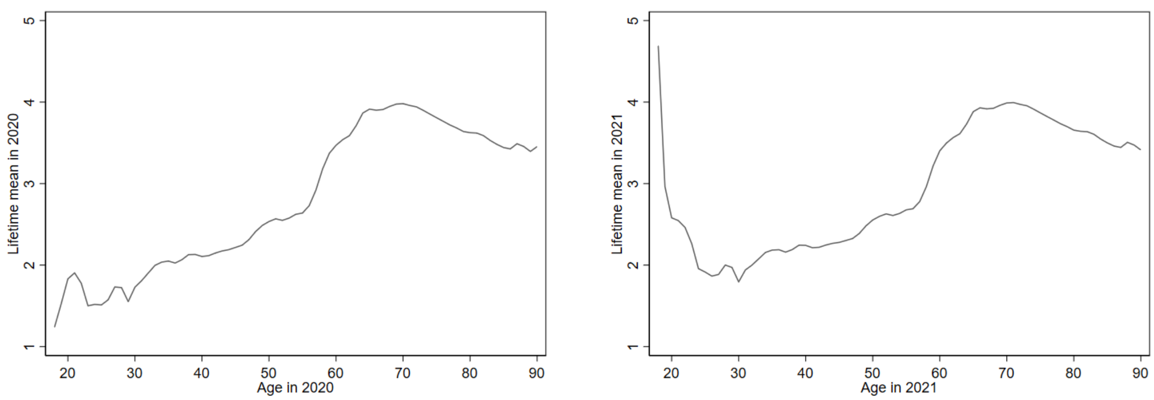
Note: The figure shows the 12-month moving average for the 10 percent and 90 percent trimmed mean for each cohort using the point forecast. We use population weights. Data goes from June 2013 to December 2021. Ages correspond to the interviewee's age at the time of the survey. The vertical line denotes March 2021.
Source: Survey of Consumer Expectations, Federal Reserve Bank of New York.

Fact 2: Inflation experiences are clustered by age.

Figure 2 plots the average lifetime inflation rate people have experienced according to their age in the years 2020 and 2021. In the US, the average lifetime inflation rates are clustered by age.

The heterogeneity of average experienced inflation rates across cohorts results from the different inflation-related events Americans have gone through. Older cohorts have experienced events such as the Great Inflation period (1965-1982), characterized by high and persistent inflation. Thus, these cohorts have a higher lifetime average inflation rate, regardless of the year we calculate. Meanwhile, intermediate cohorts have experienced periods of low and stable inflation rates throughout the 80s, 90s, 00s, and 10s. Therefore, they present lower values of lifetime average inflation rate. For older and intermediate cohorts, experiencing the high-inflation episode of 2021 did not significantly affect their lifetime average inflation rate.

Figure 2: Lifetime average inflation rate among respondents.



(a) 2020

(b) 2021

Note: The figure shows the mean of the monthly YoY inflation rate that people of the age indicated in the years 2020 and 2021 have experienced in their lifetimes, starting when they were age 18.
Source: FRED, Federal Reserve Bank of St. Louis.

In contrast, the youngest cohorts present a change between the years 2020 and 2021. Up to 2020, the youngest cohorts had not experienced high inflation, showing low lifetime average inflation rates. However, after being exposed to the high-inflation episode of 2021, their lifetime average inflation rate dramatically increased.

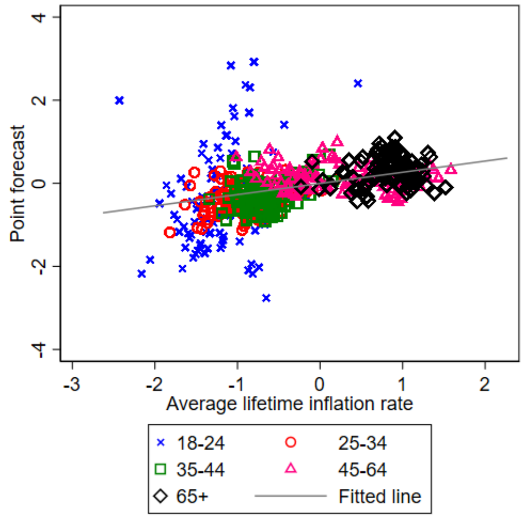
Fact 3: A higher average lifetime inflation rate is correlated with a higher point forecast.

Tying together both previous empirical facts, Figure 3 shows that people who have experienced higher average inflation rates during their lifetimes, when surveyed, tend to give a higher inflation point forecast.[7] We formally test this result in Table 1. Columns 3 and 4 of this table conclude that the inflation experienced significantly affects individuals' inflation expectations, even after controlling for the current environment and individual characteristics.

This fact provides empirical support for the literature on learning from past experiences (Malmendier and Nagel, 2016; Malmendier, 2021; Malmendier et al., 2021; Malmendier and Wachter, 2022) pointing to a possible source of heterogeneity in inflation expectations: past experiences with the aggregate inflation variable.

[7]We control for observable characteristics of the respondent except for the age and period variables.

Figure 3: Inflation point forecast and average lifetime inflation.



Note: The figure shows binned scatterplot across lifetime average inflation bins. We residualized the variables by respondent gender and commuting zone. Data goes from June 2013 to December 2021. Ages correspond to the interviewee's age at the time of the survey.
Source: Survey of Consumer Expectations, Federal Reserve Bank of New York.

While the evidence we provide here is for the US economy, in Appendix E, we find similar evidence for a panel of European countries. With this European data set, we show that the pattern (i) is present beyond the US and (ii) does not arise from cohorts' systematic characteristics but because of cross-country heterogeneous inflation experiences. In the panel of countries we use, we observe different inflation histories across countries that are not necessarily similar to the US experience. Again, we find that average inflation experience positively relates to individual inflation expectations, even after including country-time fixed effects and, more importantly, cohort fixed effects. This last set of fixed effects control for the fact that cohorts can have biases because of their age. In this sense, Hajdini et al. (2022a) find similar evidence using a survey for a panel of countries.

Fact 4: After controlling for the average lifetime inflation rate, younger cohorts do not react more strongly to inflation news than older cohorts.

We test whether younger generations react more strongly to the current economic environment after controlling for their average lifetime inflation. The idea behind this exercise is to check some results presented by Malmendier and Nagel (2016), where younger cohorts react more strongly to current events when compared to older cohorts. According to Malmendier and Nagel (2016), younger generations have fewer observations and are learning about the economy. We test this hypothesis through some individual-level regressions presented in Table 1. Similar to the results of other papers using information treatment (Hajdini et al., 2022b), column 1 of this table shows that all individuals react to current inflation events. These results also confirm the existence of a positive relationship between the inflation forecasts and average lifetime inflation rates, as we saw previously in Figure 3, even after considering current inflation.

Table 1: Effects of current and experienced inflation on inflation expectations

Dep. var.: Inflation expectations	(1)	(2)	(3)	(4)
Average lifetime inflation	0.325***	0.259***	0.293***	0.244***
	(0.026)	(0.065)	(0.024)	(0.023)
Current inflation	0.523***	0.669***		
	(0.017)	(0.119)		
Cohort 25-34		0.008		
		(0.342)		
Cohort 35-44		-0.048		
		(0.328)		
Cohort 45-64		0.082		
		(0.348)		
Cohort 65+		0.175		
		(0.352)		
Current inflation × 25-34		-0.202		
		(0.131)		
Current inflation × 35-44		-0.130		
		(0.128)		
Current inflation × 45-64		-0.119		
		(0.124)		
Current inflation × 65+		-0.187		
		(0.127)		
Time FE	No	No	Yes	Yes
Controls	No	No	No	Yes
Observations	105,413	105,413	105,413	105,399
R-squared	0.057	0.057	0.091	0.196

Note: Table shows regressions where the dependent variable is inflation expectations according to the Survey of Consumer Expectations (SCE) of the Federal Reserve Bank of New York. Column (1) shows controls by the average lifetime inflation of respondents of a given age at each period of time and the last inflation measure. Column (2) follows (1) but adds cohort fixed effects and the interaction of those cohort fixed effects with the current inflation. Column (3) follows (1) but adds time fixed effects and, hence, omits the current inflation variable. Column (4) follows (1) but adds time fixed effects and demographic controls. The demographic controls are income, gender, Hispanic origin, race, educational level, numerical proficiency, and commuting zone. Robust standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1. Standard errors clustered by age. The dependent variable is trimmed, dropping the lower and upper 10 percent of answers in each period.

To study if there are different reactions across cohorts, we run regressions that consider interactions of current inflation with a cohort indicator variable. Column 2 shows the corresponding results. After controlling for average lifetime inflation, we see that the interaction term does not have a statistically significant effect. Therefore, there are no different reactions to current inflation news across cohorts. We confirm the finding by performing a F-test where the null hypothesis is that all interactions are jointly equal to zero. The test gives a p-value of 0.39, so we cannot reject the null hypothesis. Furthermore, in Table A.6 of Appendix E, we find this result holds in a sample of European countries. Together, these results suggest that the heterogeneity across cohorts comes from the different past experiences with inflation, which contrasts with the finding of younger cohorts reacting more strongly to current events from Malmendier and Nagel (2016).

IV A simple model with heterogeneous expectations

In this section, we propose a diagnostic expectations-augmented Kalman filter (Bordalo et al. 2019) as the process by which agents form their inflation expectations. We begin with a simple model that provides a good starting point from which differences in agents' personal experiences do not imply heterogeneity in expectations. Given the absence of private information, we show that the observed heterogeneity cannot arise from a standard Kalman filter. Then, by introducing a diagnostic expectations-augmented Kalman filter, we explain how the inflation history experienced by the agents distorts their expectations, generating heterogeneity in their inflation forecasts. Moreover, we estimate the corresponding distortion parameter. Lastly, we close the section by comparing these heterogeneous rates of inflation expectations generated by our proposed framework and those observed in the data (i.e., Figure 1).

IV.1 Standard Kalman filter

IV.1.1 Setup

The economy is composed of different cohorts indexed by i. These cohorts are heterogeneous in their dates of birth and the inflation history they have experienced. Since there is no heterogeneity within cohorts, a single representative agent summarizes the situation of each one of these groups. In a given period $t+1$, the level of inflation π_{t+1} is defined according to the following random walk process[8]

$$\pi_{t+1} = \pi_t + \varepsilon_t,$$

where ε_t is a normally independent and identically distributed inflation shock. We assume that agents wish to forecast the future inflation rate π_{t+1}, but they only observe a noisy signal of this variable. In other words, the agents face a standard signal extraction problem. To simplify the analysis, in a given period t, we assume that the signal s_t is defined as

$$s_t = \zeta \pi_{t+1} + \upsilon_t,$$

where the coefficient $\zeta \geq 0$ denotes the pass-through existing between the unobserved variable π_{t+1} and its corresponding signal s_t, and υ_t is a signal noise. We assume that this noise is a normally independent and identically distributed variable. Moreover, to consider the existence of some elements causing movements in both the observed signal and the unobserved variable, we allow for a non-zero covariance between both shocks. More precisely, we assume

$$\begin{pmatrix} \varepsilon_t \\ \upsilon_t \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\varepsilon^2 & \sigma_{\varepsilon\upsilon} \\ \sigma_{\varepsilon\upsilon} & \sigma_\upsilon^2 \end{pmatrix} \right).$$

As a further simplification, we assume that there is no private information in the model. In other words, all of the agents receive exactly the same signal. Since the agents face a standard signal extraction problem, we assume that they generate a forecast of the inflation variable using the corresponding conditional expected value of the variable. More precisely, given their information set in period t, the agents apply a linear Kalman filter to forecast inflation in period $t+1$. Therefore, the predicted value of the inflation variable is given by

$$\mathbb{E}_{i,t}^{KF}\left[\pi_{t+1}\right] = (1 - \zeta K)\,\mathbb{E}_{i,t-1}^{KF}\left[\pi_{t+1}\right] + K s_t, \tag{1}$$

[8] We opt for a random walk process instead of an AR(1) specification because the data cannot reject the hypothesis that the monthly inflation rate has a unit root. We provide a more thorough discussion in Appendix C. Also see Pivetta and Reis (2007) for a discussion on the very high persistence of the (quarterly) inflation rate in the US. To complete the analysis, in Appendix D, we show the model's results when the inflation series follows an AR(1) process. These results are very similar to those found under the random walk assumption.

where K denotes the Kalman gain.[9] The Kalman filter approach allows us to characterize the forecasted distribution of the unobserved variable π_{t+1} in any period t conditional on agents' past and current signals $\left\{s_j\right\}_{j\in[0,t]}$. Notice that when the signal is perfectly revealing about the true state of the variable π_{t+1}, we have $\zeta = 1$, $\upsilon_t = 0$ in every period t, and $\sigma_{\varepsilon\upsilon} = 0$. As a conclusion, we obtain $K = \zeta K = 1$ and $\mathbb{E}_{i,t}^{KF}\left[\pi_{t+1}\right] = s_t = \pi_{t+1}$. The presence of a signal noise induces $K, \zeta K \in [0,1)$ even without a correlation between both error terms.

Regarding long-run values of inflation expectations, from the Kalman-based prediction equation and using the random walk structure associated with the inflation variable, we conclude that given $h \geq 1$, we must have

$$\mathbb{E}_{i,t}^{KF}\left[\pi_{t+h}\right] = \mathbb{E}_{i,t}^{KF}\left[\pi_{t+1}\right].$$

Finally, and considering $\gamma = (1-\zeta K) \in [0,1]$, the Kalman filter prediction can be written recursively as

$$\mathbb{E}_{i,t}^{KF}\left[\pi_{t+1}\right] = \gamma^{t+1}\mathbb{E}_{i,-1}^{KF}\left[\pi_0\right] + K\sum_{j=0}^{t}\gamma^{t-j}\left(\zeta\pi_{j+1} + \upsilon_j\right)$$

Therefore, using this simple version of the model, we conclude that higher values of past inflation imply a higher forecasting value of this same variable. However, agents' personal experiences are not associated with heterogeneity in expectations. According to this model, in any period t, agents who lived through episodes of high inflation forecast an inflation value identical to those who lived through episodes of low inflation. Given a starting point assumption where every agent observes the initial level of inflation, i.e. $\mathbb{E}_{i,-1}^{KF}\left[\pi_0\right] = \pi_0$ for every agent i, we conclude that $\mathbb{E}_{i,t}^{KF}\left[\pi_{t+1}\right] = \mathbb{E}_{t}^{KF}\left[\pi_{t+1}\right]$ must hold for every agent. In what follows, to simplify the analysis, we assume $\zeta = 1$.

[9] As usual, this signal-to-noise ratio is defined such that it minimizes the variance of the prediction error associated with the unobserved variable, i.e., $\pi_{t+1} - \mathbb{E}_{i,t}^{KF}\left[\pi_{t+1}\right]$. The Kalman gain that solves this optimization problem is a function of the covariance existing between the error associated with the observed signal and the unobserved variable, and the constant $\Sigma_{t+1|t-1} = \mathbb{V}\mathrm{ar}\left[\pi_{t+1} - \mathbb{E}_{i,t-1}^{KF}\left[\pi_{t+1}\right]\right]$ (see Cheung 1993 for an example of a Kalman gain that considers these terms). Regarding the constant $\Sigma_{t+1|t-1}$, it satisfies

$$\left(\Sigma_{t+1|t-1} - \sigma_\varepsilon^2\right)\left(\zeta^2\Sigma_{t+1|t-1} + \sigma_\upsilon^2 + 2\zeta\sigma_{\varepsilon\upsilon}\right) - \left(\sigma_\upsilon^2\Sigma_{t+1|t-1} - \sigma_{\varepsilon\upsilon}^2\right) = 0.$$

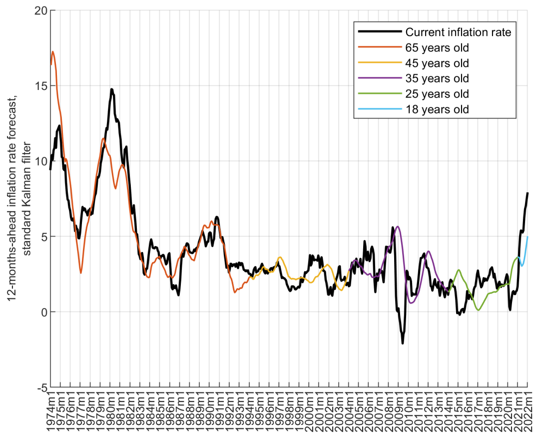
To identify an appropriate signal for the empirical counterpart of the expectations formation model, we follow the evidence presented in D'Acunto et al. (2021), who show that agents use their consumption experience to form expectations. More specifically, Campos et al. (2022), using the University of Michigan's Survey of Consumers (MSC), conclude that consumers weigh the food components of the CPI highly when forming inflation expectations. Dietrich (2022) finds similar evidence using different data sources. Therefore, we use the food component of the CPI as a shared inflation signal for consumers. More precisely, because of timing issues, we use the lagged inflation of the food component of the CPI. With that, consumers use the past changes in food prices as a signal to forecast inflation in the future. Note the forecast agents make is about a future period, where current prices are the base for the projected change. For example, if in December an agent forecasts aggregate inflation for the next 12 months, and we presume that consumers make this prediction at the beginning of the month, then we assume this agent considers November's food inflation to make her forecast. Based on this, for the empirical counterpart of the expectations formation model we assume $s_t = \pi_{t-1}^{food}$ where π_{t-1}^{food} denotes food inflation in period $t-1$.

Using the data of inflation and food inflation, we obtain $\sigma_\varepsilon^2 = 0.15$, $\sigma_\upsilon^2 = 4.09$ and $\sigma_{\varepsilon\upsilon} = -0.03$ from monthly data on the inflation and food inflation, with which we obtain $K = 0.1751$. We give more detail on how we obtain this calibration in Appendix C.

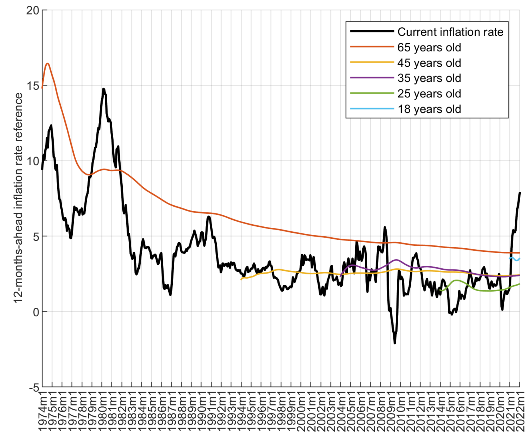
IV.1.2 Forecasting exercise

We now perform a forecasting exercise using monthly inflation data and distinguishing agents by cohorts. Given the recursive structure of the Kalman filter, and to initialize the forecasting process of each cohort, we assume that in the period in which the cohort representative agent reaches adulthood and begins forecasting, she uses the previous period's Kalman filter expected value as a starting point. We denote the period when agent i starts forecasting as period k_i. Given the starting point assumption where the initial level of inflation is common knowledge, we have $\mathbb{E}_{i,k_i-1}^{KF}[\pi_{k_i}] = \mathbb{E}_{k_i-1}^{KF}[\pi_{k_i}]$ for every agent i. Panel (a) of Figure 4 presents the 12-months-ahead inflation forecasts by different cohorts according to the standard Kalman filter. This figure plots the actual inflation rate and the forecast made by different selected cohorts.

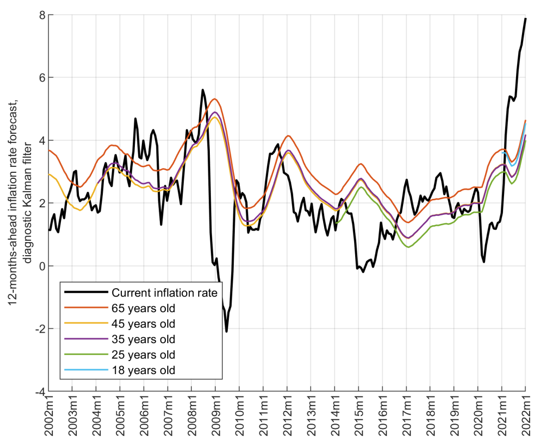
Figure 4: Diagnostic Kalman-filter-based inflation forecasts by cohort



(a) Standard Kalman-filter-based inflation forecasts



(b) Inflation rate reference



(c) Diagnostic Kalman-filter-based inflation forecasts

Note: Panel (a) shows the Kalman filter forecast for the common component for selected cohorts, differentiated by their age in 2021. Panel (b) shows the references for selected cohorts obtained according to the Kalman filter and given the history of inflation experienced by the corresponding age group. Panel (c) shows forecasts for selected cohorts according to the Kalman-filter-augmented expectations and considering the estimate for θ from Column 1 of Table 2. Selected cohorts are differentiated by their age in 2021. We further assume that each cohort starts forecasting when they become 18 years old.

As expected, the standard Kalman filter does not generate the heterogeneous pattern in inflation expectations observed in the data (i.e., Figure 1). In other words, the rate of inflation expectations evolves following an identical process across cohorts. We need to move to a more sophisticated framework to replicate the facts observed in the data.

IV.2 Diagnostic Kalman filter

IV.2.1 Setup

In this section, we depart from the standard Kalman filter framework by adopting the model of non-Bayesian beliefs known as diagnostic expectations. Following Bordalo et al. (2018), Bordalo et al. (2019) and Bordalo et al. (2020), we denote the true conditional distribution of the unknown inflation variable in a given period t as $f\left(\pi_{t+1} \mid \mathscr{I}_{i,t}\right)$. The term $\mathscr{I}_{i,t}$ denotes the information available to agent i up to the current period t. Given this definition, we assume that the diagnostic belief distribution of inflation for agent i is given by

$$f^{\theta}_{i,t}\left(\pi_{t+1}\right) = f\left(\pi_{t+1} \mid \mathscr{I}_{i,t}\right) D^{\theta}_{i,t}\left(\pi_{t+1}\right) Z_{i,t}, \tag{2}$$

where

$$D^{\theta}_{i,t}\left(\pi_{t+1}\right) = \left[\frac{f\left(\pi_{t+1} \mid \mathscr{I}_{i,t}\right)}{f\left(\pi_{t+1} \mid \mathscr{I}^{ref}_{i,t}\right)^{t-k_i}}\right]^{\theta}.$$

In this setup, the diagnostic parameter $\theta \in \mathbb{R}$ governs the level of distortion that the likelihood ratio $\left[D^{\theta}_{i,t}\left(\pi_{t+1}\right)\right]^{\frac{1}{\theta}}$ introduces into agents' beliefs. The normalizing parameter $Z^{-1}_{i,t} = \int f^{\theta}_{i,t}\left(\pi_{t+1}\right) d\pi_{t+1}$ is a constant that ensures that the diagnostic distribution $f^{\theta}_{i,t}\left(\pi_{t+1}\right)$ integrates to one in every period t and for every agent i. In this setup, agent i compares hers current information set $\mathscr{I}_{i,t}$ against a referential information set $\mathscr{I}^{ref}_{i,t}$. Later we show that this referential information set relates to agents' i past inflation experiences. As mentioned above, the parameter θ captures the level of distortion associated with the model. Under a standard Kalman filter framework, we have $\theta = 0$, which implies $D^{\theta}_{i,t}\left(\pi_{t+1}\right) = 1$. In this case, there is no distortion in beliefs, and $f^{\theta}_{i,t}\left(\pi_{t+1}\right) = f\left(\pi_{t+1} \mid \mathscr{I}_{t}\right)$. When $\theta \neq 0$, beliefs are distorted.

Notice that the no private information assumption implies that the set of information associated with the true conditional distribution is equal for everyone. In other words, we have $\mathscr{I}_{i,t} = \mathscr{I}_{t}$ for every agent i. Therefore, under the proposed signal extraction framework, in any period t, the expected value associated with the true conditional distribution of the unknown inflation variable is common to every agent and corresponds to $\mathbb{E}^{KF}_{t}\left[\pi_{t+1}\right]$. Given the normality assumption on the error term of the inflation process, the true conditional distribution satisfies $f(\pi_{t+1} \mid \mathscr{I}_t) \sim \mathscr{N}\left(\mathbb{E}_t^{KF}[\pi_{t+1}], \sigma_\pi^2\right)$. As we explain below, this normality result implies that the distribution associated with the referential information set $\mathscr{I}_{i,t}^{ref}$ is normal too. Given both normality results, from Equation 2, we can show that the diagnostic distribution of agent i is $f_{i,t}^{\theta}(\pi_{t+1}) \dot{\sim} \mathscr{N}\left(\mathbb{E}_{i,t}^{\theta}[\pi_{t+1}], \sigma_\pi^2\right)$, where the mean value of this distribution has the following linear structure

$$\mathbb{E}_{i,t}^{\theta}[\pi_{t+1}] = \mathbb{E}_t^{KF}[\pi_{t+1}] + \theta\left(\mathbb{E}_t^{KF}[\pi_{t+1}] - \mathbb{E}_{i,t}^{ref}[\pi_{t+1}]\right), \tag{3}$$

where $\mathbb{E}_{i,t}^{\theta}[\pi_{t+1}]$ is the diagnostic-distorted forecast associated with the diagnostic belief distribution $f_{i,t}^{\theta}(\pi_{t+1})$, and $\mathbb{E}_{i,t}^{ref}[\pi_{t+1}]$ is the expected value obtained according to the distribution associated with the referential information set $\mathscr{I}_{i,t}^{ref}$.[10] We define this linear composition of the standard Kalman filter as our diagnostic-augmented Kalman filter.

Until this point, our definitions have been history-independent. The standard Kalman filter is Markovian in the sense that it only needs the belief from the previous period, but this mechanism is not able to reproduce the empirical facts. Thus, we now introduce the role of the past through the reference term $\mathbb{E}_{i,t}^{ref}[\pi_{t+1}]$. For a representative agent of cohort i, we define the reference term as

$$\mathbb{E}_{i,t}^{ref}[\pi_{t+1}] = \frac{\sum_{j=1}^{t-k_i} \mathbb{E}_{i,t-j}^{KF}[\pi_{t+1}]}{t-k_i}, \tag{4}$$

where k_i is the period in which cohort i reaches adulthood and starts forecasting. This way, the reference term contains all the expectations agent i had in the past about the future inflation rate. In Panel (b) of Figure 4 we show how the inflation rate reference defined in Equation 4 evolves for different cohorts. We see that older cohorts, which have gone through episodes of higher inflation in their lifetimes, have higher reference points when compared to the younger cohorts, which have not experienced inflationary episodes.

Now, we turn to discuss the diagnosticity parameter θ. When $\theta > 0$, agents "overreact" to the current information they receive relative to their prior beliefs. For instance, if agents receive news about inflation being different that what they expected in the past, they will overreact to that news,

[10] See Appendix B for the derivation.

by putting more weight on the current information and underweighting their memory. If $\theta < 0$, agents "underreact" to the information just received, placing more weight on their references with respect to the current information. In this case, if agents see inflation being different than their priors, they will tend to anchor their current expectations to their past beliefs. As we discussed in Section III, recent inflation developments show that when inflation was low (2010-2020), older consumers had higher inflation expectation. Meanwhile, when inflation was high in 2021, consumers tended to react in a moderate way, not increasing their expectations by much. This is in line with agents anchoring their expectations to their experience or, in our framework, with $\theta < 0$.

Lastly, when $\theta = 0$, there are no distortions in beliefs, and we conclude $\mathbb{E}_{i,t}^{\theta}[\pi_{t+1}] = \mathbb{E}_{t}^{KF}[\pi_{t+1}]$ for every cohort i. After agent i forecasts inflation for period $t+1$, the next step is to forecast its future values. Given the random walk structure associated with the inflation variable, and considering $h \geq 1$, we concluded $\mathbb{E}_{t}^{KF}[\pi_{t+h}] = \mathbb{E}_{t}^{KF}[\pi_{t+1}]$. Since the reference term is a linear composition of expected values associated with the true conditional distribution, we must have $\mathbb{E}_{i,t}^{ref}[\pi_{t+h}] = \mathbb{E}_{i,t}^{ref}[\pi_{t+1}]$. Therefore, when $h \geq 1$, we observe $\mathbb{E}_{i,t}^{\theta}[\pi_{t+h}] = \mathbb{E}_{i,t}^{\theta}[\pi_{t+1}]$.

IV.2.2 Estimation and forecasting exercise

Before performing a forecasting exercise based on the diagnostic Kalman filter, we need to know the value of the diagnostic parameter θ. In this section we propose a way of estimating this directly from the data.

We begin with the diagnostic Kalman filter from Equation 3, rewritten for the forecast agent i makes for period $t+12$, so

$$\mathbb{E}_{i,t}^{\theta}[\pi_{t+12}] = \mathbb{E}_{i,t}^{KF}[\pi_{t+12}] + \theta\left(\mathbb{E}_{i,t}^{KF}[\pi_{t+12}] - \mathbb{E}_{i,t}^{ref}[\pi_{t+12}]\right).$$

However, we know from Section IV.1 that under our current assumptions it is reasonable to assume that $\mathbb{E}_{i,t}^{KF}[\pi_{t+12}] = \mathbb{E}_{t}^{KF}[\pi_{t+12}]\ \forall i$. Then, the diagnostic Kalman filter becomes

$$\mathbb{E}_{i,t}^{\theta}[\pi_{t+12}] = \mathbb{E}_{t}^{KF}[\pi_{t+12}] + \theta\left(\mathbb{E}_{t}^{KF}[\pi_{t+12}] - \mathbb{E}_{i,t}^{ref}[\pi_{t+12}]\right).$$

Rearranging terms, we obtain

$$\mathbb{E}_{i,t}^{\theta}\left[\pi_{t+12}\right] = (1+\theta)\,\mathbb{E}_{t}^{KF}\left[\pi_{t+12}\right] - \theta\mathbb{E}_{i,t}^{ref}\left[\pi_{t+12}\right]. \tag{5}$$

We now explain how to take Equation 5 to the data. First, $(1+\theta)\,\mathbb{E}_{t}^{KF}\left[\pi_{t}\right]$ is common across all cohorts, so it can be captured by a time fixed effect γ_t. Second, for the distorted inflation expectation $\mathbb{E}_{i,t}^{\theta}\left[\pi_{t+12}\right]$ we use the 12-months-ahead forecasts for each agent m from cohort i from the SCE $\mathbb{E}_{m,i,t}^{SCE}\left[\pi_{t+12}\right]$. Lastly, for $\mathbb{E}_{i,t}^{ref}\left[\pi_{t+12}\right] = \frac{\sum_{j=1}^{k_i}\mathbb{E}_{i,t-j}^{KF}\left[\pi_{t+12}\right]}{t-k_i}$ we go back to the standard Kalman filter from Section IV.1 and recover the terms $\mathbb{E}_{i,t-j}^{KF}\left[\pi_{t+12}\right]$, which are the optimal forecasts under the given setup.

With this, we regress

$$\mathbb{E}_{m,i,t}^{SCE}\left[\pi_{t+12}\right] = \gamma_t + \varphi\mathbb{E}_{i,t}^{ref}\left[\pi_{t+12}\right] + \varepsilon_{m,i,t}. \tag{6}$$

We present the results in Table 2. Column 1 shows the main specification, from which we obtain $\theta = -\widehat{\varphi} = -0.317$. Because the diagnostic parameter $\theta < 0$, when agents make their forecasts, they put more weight on their reference sets (i.e. their past history, their priors) than on the news they receive in the current period. In the other columns of Table 2 we show that the negative coefficient is robust to the inclusion of control variables in the regression.

The result contrasts with the diagnostic expectations literature (for instance, Bordalo et al. 2020), where a positive θ is the usual result. However, this positive θ result is based on a diagnostic expectations formation process with a different reference point: instead of using the memory of the agents, they use the one-period lagged information set. Moreover, this finding is also conceptually different from ours, as most of the previous empirical literature on diagnostic expectations relies on surveys of professional forecasters, who are better informed about the economy than the households surveyed in the SCE.

Table 2: Diagnostic parameter estimation

	(1)	(2)	(3)	(4)
$\mathbb{E}^{ref}_{i,t}\left[\pi_{t+12}\right]$	0.317*** (0.030)	0.354*** (0.028)	0.260*** (0.030)	0.224*** (0.026)
Time FE	Yes	Yes	Yes	Yes
Controls	No	Gender, commuting zone	Gender, commuting zone, HH income	Gender, commuting zone, HH income, educational degree
Observations	101,262	101,245	101,245	101,245
R-squared	0.092	0.148	0.169	0.183

Note: Table shows results of Regression (6). $\mathbb{E}^{ref}_{i,t}\left[\pi_{t+12}\right]$ is the reference constructed for a respondent of age i as explained in the main text. Column (1) has only a time fixed effect as an additional control. Columns (2), (3) and (4) add different levels of controls. Robust standard errors in parentheses. Standard errors clustered by age. Dependent variable trimmed at 10 percent and 90 percent in each period. *** p<0.01, ** p<0.05, * p<0.1.

The value of the diagnostic parameter θ is also conditional on the expectation formation process. In Bordalo et al. (2019), current news are processed with a Kalman filter so, in a sense, current forecasts are partly processed with past information. With that modeling approach, surprises with respect to a reference based on the previous period are also coming from a reference based on past memory, as suggested by Kahneman and Tversky (1972).

However, this result changes when we consider other forms for current expectations instead of a Kalman filter, such as rational expectations (for instance, Bianchi et al. 2021 and L'Huillier et al. 2021). In that case, agents know the model and all of its features, so their memory is based on everything that is possible within the model. Given the assumption of full information, agents will overreact to any shock that was not expected in the previous period, as agents understand the shock and its effects as soon as they see them.

We take a different approach, as we explicitly model the memory of the agents given their cohort. Agents also optimally generate a forecast with a Kalman filter and a signal we provide. Then, they combine this optimal forecast with their past experience to produce their final diagnostic forecasts. Our framework allows for a common component, that might come from the news (Carroll, 2003), common price experiences (D'Acunto et al., 2021), among other reasons. While the time fixed effect would capture any of those factors, we choose to model it with a common adjustment to their price experience. Then, we explicitly model the reference given their history. Our findings are consistent with a heuristic representation: people use their memory to positively weigh them given the current developments in prices. When events deviate from what an agent has experienced, they tend to revise their expectation toward their memory. Our empirical results strongly point in that direction.

With this estimate we perform a forecasting exercise using the diagnostic Kalman filter. Panel (c) of Figure 4 shows the 12-months-ahead inflation forecasts for different cohorts. We can see that the diagnostic Kalman filter is able to generate a heterogeneous pattern across cohorts, similar to the one we saw in the data in Figure 1.[11] First, older cohorts show higher inflation expectations than the rest of the cohorts throughout most of the sample, based on their experiences of high inflation in the 60s, 70s, and early 80s. Second, the intermediate cohorts show low inflation expectations when compared to the other cohorts, because they went through the stable and low inflation rates of the 90s, 00s, and 10s. The youngest cohort shows the highest inflation expectations after being exposed to the high inflation rates of 2021.

As a way of checking the external validity of our results, in Appendix E we perform the same exercises but using data from the Consumer Expectations Survey of the European Central Bank, which contains observations for six countries: Belgium, France, Germany, Italy, the Netherlands, and Spain. We find evidence that supports our main findings. We find that inflation expectations are also heterogeneous in Europe and are explained partially by the history of consumers' inflation experiences. In addition, we also find support for the use of a diagnostic Kalman filter as a way of modeling heterogeneous inflation expectations.

Moreover, thanks to the cross-country panel structure, we are able to control for a common cohort fixed effect in this exercise, as in Hajdini et al. (2022a). This is important, as common cohort characteristics (i.e. different patterns of inflation exposure over the life cycle) could affect our results. By controlling for cohort fixed effect we rule out those common cohort characteristics and exploit differences in the inflation experienced by the different cohorts across different countries. We find similar results after adding these controls, implying that the heterogeneity does not stem from common cohort characteristics, but from different inflation experiences in different countries.

[11] We show the diagnostic forecasts for the full sample in Figure A.1 in Appendix A.

Thus, we conclude that our findings are also valid for Europe.

IV.2.3 Goodness of fit

In this subsection we show how our proposed diagnostic Kalman filter expectations, which combines time variation and individual variation, compares to the data. We run a regression where the dependent variable is the survey individual inflation expectation and the independent variable is diagnostic Kalman filter expectations. As explained before, our formulation has two components: one that is coming from the Kalman filter, with a signal coming from common food inflation data, and the second coming from the past references of cohorts and a diagnostic coefficient θ that is estimated using survey data. In that sense, the time variation from our inflation expectations measure is not being informed by the individual data, as we use a time fixed effect for the coefficient estimation. Column 1 of Table 3 shows that the slope between the survey forecast and the forecast produced by the diagnostic Kalman filter is 0.888. This confirms that our diagnostic formulation for inflation expectations is effective in forecasting consumers' inflation expectations. After considering the time and cross-section variation, our estimate is able to provide a good prediction of heterogeneous inflation expectations.

Table 3: Goodness of fit

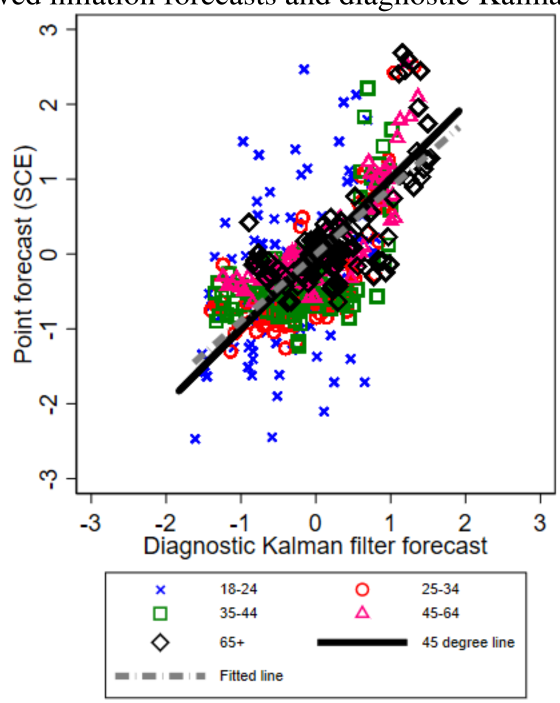
	(1)	(2)	(3)
$\mathbb{E}^{\theta}_{i,t}\left[\pi_{t+12}\right]$	0.888***		0.878***
	(0.037)		(0.038)
$\overline{\pi_{i,t}}$		0.224***	0.029
		(0.027)	(0.029)
Observations	101,262	101,262	101,262
R-squared	0.036	0.004	0.036

Note: Table shows results of a regression where the dependent variable is consumers' inflation expectations according to the Survey of Consumer Expectations (SCE) of the Federal Reserve Bank of New York. $\mathbb{E}^{\theta}_{i,t}\left[\pi_{t+12}\right]$ is our estimated measure of inflation expectations. $\overline{\pi_{i,t}}$ is average inflation expectations. Standard errors clustered at the date-of-birth level in parentheses. Dependent variable trimmed at 10 percent and 90 percent in each period. *** p<0.01, ** p<0.05, * p<0.1.

An alternative measure to explain the heterogeneous inflation expectations from the data is the lifetime average inflation rate by cohort $\overline{\pi_{i,t}} = \frac{\sum_{j=0}^{k_i} \pi_{t-j}}{t-k_i+1}$. Column 2 of Table 3 shows that the history of inflation by cohort, by itself, is also able to predict part of the variation in the data. We can make the diagnostic Kalman filter compete with the lifetime average inflation rate to see which measure better predicts the forecasts we see in the data by estimating a regression with both variables. Column 3 of Table 3 shows that in a horserace the diagnostic forecast is superior to the lifetime average inflation rate for explaining the observed heterogeneous inflation expectations we find in the data. The coefficient for our diagnostic measure is close to one and statistically significant, while the coefficient for the history of inflation by cohort goes close to zero and becomes statistically insignificant.

Figure 5 visually presents the results of Column 1 in Table 3. We can see that the slope between the regression and a 45-degree line are very close. Our diagnostic measure can effectively model the time and cross sectional variation of consumers' inflation expectations.

Figure 5: Observed inflation forecasts and diagnostic Kalman filter forecasts



Note: Figure shows binned scatterplot across diagnostic Kalman filter forecasts (x-axis) and point forecast inflation expectations according to the Survey of Consumer Expectations (SCE) of the Federal Reserve Bank of New York (y-axis). Variables demeaned by the intercept. Data go from June 2013 to December 2021. SCE variable trimmed at 10 percent and 90 percent in each period.

Overall, we show that our diagnostic measure shows a very good fit with the data and that we are able to replicate heterogeneous inflation expectations at the individual level, a complicated object, with a relatively simple model of expectations. Thus, we find that following a simple model of current inflation and heterogeneous histories can effectively model the time and cross-sectional variation on individual inflation expectations we see in the data.

V Aggregate implications of heterogeneous expectations

We present an overlapping generations monetary model that replicates the heterogeneity in the observed inflation expectations (i.e., Figure 1). We assume that agents follow the diagnostic Kalman filter introduced in Section IV.2 when forecasting future variables as similarly proposed in Bianchi et al. (2021) and L'Huillier et al. (2021). The long memory inherent in this approach allows for different past experiences to shape different inflation expectations across cohorts.

V.1 Households

On the demand side, we assume that the economy is populated by an infinite number of cohorts. Every cohort is composed of a continuum of households, all of which can be summarized by a representative agent. The cohorts are heterogeneous in their age and past inflation experiences. For modeling the different cohorts, we follow the perpetual youth approach of Blanchard (1985) and Yaari (1965). This means that households are uncertain about the date on which they will die. All they know is that they face a rate of mortality λ every period. At the same time, every period a new cohort of size λ is born. Therefore, in a given period t, the size of a cohort born in period k is $\lambda\,(1-\lambda)^{t-k}$.

We assume that households form their expectations using the diagnostic Kalman filter from Equation 3. Therefore, their expectations will be influenced by their past experiences. This also means that all the assumptions from Section IV.2 apply here. First, agents do not fully understand the model that governs the economy, so they assume that both the output gap and the inflation rate behave as a random walk. Second, agents cannot directly observe either the current output gap or the current inflation rate, but they instead receive a signal. With this, agents form diagnostic forecasts about the output gap and the inflation rate. The representative household from cohort i consumes a consumption basket $C_{i,t}$, composed of a continuum of $C_{i,t}\,(j)$ goods indexed by j. This basket is defined as

$$C_{i,t}=\left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}}dj\right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where ε is the elasticity of substitution in the CES basket.

A representative household of cohort i solves

$$\max\left[\frac{C_{i,t}^{1-\sigma}}{1-\sigma}-\frac{L_{i,t}^{1+\eta}}{1+\eta}\right]+\sum_{j=1}^{\infty}\beta^{j-t}(1-\lambda)^{j-t}\mathbb{E}_{i,t}^{\theta}\left[\frac{C_{i,t+j}^{1-\sigma}}{1-\sigma}-\frac{L_{i,t+j}^{1+\eta}}{1+\eta}\right],$$

subject to

$$P_tC_{i,t}+(1-\lambda)\frac{B_{i,t+1}}{(1+i_t)}=W_tL_{i,t}+B_{i,t}+T_{i,t},$$

where $C_{i,t}$ is consumption, $L_{i,t}$ is the labor supply, $B_{i,t}$ are nominal savings, P_t is the price level, W_t are the nominal wages, $T_{i,t}$ are transfers, and i_t is the nominal interest rate. Also, β is the discount factor, σ is the intertemporal elasticity of substitution, and η is the inverse of the Frisch elasticity.

The transfers $T_{i,t}$ are crucial to our model, as they incorporate two different mechanisms. First, as in Blanchard (1985) and Yaari (1965) we assume that households insure themselves to receive a flow of income every period they are alive. Then, when they die, the insurance company takes away any wealth residual. Thus, we do not have to worry about accidental bequests. Second, as in Mankiw and Reis (2006), we assume that the flow of income households receive each period from the insurance company is such that households start each period with the same wealth and that the nominal savings market clears. Therefore, we do not have to worry about the wealth distribution. Lastly, as a way of closing the model, the transfers also incorporate the benefits coming from firms that produce intermediate goods.

As in Bianchi et al. (2021) and L'Huillier et al. (2021) we introduce diagnostic expectations in a general equilibrium setting. The expectations operator households use is $\mathbb{E}_{i,t}^{\theta}[.]$, which works under the assumptions of Section IV.2. We also assume that for any current variable X_t we have that $\mathbb{E}_{i,t}^{\theta}[X_t]=X_t$ and that for any lagged variable X_{t-h} we have that $\mathbb{E}_{i,t}^{\theta}[X_{t-h}]=X_{t-h}$. The first-order conditions are

$$\frac{1}{C_{i,t}} = \beta\left(1+i_t\right)\mathbb{E}^{\theta}_{i,t}\left[\frac{1}{\left(1+\pi_{t+1}\right)C_{i,t+1}}\right], \tag{7}$$

$$L^{\eta}_{i,t} = \frac{W_t}{P_t C_{i,t}}, \tag{8}$$

where the first equation is the Euler equation and the second one denotes the labor supply. Additionally, the transversality condition is

$$\lim_{T\to\infty}\frac{(1-\lambda)^T}{\Pi_{h=0}^{T-1}\left(1+i_{t+h}\right)}B_{i,t+T} = 0.$$

The log-linearization of Equation 7, following the diagnostic Kalman filter from Equation 3, gives

$$c_{i,t} = \begin{array}{c} \left\{\mathbb{E}^{KF}_t\left[c_{i,t+1}\right] - \frac{1}{\sigma}\left(i_t - \mathbb{E}^{KF}_{i,t}\left[\pi_{t+1}\right]\right)\right\} \\ +\theta\left\{\left(\mathbb{E}^{KF}_t\left[c_{i,t+1}\right] - \mathbb{E}^{ref}_{i,t}\left[c_{i,t+1}\right]\right) + \dfrac{1}{\sigma}\left(\mathbb{E}^{KF}_t\left[\pi_{t+1}\right] - \mathbb{E}^{ref}_{i,t}\left[\pi_{t+1}\right]\right)\right\}, \end{array} \tag{9}$$

where the lowercases denote deviations from the steady state.[12] This is the IS curve for a given cohort i.

V.1.1 Aggregation

In our log-linearized economy, the aggregate consumption gap c_t is defined as the weighted sum of all the cohort-level consumption gaps, so

$$c_t = \lambda\sum_{k=0}^{\infty}\left(1-\lambda\right)^k c_{k,t}. \tag{10}$$

Incorporating Equation 9 into Equation 10 we find

[12] An intermediate step in the log-linearization of Equation 7 results in

$$c_{i,t} = \begin{array}{c} \left\{\mathbb{E}^{KF}_t\left[c_{i,t+1}\right] - \frac{1}{\sigma}\left(i_t - \mathbb{E}^{KF}_{i,t}\left[\pi_{t+1}\right]\right)\right\} \\ +\theta\left\{\left(\mathbb{E}^{KF}_t\left[c_{i,t+1}\right] - \mathbb{E}^{ref}_{i,t}\left[c_{i,t+1}\right]\right) + \dfrac{1}{\sigma}\left(\mathbb{E}^{KF}_t\left[\pi_{t+1}\right] - \mathbb{E}^{ref}_{i,t}\left[\pi_{t+1}\right]\right)\right\} \\ +\dfrac{\theta}{\sigma}\sum_{j=0}^{k_i+1}\left(\mathbb{E}^{\theta}_{i,t}\left[\pi_{t-j}\right] - \pi_{t-j}\right), \end{array}$$

where the last term results from the fact that $\mathbb{E}^{\theta}_{i,t}\left[X_{t+1}Z_t\right] \neq \mathbb{E}^{\theta}_{i,t}\left[X_{t+1}\right]Z_t$ (see, for instance, L'Huillier et al. 2021). Because $\mathbb{E}^{\theta}_{i,t}\left[\pi_{t-j}\right] = \pi_{t-j}$ we drop the last term and obtain Equation 9.

$$c_t = \begin{array}{c} \left\{-\frac{i_t}{\sigma} + \mathbb{E}_t^{KF}\left[c_{t+1}\right] + \mathbb{E}_t^{KF}\left[\frac{\pi_{t+1}}{\sigma}\right]\right\} + \theta\left\{\mathbb{E}_t^{KF}\left[c_{t+1}\right] + \mathbb{E}_t^{KF}\left[\frac{\pi_{t+1}}{\sigma}\right]\right\} \\ -\theta\lambda\sum_{k=0}^{\infty}(1-\lambda)^k\left\{\mathbb{E}_{k,t}^{ref}\left[c_{k,t+1}\right] + \mathbb{E}_{k,t}^{ref}\left[\frac{\pi_{t+1}}{\sigma}\right]\right\}. \end{array}$$

Here, we further assume that household k, when forecasting its future individual consumption gap, believes that all the other households will behave in a similar way such that $\mathbb{E}_{k,t}^{ref}\left[c_{k,t+1}\right] = \mathbb{E}_{k,t}^{ref}\left[c_{t+1}\right]$ and

$$c_t = \begin{array}{c} \left\{-\frac{i_t}{\sigma} + \mathbb{E}_t^{KF}\left[c_{t+1}\right] + \mathbb{E}_t^{KF}\left[\frac{\pi_{t+1}}{\sigma}\right]\right\} + \theta\left\{\mathbb{E}_t^{KF}\left[c_{t+1}\right] + \mathbb{E}_t^{KF}\left[\frac{\pi_{t+1}}{\sigma}\right]\right\} \\ -\theta\lambda\sum_{k=0}^{\infty}(1-\lambda)^k\left\{\mathbb{E}_{k,t}^{ref}\left[c_{t+1}\right] + \mathbb{E}_{k,t}^{ref}\left[\frac{\pi_{t+1}}{\sigma}\right]\right\}. \end{array}$$

Further assuming that in equilibrium the output gap $y_t = c_t$, then

$$y_t = \begin{array}{c} \left\{-\frac{i_t}{\sigma} + \mathbb{E}_t^{KF}\left[y_{t+1}\right] + \mathbb{E}_t^{KF}\left[\frac{\pi_{t+1}}{\sigma}\right]\right\} + \theta\left\{\mathbb{E}_t^{KF}\left[y_{t+1}\right] + \mathbb{E}_t^{KF}\left[\frac{\pi_{t+1}}{\sigma}\right]\right\} \\ -\theta\lambda\sum_{k=0}^{\infty}(1-\lambda)^k\left\{\mathbb{E}_{k,t}^{ref}\left[y_{t+1}\right] + \mathbb{E}_{k,t}^{ref}\left[\frac{\pi_{t+1}}{\sigma}\right]\right\}. \end{array} \tag{11}$$

Equation 11 is the diagnostic IS curve in our model. It is equal to the standard IS curve plus two distortion terms. In this version of the IS curve, the past matters in the sense that current realizations are affected by the memory of the cohorts.

V.2 Firms

On the supply side, there is a final goods producer that operates in a perfectly competitive market, which produces using a continuum of intermediate goods as inputs. There is a continuum of intermediate goods producers, each operating under monopolistic competition. These intermediate goods producers are subject to Calvo pricing frictions. We assume that these firms follow rational expectations when setting their prices, in the sense that they are model consistent. Thus, we follow the usual derivations for firms in a New Keynesian setting, such that we obtain the standard New Keynesian Phillips curve.[13]

V.3 Monetary policy

The central bank sets the interest rate following a standard Taylor rule. Then, we have

[13] We present the derivations in Appendix F.

$$\frac{(1+i_t)}{(1+\bar{i})} = \left[\frac{(1+\pi_t)}{(1+\overline{\pi})}\right]^{\chi_\pi} \left[\frac{Y_t}{\overline{\overline{Y}}}\right]^{\chi_y}, \tag{12}$$

where the bars denote steady state values and χ_π and χ_y represent the central bank's reaction to deviations from the steady state of the inflation rate and output, respectively.

After log-linearizing, the model is summarized by

$$y_t = \begin{array}{c} \left\{-\frac{i_t}{\sigma} + \mathbb{E}_t^{KF}\left[y_{t+1}\right] + \mathbb{E}_t^{KF}\left[\frac{\pi_{t+1}}{\sigma}\right]\right\} + \theta\left\{\mathbb{E}_t^{KF}\left[y_{t+1}\right] + \mathbb{E}_t^{KF}\left[\frac{\pi_{t+1}}{\sigma}\right]\right\} \\ -\theta\lambda\sum_{k=0}^{\infty}(1-\lambda)^k\left\{\mathbb{E}_{k,t}^{ref}\left[y_{t+1}\right] + \mathbb{E}_{k,t}^{ref}\left[\frac{\pi_{t+1}}{\sigma}\right]\right\} + u_t^{taste}, \end{array} \tag{13}$$

$$\pi_t = \frac{(1-\phi)(1-\phi\beta)}{\phi}(\sigma+\eta)\,y_t + \beta\mathbb{E}_t\left[\pi_{t+1}\right] + u_t^{cost}, \tag{14}$$

$$i_t = \chi_\pi\pi_t + \chi_y y_t, \tag{15}$$

where Equation 13 is the diagnostic dynamic IS curve augmented with a taste shock, Equation 14 is the Phillips curve, and Equation 15 is the Taylor rule. Notice the Phillips curve follows the rational expectations operator $E_t[.]$, while the diagnostic IS curve results from following the diagnostic Kalman filter operator $E_t^{\theta}[.]$.

We consider a cost shock u_t^{cost} and a taste shock u_t^{taste} that behave as an AR(1) process, such that

$$u_t^{cost} = \rho_{cost}u_{t-1}^{cost} + \varepsilon_t^{cost}, \tag{16}$$

$$u_t^{taste} = \rho_{taste}u_{t-1}^{taste} + \varepsilon_t^{taste}, \tag{17}$$

where ρ_{cost} and ρ_{taste} are the persistence parameter and ε_t^{cost} and ε_t^{taste} are the unexpected innovations.

V.4 Calibration

The model is calibrated to a monthly frequency. The parameters from Table A.1 in Appendix A show a fairly standard calibration. We calibrate the price stickiness parameter ϕ so that the expected duration of a given price quote is 12 months. We also calibrate the mortality rate λ so that the expected life span is 80 years.[14]

Regarding the diagnostic Kalman filter, we need to calibrate the Kalman gain K and the diagnostic parameter θ. We calibrate both according to the results from Sections IV.1 and IV.2. We must make an additional assumption around these two parameters. While we only used inflation rate data in the previous sections, here we assume that these parameters also hold true for the output gap.

V.5 Simulations

We compare three different cases: (i) households form their expectations according to full information rational expectations (FIRE), (ii) households form their expectations according to the standard diagnostic expectations operator with overreaction,[15] and (iii) households form their expectations according to the diagnostic Kalman filter from Equation 3.[16]

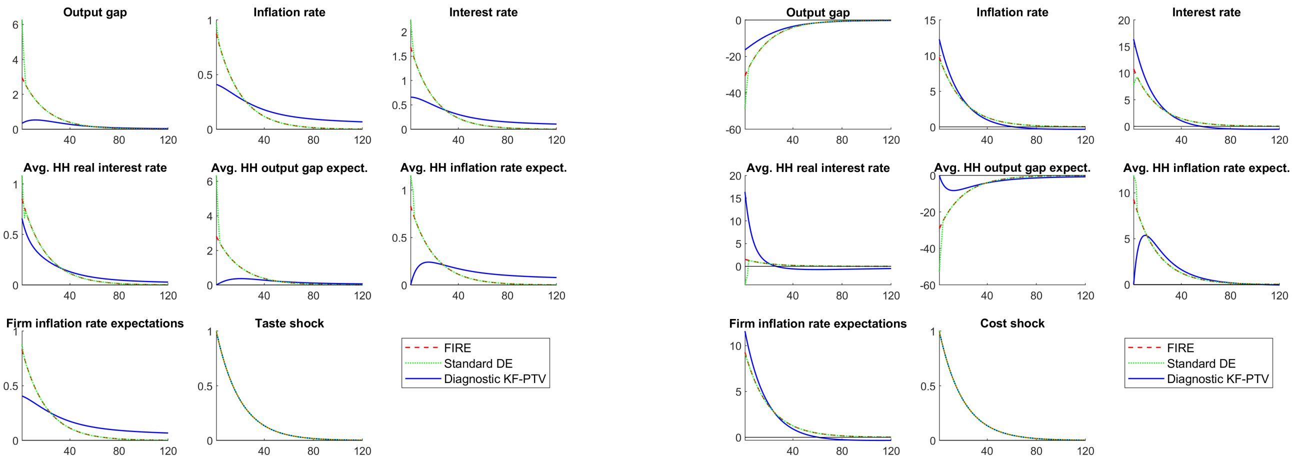
Figure 6 presents the impulse response functions to a taste shock and a cost shock. In Panel (a) we can see that after a taste shock, inflation and the output gap increases. The FIRE case shows the usual reaction and the standard diagnostic expectations model presents an overreaction that lasts a three periods, consistent with the reference used. In the case of the diagnostic Kalman filter model, we first see a milder reaction in terms of inflation and output gap. Agents remember the past (in this case, the steady state), so their expectations tend to stay closer to such value. This allows the central bank to reduce the size of its hike in the interest rate, resulting in a slightly lower real interest rate. In addition, firms reduce the size of their price increase. Overall, under the diagnostic Kalman filter, agents anchor their expectations to the past, reducing the magnitude of the responses on impact.

[14]Because we assume that agents become economically active and relevant at age 18, this means agents expect to consume and work for 62 years

[15]Following the standard diagnostic expectations literature, we define the standard diagnostic expectations operator in this case equal to 0.317, such that there is overreaction. See Bianchi et al. (2021); L'Huillier et al. (2021)

[16]For our diagnostic Kalman filter operator we assume each cohort differs in its beliefs. In Appendix G we analyze the consequences of dropping this assumption and having all cohorts follow the same beliefs while keeping the diagnostic Kalman filter structure.

Figure 6: Impulse response functions



(a) Taste shock

(b) Cost shock

Note: Figure shows impulse response functions for a selected group of variables after the mentioned shocks. The red dashed line shows the results for the case of the full information rational expectations model (FIRE), the green dotted line shows the results of a standard diagnostic expectations operator and the solid blue line shows the diagnostic Kalman filter model. For the standard diagnostic expectations case we assume that agents use the expectations operator $\mathbb{E}_t^{\theta,s}[X_{t+h}] = \mathbb{E}_t[X_{t+h}] + \varpi(\mathbb{E}_t[X_{t+h}] - \mathbb{E}_{t-3}[X_{t+h}])$ with $\varpi > 0$. Horizontal axis denotes months after the shock.

While inflation is lower on impact with the diagnostic Kalman filter, it takes longer to return to its steady state values when compared to the other cases (the FIRE and standard diagnostic expectations cases have inflation going back to steady state at the same pace of the persistence of the shock). This is because, with the diagnostic Kalman filter, agents remember the high inflation period. This effect is exacerbated by new cohorts that have only experienced inflation above the steady state.

Panel (b) of Figure 6 shows the responses for a cost shock. In the diagnostic Kalman filter case there are two forces going in opposite directions. There is a high persistence in inflation; but we also have consumers that remember the zero output gap of the steady state, which reduces the pressure on prices. Thus, with the diagnostic Kalman filter the IS curve becomes more inelastic to the shock. Then, in this diagnostic economy, rational firms are able to raise prices by more than they would under FIRE. This is followed by a central bank that must raise the interest rate more strongly than in the rational economy.

One thing to notice in the diagnostic Kalman filter case is that while the household inflation rate expectations have a hump shape, actual inflation does not. This is because firms are always rational. Thus, their expectations follow the shock very closely (no hump shape because of the

AR(1) nature of the shock) and they set prices accordingly. The hump shaped expectations are consistent with evidence provided by Angeletos et al. (2021). In our setting, consumers underreact to the inflationary shock, as their memories are tied to the steady state. After the inflationary shock, they incorporate the inflationary episode in their memories, overextrapolating the shock.

Figures A.2 and A.3 of Appendix A present the heterogeneity in expectations across cohorts under the diagnostic Kalman filter. Overall, we see higher persistence in the expectations with respect to the FIRE case, which extends the effects of the shock in the economy.

VI Optimal Taylor rules

Now we analyze the use of an optimal Taylor rule in each of the different cases from the previous section. The Taylor rule we use in this section is

$$i_t = \chi_\pi^* \pi_t + \chi_y^* y_t. \tag{18}$$

We assume that the central bank chooses the time-invariant parameters χ_π^* and χ_y^* such that it solves the problem

$$\min \mathbb{E}_t \left[\pi_t^2 + \vartheta y_t^2 \right],$$

subject to the equations of the model in Section V and ϑ is the weight of the output gap in the objective function.[17] That is, the central bank, given the model setup, seeks to minimize the volatility of both the inflation rate and the output gap. Notice that we assume that the central bank has rational expectations.

The optimal parameters are dependent on which shocks exist in the model (cost or taste). Therefore, we will have two sets of parameters, one for each shock.[18]

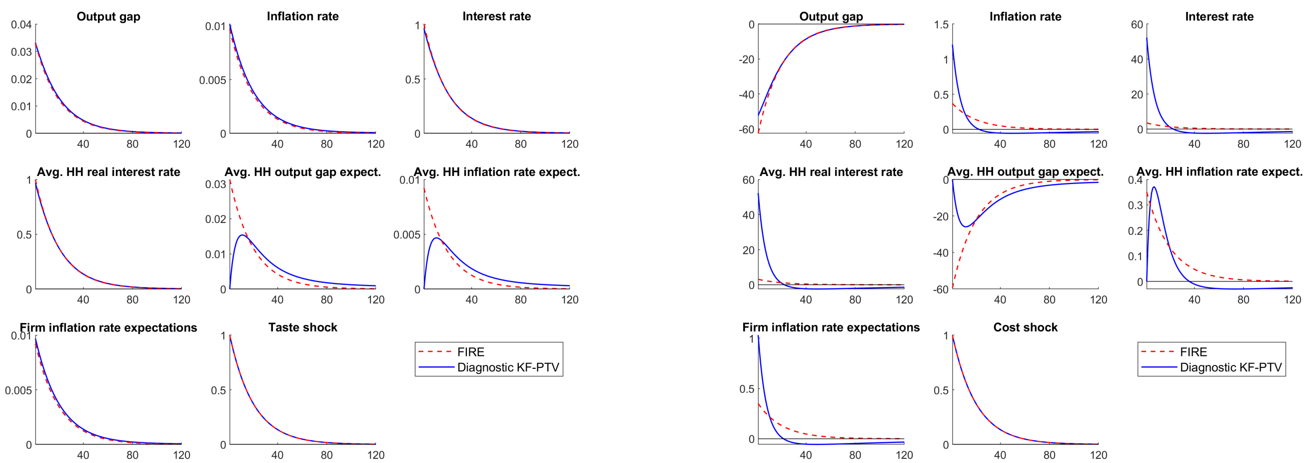
We start by analyzing the response to a cost shock under the optimal Taylor rule. Panel (b) of Figure 7 shows that, when responding to this shock, the central bank faces the usual trade-off between the output gap and the inflation rate. After the cost shock, the inflation rate goes up and

[17] Following Gali (2015) we define $\vartheta = \frac{(1-\phi)(1-\phi\beta)(\sigma+\eta)}{\phi\varepsilon} = 0.0017$.

[18] Tables A.2 and A.3 in Appendix A show the optimal parameters.

the output gap goes down after the interest rate hikes. In this particular exercise we will have that, given the relative importance of the output gap in the objective function, the central bank will favor reducing the volatility of the inflation rate.

Figure 7: Impulse response functions, optimal Taylor rule



(a) Taste shock (b) Cost shock

Note: Figure shows impulse response functions for a selected group of variables after the mentioned shocks under an optimal Taylor rule. The red dashed line shows the results for the case of the full information rational expectations model (FIRE) and the solid blue line shows the diagnostic Kalman filter model. Horizontal axis denotes months after the shock.

After the shock hits, the central bank becomes more active in the diagnostic Kalman filter case when compared to the FIRE case. The reason behind this behavior is that memory plays a role when there are diagnostic expectations. The central bank knows that people will remember the current shock far into the future, affecting future inflation expectations. By being more active under the diagnostic Kalman filter case, we see the central bank can very quickly lower the inflation expectations. While in the baseline results from Figure 6 the inflation expectations remained high for a long period, the optimal Taylor rule brings them down and even generates deflation expectations that later spill over to the observed inflation rate.

Panel (a) of Figure 7 shows the impulse response functions to a taste shock and an optimal response from the central bank. As it is well known in the literature, upon a taste shock, the optimal response of the central bank is to strongly raise the interest rate. What follows is that the central bank manages to bring down both the output gap and the inflation rate to their steady state values.

We find no significant difference in the response of the central bank between the FIRE and diagnostic cases. The optimal Taylor rule case says that the central bank should always be active when facing a taste shock, no matter the type of expectations agents have. This way, the central bank is able to close the output gap and lower the inflation rate more quickly than with the baseline results. Moreover, with the active stance recommended by the optimal Taylor rule, both the output gap and inflation expectations are positive but very close to zero in all of the cases.

VII Analyzing an episode of high inflation

In this section we analyze the behavior of the model after the high-inflation episode of 2021.[19] To do so, we feed the model from Section V with actual monthly data on the output gap, inflation rates, and interest rates up to December 2021.[20] Afterward, we produce forecasts using the different versions of the model.[21]

We first analyze the diagnostic Kalman filter case. Figure A.4 of Appendix A shows the inflation rate diagnostic expectations by cohort according to our model and the data. Before 2021 we see that older cohorts had the highest inflation expectations. This is because older cohorts experienced the high-inflation episodes of the 60s, 70s, and early 80s. They are followed by the intermediate cohorts and finally by the youngest cohorts, who experienced low and stable inflation rates from the 90s to the 10s. With the inflationary shock, things changes, as newer cohort experience a significant part of their live in a high inflation environment

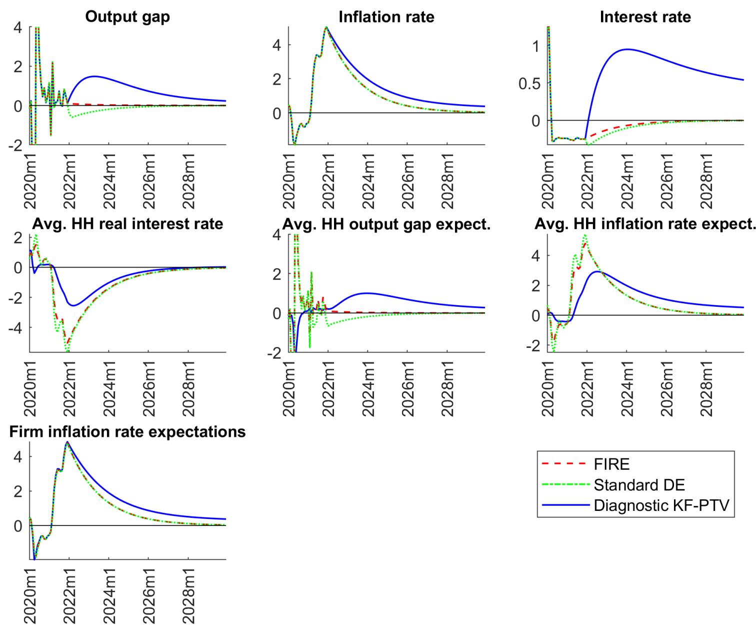
Figure 8 presents how variables evolve based on our model and the data. After 2021, when we allow for a diagnostic Kalman filter, average inflation expectations are higher and more persistent with respect to other cases. This is because agents remember and anchor their expectations to what they experienced in the past. Hence, the central bank must react more strongly.

[19]In this part we go back to the basic calibration of Table A.1.

[20]We use monthly series from March 1967 to December 2021. We go as far as the data allow to build the memory that agents use as a reference. For the output gap we use the National Activity Index (CFNAI) from the Federal Reserve Bank of Chicago. For the interest rate we use the effective federal funds rate. For the inflation rate we use the CPI 12-month percentage change.

[21]We present the shocks that, according to our model, explain the observed data in Figure A.5 in Appendix A.

Figure 8: Impulse response functions, forecast



Note: Figure shows impulse response functions for a selected group of variables according to the model and the data (up to December 2021). The red dashed line shows the results for the case of the full information rational expectations model (FIRE), the green dotted line shows the results of a standard diagnostic expectations operator and the solid blue line shows the diagnostic Kalman filter model. For the standard diagnostic expectations case we assume that agents use the expectations operator $\mathbb{E}_t^{\theta,s}[X_{t+h}] = \mathbb{E}_t[X_{t+h}] + \varpi(\mathbb{E}_t[X_{t+h}] - \mathbb{E}_{t-3}[X_{t+h}])$ with $\varpi > 0$. Horizontal axis denotes months.

Because of memory, agents that follow a diagnostic Kalman filter remember the high inflation episode far into the future when compared to the other cases case. As a consequence, the observed inflation rate and the interest rate also remain higher for longer.

VIII Conclusions

This paper studies the macroeconomic consequences of heterogeneous inflation expectations. We first show that inflation expectations are heterogeneous across cohorts. Based on Bordalo et al. (2020), we introduce a Kalman filter augmented with diagnostic expectations to model the inflation forecast formation process. We structurally estimate the relevant diagnostic parameter, concluding that individuals effectively consider their past inflation histories when forecasting.

Our expectation formation process includes two known and relevant aspects of consumers' expectations processes. On one side, we consider the history of inflation each cohort has experienced; and on the other side, we take into account the current inflation experience, in particular with salient prices such as grocery prices. Using both components, we show that we are able to predict consumers' inflation expectation only using monthly inflation data and a constant diagnostic parameter informed by survey data. This way, we show that the usually noisy consumer inflation expectation data can be modeled, predicted and has meaningful information.

Our modeling approach also has the advantage of being flexible such that we incorporate it in a general equilibrium model. We find that this heterogenous expectation process has relevant aggregate implications. Heterogeneous expectations anchor aggregate response to agents' memories while increasing the persistence of the effect of the shocks.

This result has implications for monetary policy: the optimal response of monetary authorities when inflation starts rising is to take an active stance, as agents have a long memory and remember current shocks far into the future. An energetic response of the central bank under inflationary pressures prevents current inflation from rising and, more importantly, prevents agents from incorporating high-inflation episodes into their memories, thus preventing higher inflation expectations in the future.

These results also have relevant implications for the current macroeconomic environment. The model suggests that the 2021 high-inflation episode, even though it may be transitory, could have long-lasting effects: new cohorts incorporate the high-inflation episode into their memories of inflation, adjusting future inflation expectations upwards.

References

Afrouzi, H. (2020). Strategic inattention, inflation dynamics, and the non-neutrality of money.

Angeletos, G.-M., Huo, Z., and Sastry, K. A. (2021). Imperfect macroeconomic expectations: Evidence and theory. *NBER Macroeconomics Annual*, 35(1):1–86.

Bianchi, F., Ilut, C. L., and Saijo, H. (2021). Implications of diagnostic expectations: Theory and applications.

Blanchard, O. J. (1985). Debt, deficits, and finite horizons. *Journal of Political Economy*, 93(2):223–247.

Blanchard, O. J., L'Huillier, J.-P., and Lorenzoni, G. (2013). News, noise, and fluctuations: An empirical exploration. *American Economic Review*, 103(7):3045–70.

Bordalo, P., Gennaioli, N., Ma, Y., and Shleifer, A. (2020). Overreaction in macroeconomic expectations. *American Economic Review*, 110(9):2748–82.

Bordalo, P., Gennaioli, N., Porta, R. L., and Shleifer, A. (2019). Diagnostic expectations and stock returns. *Journal of Finance*, 74(6):2839–2874.

Bordalo, P., Gennaioli, N., and Shleifer, A. (2018). Diagnostic expectations and credit cycles. *Journal of Finance*, 73(1):199–227.

Branch, W. A. and McGough, B. (2010). Dynamic predictor selection in a new keynesian model with heterogeneous expectations. *Journal of Economic Dynamics and Control*, 34(8):1492–1508.

Campos, C., McMain, M., and Pedemonte, M. (2022). Understanding which prices affect inflation expectations. *Economic Commentary*, (2022-06).

Candia, B., Coibion, O., and Gorodnichenko, Y. (2022). The macroeconomic expectations of firms.

Carroll, C. D. (2003). Macroeconomic expectations of households and professional forecasters. *Quarterly Journal of Economics*, 118(1):269–298.

Cheung, Y.-W. (1993). Exchange rate risk premiums. *Journal of International Money and Finance*, 12(2):182–194.

Coibion, O., Georgarakos, D., Gorodnichenko, Y., and Van Rooij, M. (2019). How does consumption respond to news about inflation? Field evidence from a randomized control trial.

Coibion, O. and Gorodnichenko, Y. (2015). Information rigidity and the expectations formation process: A simple framework and new facts. *American Economic Review*, 105(8):2644–78.

Coibion, O., Gorodnichenko, Y., and Kumar, S. (2018). How do firms form their expectations? New survey evidence. *American Economic Review*, 108(9):2671–2713.

Coibion, O., Gorodnichenko, Y., and Ropele, T. (2020). Inflation expectations and firm decisions: New causal evidence. *Quarterly Journal of Economics*, 135(1):165–219.

Collin-Dufresne, P., Johannes, M., and Lochstoer, L. A. (2017). Asset pricing when 'this time is different'. *Review of Financial Studies*, 30(2):505–535.

D'Acunto, F., Malmendier, U., Ospina, J., and Weber, M. (2021). Exposure to grocery prices and inflation expectations. *Journal of Political Economy*, 129(5):1615–1639.

D'Acunto, F., Malmendier, U., and Weber, M. (2022). What do the data tell us about inflation expectations?

De Grauwe, P. (2011). Animal spirits and monetary policy. *Economic Theory*, 47(2):423–457.

Dietrich, A. M. (2022). Consumption categories, household attention, and inflation expectations: Implications for optimal monetary policy.

Evans, G. W. and Honkapohja, S. (2001). *Learning and Expectations in Macroeconomics*. Princeton University Press.

Gabaix, X. (2020). A behavioral New Keynesian model. *American Economic Review*, 110(8):2271–2327.

Gali, J. (2015). *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications*. Princeton University Press.

Gali, J. (2021). Monetary policy and bubbles in a new keynesian model with overlapping generations. *American Economic Journal: Macroeconomics*, 13(2):121–67.

Gáti, L. (2020). Monetary policy & anchored expectations: An endogenous gain learning model.

Hajdini, I., Knotek II, E. S., Leer, J., Pedemonte, M., Rich, R., and Schoenle, R. (2022a). Indirect consumer inflation expectations: Theory and evidence. *Federal Reserve Bank of Cleveland Working Paper*.

Hajdini, I., Knotek II, E. S., Leer, J., Pedemonte, M., Rich, R. W., and Schoenle, R. S. (2022b). Low passthrough from inflation expectations to income growth expectations: Why people dislike inflation. Working Paper 22-21, Federal Reserve Bank of Cleveland.

Hajdini, I., Knotek II, E. S., Pedemonte, M., Rich, R., Leer, J., and Schoenle, R. (2022c). Indirect consumer inflation expectations. *Economic Commentary*, (2022-03).

Jump, R. C., Hommes, C., and Levine, P. (2019). Learning, heterogeneity, and complexity in the New Keynesian model. *Journal of Economic Behavior & Organization*, 166:446–470.

Kahneman, D. and Tversky, A. (1972). Subjective probability: A judgment of representativeness. *Cognitive psychology*, 3(3):430–454.

Kaplan, G., Moll, B., and Violante, G. L. (2018). Monetary policy according to HANK. *American Economic Review*, 108(3):697–743.

Kuchler, T. and Zafar, B. (2019). Personal experiences and expectations about aggregate outcomes. *Journal of Finance*, 74(5):2491–2542.

L'Huillier, J.-P., Singh, S. R., and Yoo, D. (2021). Diagnostic expectations and macroeconomic volatility.

Malmendier, U. (2021). Exposure, experience, and expertise: Why personal histories matter in economics. *Journal of the European Economic Association*, 19:2857–2894.

Malmendier, U. and Nagel, S. (2016). Learning from inflation experiences. *Quarterly Journal of Economics*, 131(1):53–87.

Malmendier, U., Nagel, S., and Yan, Z. (2021). The making of hawks and doves. *Journal of Monetary Economics*, 117:19–42.

Malmendier, U. and Wachter, J. A. (2022). Memory of past experiences and economic decisions.

Mankiw, N. G. and Reis, R. (2002). Sticky information versus sticky prices: A proposal to replace the New Keynesian Phillips curve. *Quarterly Journal of Economics*, 117(4):1295–1328.

Mankiw, N. G. and Reis, R. (2006). Pervasive stickiness. *American Economic Review*, 96(2):164–169.

Mankiw, N. G. and Reis, R. (2011). Imperfect information and aggregate supply. *Handbook of Monetary Economics*, 3:183–229.

Marcet, A. and Sargent, T. J. (1989). Convergence of least squares learning mechanisms in self-referential linear stochastic models. *Journal of Economic Theory*, 48(2):337–368.

Pivetta, F. and Reis, R. (2007). The persistence of inflation in the United States. *Journal of Economic Dynamics and Control*, 31(4):1326–1358.

Roth, C. and Wohlfart, J. (2020). How do expectations about the macroeconomy affect personal expectations and behavior? *Review of Economics and Statistics*, 102(4):731–748.

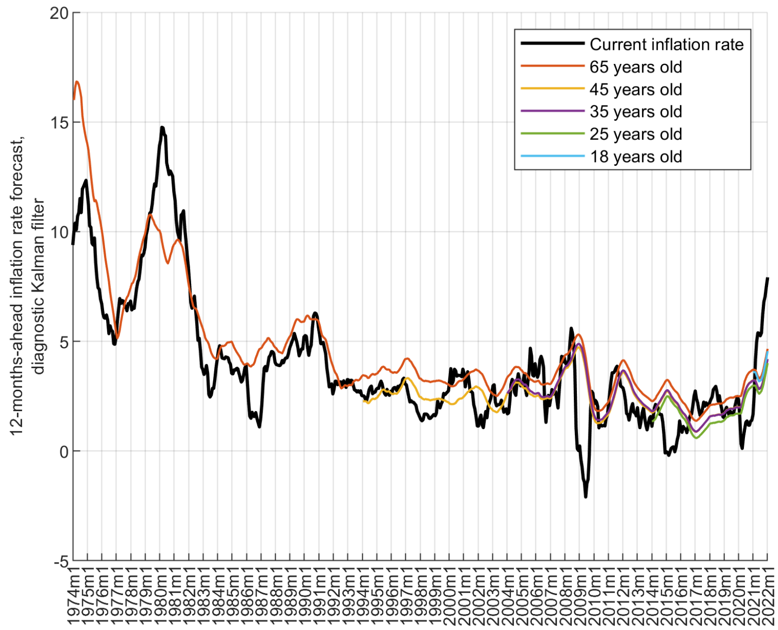
Woodford, M. (2001). Imperfect common knowledge and the effects of monetary policy.

Yaari, M. E. (1965). Uncertain lifetime, life insurance, and the theory of the consumer. *Review of Economic Studies*, 32(2):137–150.

Online Appendix

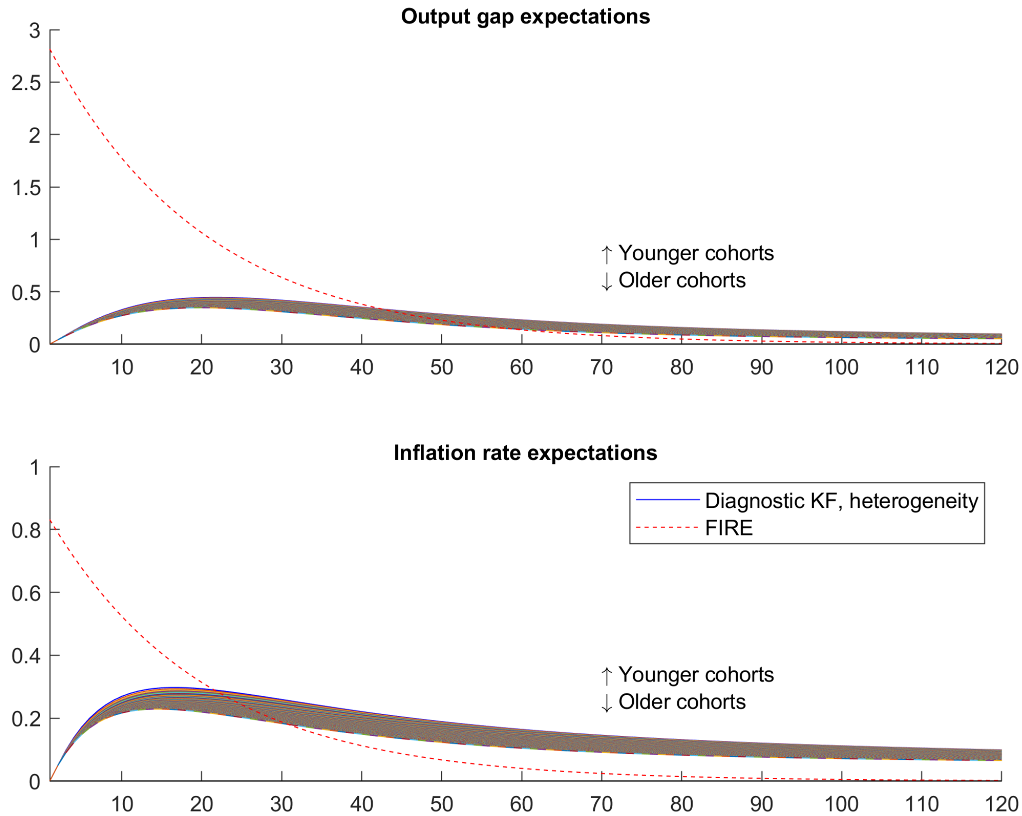
A Additional figures and tables

Figure A.1: Diagnostic Kalman-filter-based inflation forecasts by cohort, full sample



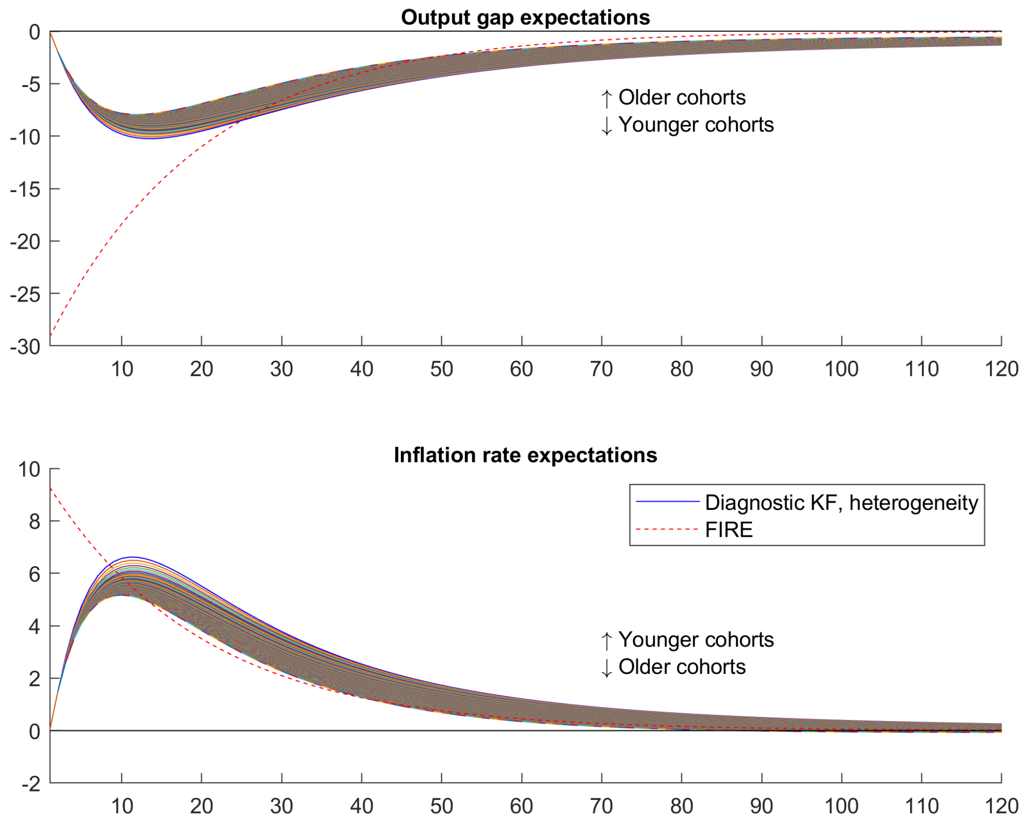
Note: Figure shows forecasts for selected cohorts according to the Kalman-filter-augmented expectations and considering the estimate for θ from Column 1 of Table 2. Selected cohorts differentiated by their age in 2021. We further assume that each cohort starts forecasting when they become 18 years old.

Figure A.2: Impulse response functions, inflation rate diagnostic expectations by cohort, taste shock



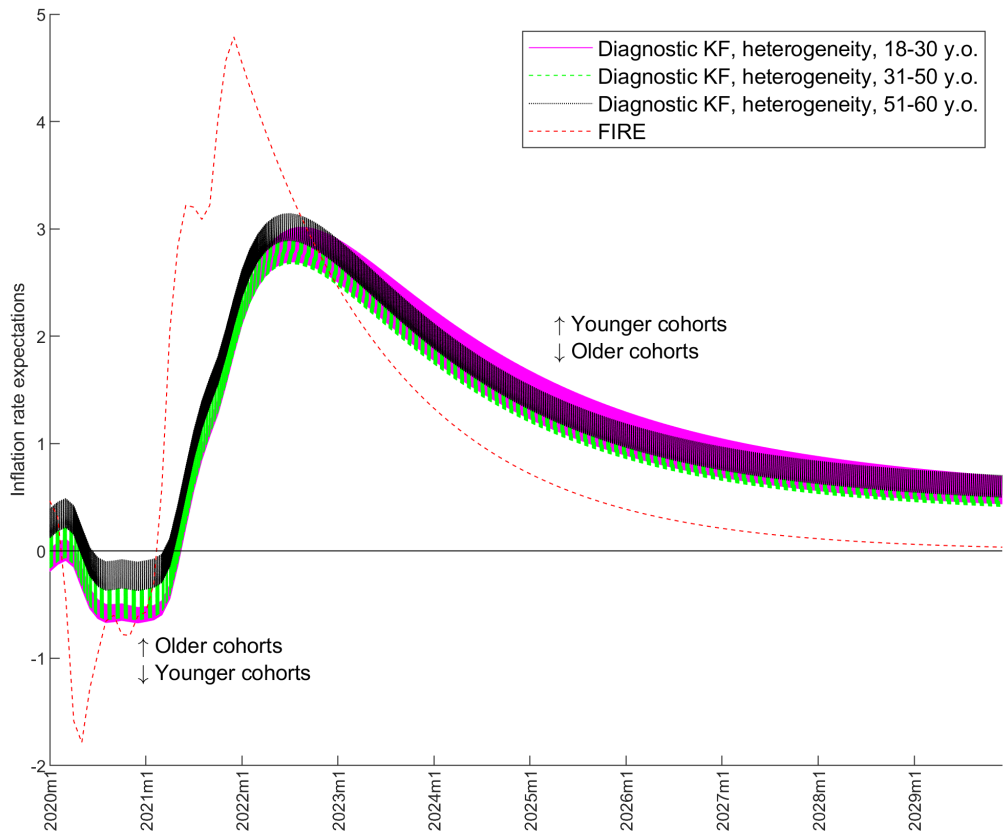
Note: Figure shows the heterogeneous expectations generated by the diagnostic Kalman filter. Cohorts denote age at the time of the shock.The solid lines represent different cohorts in the diagnostic Kalman filter model. The dashed red line is the full information rational expectations model. Horizontal axis denotes months after the shock.

Figure A.3: Impulse response functions, inflation rate diagnostic expectations by cohort, cost shock



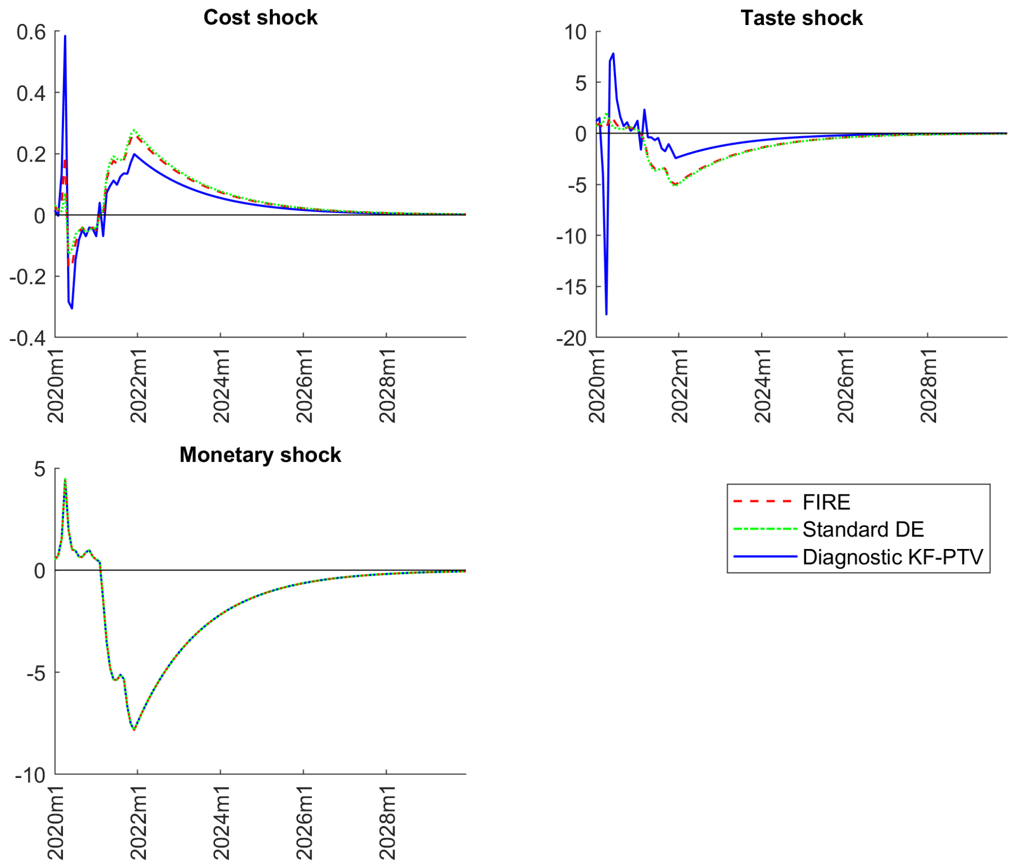
Note: Figure shows the heterogeneous expectations generated by the diagnostic Kalman filter. Cohorts denote age at the time of the shock. The solid lines represent different cohorts in the diagnostic Kalman filter model. The dashed red line is the full information rational expectations model. Horizontal axis denotes months after the shock.

Figure A.4: Impulse response functions, inflation rate diagnostic expectations by cohort, forecast



Note: Figure shows the heterogeneous expectations generated by the diagnostic Kalman filter and the data (up to December 2021). Cohorts denote age in 2021. Horizontal axis denotes months.

Figure A.5: Impulse response functions, shocks, forecast



Note: Figure shows the paths shocks follow according to the model and the data (up to December 2021). The red dashed line shows the results for the case of the full information rational expectations model (FIRE), the green dotted line shows the results of a standard diagnostic expectations operator and the solid blue line shows the diagnostic Kalman filter model. For the standard diagnostic expectations case we assume that agents use the expectations operator $\mathbb{E}_t^{\theta,s}[X_{t+h}] = \mathbb{E}_t[X_{t+h}] + \zeta(\mathbb{E}_t[X_{t+h}] - \mathbb{E}_{t-3}[X_{t+h}])$ with $\zeta > 0$. Horizontal axis denotes months.

Table A.1: Model calibration

Parameter	Value	Parameter	Value
β	0.9967	χ_y	0.125
η	1	ρ^{cost}	0.9
ϕ	0.9167	ρ^{taste}	0.9
σ	1	λ	0.001
ε	9	K	0.1751
χ_π	1.5	θ	-0.317

Note: Table shows the parameters used for the model. We follow a standard monthly calibration.

Table A.2: Optimal Taylor rule parameters, cost shock

	χ_π^*	χ_y^*
FIRE	582.33	3.38
Diagnostic KF-PTV	43.28	0.00

Note: Table shows the Taylor rule parameters that minimize objective function $\mathbb{E}_t\left[\pi_t^2 + \vartheta y_t^2\right]$ when an unexpected cost shock hits the economy.

Table A.3: Optimal Taylor rule parameters, taste shock

	χ_π^*	χ_y^*
FIRE	8.73	28.10
Diagnostic KF-PTV	9.24	26.37

Note: Table shows the Taylor rule parameters that minimize objective function $\mathbb{E}_t\left[\pi_t^2 + \vartheta y_t^2\right]$ when an unexpected taste shock hits the economy.

B Normality of diagnostic expectations

Let $f\left(\pi_{t+1}|\mathscr{I}_t\right)$ be the true distribution of the future inflation rate conditional on information set $\mathscr{I}_t$. We assume this behaves as

$$f\left(\pi_{t+1}|\mathscr{I}_t\right) \sim N\left(\mathbb{E}_t^{KF}\left[\pi_{t+1}\right], \sigma_\pi^2\right),$$

where $\mathbb{E}_t^{KF}\left[\pi_{t+1}\right]$ is the expectation according to the standard Kalman filter and σ_π^2 is the variance. We further assume this distribution to be true across all cohorts i.

Let $f\left(\pi_{t+1}|\mathscr{I}_{i,t}^{ref}\right)$ be the distribution of the inflation rate conditional on the referential information set for cohort i. This distribution behaves as

$$f\left(\pi_{t+1}|\mathscr{I}_{i,t}^{ref}\right) \sim N\left(\mathbb{E}_{i,t}^{ref}\left[\pi_{t+1}\right], \sigma_\pi^2\right),$$

where

$$\mathbb{E}_{i,t}^{ref}\left[\pi_{t+1}\right] = \frac{\sum_{j=1}^{t-k_i} \mathbb{E}_{i,t-j}^{KF}\left[\pi_{t+1}\right]}{t-k_i}.$$

Given these two elements, we define the diagnostic distribution as

$$f_{i,t}^{\theta}\left(\pi_{t+1}\right) = f\left(\pi_{t+1}|\mathscr{I}_t\right) D_{i,t}^{\theta}\left(\pi_{t+1}\right),$$

with

$$D_{i,t}^{\theta}\left(\pi_{t+1}\right) = \left[\frac{f\left(\pi_{t+1}|\mathscr{I}_t\right)}{f\left(\pi_{t+1}|\mathscr{I}_{i,t}^{ref}\right)^{t-k_i}}\right]^{\theta} Z_{i,t},$$

where $Z_{i,t}$ is a term that ensures that $f_{i,t}^{\theta}\left(\pi_{t+1}\right)$ integrates to 1.

Therefore, the PDF of the diagnostic distribution is[22]

[22]Note that π_{t+1} denotes the future inflation rate, while π denotes the constant equal to 3.14.

$$f_{i,t}^{\theta}\left(\pi_{t+1}\right)=\frac{\left[\frac{1}{\sigma_{\pi}\sqrt{2\pi}}\exp\left\{-\frac{\left(\pi_{t+1}-\mathbb{E}_{i,t}^{KF}\left[\pi_{t+1}\right]\right)^{2}}{2\sigma_{\pi}^{2}}\right\}\right]^{(1+\theta)}}{\left[\frac{1}{\sigma_{\pi}\sqrt{2\pi}}\exp\left\{-\frac{\left(\pi_{t+1}-\mathbb{E}_{i,t}^{ref}\left[\pi_{t+1}\right]\right)^{2}}{2\sigma_{\pi}^{2}}\right\}\right]^{\theta}}Z_{i,t},$$

where we define $Z_{i,t}^{-1}=\int f_{i,t}^{\theta}\left(\pi_{t+1}\right)d\pi_{t+1}$.

We can make the following approximation:

$$f_{i,t}^{\theta}\left(\pi_{t+1}\right)\approx\frac{1}{\sigma_{\pi}\sqrt{2\pi}}\exp\left\{-\frac{\left(\pi_{t+1}-\mathbb{E}_{i,t}^{\theta}\left[\pi_{t+1}\right]\right)^{2}}{2\sigma_{\pi}^{2}}\right\}Z_{i,t},$$

where

$$\mathbb{E}_{i,t}^{\theta}\left[\pi_{t+1}\right]=\mathbb{E}_{i,t}^{KF}\left[\pi_{t+1}\right]+\theta\left(\mathbb{E}_{i,t}^{KF}\left[\pi_{t+1}\right]-\mathbb{E}_{i,t}^{ref}\left[\pi_{t+1}\right]\right).$$

Thus, we conclude that

$$f_{i,t}^{\theta}\left(\pi_{t+1}\right)\sim N\left(\mathbb{E}_{i,t}^{\theta}\left[\pi_{t+1}\right],\sigma_{\pi}^{2}\right).$$

C Monthly inflation as a random walk

Throughout the paper we consider the monthly inflation to be a random walk process. The reason behind is that, with monthly inflation data, we cannot reject the hypothesis of unit root. To further expand on this, in Table A.4 we present the results of AR(1) regressions on the monthly US inflation rate from January 1960 to March 2022. Column 1 does not consider a constant while Column 2 does.

Table A.4: AR(1) regression for monthly inflation

	(1)	(2)
π_{t-1}	0.999*** (0.993 - 1.005)	0.992*** (0.982 - 1.002)
Observations	747	747
R-squared	0.993	0.981

Note: Table shows the results of AR(1) regressions with the monthly inflation rate in the US from January 1960 to March 2022. Column 1 does not consider a constant while Column 2 does. 95 percent confidence intervals in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

We see that in both specifications the autorregresive coefficient is very close to 1. Furthermore, the value of 1 falls within the 95 percent confidence intervals. Also, the F-test in which the null hypothesis is that the autorregresive coefficient is equal to 1 gives a p-value of 0.66 when we do not consider a constant and a p-value of 0.12 when we consider a constant, such that we cannot reject the null hypothesis in any of the two cases. Lastly, an augmented Dickey-Fuller test on the monthly inflation gives a p-value of 0.52, so that we cannot reject the null hypothesis of a unit root. These findings are in line with those of Pivetta and Reis (2007), who find that there is a very high persistence in the quarterly inflation rate in the US.

Given that we take the inflation rate to be a random walk process, we now turn to the calibration of the Kalman filter of section IV.1. There, we defined the agents assume that inflation behaves as

$$\pi_{t+1} = \pi_t + \varepsilon_t,$$

where ε_t is a normally independent and identically distributed inflation shock.

Agents also receive a signal s_t given by

$$s_t = \pi_{t+1} + \upsilon_t,$$

where υ_t is a normally independent and identically distributed signal noise.

Furthermore, we assume that the signal is $s_t = \pi_{t-1}^{food}$ where π_{t-1}^{food} denotes the inflation in period $t-1$ of the food component of the CPI. By manipulating the equations we get

$$\varepsilon_t = \pi_t - \pi_{t+1},$$

$$\upsilon_t = s_t - \pi_{t+1} = \pi_{t-1}^{food} - \pi_{t+1}.$$

Then, armed with the US monthly inflation and food inflation series from January 1960 to March 2022 we calculate the variances σ_ε^2, σ_υ^2 and the covariance $\sigma_{\varepsilon\upsilon}$.

D Diagnostic Kalman filter with AR(1) assumption

In this section we repeat the forecasting exercise from Section IV but replacing the random walk assumption with an AR(1) specification. Therefore, agents assume that inflation behaves as

$$\pi_{t+1} = \rho_\pi \pi_t + \varepsilon_t,$$

where the coefficient $\rho_\pi \in [0,1]$ captures the mean-reversion of the inflation variable. Here, we assume the inflation rate has been properly demeaned.

As before, we assume that the signal is given by

$$s_t = \zeta \pi_{t+1} + \upsilon_t.$$

The forecasted value of the inflation variable is

$$\mathbb{E}^{KF}_{i,t}[\pi_{t+1}] = (1-\zeta K)\,\mathbb{E}^{KF}_{i,t-1}[\pi_{t+1}] + K s_t,$$

where the difference now lies in the fact that agents use the AR(1) assumption to forecast the inflation rate such that

$$\mathbb{E}^{KF}_{i,t}[\pi_{t+h}] = \rho_\pi^{h-1}\mathbb{E}^{KF}_{i,t}[\pi_{t+1}].$$

In this section we assume $\zeta = 1$, $\rho = 0.99$, $\sigma_\varepsilon = 0.15$, $\sigma_\nu = 4.09$ and $\sigma_{\varepsilon\nu} = -0.06$.[23] This gives $K = 0.175$.

For the different cohorts Panel (a) of Figure A.6 presents the standard Kalman filter forecast, while Panel (b) of Figure A.6 presents the reference.[24]

Table A.5 presents the result of the diagnostic parameter estimation. In this case, $\theta = -0.526$. Armed with this coefficient, Panel (c) of Figure A.6 shows the heterogeneous diagnostic forecasts across cohorts.

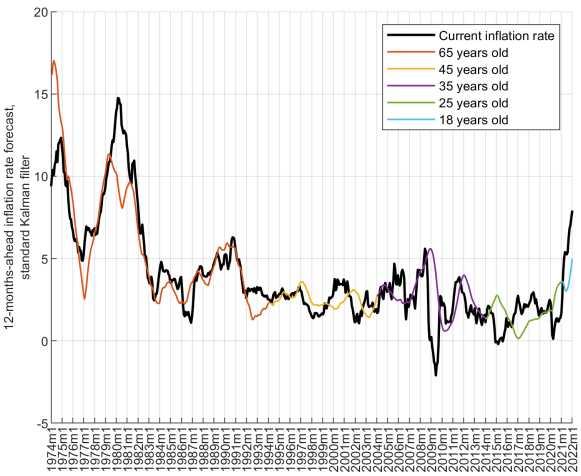
Finally, Figure A.7 presents the comparison of the diagnostic forecasts with the AR(1) assump-

[23] We calibrate the variances and covariance according to Appendix C.

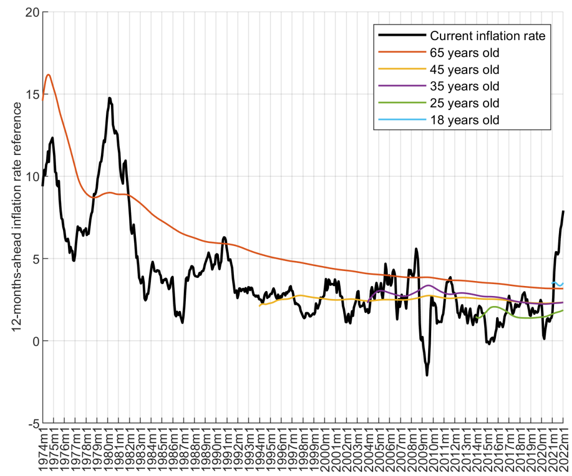
[24] We return the mean to the data before plotting the graphs, where the long-run mean of the inflation rate is 2 percent.

tion and the observed forecasts in the data. We find that, as with the random walk process, this version of the diagnostic forecast based on an AR(1) assumption provides a good fit to the data.

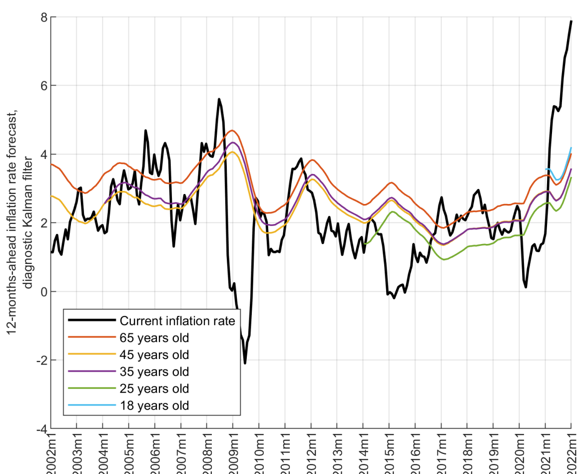
Figure A.6: Diagnostic Kalman-filter-based inflation forecasts by cohort, AR(1)



(a) Standard Kalman-filter-based inflation forecasts



(b) Inflation rate reference



(c) Diagnostic Kalman-filter-based inflation forecasts

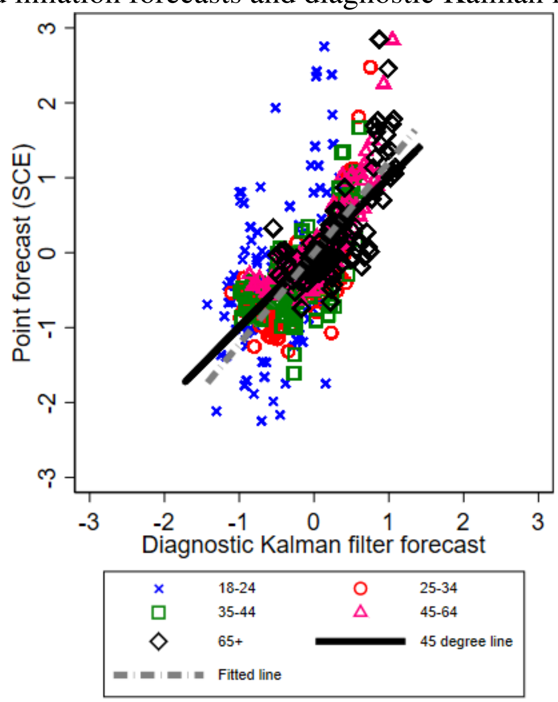
Note: Panel (a) shows the Kalman filter forecast for the common component for selected cohorts, differentiated by their age in 2021. Panel (b) shows the references for selected cohorts obtained according to the Kalman filter and given the history of inflation experienced by the corresponding age group. Panel (c) shows forecasts for selected cohorts according to the Kalman-filter-augmented expectations and considering the estimate for θ from Column 1 of Table 2. Selected cohorts are differentiated by their age in 2021. We further assume that each cohort starts forecasting when they become 18 years old.

Table A.5: Diagnostic parameter estimation, AR(1)

	(1)
$\mathbb{E}_{i,t}^{ref}\left[\pi_{t+12}\right]$	0.526***
	(0.047)
Time FE	Yes
Observations	101,262
R-squared	0.092

Note: Table shows results of Regression (6). $\mathbb{E}_{i,t}^{ref}\left[\pi_{t+12}\right]$ is the reference constructed for a respondent of age i as explained in the main text. Column (1) has only a time fixed effect as an additional control. Robust standard errors in parentheses. Standard errors clustered by age. Dependent variable trimmed at 10 percent and 90 percent in each period. *** p<0.01, ** p<0.05, * p<0.1.

Figure A.7: Observed inflation forecasts and diagnostic Kalman filter forecasts, AR(1)



Note: Figure shows binned scatterplot across diagnostic Kalman filter forecasts (x-axis) and point forecast inflation expectations according to the Survey of Consumer Expectations (SCE) of the Federal Reserve Bank of New York (y-axis). Variables demeaned by the intercept. Data go from June 2013 to December 2021. SCE variable trimmed at 10 percent and 90 percent in each period.

E External validity: European data

We check the external validity of our results using data from the Consumer Expectations Survey (CES) of the European Central Bank. It contains monthly data between April 2020 and September 2022 for six countries: Belgium, France, Germany, Italy, the Netherlands, and Spain.[25]

We see that in Europe, lifetime experiences with the inflation rate are also heterogeneous across cohorts, as we show in Figure A.9. Moreover, we see that by 2020 the youngest cohorts had not been exposed to high inflation rates, but this changes after the high inflation rate episode of 2021 and 2022. After this, the youngest cohorts are the ones that show the highest lifetime average for the inflation rate, even larger than that of the people who experienced the high inflation rates of the 80s.

In Figure A.10 we relate the two previous facts and find that in Europe, similar to the US, the larger the inflation rate individuals have experienced in their lifetimes, the higher their inflation expectations.

Table A.6 shows that in Europe, as happened in the US, after controlling for the average lifetime inflation rate, younger generations do not react more strongly to inflation news than older cohorts.[26]

We now turn to the diagnostic Kalman filter of Section IV.1. Table A.7 shows the parameters that go into the Kalman filter calibration after using European inflation rate and food inflation rate. Then, we estimate the diagnostic parameter according to Equation 6.[27] In Table A.8 in our baseline specification of Column 1 we find $\theta^{eur} = -0.156$, a parameter that suggests underreaction to current news. With this parameter, in Figure A.11 we plot inflation expectations according to our diagnostic Kalman filter, across cohorts and in each of the six countries in our sample. We find that the oldest cohorts have the highest inflation expectations before 2021. Then, after 2021 the

[25]There is a relevant difference between the data sets of US and Europe. In the former we have the exact age of the respondents. In the latter we do not have detailed information on the age of the respondents, as they are classified in 4 age groups: 18-34, 35-49, 50-70 and 71+.

[26]We confirm the finding with a F-test where the null hypothesis is that all of the interactions are jointly equal to zero. The test gives a p-value of 0.35, so we cannot reject the null hypothesis.

[27]Because we do not know the exact age of the respondents, we do not know which are the exact lifetime average inflation rates they have experienced. Therefore, for this estimation, we assume that every agent in cohort 18-34 has the lifetime average inflation rate of a 25-year-old, every agent in cohort 35-49 has the lifetime average inflation rate of a 35-year-old, every agent in cohort 50-70 has the lifetime average inflation rate of a 50-year-old and every agent in cohort 71+ has the lifetime average inflation rate of a 71-year-old. On the signals used, because the series on the inflation rate of the food component of the CPI have varying starting dates in the different countries, we replace the missing values with the observed inflation rate in order to make the starting dates of all countries uniform.

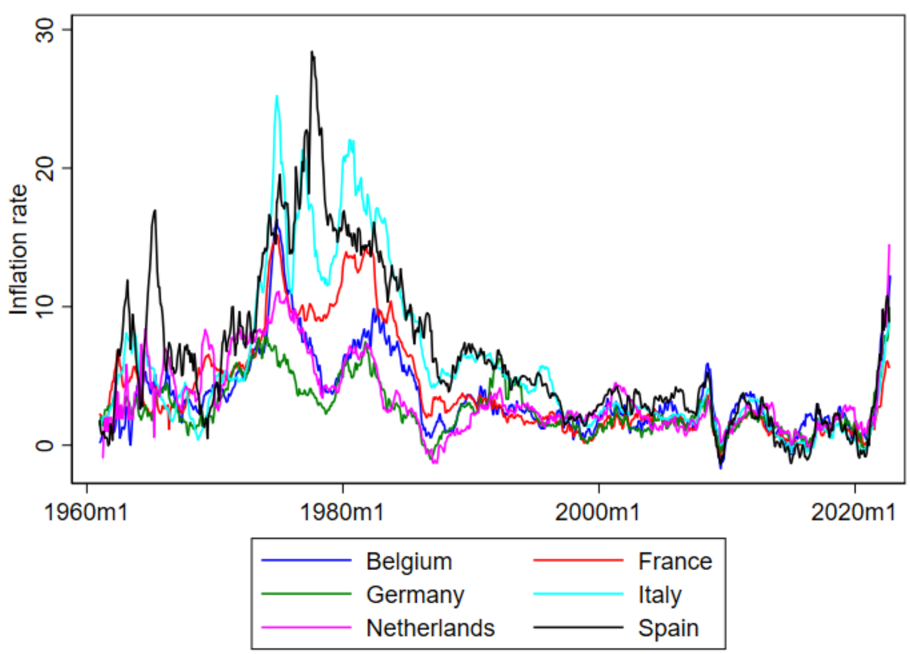
youngest cohorts start catching up with the oldest ones and even surpass them in some countries.

Table A.8 also shows additional specifications of the estimation of Equation 6. We find that after controlling for cohort and country fixed effects, we still find that agents underreact to current news when forming their expectations. These additional specifications also tell us that the heterogeneity in expectations across cohorts is not due to people of different ages or from different countries facing different consumption bundles or having different preferences, but to the proposed anchoring-to-the-past mechanism. Thus, it is past experiences that define expectations, not the age or the geographic location per se.[28]

Lastly, in Figure A.12 we compare the inflation expectations generated by our diagnostic Kalman filter to the survey data. We see that we have a decent fit to the data.

We conclude that our findings from the main text are also valid for Europe. We find evidence that supports the claim that (i) inflation expectations are also heterogeneous in Europe and (ii) can also be modeled by a diagnostic Kalman filter with underreaction to current news and over-weighting to the reference term.

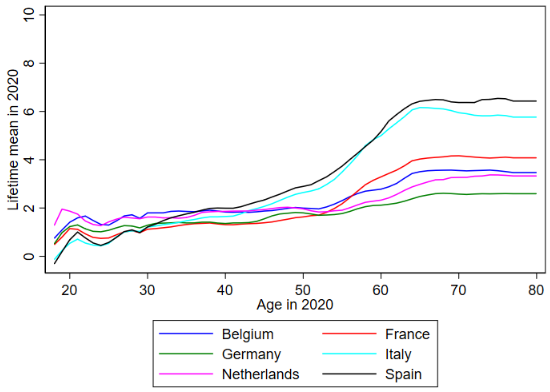
Figure A.8: Inflation rate, Europe



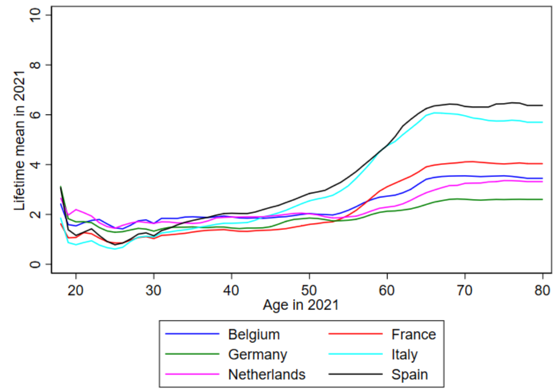
Source: FRED.

[28]See Hajdini et al. (2022a) for a further discussion on this.

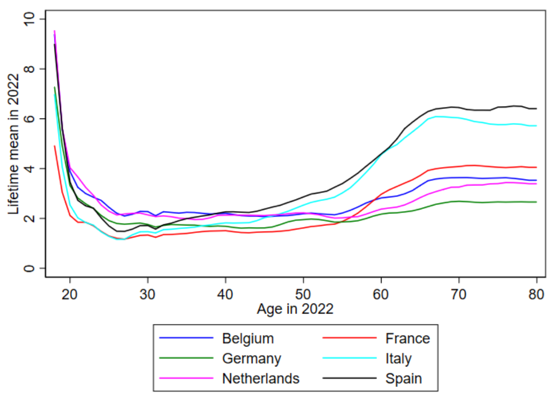
Figure A.9: Lifetime average inflation rate among respondents, Europe



(a) 2020



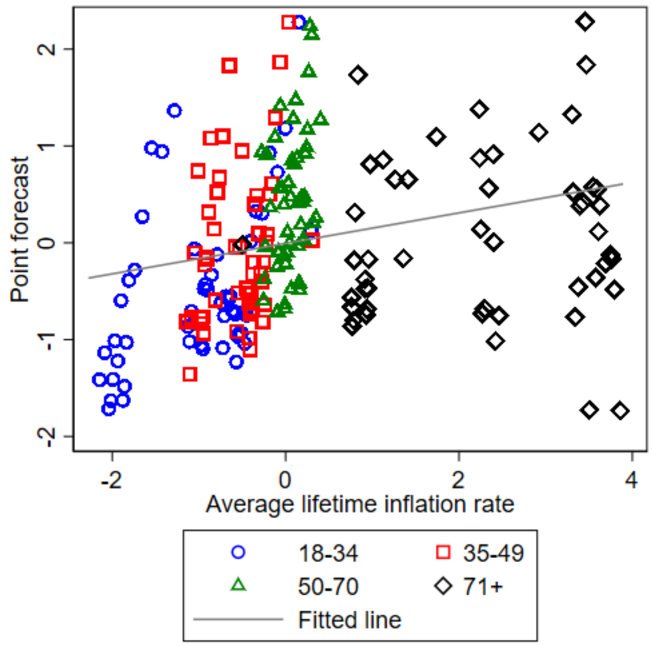
(b) 2021



(c) 2022

Note: Figure shows the mean of the monthly YoY inflation rate that people of the age shown in 2020, 2021, and 2022 have experienced in their lifetimes, starting when they were age 18.
Source: FRED.

Figure A.10: Inflation point forecast and average lifetime inflation rate, Europe



Note: Figure shows binned scatterplot across lifetime average inflation rate bins. Variables residualized by respondent gender and commuting zone. Data go from April 2020 to September 2022. Ages correspond to the interviewee's age at the time of the survey.
Source: Consumer Expectations Survey.

Table A.6: Effects of current and experienced inflation rates on inflation expectations

Dep. var.: Inflation expectations	(1)	(2)	(3)	(4)
Average lifetime inflation rate	0.276*	0.244**	0.301*	0.252*
	(0.132)	(0.100)	(0.149)	(0.129)
Current inflation	0.351***	0.275***		
	(0.033)	(0.057)		
Cohort 35-49		0.155		
		(0.090)		
Cohort 50-70		0.203*		
		(0.112)		
Cohort 71+		-0.401		
		(0.348)		
Current inflation × 35-49		0.074		
		(0.074)		
Current inflation × 50-70		0.122		
		(0.069)		
Current inflation × 71+		0.124		
		(0.083)		
Time FE	No	No	Yes	Yes
Controls	No	No	No	Yes
Observations	294,232	294,232	294,232	294,232
R-squared	0.128	0.135	0.152	0.164

Note: Table shows regressions where the dependent variable is inflation expectations according to the Consumer Expectations Survey (CES) of the European Central Bank. Column (1) shows controls for the average lifetime inflation of respondents of a given age at each period in time and the last inflation measure. Column (2) follows (1) but adds cohort fixed effects and the interaction of those cohort fixed effects with the current inflation. Column (3) follows (1) but adds time fixed effects and, hence, omits the current inflation variable. Column (4) follows (1) but adds time fixed effects and demographic controls. The demographic controls are income, gender, educational level, and country. Robust standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1. Standard errors clustered by age. The dependent variable is trimmed, dropping the lower and upper 10 percent of answers in each period.

Table A.7: Diagnostic Kalman filter parameters, Europe

	σ_ε^2	σ_υ^2	$\sigma_{\varepsilon\upsilon}$	K
Belgium	0.16	2.51	-0.28	0.24
France	0.09	2.63	-0.14	0.17
Germany	0.12	3.13	-0.15	0.18
Italy	0.23	3.38	-0.38	0.24
Netherlands	0.13	3.54	-0.18	0.18
Spain	0.45	5.83	-0.65	0.26

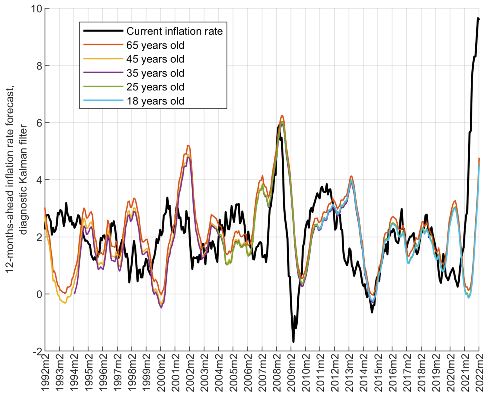
Note: We obtain this calibration for each country following the steps outlined in Appendix C. The data for these calculations goes from January 1971 to October 2022.

Table A.8: Diagnostic parameter estimation, Europe

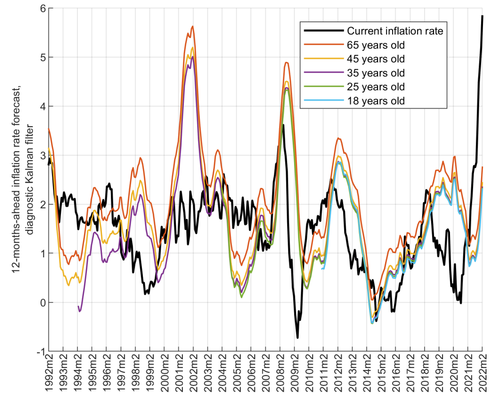
	(1)	(2)	(3)	(4)
$\mathbb{E}_{i,t}^{ref}\left[\pi_{t+12}\right]$	0.156*** (0.025)	0.208*** (0.044)	0.094*** (0.019)	0.060* (0.030)
Time FE	Yes	Yes	Yes	Yes
Controls	No	Cohort	Country	Cohort, country
Observations	271,311	271,311	271,311	271,311
R-squared	0.122	0.130	0.132	0.140

Note: Table shows results of Regression (6), but changing the dependent variable for inflation expectations according to the Consumer Expectations Survey (CES) of the European Central Bank. The independent variable $\mathbb{E}_{i,t}^{ref}\left[\pi_{t+12}\right]$ is the reference constructed for a respondent of age i as explained in the main text. Column (1) considers a time fixed effect as a control. Column (2) has time and cohort fixed effects. Column (3) has time and country fixed effects. Column (4) has time, cohort, and country fixed effects. Standard errors clustered by age in parentheses. Dependent variable trimmed at 10 percent and 90 percent in each period. We use population weights. *** p<0.01, ** p<0.05, * p<0.1.

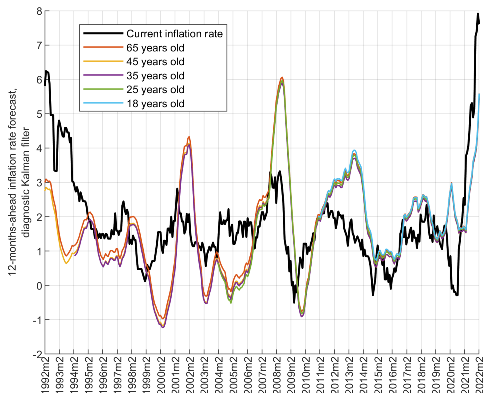
Figure A.11: Diagnostic Kalman-filter-based inflation forecasts by cohort, Europe



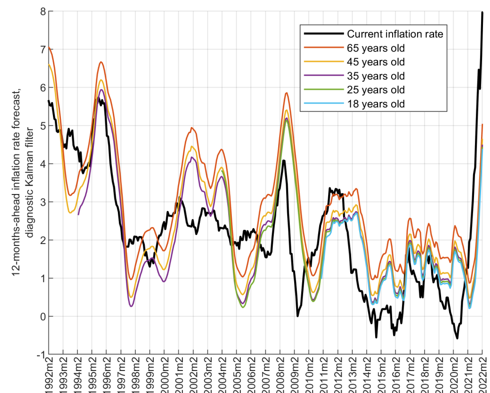
(a) Belgium



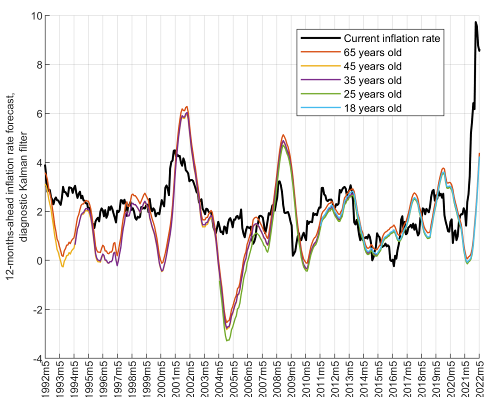
(b) France



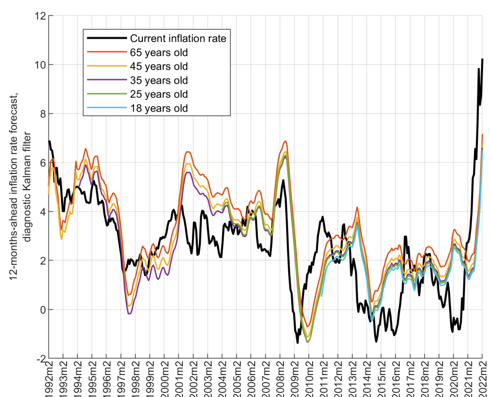
(c) Germany



(d) Italy



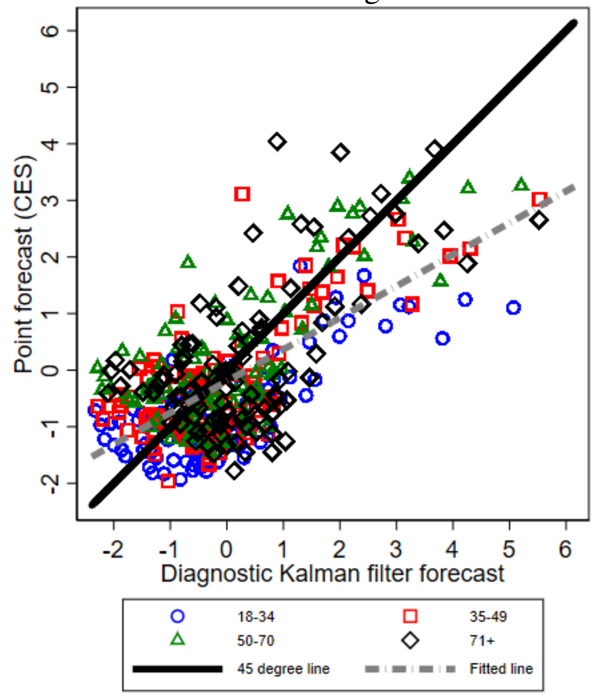
(e) Netherlands



(f) Spain

Note: Figure shows forecasts for selected cohorts according to the Kalman-filter-augmented expectations and considering the estimate for θ from Column 1 of Table A.8. Selected cohorts differentiated by their age in 2021. We further assume that each cohort starts forecasting when they become 18 years old.

Figure A.12: Observed inflation forecasts and diagnostic Kalman filter forecasts, Europe



Note: Figure shows binned scatterplot across diagnostic Kalman filter forecasts (x-axis) and point forecasts of inflation expectations according to the Consumer Expectations Survey (CES) of the European Central Bank (y-axis). Variables demeaned by the intercept. Data go from April 2020 to September 2022. SCE variable trimmed at 10 percent and 90 percent in each period.

F Derivations for firm block

F.1 Final good producer

The final good producer operates in a perfectly competitive market. It produces the final good Y from a CES basket composed by a continuum of intermediate goods $Y(j)$ with $j \in [0,1]$. The maximization problem of this firm is

$$\max_{Y_t(j)} P_t Y_t - \int_0^1 P_t(j) Y_t(j)\, dj,$$

subject to

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where $P_t(j)$ is the price of intermediate good j and ε is the elasticity of substitution in the CES basket.

The first-order condition gives

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon} Y_t,$$

which represents the demand for good j.

For the CES basket, we also get a corresponding aggregate price level expression of

$$P_t = \left(\int_0^1 P_t(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}.$$

F.2 Intermediate good producers

Any given intermediate good producer j will produce the intermediate good $Y(j)$ according to

$$Y_t(j) = A_t L_t(j),$$

where A_t is a process that represents technology and $L_t(j)$ is the labor supplied to firm j. The intermediate good producer will pay a nominal wage w to workers.

The problem of an intermediate good producer indexed by j is

$$\min w_t L_t\left(j\right),$$

subject to

$$Y_t\left(j\right) = A_t L_t\left(j\right),$$

$$Y_t\left(j\right) = \left(\frac{P_t\left(j\right)}{P_t}\right)^{-\varepsilon} Y_t.$$

The first-order conditions are

$$mc_t = \frac{w_t}{A_t},$$

where mc is the real marginal cost the firm faces.

F.3 Price setting

We assume that, additionally, intermediate good producers face price rigidities à la Calvo. In any given period, a firm has a probability $1-\phi$ of adjusting its price. That is to say, with probability ϕ this firm will have to keep the price it chose in the previous period. The standard derivation for an optimal reset price results in

$$P_t^* = \frac{\varepsilon}{\varepsilon-1}\frac{X_{1,t}}{X_{2,t}},$$

$$X_{1,t} = \Lambda_t mc_t P_t^{\varepsilon} Y_t + \phi\beta\mathbb{E}_t\left[X_{1,t+1}\right],$$

$$X_{2,t} = \Lambda_t mc_t P_t^{\varepsilon'1} Y_t + \phi\beta\mathbb{E}_t\left[X_{2,t+1}\right],$$

where P^* is the optimal reset price, X_1 and X_2 are auxiliary variables and Λ=$u_C\left(C\right)$.

The definition of the two auxiliary variables can be rewritten in real terms as

$$x_{1,t} = \Lambda_t mc_t Y_t + \phi \beta \mathbb{E}_t \left[(1+\pi_{t+1})^{\varepsilon} x_{1,t+1} \right],$$

$$x_{2,t} = \Lambda_t Y_t + \phi \beta \mathbb{E}_t \left[(1+\pi_{t+1})^{\varepsilon-1} x_{2,t+1} \right],$$

where

$$x_{1,t} = \frac{X_{1,t}}{P_t^{\varepsilon}},$$

$$x_{2,t} = \frac{X_{2,t}}{P_t^{\varepsilon-1}}.$$

Then, from the reset price definition, we define the reset price inflation rate as

$$(1+\pi_t^*) = \frac{\varepsilon}{\varepsilon-1} (1+\pi_t) \frac{x_{1,t}}{x_{2,t}},$$

where π_t^* is the reset price inflation rate.

Moreover, we can rewrite the price index definition in terms of the inflation rate as

$$(1+\pi_t)^{1-\varepsilon} = (1-\phi)(1+\pi_t^*)^{1-\varepsilon} + \phi.$$

G Heterogeneous cohorts

In the baseline exercises we assumed a diagnostic Kalman filter operator where memory varies by cohort, as in Equation 3. In this section we analyze a variation of such operator, where we assume that memory is fixed across all cohorts. We define this alternative diagnostic Kalman filter operator as

$$E_t^{\theta,alt}\left[X_{t+h}\right] = E_t^{KF}\left[X_{t+h}\right] + \theta\left(E_t^{KF}\left[X_{t+h}\right] - \sum_{j=1}^{J} \frac{E_{t-j}^{KF}\left[X_{t+h}\right]}{J}\right),$$

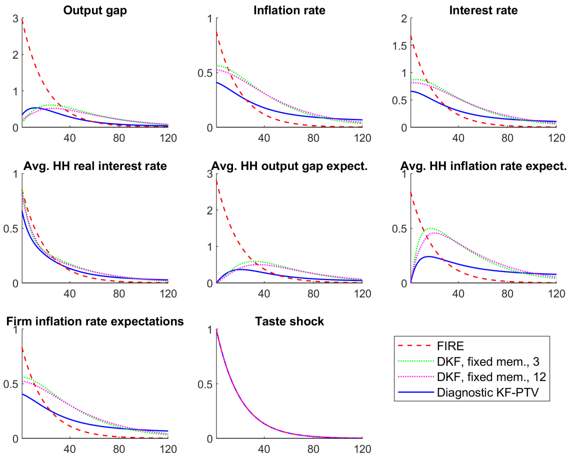
where agents remember what occurred in the last J periods. As in the baseline case, we set $\theta = -0.317$. This means there is underreaction.

Figure A.13 shows the impulse response functions to a cost shock and a taste shock according to our model of Section V. Besides the responses coming from the FIRE and diagnostic Kalman filter cases with varying memory across cohorts, we also consider the two cases for the alternative diagnostic Kalman filter operator: fixing the memory of all cohorts to the last 3 periods and fixing the memory of all cohorts to the last 12 periods.

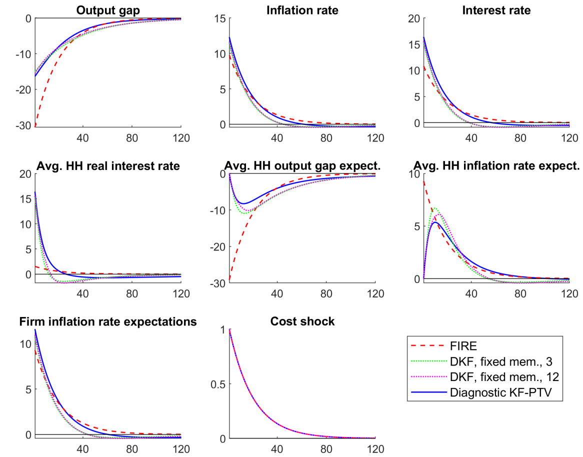
We see that the three diagnostic cases follow the same pattern when compared to the FIRE case. However, the expectations under the fixed memory diagnostic cases show a stronger reaction to the shocks. This is because the memory span in these two alternative diagnostic cases is shorter than in the full-fledged diagnostic case. While the latter remembers and is pegged to the steady state for longer, the former cases do not.

It could be argued that if we choose a sufficiently long memory with the alternative diagnostic Kalman filter operator, then we could closely replicate the results coming from the full-fledged diagnostic case with different cohorts. However, we prefer the full-fledged diagnostic case with different cohorts as it picks up the richness of the data and introduce it in the model: there are different cohorts living at the same time, each has had different life experiences and each has different beliefs.

Figure A.13: Impulse response functions, comparison with alternative diagnostic Kalman filter operator



(a) Taste shock



(b) Cost shock

Note: Figure shows impulse response functions for a selected group of variables after the mentioned shocks. The red dashed line shows the results for the case of the full information rational expectations model (FIRE), the green and magenta dotted lines show the results of diagnostic Kalman filter model with fixed memory and the solid blue line shows the diagnostic Kalman filter model where memory varies by cohort. Horizontal axis denotes months after the shock.