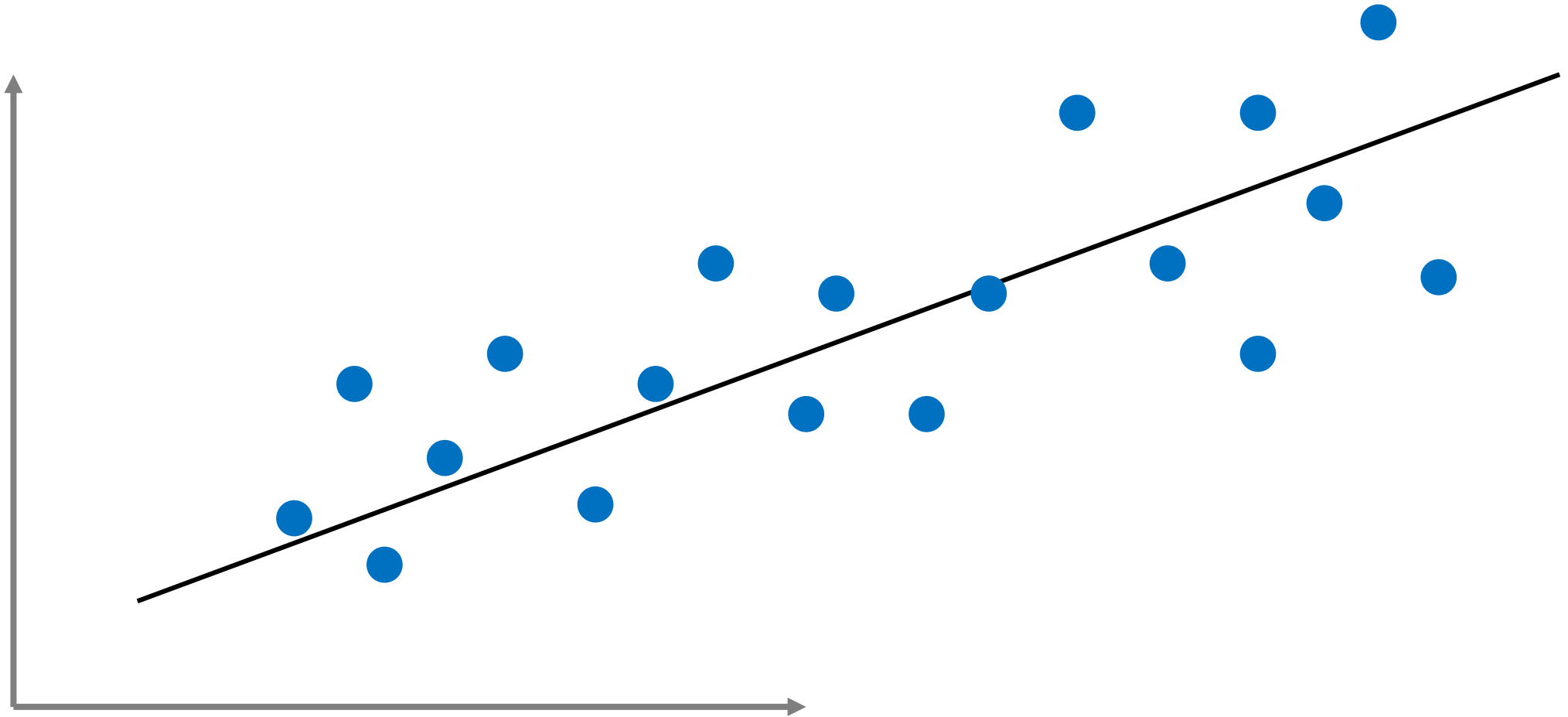


# Linear models

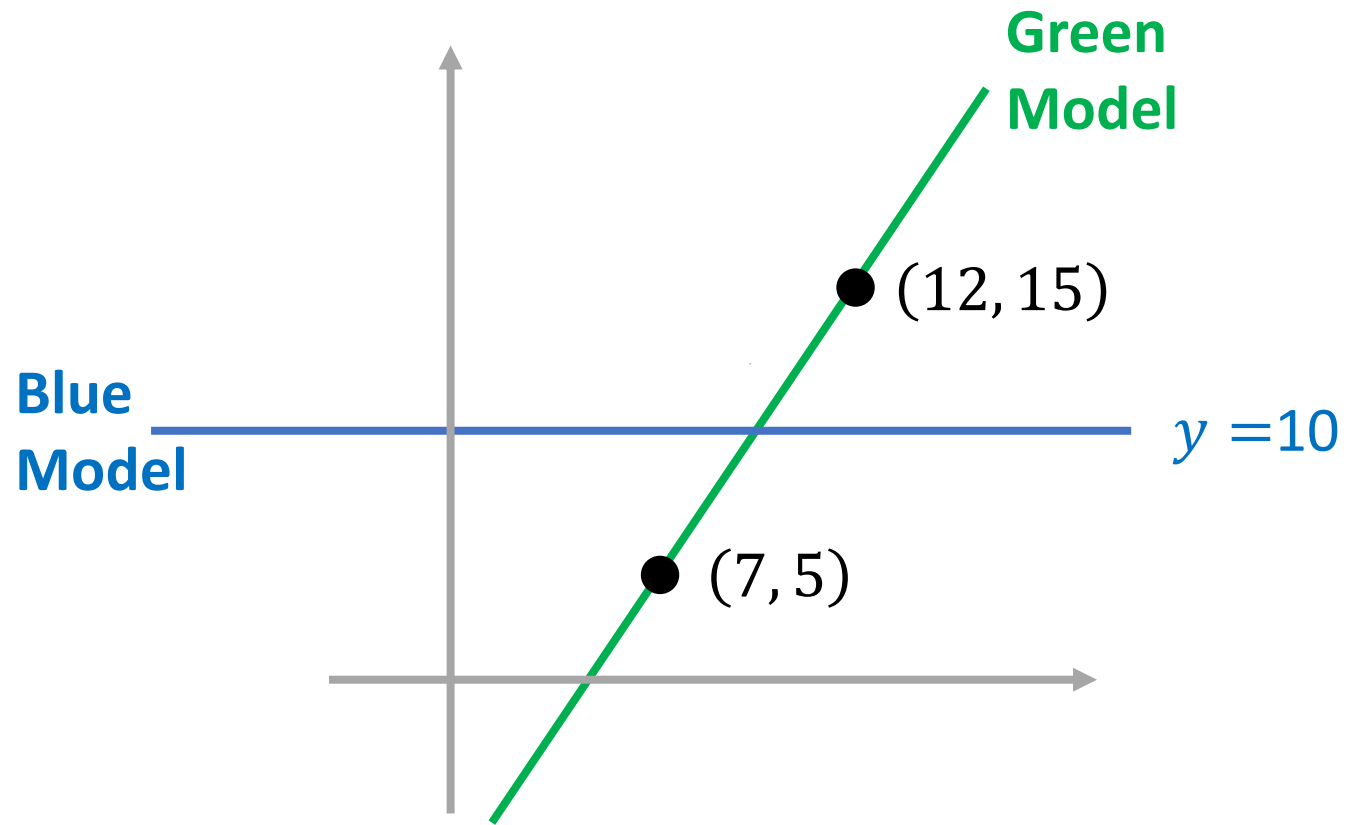
# Linear regression



# Error definition

$y_i$  = Instance

$\hat{y}_i$  = Model

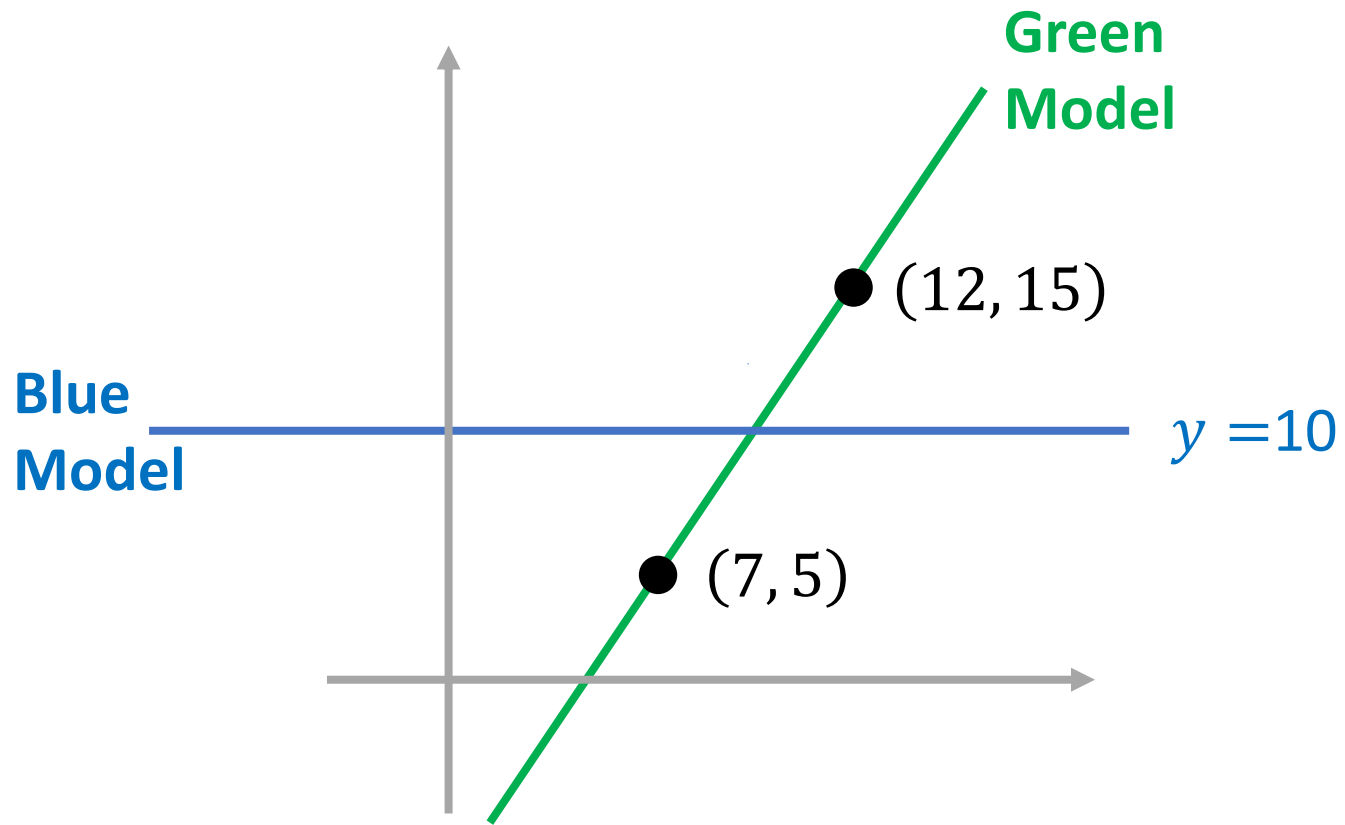


$$\varepsilon \propto \hat{y}_i - y_i$$

$$\varepsilon = 0$$

$$\varepsilon = 0$$

# Error definition



$$\varepsilon \propto \hat{y}_i - y_i$$

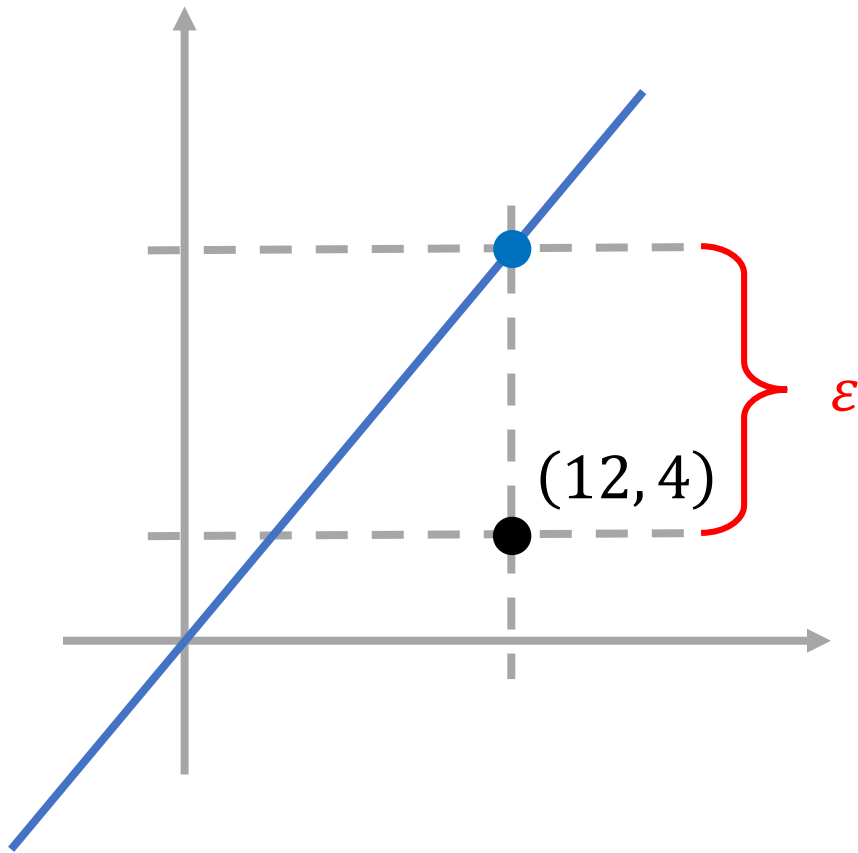
$$\varepsilon = 0$$

$$\varepsilon = 0$$

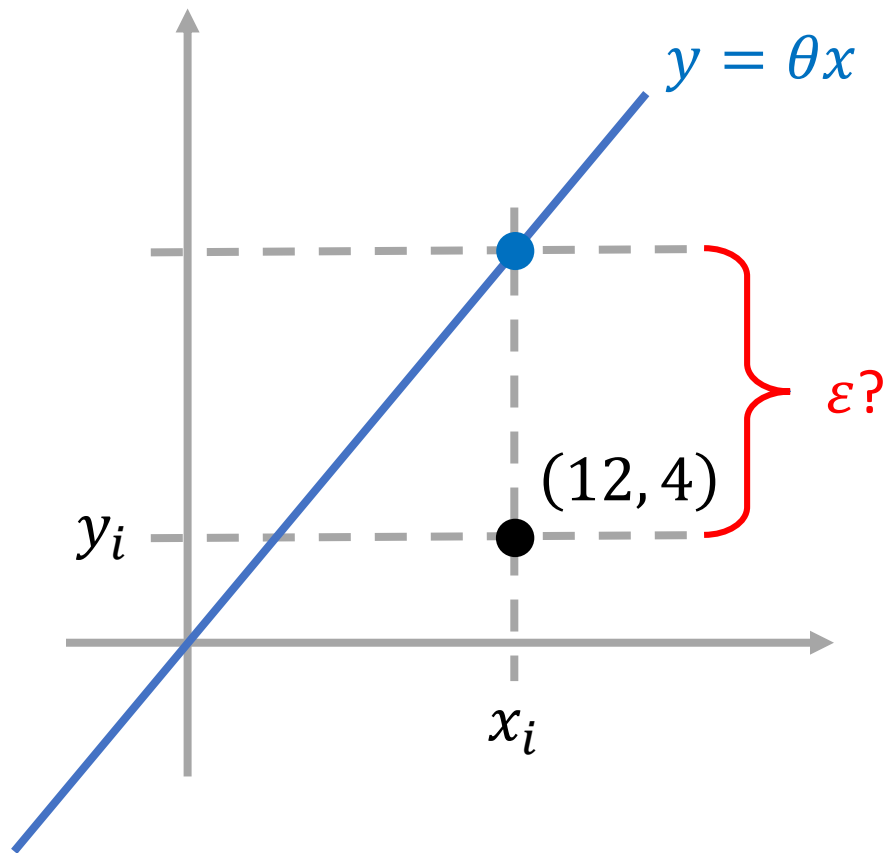
Solution:

$$\varepsilon := \sum_{i=1}^m (\hat{y}_i - y_i)^2$$

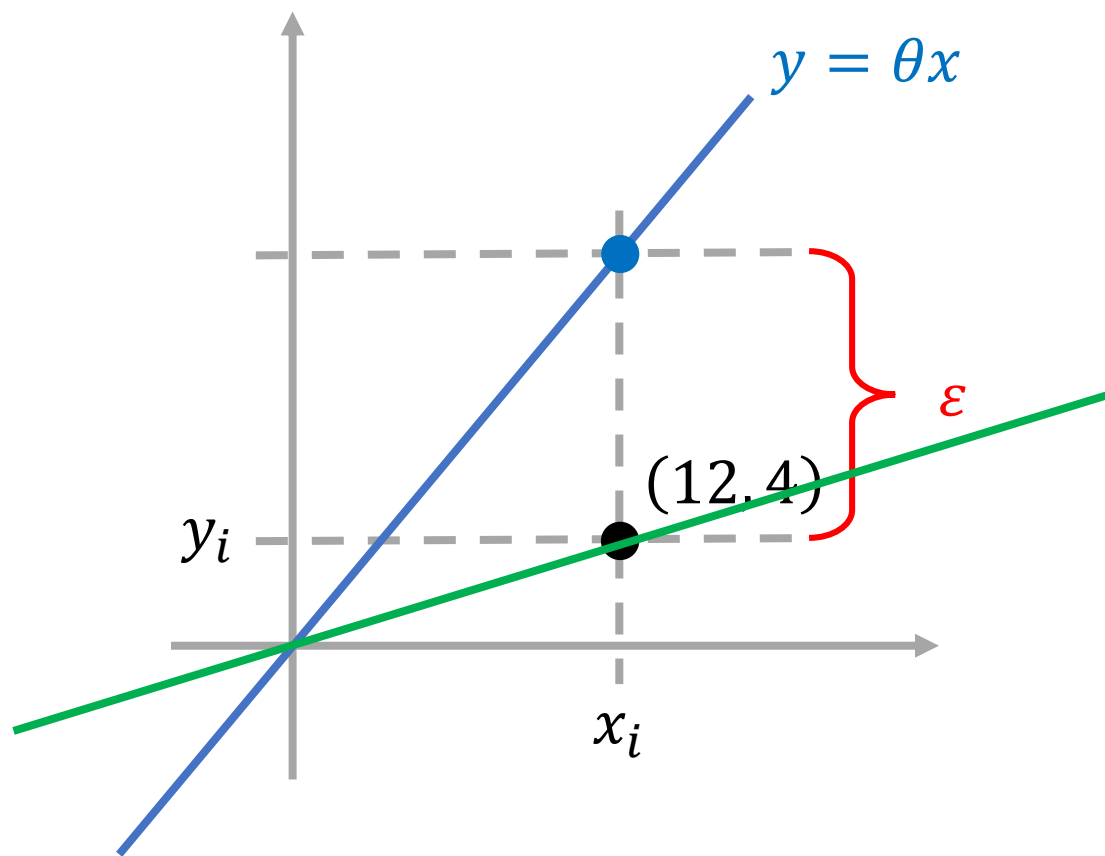
$$y = \theta x \quad m = 1$$



$$y = \theta x \quad m = 1$$



$$y = \theta x \quad m = 1$$



$$\begin{aligned} \epsilon &= (y - 4)^2 \\ &= (12\theta - 4)^2 \\ &= (x_i\theta - y_i)^2 \end{aligned}$$

$$\frac{d\epsilon}{d\theta} = 0 \quad \Rightarrow \quad \theta = 4/12 = y_i/x_i$$

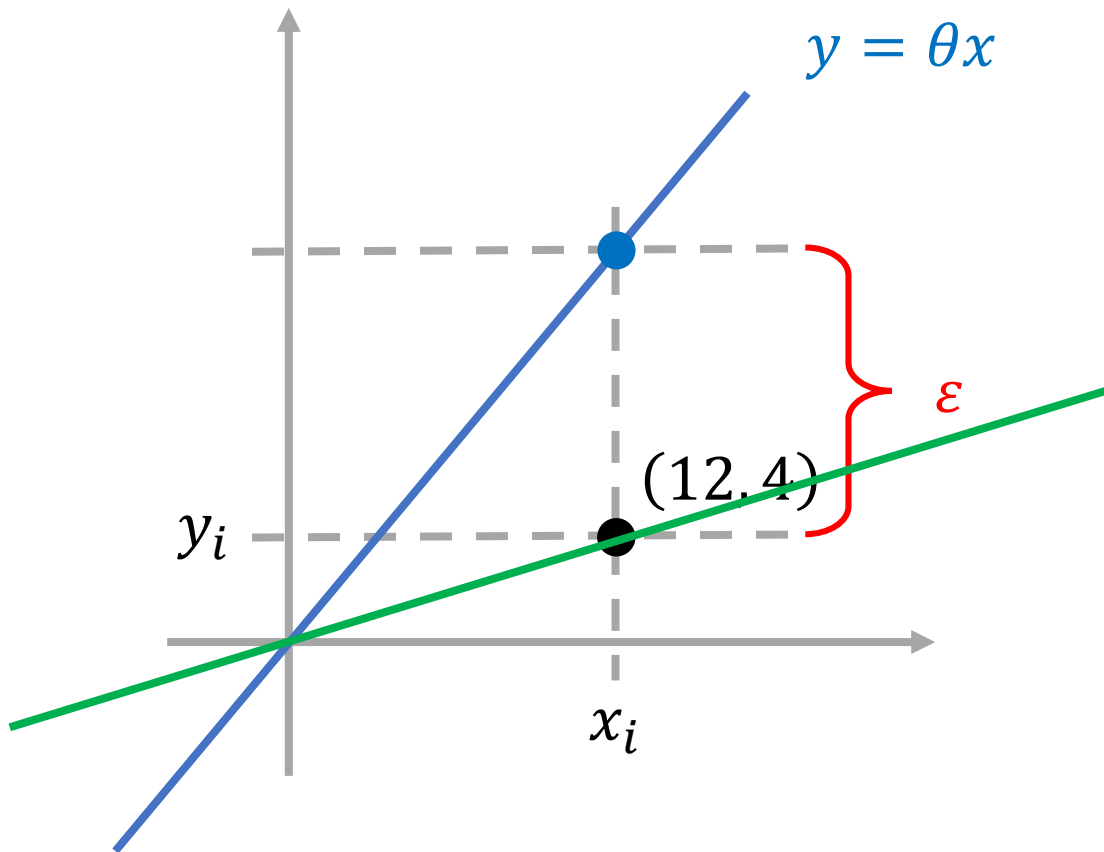
$$\therefore y = \frac{4}{12}x$$

$$\begin{aligned} y &= f(\theta) \\ \epsilon &= g(y) \end{aligned} \quad \Rightarrow \quad \epsilon = h(\theta)$$

$$y = \theta x \quad m = 1$$

>\_ Code

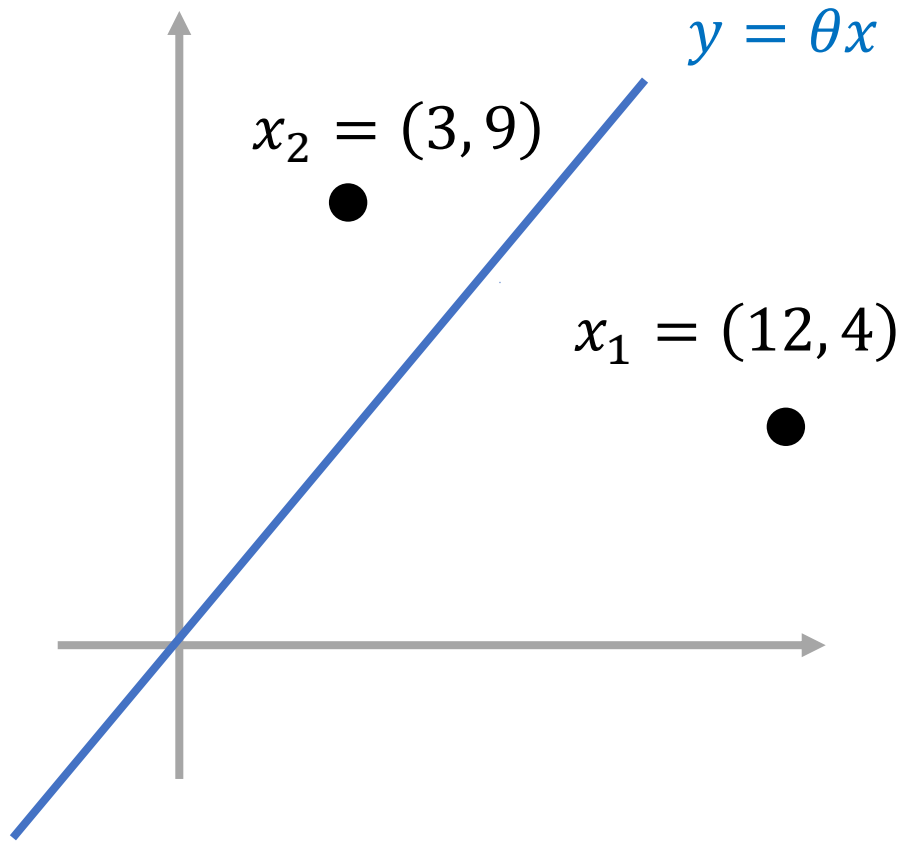
$$\therefore y = \frac{4}{12}x$$



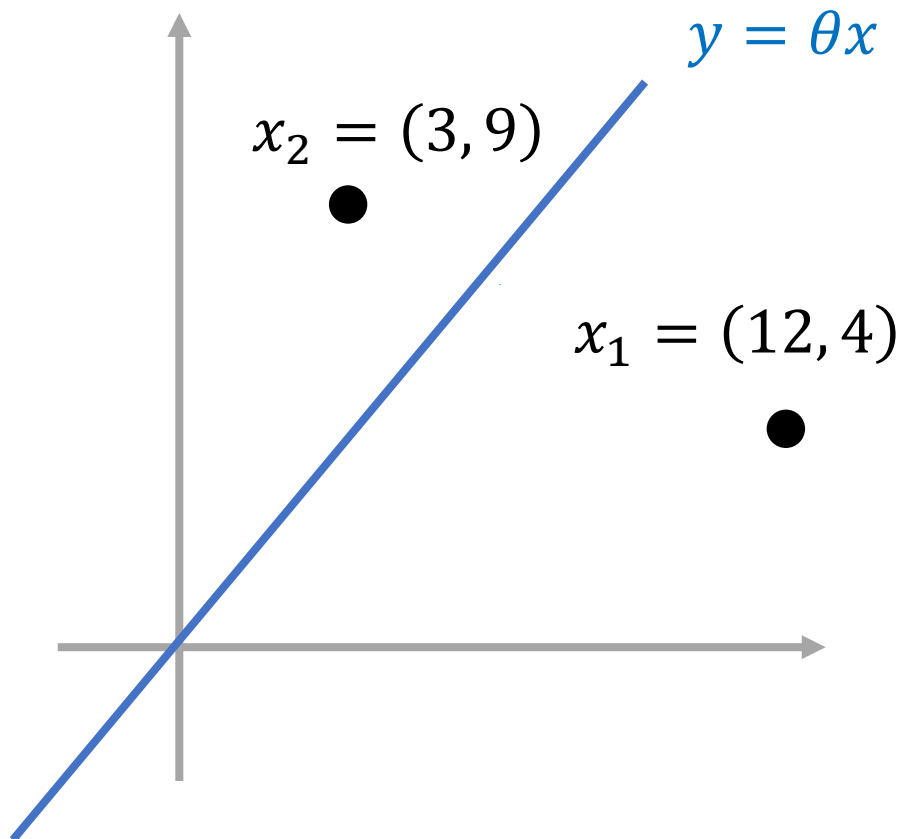
```
import matplotlib.pyplot as plt
xi = 12 # data points o o o o o
yi = 4
theta = yi/xi
plt.scatter(xi,yi)
# model
x = np.linspace(10, 14, num = 5)
y = theta * x
plt.plot(x, y)
plt.axis('equal')
plt.show()
```



$$y = \theta x \quad m = 2$$

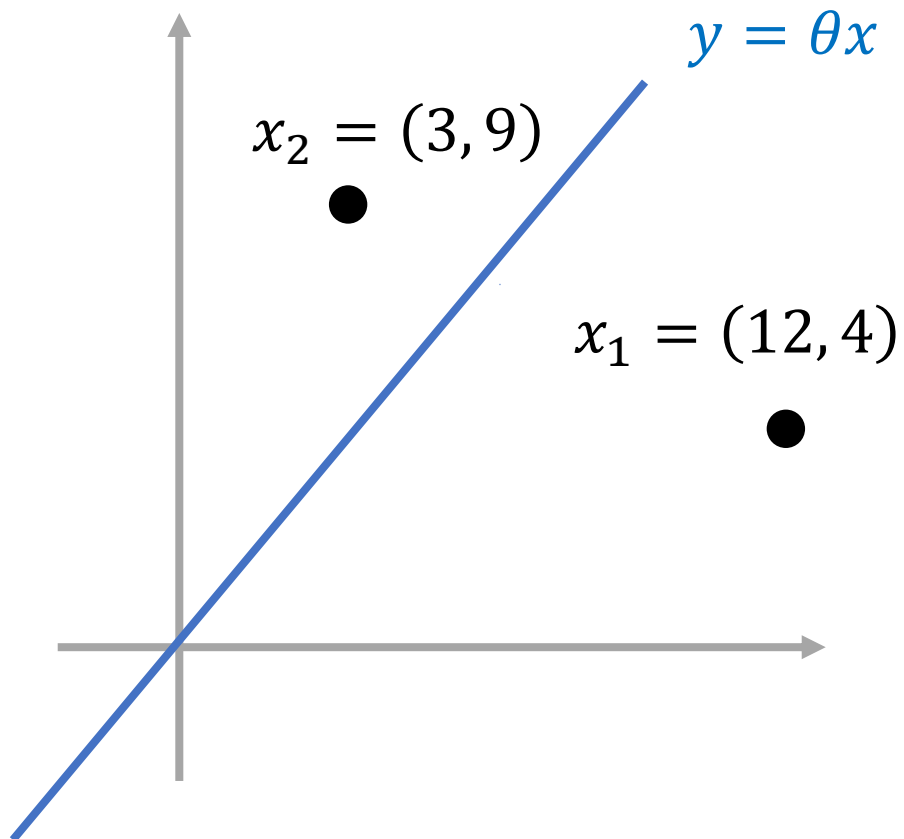


$$y = \theta x \quad m = 2$$



$$\varepsilon = (\theta x_1 - y_1)^2 + (\theta x_2 - y_2)^2$$

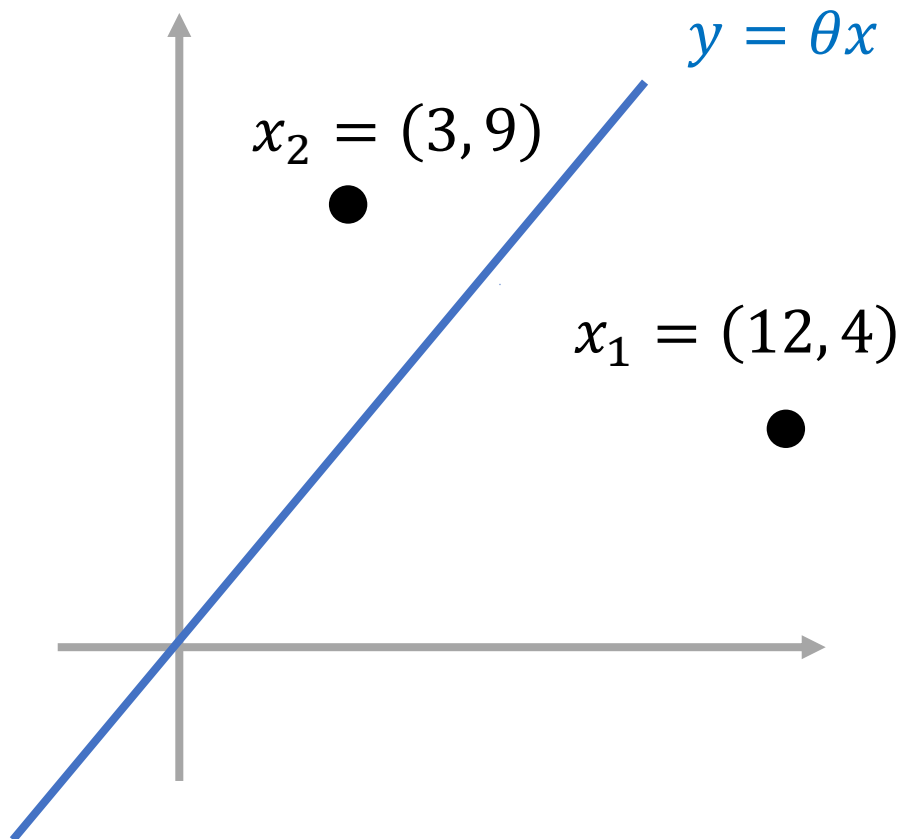
$$y = \theta x \quad m = 2$$



$$\varepsilon = (\theta x_1 - y_1)^2 + (\theta x_2 - y_2)^2$$

$$\frac{d\varepsilon}{d\theta} = 0 \quad \Rightarrow \quad \theta = \frac{x_1 y_1 + x_2 y_2}{x_1^2 + x_2^2}$$

$$y = \theta x \quad m = 2$$



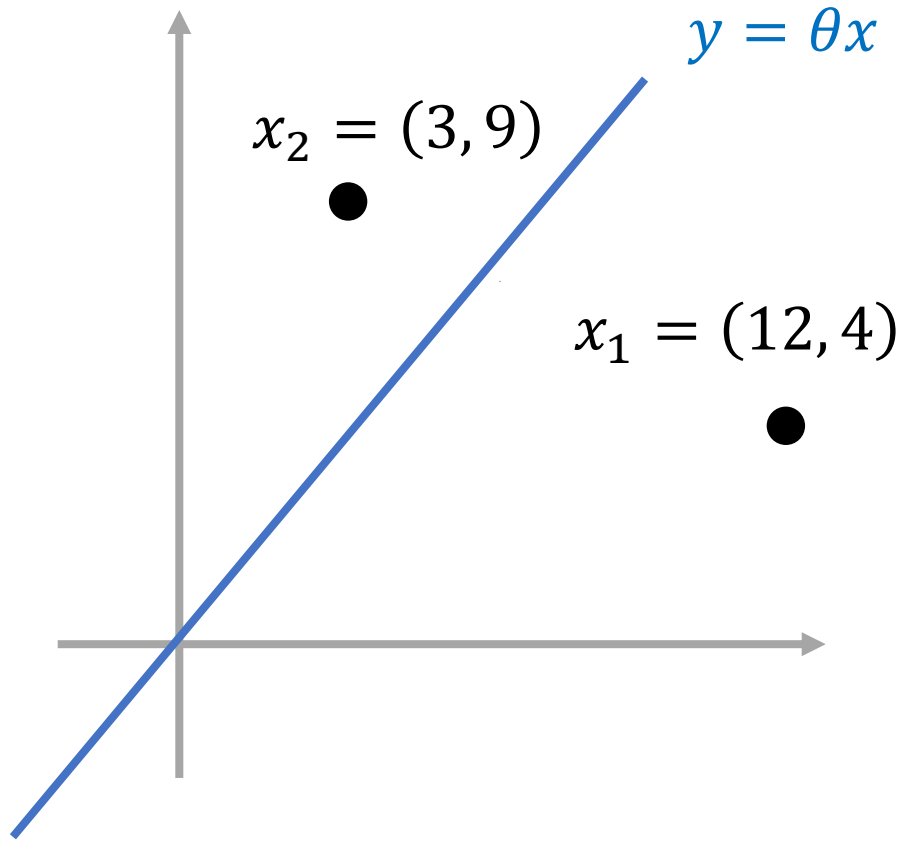
$$\varepsilon = (\theta x_1 - y_1)^2 + (\theta x_2 - y_2)^2$$

$$\frac{d\varepsilon}{d\theta} = 0 \Rightarrow \theta = \frac{x_1 y_1 + x_2 y_2}{x_1^2 + x_2^2}$$

Matrix form for fast vectorized computations

$$\begin{pmatrix} \boxed{\phantom{00}} & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \boxed{\phantom{00}} \end{pmatrix}$$

$$y = \theta x \quad m = 2$$



$$\varepsilon = (\theta x_1 - y_1)^2 + (\theta x_2 - y_2)^2$$

$$\frac{d\varepsilon}{d\theta} = 0 \quad \Rightarrow \quad \theta = \frac{x_1 y_1 + x_2 y_2}{x_1^2 + x_2^2}$$

Matrix form for fast vectorized computations

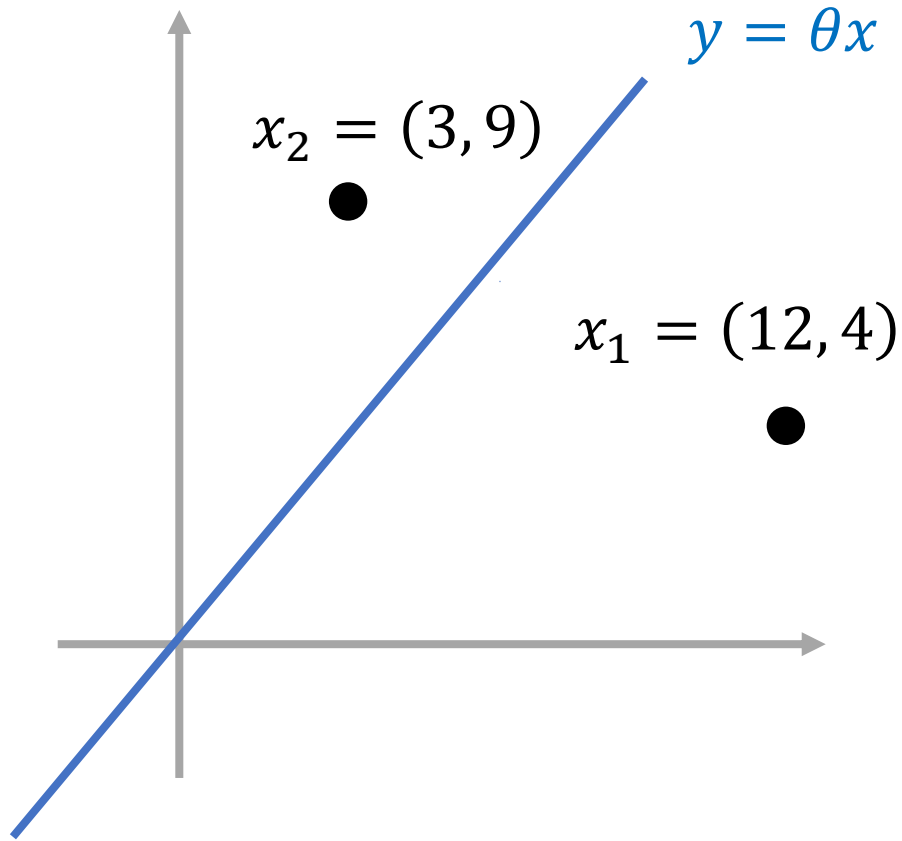
$$\begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix} \quad \mathbf{x} := \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad \mathbf{y} := \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$$\Rightarrow \theta = \frac{(x_1, x_2)^T \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}}{(x_1, x_2)^T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}} = \mathbf{x}^T \mathbf{y} / \mathbf{x}^T \mathbf{x}$$

$$y = \theta x \quad m = 2$$

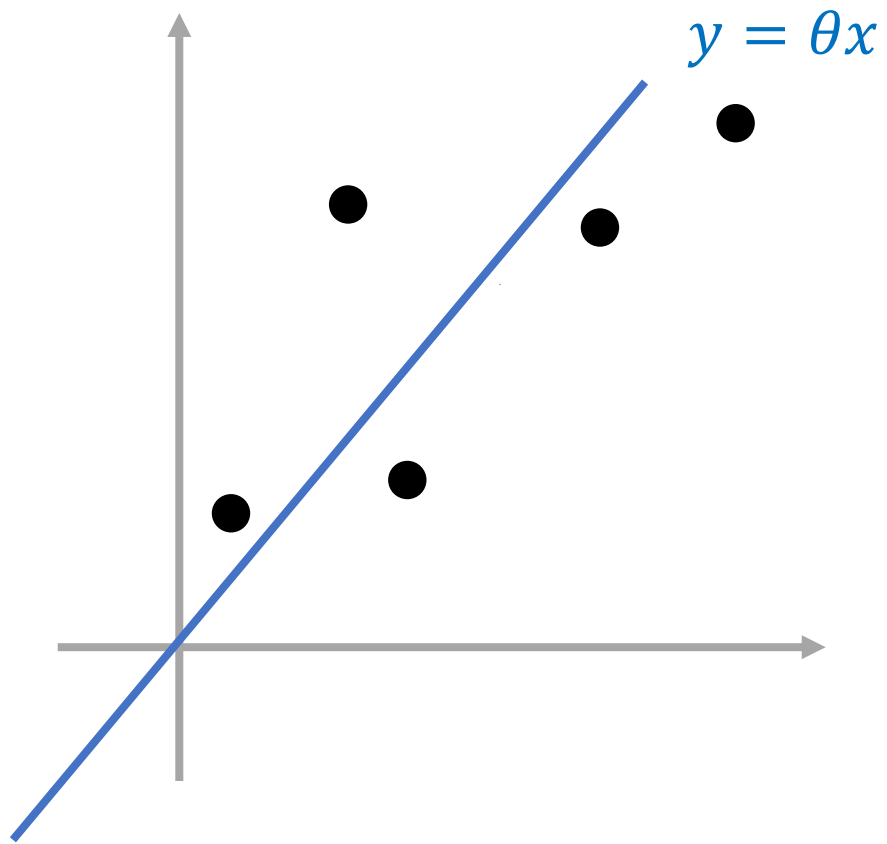
$$\theta = \frac{(x_1, x_2)^T \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}}{(x_1, x_2)^T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}} = \mathbf{x}^T \mathbf{y} / \mathbf{x}^T \mathbf{x}$$

>\_ Code

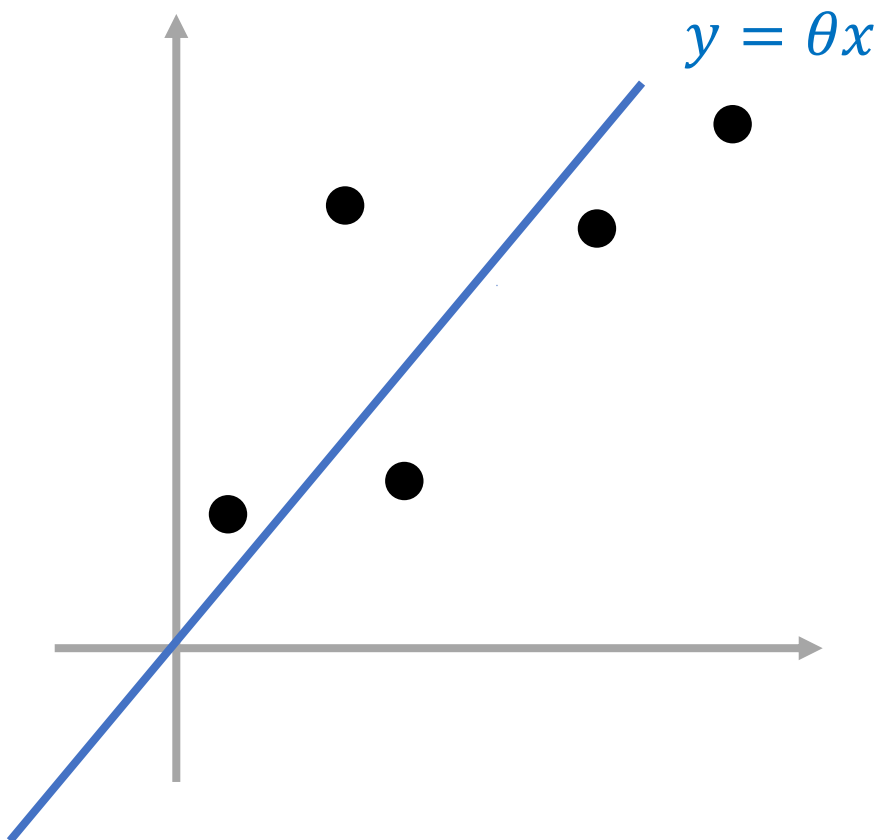


```
import numpy as np
import matplotlib.pyplot as plt
xi = np.array([3, 12]) # data points o o o
yi = np.array([9, 4])
theta = sum(xi * yi) / sum(xi**2)
plt.scatter(xi, yi)
# model
x = np.linspace(min(xi), max(xi), num = 5)
y = theta * x
plt.plot(x, y)
plt.axis('equal')
plt.show()
```

$$y = \theta x \quad m$$



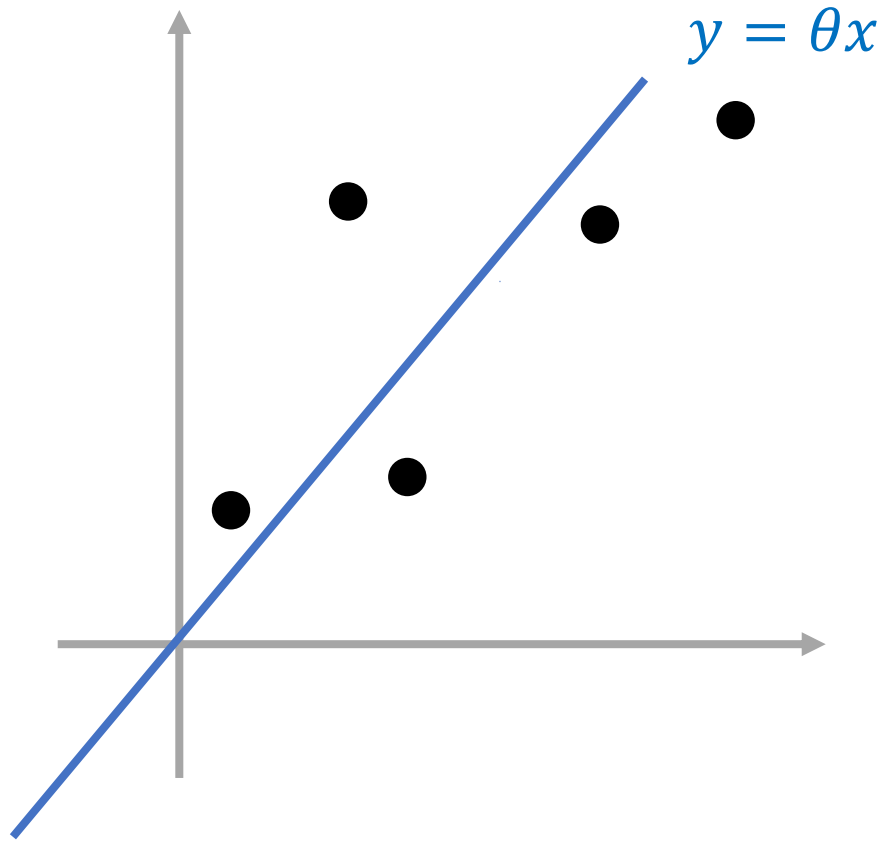
$$y = \theta x \quad m$$



$$\begin{aligned} \varepsilon = & (\theta x_1 - y_1)^2 \\ & + (\theta x_2 - y_2)^2 \\ & \vdots \\ & + (\theta x_m - y_m)^2 \end{aligned}$$



$$y = \theta x \quad m$$



$$\begin{aligned} \varepsilon = & (\theta x_1 - y_1)^2 \\ & + (\theta x_2 - y_2)^2 \\ & \vdots \\ & + (\theta x_m - y_m)^2 \end{aligned}$$

$$\frac{d\varepsilon}{d\theta} = 0 \quad \Rightarrow \quad \theta = \frac{\sum_{i=1}^m x_i y_i}{\sum_{i=1}^m x_i^2}$$

Matrix form for fast vectorized computations

$$\begin{pmatrix} \boxed{\phantom{0}} & \boxed{\phantom{0}} \\ \boxed{\phantom{0}} & \boxed{\phantom{0}} \end{pmatrix} \quad \mathbf{x} := \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix} \quad \mathbf{y} := \begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix}$$

$$\Rightarrow \quad \theta = \mathbf{x}^T \mathbf{y} / \mathbf{x}^T \mathbf{x}$$

# Your turn

1.  $y = \theta x + 5$

2.  $y = A \sin(x)$

