Numerical Methods

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Reflection About a Line

Suppose there are given a point $A = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$ and line $l : y = ax + b$. Now we want to reflex point A about line l and get the point A'. For this first use a transformation to transform line to origin

$$T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y - b \end{bmatrix}$$

then rotate the line to fix it on $x - axis$, by usnig rotate matrix

$$R_\theta = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$$

Since the first rotate is clockwise use $R_{-\alpha}$ where $\alpha = arctan(a)$ then use reflex matrix about $x - axis$ called $X_r = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$,

$$X_r \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ -y \end{bmatrix}$$

and finally R_α, then shift the inverse of the transformation.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = R_\alpha \cdot X_r \cdot R_{-\alpha}(\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ -b \end{bmatrix}) + \begin{bmatrix} 0 \\ b \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = R_\alpha \cdot X_r \cdot \begin{bmatrix} cos\alpha & sin\alpha \\ -sin\alpha & cos\alpha \end{bmatrix} \cdot \begin{bmatrix} x \\ y - b \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} xcos^2\alpha + (y-b)sin\alpha cos\alpha - xsin^2\alpha + (y-b)sin\alpha cos\alpha \\ xsin\alpha cos\alpha + (y-b)sin^2\alpha + xsin\alpha cos\alpha - (y-b)cos^2\alpha \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} xcos2\alpha + (y-b)sin2\alpha \\ xsin2\alpha - (y-b)cos2\alpha \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} cos2\alpha & sin2\alpha \\ sin2\alpha & -cos2\alpha \end{bmatrix} \cdot \begin{bmatrix} x \\ y - b \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix}$$

Express $sin2\alpha, cos2\alpha$ by $tan\alpha$

$$cos2\alpha = \frac{1-a^2}{1+a^2},\ sin2\alpha = \frac{2a}{1+a^2}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x\frac{1-a^2}{1+a^2} + (y-b)\frac{2a}{1+a^2} \\ x\frac{2a}{1+a^2} + (y-b)\frac{a^2-1}{1+a^2} + b \end{bmatrix}$$