400A - Nuclear cycles

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Materials: Onno Pol's lecture notes Chapter 6.1 and 6.2, Kippenhahn book Chapter 9 and 18, Cox & Giuli vol. I, Chapter 17.7, Clayton, Chapter 5, Bethe 1939

Nuclear burning cycles

Heuristic summary and S-factor

In the previous lecture we have developed some idea of the physics involved in thermonuclear reactions. We can have a heuristic derivation of the main results we obtained for a generic charged particle reaction X(a,b)Y by writing the cross section for the reaction as:

$$\sigma = \pi\lambda_X^2 \times \{\text{Probability of Tunneling}\} \times \{\text{Probability of C}^\star \text{ decay in Y} + \text{b}\} \ , \quad (1)$$

where the first term is akin to a geometric cross section using the de Broglie wavelength of the target nucleus X. The product of the two terms in curly brackets appears because of Bohr's approximation (so that the two probabilities are independent of each other). Since $\lambda_\mathrm{X} = \mathrm{h/p} = \mathrm{h/E}^{1/2} \Rightarrow \lambda_\mathrm{X}{}^2 \propto \mathrm{E}^{-1}$ and thus:

$$\sigma \equiv \sigma(E) \propto \frac{1}{E}\exp\left(\frac{-b}{\sqrt{E}}\right) S(E) \ \ , \quad (2)$$

where S(E) is the so called "*astrophysical S-factor*" that contains the intrinsic cross section that depends on the shape of the nuclear potential well. This can be influenced strongly by nuclear resonances (when E $\sim$ energy of a metastable state of $\mathrm{C}^\star$).

In practice, often one relies on laboratory measurements (at high E$\gg$ thermal energy in stars), and extrapolation to low energies to obtain S(E), which is a risky business precisely because of the resonances! In absence of resonances though, one expects S(E) to be only slowly varying (unlike σ(E) which depends on the Coulomb penetration probability).

To obtain a *thermo*-nuclear reaction rate, the cross section above has to be averaged over a Maxwell-Boltzmann distribution. The product between the tunneling probability and the exponential factor of the Maxwell-Boltzmann distribution produces the "Gamow peak" making thermonuclear reaction rates strongly peaked functions of the temperature whether there are resonances (included in the astrophysical factor S(E)) or not.

We already know from the B/A vs. A curve that the most energy is released by hydrogen burning into helium (because α particles have such a high binding energy), and using conservation of charge and leptonic number we have already written the overall reaction as:

$$4p \rightarrow ^4\mathrm{He} + 2e^+ + 2\nu_e \quad . \tag{3}$$

And we know that a fraction ϕ=0.007 of the rest mass of the 4 incoming protons (i.e., hydrogen positive ions) is converted into energy, corresponding to a total energy released is $Q_{\mathrm{H\ burn}}$= 26.5 MeV.

From the overall reaction 3 we know that the conversion of hydrogen nuclei into helium requires two weak reactions to explain the two ν_{e}, necessary to conserve leptonic number. This is needed since we need to produce two positrons e^+ to conserve the charge.

As you may expect from the name the weak interactions are weak, meaning the coupling constant is small compared to the coupling constant for the strong force and electromagnetic force: *weak interactions are the bottleneck for nuclear burning in stars.*

Moreover, since the nucleus of helium (i.e., α particle) is made of 4 nucleons (2p and 2n), the reaction 3 seems to require the encounter of 4 protons (i.e., hydrogen ions) in one point in space (within $r_{\mathrm{nuc}} \sim 10^{-13}$ cm) and with the right relative energies. The statistical probability of this is vanishingly small, and that *cannot* be how the process works in nature, the rate would effectively be zero. Hans Bethe was the person who worked out the way this could actually happen.

The solution to the prohibitively small probability that nature finds in stars is to go through intermediate steps instead of doing the reaction 3 in one single step, in other words, to use *thermonuclear reaction chains.*This theorized solution has been directly confirmed through solar neutrino observations (which also lead to the development of new neutrino physics), and chemical observations across astronomical sources.

N.B.: all the reactions we will write down today can be inferred using the conservation law for baryonic number, leptonic number, and four-momentum (i.e., energy and momentum).

Hydrogen burning

pp chain

The "pp chain" is the dominant hydrogen burning mode in low mass stars, like the Sun:

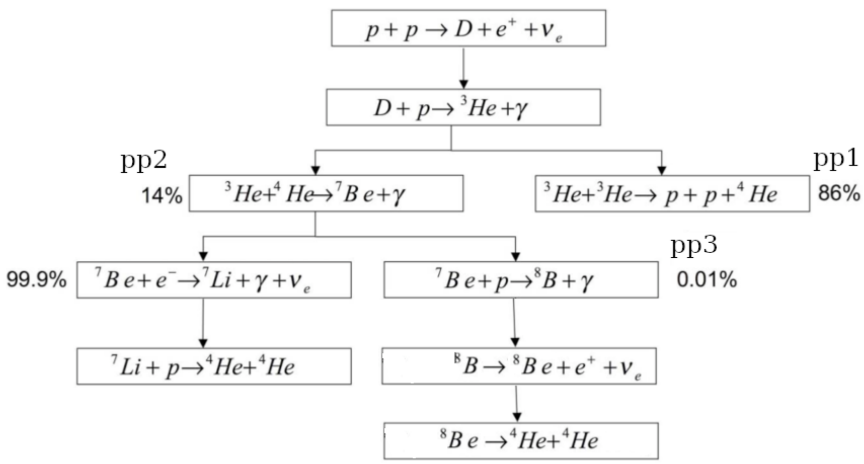

Figure 1: Schematic representation of the pp chain. The right branch is the so called pp1, the rarer left branch is the so called pp2, which further has a rarer pp3 branch ending up with the formation and decay of a ^{8}Be nucleus.

Important thing to notice:

- there are three branches (pp1, pp2 which further branches off into pp3) with different "branching ratios" (86% and 14% for pp1 and pp2, respectively, with 0.01% of pp2 ending in pp3) determined by the resonances in the daughter nuclei $C^{\star}$ which are ^{7}Be, ^{7}Li, and ^{8}B$ respectively here.

- the overall reaction produces two neutrinos ν_{e}, as expected from the overall reaction 3.

- the first reaction, the weak reaction producing Deuterium (D) is the main bottleneck, which can also be understood in terms of the nuclear physics: it requires the interaction of two protons (A=1 $\Rightarrow$ $r_{nuc} = r_0 = 1.5\times10^{-13}$ cm) with a Coulomb barrier.

- the γ particle produced in certain reactions (i.e., photons needed for the conservation of the four-momentum, which are γ rays because of the 1-10MeV energy scale of nuclear interactions) will quickly scatter around off electrons and "thermalize" providing the energy to the star

- viceversa, the ν_{e} leave directly the star. In fact the Earth is constantly bombarded by a flux of ν_{e} from (primarily) the Sun, with a flux of $\sim 10^9$ neutrinos per s^{-1} cm^{-2}.

- the $^3\mathrm{He} + ^3\mathrm{He} \to ^4\mathrm{He} + p + p$ turns two nuclei into 3, this will impact the number of particles per unit baryonic mass (which remains $\sim 6\mathrm{m_u}$ at zeroth-order, neglecting the small fraction ϕ going into energy): this will impact the mean molecular μ and thus the temperature gradient and the mixing!

- note from the pp1 cycle we get two 2 protons out at the end as well (but 6 went in, 4 went into the α particle as expected in the overall reaction 3).

Fitting the temperature dependence for the overall cycle one obtains $\varepsilon_{\mathrm{nuc}} \propto \langle\, \sigma(\mathrm{v})\mathrm{v}\, \rangle \propto \mathrm{T}^4$. This is a fairly steep powerlaw, therefore one should expect the nuclear burning to be *very concentrated* inside the star in temperature coordinate.

CN-NO bi-cycle

When the luminosity of a star is high (recall for a fully radiative star $\mathrm{L} \propto \mathrm{M}^{\mathrm{x}}\mu^4$ with $\mathrm{x} \equiv \mathrm{x(M)} \geq 0$), the bottleneck reaction $\mathrm{p+p} \to \mathrm{D+e^+} + \nu_{\mathrm{e}}$ prevents the stars from reaching $\mathrm{L{=}L_{nuc}}$ through the pp chain: its rate is too low. Another mechanism to achieve the overall reaction 3 is needed.

Therefore, for masses larger than a certain threshold, hydrogen core burning occurs through the CN-NO bi-cycle, where the "trick" to bypass the bottleneck of the weak reaction in the pp chain is to use metal ions as catalysts for the reactions.

Important things to notice:

- C, N, and O are not *destroyed* here, they just act as nuclear catalyst. Protons momentarily "stick" to them, and then α particles are produced re-making the original C, N, and O

- the bottleneck of the weak reactions is now bypassed by forming an unstable isotope of a metal and having that decay: in a sense *massive*

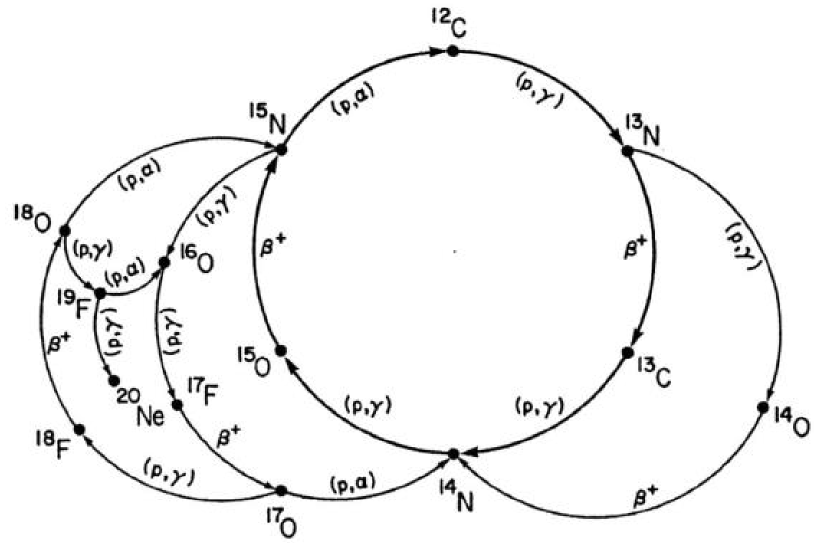

Figure 2: Schematic representation of the CN-NO bi cycle.

stars hack nuclear physics to make the weak reaction not happen in an isolated proton+proton reaction which is hard, but within nuclei.

- There are really two main cycles, the CN cycle and the NO (the figure shows also minor branching out points). The second involves oxygen, which has a higher Z and slightly higher Coulomb barriers, so it kicks in at slightly higher temperatures, but typically a star going through the CN cycle will also do the NO cycle, and they are often referred to jointly as CNO cycle.

- ^{14}N is a stable isotope (N=Z=A/2), and the $\$^{14}N\$$+p reaction is the bottleneck of the bi-cycle: since at equilibrium all these reactions occur at the same rate set by the slowest reaction, this means that C and O are converted into ^{14}N in the core of massive stars.

Because of the higher Coulomb barriers involved one can expect a steeper temperature dependence of this hydrogen burning mode, and in fact one finds for the CNO cycle $\varepsilon_{\rm nuc} \propto \mathrm{T}^{16}$.

This very steep energy dependence produces a very large temperature gradient in the core of massive stars (by releasing energy in a very small region in mass): the cores of stars burning through the CNO cycle are **convective**.

N.B.: This is also how hydrogen burns in a shell or at the surface of a white dwarf during a nova explosion: in those cases the temperature of the gas is set by the structure of the star independently of the requirement of thermonuclear burning! For novae explosions, the burning is not even hydrostatic: can have the "hot CNO" cycle, where reaction rates do *not* need to wait for β-decays to occur. This is a general feature of explosive burning: in that case we don't need to wait for the slowest nuclear reaction in the cycle, as the high temperatures can allow to bypass Coulomb barriers more easily.

- **Q**: How did population III stars do this? This is the topic of the honors project!

pp → CNO transition

Because of the higher Coulomb barriers involved in the CNO cycle (i.e. the higher charge of ^{12}C), it has a more sensitive temperature dependence:

- $\varepsilon_{\mathrm{nuc,\ pp}} \propto \mathrm{T}^4$
- $\varepsilon_{\mathrm{nuc,\ CNO}} \propto \mathrm{T}^{16}$

However, the proportionality constant is larger for the pp cycle (see figure), therefore, for stars with lower mass M, that is lower $\langle$ T $\rangle$ by the virial theorem, hydrogen burning through the pp-chain will dominate. However, increasing M (and thus $\langle$ T $\rangle$), at some point the CNO cycle takes over as dominant energy production mechanism.

Because of the steepness of the $\varepsilon_{\mathrm{nuc,\ CNO}}$ we expect that at higher initial total mass M the core will be *convective* (recall that if $\nabla_{\mathrm{rad}} \propto \kappa\ \mathrm{L} > \nabla_{\mathrm{ad}}$ we expect convection), while if M is sufficiently low that the pp chain dominates, the core is stable against convection and remains radiative. This is something that is *indirectly* confirmed by studying the eigenfrequencies of stars that can be observed as pulsational frequencies (i.e., asteroseismology), stellar lifetimes inferred from population analyses, etc.

The initial mass M at which the transition happens needs to be determined with stellar models and is around $\mathrm{M}_{\mathrm{pp} \rightarrow \mathrm{CNO}} \sim$ 1.1-1.2$\mathrm{M}_\odot$, where the imprecision comes from the systematic uncertainties in stellar models (in terms of input physics and algorithmic representation of the processes in the code, that we will discuss in more detail in the next lecture). To some extent, this is a physically motivated threshold to *define* what a "massive" star is (for example this is typical in asteroseismology context), though other

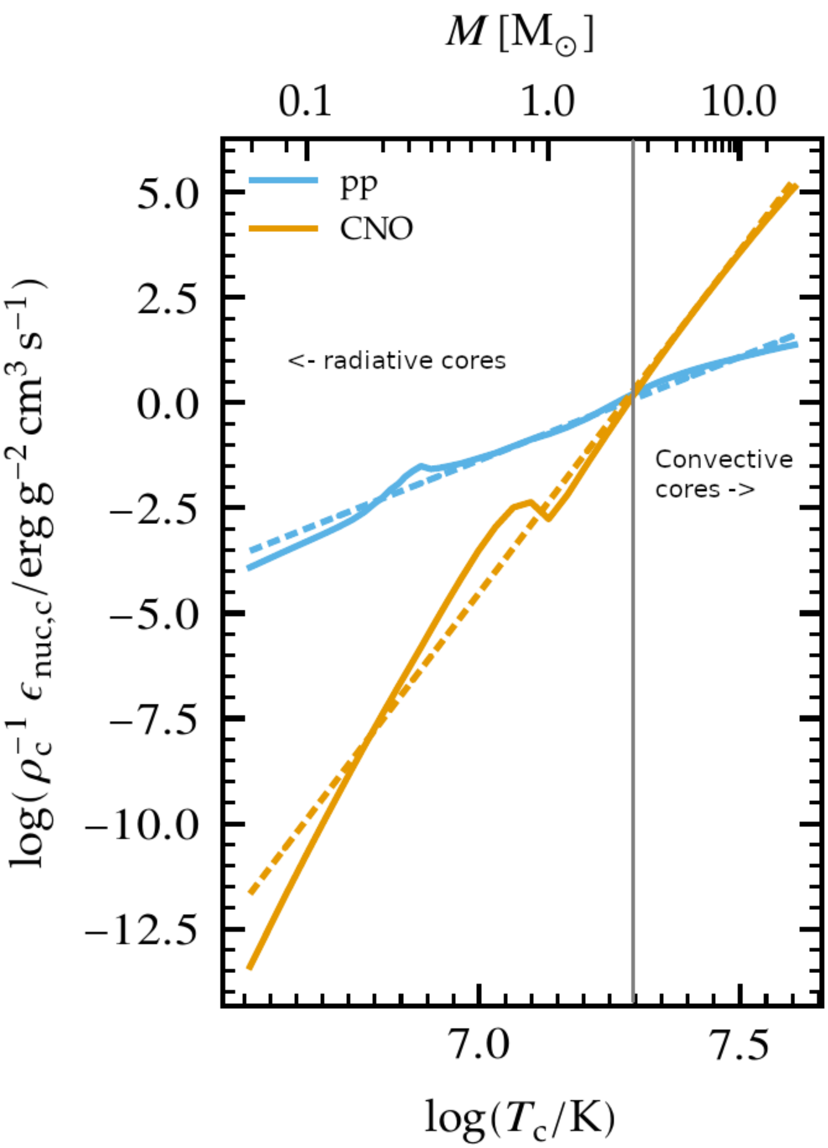

Figure 3: ε_{nuc}/ρ as a function of central temperature T_c (bottom axis) or initial mass (top axis) for the pp chain (blue) and CNO cycle (orange) based on a grid of MESA models at the onset of hydrogen core burning. Dashed lines are linear fits. Modified from a figure by R. Townsend.

meaningful threshold exist: the term "massive star" is typically context dependent.

Helium burning

Once hydrogen fuel runs out in the core of a star (i.e., anywhere with mass coordinate $m \leq f_{burn} M_{tot}$), the next fuel to burn is going to be helium. First of all, this is always available since it was produced in the Big Bang and also *locally brewed* by the burning of hydrogen that just ended, and secondly, it's the energetically second-best fuel in terms of energy release per nucleon.

However, there is a nuclear physics problem: there are no stable nuclei with A=5 and A=8 and the few A=7 produced during hydrogen burning are very fragile and easy to break, so they don't survive the hydrogen burning phase either. So how can one do thermonuclear reactions of helium?

This was solved by Fred Hoyle, who predicted that there would be a metastable state of carbon (as a $C^\star$ compound nucleus in Bohr's approximation) that would allow to consume helium, using astrophysics to understand nuclear structure!

"Triple α" reaction

The easiest way to consume helium is through the so called "tiple α reaction", which is actually a *compound* reaction with a very short lived intermediate state (**N.B.:** the probability of 3 alpha particles to meet in the same place within their nuclear radii because of thermal motion is negligibly small). The $3\alpha \rightarrow^{12}$ C compound reaction is actually: $\alpha + \alpha \leftrightarrow^{8}$ Be $+ \gamma$
^{8}Be $+ \alpha \rightarrow^{12}$ C$^\star \rightarrow^{12}$ C $+ \gamma$

As mentioned above, the A=8 nucleus of Beryllium is unstable, and decays with a half life of $\tau_{Be} \sim 8\times10^{-17}$ seconds (**N.B.:** this is still much longer than the light crossing time $\sim 2\times10^{-23}$sec, so Bohr's approximation holds!). Therefore, the first reaction can go both ways, with the beryllium decaying back in two α particles. However, if the temperature is such that *before* it can decay, a third α particle will interact with the beryllium, then one can consume three nuclei of helium to make a carbon. This requires $T \sim 10^8$ K $\gg T_{center,\odot} \simeq 10^7 \sim T_{H\ burn}$.

The insight of F. Hoyle was that this would happen, and for this to happen there *needs* to be an excited (actually doubly-excited) state of carbon 12 which was theoretically predicted from the astrophysical evidence that carbon is the next most abundant element after H and He: stellar observations were used to correctly predict something on the structure of nuclei!

N.B.: In practice ^{8}Be is so short lived that often it is not included in stellar evolution simulation and what we use is a reaction rate for the *compound* 3α process with a temperature dependence that makes the rate non-zero only for $T \geq 10^{8}$K allowing $\alpha+^{8}$Be to occur. This allows to not have to track the isotope of ^{8}Be saving computational time.

The nuclear astrophysics "holy grail": $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$

Once some carbon is produced, a new avenue for consuming α particles opens up, the (in)famous $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$ reaction. This still consumes α, but of course requires overcoming a higher Coulomb barrier ($\propto \mathrm{Z_c Z_{He}} > \mathrm{Z_{He}}^2$): it becomes convenient for the star when α particles are getting scarse towards the end of helium core burning.

This reaction regulates the C/O ratio in stars, and ultimately in the Universe, clearly something of interest for biology (among other things). However, its rate is notoriously uncertain, because of the possible presence of unknown resonances in the compound nucleus. In fact, here elements are getting sufficiently heavy that the energy gap between laboratory experiments and the stellar conditions grows and extrapolation of the astrophysical S-factor gets more and more uncertain.

This reaction ultimately regulates the composition of the most common white dwarfs, and also the masses of the most massive black holes, and it is actively studied in laboratories *and* in stellar context (see for example Fields et al. 2016 on the structure and composition of white dwarfs, Farmer et al. 2020 on black hole masses, Shen et al. 2023 for the most recent lab measurements).

Heavier burning

Once helium fuel is exhausted, helium core burning cannot provide the energy to compensate for the losses, and the core of the star contracts further, resulting in an increase in temperature until the next fuel can ignite.

Because the end of helium core burning occurs through the $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$ reaction, the composition of the core is now a mixture of carbon and oxygen (plus some primordial metals there since formation, which have so far been untouched by nuclear processes, except maybe conversion of some primordial C and O into N by the CNO cycle). The next fuel is thus carbon, because it has the lowest Coulomb barrier.

N.B.: Recall that not all stars need to do all burning phases possible: if electron degeneracy kicks in before $\mathrm{T_{center}}$ is high enough to start reactions, the gravothermal collapse imposed by the virial theorem interrupts because of the EOS. Since we also know from the virial theorem that $\langle$ T $\rangle \propto$ M, we know that lower mass stars are on average cooler and will stop burning earlier.

Carbon burning

The carbon+carbon reaction can have several branching ratios with positive Q values (thus exo-energetic and of interest for the star to sustain itself):

$^{12}\mathrm{C} +^{12}\mathrm{C} \rightarrow^{24}\mathrm{Mg} + \gamma$ (Q$\sim$13.93MeV)
$^{12}\mathrm{C} +^{12}\mathrm{C} \rightarrow^{20}\mathrm{Ne} + \alpha$ (Q$\sim$4.616MeV)
$^{12}\mathrm{C} +^{12}\mathrm{C} \rightarrow^{23}\mathrm{Na} + p$ (Q$\sim$2.238MeV)
...

These require T$\geq 10^{8.5}$ K, and produce a mixture of Neon, Magnesium (**N.B.:** an α-nucleus!) and Sodium.

N.B.: the α particle released by the second reaction listed, and the proton released by the third will immediately start reacting with the other particles present at the temperatures necessary for $^{12}\mathrm{C}+^{12}\mathrm{C}$ to be activated. Actually at these T, the reactions rates for reactions involving these light particles are going to be extremely high!

Neon ignition and nuclear "magic numbers"

Because of carbon burning, a significant amount of neon is produced. Although neon has A=20 and Z=10 so it is heavier and more charged than oxygen (produced by the end of helium core burning and still untouched by carbon burning at this point), it will ignite before.

The reason has to do with a nuclear physics property: one can make a direct analogy between nuclei and atoms, and like atoms of "noble gases" which have electrons in all their shells are very stable and don't like to do chemical interactions with other atoms/molecules, nuclei with nucleons filling all their shells are also particularly stable. This gives the nuclear "magic numbers" of nucleons that are particularly stable: Z or N = 2, 8, 20, 28, 52 (and there are higher numbers theoretically predicted from nuclear structure calculations).

^{16}O has Z=8 and N=8, so it is a "double magic nucleus", that from the nuclear interaction perspective is like a noble gas from the chemical perspective: it is extremely stable and does not want to interact.

N.B.: α particles have Z=N=2 and are also a "double magic" nucleus, and in fact they have an extremely high binding energy per nucleon! ^{16}O is the second-lightest double magic nucleus.

Therefore, the gravothermal collapse reaches temperatures sufficient to *photodisintegrate* the heavier (but not magic) nucleus of ^{20}Ne before oxygen burns: photodisintegration reactions are in fact not affected by the Coulomb barriers. This photodisintegration produce α particles and protons that stick to the existing nuclei changing the composition typically increasing the mass fractions of α-nuclei (^{16}O and ^{24}Mg, primarily). While unimportant for the energy generation, secondary reactions producing ^{22}Ne can be important for the synthesis of elements heavier than iron through the s-process.

However, the fact that the photodisintegration of neon is easy relative to oxygen ignition is a direct consequence that the nucleus of neon is not very bound (compared to oxygen, or carbon) and its burning does not release very much energy, resulting in a brief phase of stellar evolution only.

Oxygen

As T further increases because of the gravothermal collapse the oxygen finally ignites. The dominant reaction has multiple branching ratios with positive Q value that produce sulfur and silicon primarily:

$^{16}\mathrm{O} +^{16}\mathrm{O} \rightarrow^{32}\mathrm{S} + \gamma$ (Q∼16.53MeV)
$^{16}\mathrm{O} +^{16}\mathrm{O} \rightarrow^{28}\mathrm{Si} + \alpha$ (Q∼9.593MeV)
$^{16}\mathrm{O} +^{16}\mathrm{O} \rightarrow^{31}\mathrm{P} + p$ (Q∼7.676MeV)
...

N.B.: as before all the light produced will immediately react with the present mixture at this temperature!

Silicon core burning

Finally, if a star has reached this point, gravity will compress its core until it burns all the way to the most bound nucleus (iron/nickel): stopping somehow the gravothermal collapse at this point would require extreme fine-tuning, and by now the core density is so high that the gravothermal collapse (of the core) is driven by neutrino emission (as we will discuss in the next lecture) rather than photon losses at the surface.

Post core oxygen burning, the core is made of a mixture of silicon and sulfur (**N.B.:** this is now very hard to probe directly observationally because of the very short evolutionary timescales, but it matches well the nuclear data and abundance patterns in the Universe!).

The next burning phase is typically referred to as "silicon burning", although it physically proceeds in a slightly different fashion than all the burning phases we have seen so far.

It typically requires T$\sim$2-5$\times 10^9$ K and densities $\rho \sim 10^7$-10^{10} g cm^{-3} and only lasts order of $\tau_{\mathrm{nuc,\ Si}}\sim$ days-weeks since the energy release per nucleon is *only* 0.1MeV/nucleon (cf. $\sim$ Q$_{\mathrm{H\text{-}burn}}/4\simeq$6.6 MeV/nucleon for H burning!).

At such temperatures, we reach a "quasi statistical equilibrium" between nuclei: the silicon, sulfur, and other elements (referred to as "silicon group elements" with A$\sim$28 and Z$\sim$14) are photodisintegrated and re-created at very high and nearly canceling rates:

$\gamma +^A Z \rightarrow^{A'} Z' + p, n, \alpha$

This produces also a variety of light particles (protons, neutrons, and α particles), which can be captured on the silicon group elements to form heavier "iron group nuclei" (which are also photodisintegrated and recreated constantly):

$\{p, n, \alpha\} + \{^A Z, ^{A'} Z'\} \rightarrow \{\text{Fe group nuclei}\} + \dots$

Moreover, many $^{A'}Z'$ nuclei produced by photo-disintegrations and particles captures are neutron or proton rich, therefore a lot of weak reaction such as $\beta^{\pm}$-decays and electron captures happen too (while positron captures are always negligible for stars with M $\leq$ 40 M$_{\odot}$, and positrons prefer to annihilate with an electron producing γ rays that quickly thermalize in the plasma see Arnett et al. 1977).

This process is computationally very challenging, since there are many forward and reverse reactions happening at very high rates but canceling each other out, resulting in a very *stiff* set of equations to solve for the evolution of the chemical composition. In this situation, the truncation errors in the floating point algebra of computers can easily become problematic.

The rates are so high that the Quasi Statistical Equilibrium (QSE) regime is achieved: two distinct groups of isotopes in equilibrium are formed around silicon and iron and only few reactions linking the two groups are out of balance with their reverse.

Within each "equilibrium group", the abundances of each isotope stay roughly constant, because production and destruction reactions involving only isotopes of that group cancel out almost exactly. This means that within each group, Nuclear Statistical Equilibrium (NSE) is reached, an assumption that can simplify the calculations that we will use in atomic (rather than nuclear) context later.

Note however that *weak reaction are never balanced by their reverse reaction*: the cross section for neutrino captures is too small at this stage. Strong and electromagnetic mediated nuclear reactions need to compensate also the weak reactions for the isotopes that can β-decay or capture electrons. Therefore this is not a true statistical equilibrium regime, and the "principle of

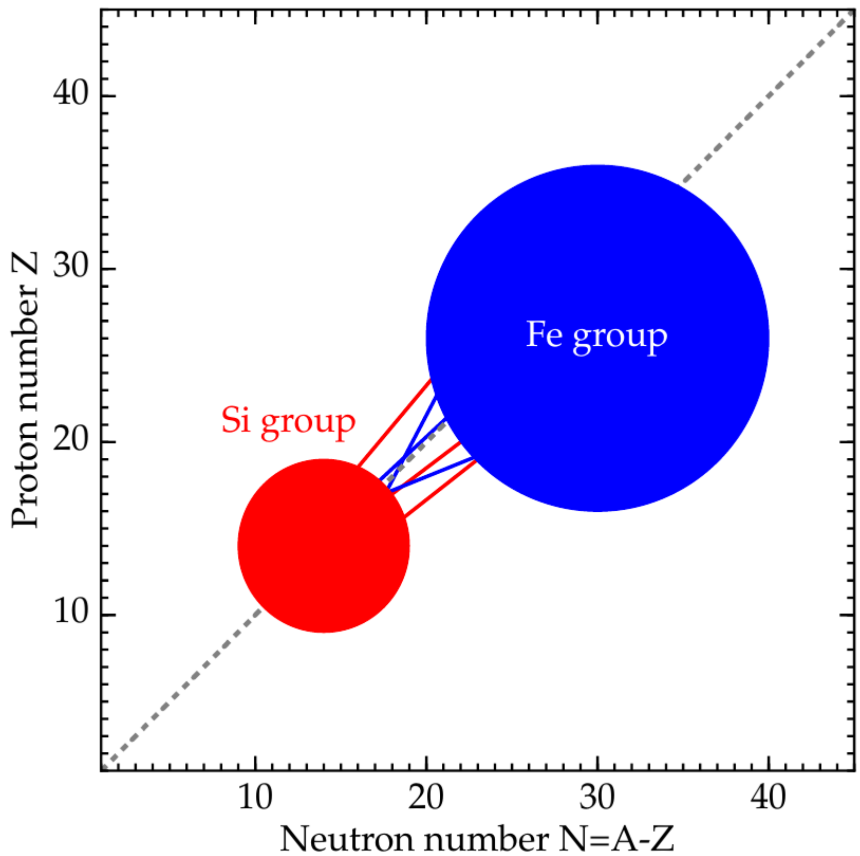

Figure 4: Schematic representation of quasi statistical equilibrium on the nuclear chart. The two filled circle represent the Si (red) and Fe (blue) groups. The abundance of nuclei within each group reach NSE. The links connecting specific isotopes within each group represent the few reactions out of equilibrium, which progressively result in the depletion of the number of isotopes in the Si group in favor of those in the Fe group.

detailed balance" does not hold strictly. Some widely used stellar evolution codes therefore do not rely on the approximation of "quasi equilibrium" and instead calculate directly all the reactions.

This process ends with the formation of an Iron core, made of nuclei at the maximum of the B/A vs. A curve that the star cannot burn to sustain itself: at this point gravity wins, and we get a core-collapse event resulting in (possibly) a supernova explosion and the formation of a neutron star or a black hole, which we will discuss in a future lecture.

Summary of energy scaling

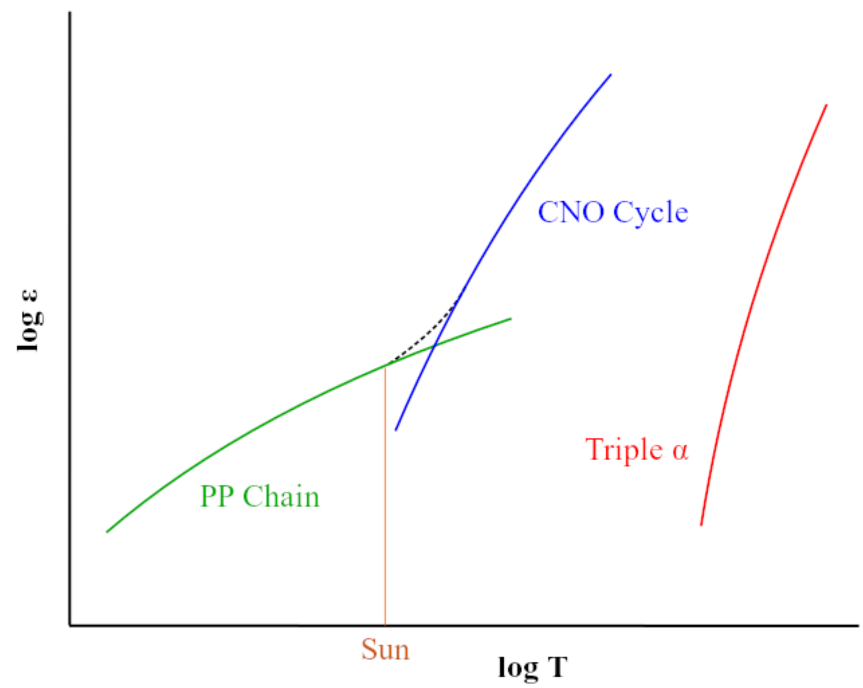

Figure 5: $\varepsilon_{\rm nuc} \equiv \varepsilon_{\rm nuc}(\rm T)$ dependence on a log-log plot for the burning cycles that cover $\geq 99\%$ of the stellar lifetime (H and He core burning). From wikipedia.

N.B.: The central temperature of the Sun correspond to the higher range where the pp chain dominates the energy production. This means that in the Sun we expect some CNO burning at a sub-dominant level, and indeed $\nu_{\rm e}$

from the decay of ^{13}N have been experimentally detected (one can recognize them from their spectrum), see Borexino collaboration 2020.

Because of the higher and higher Coulomb barriers, the temperature dependence of $\varepsilon_{\rm nuc}$ gets steeper for heavier nuclear fuel, so post-helium core burning $\varepsilon_{\rm nuc}$ is more and more concentrated towards the center, leaving the outer layers unburned and allowing for the so called "onion layer" structure of stars:

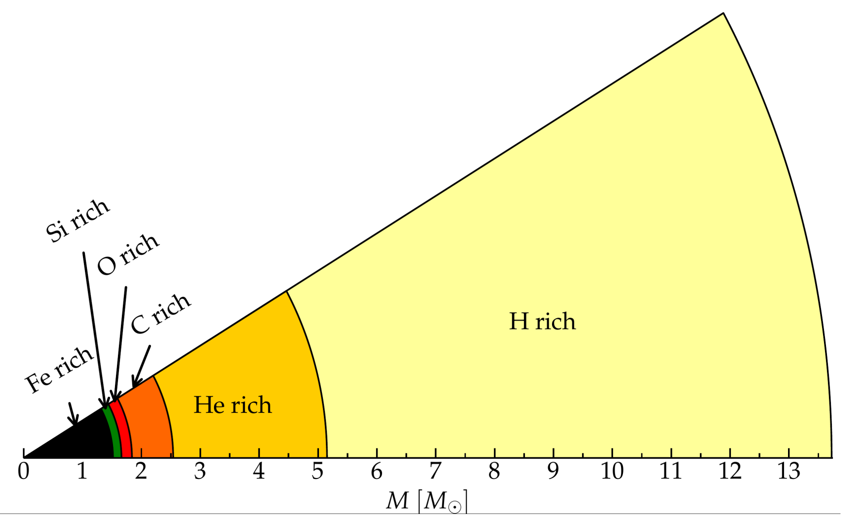

Figure 6: Layer structure for an initially $15\mathrm{M}_\odot$ star at the end of its nuclear burning lifetime in mass coordinate from Renzo 2015. Note the final total mass is *not* $15\mathrm{M}_\odot$ because stars this massive lose mass through radiatively driven winds.

How much burns, that is, how "thick" in mass coordinate is each layer of the onion, depends on the mixing processes connecting the burning layer with the fuel reservoir. At the outer edge of each shell of the "onion" there can be (and typically there is) an off-center burning region, so called "shell burning". The outer layer of the He core/inner layer of the H envelope (the exact boundary depends on the definition one adopts) is sufficiently hot to burn the remaining fuel there, and this burning sustains the outer layer above it. Because of the presence of burning shells, the core burning only needs to sustain the matter inside the shell, which slightly complicates the *gravothermal* argument we have used.

Each layer of heavier material requires a higher T to burn (to have a non-zero tunneling probability), so it is more centralized in mass and radius.

However, moving inward $\varepsilon_{\rm nuc}$ depends more steeply on T, which implies the dT/dr becomes steeper, and thus we have convection, the extent of which determines where fuel is depleted. At high masses (M$\geq$20M$_\odot$), the inner layers start cooling through neutrinos, which may take away enough energy to prevent convection from occurring (**N.B.:** convection kicks in only if needed to transport the energy flux).

The interplay between convection, neutrino cooling, and nuclear burning ultimately decides the core structure of massive stars at the end of their life, and whether they form a neutron star or a black hole, in ways that are still poorly understood (see for example Sukhbold & Woosley 2014, Laplace et al. 2024).

Duration of each burning phase

As we have seen (cf. virial theorem lecture) stars shine as anything with a finite temperature does. This eats at their internal thermal energy, which by the virial theorem is related to their gravitational potential and drives a decrease in the radius. All this happens (by definition) at the Kelvin-Helmholtz timescale if one assumes constant luminosity.

However, since the virial theorem implies $\langle$ T $\rangle \propto$ R^{-1} as the star loses energy at the surface, it's average temperature must increase: this is often phrased by saying that self-gravitating bodies have a *negative heat capacity*. It is because of this temperature increase that nuclear burning must kick in: *stars don't shine because they burn, viceversa, they burn because they shine.*

Therefore, thermonuclear burning in stars exists only to compensate the energy losses (to photons at the surface and to neutrinos throughout the volume that can emit neutrinos), and at equilibrium L$_{\rm nuc}\equiv$ L making the nuclear burning a self-regulating process.

Under the assumption that L$_{\rm nuc}$ = L, we can ask how long does the consumption of a given fuel take in a star, that is the nuclear timescale for a given fuel:

$$\tau_{\rm nuc} = \varphi f_{\rm burn}\frac{Mc^2}{L} \quad . \tag{4}$$

As we saw in the the previous lecture, f$_{\rm burn}$ is a quantity that requires computing full stellar evolution models (we now have all the equations to do so under the classic approximations, it's just a matter to tell a computer how to solve them!), but clearly 0 < f$_{\rm burn}$<1. For a give star of mass M, the important factor here is ϕ, which we can estimate from the nuclear binding energy per nucleon.

Hydrogen burning into helium releases a lot of energy (26.5MeV/4 protons $\sim$ 6.625 MeV/nucleon) because if forms one of the most bound nuclei in nature, the α particle (which is also a double-magic nucleus!). This large energy release, means a large ϕ and long nuclear burning timescale. In fact, pretty much for any star, *hydrogen core burning covers ~90% of the stellar lifetime*. This independently on whether H burning happens through the pp chain or CNO cycle. For this reason, while the "main sequence" is technically an observationally defined feature on the color-magnitude diagram, it is common to refer to hydrogen core burning models as "main sequence" models: observed stars on the observed main sequence are so numerous because they are in the by far longest phase of their evolution, during which they are burning hydrogen in their core.

The burning of Helium into a mixture of carbon and oxygen is the second most energetic burning: this is the reason why it occurs once a star runs out of hydrogen fuel. Again, we get a high ϕ factor because helium fusion climbs the B/A vs. A curve, and helium core burning usually last 10% of the hydrogen core burning time ($\phi_{\mathrm{He}} \simeq 0.1\ \phi_{\mathrm{H}}$), so *hydrogen and helium core burning together cover 99% of the stellar lifetime*.

All the other burning phases *collectively* are only $\leq 1\%$ of a star lifetime! Although they are *crucial* for the chemical evolution of the Universe, they are only a "blip" in the lifetime of the stars, because B/A vs. A roughly flattens (and f_{burn} also decreases): each subsequent fuel produces less and less energy per barion, thus the burning must be faster and faster to compensate for the surface losses (and L also typically increases). This also makes these phases more rare to observe and thus harder to study.

For instance, the timescale for the last possible burning phase in massive stars, silicon $\rightarrow$ iron only lasts order of *days*.

Energetically unimportant but observationally puzzling Lithium

Lithium is a relatively rare element which has a low nuclear binding energy and is thus easily broken without releasing much energy. Therefore, lithium burning is never energetically important.

N.B.: similarly deuterium (D=hydrogen with an extra neutron) is very loosely bound and its burning is not energetically important, to the point that both can happen in sub-stellar mass objects such as brown-dwarfs

Table 1.1: Indicative duration of several core burning phases for different initial mass M_{ZAMS}. I consider each phase to begin at the end of the previous one (i.e. the shell burning/inert core phase duration is included in the next burning phase) and to end when the abundance of all the isotopes of the burning specie drops below 0.01. The data come from MESA (see §2.1 and references therein) simulations using a 21-isotope nuclear network (`approx21.net`). See also §1.3.2, and §B for more details.

Core Burning	duration [yrs]			
M_{ZAMS} $[M_\odot]$	15	20	25	30
H	$\sim 1.29\times10^{7}$	$\sim 8.93\times10^{6}$	$\sim 7.05\times10^{6}$	$\sim 5.98\times10^{6}$
He	$\sim 1.18\times10^{6}$	$\sim 8.63\times10^{5}$	$\sim 7.00\times10^{5}$	$\sim 6.06\times10^{5}$
C	$\sim 4.04\times10^{4}$	$\sim 2.58\times10^{4}$	$\sim 2.13\times10^{4}$	$\sim 1.73\times10^{4}$
Ne	$\sim 1.76\times10^{2}$	$\sim 2.89\times10^{1}$	$\sim 1.14\times10^{1}$	$\sim 3.96\times10^{0}$
O	$\sim 1.25\times10^{0}$	$\sim 4.36\times10^{-1}$	$\sim 5.56\times10^{-3}$	$\sim 3.14\times10^{-1}$
Si	$\sim 1.39\times10^{-1}$	$\sim 2.66\times10^{-2}$	$\sim 3.61\times10^{-2}$	$\sim 1.03\times10^{-1}$
Total	$\sim 1.41\times10^{7}$	$\sim 9.82\times10^{6}$	$\sim 7.78\times10^{6}$	$\sim 6.60\times10^{6}$

Figure 7: example of durations of burning phases for a few massive stars from Renzo 2015.

(where the virial theorem imposes a collapse, but $\langle$ T $\rangle$ is never high enough to generate enough energy through nuclear reaction to sustain the structure and their collapse is interrupted by degeneracy pressure).

Traces of Lithium are produced in the Big Bang nucleosynthesis, so some of it exists in stars at the beginning of their evolution. Because of its fragility, Lithium is a sensitive tracer of temperature in stars, and in evolved stars it should not be found because of the $^7\mathrm{Li}+p \rightarrow 2^4\mathrm{He}$ reaction. However, lithium can be observed in some stars atmospheres! This posed the problem of how lithium can be produced again once the star evolves. Various mechanisms have been proposed:

- spallation of cosmic rays (but generally one expects the cosmic ray flux to be too low to explain the amount of lithium found)
- "Cameron-Fowler mechanism" (Cameron & Fowler 1971): mixing at the bottom of the envelope can lead to $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ and the produced ^{7}Be is then mixed outwards where it may decay into ^{7}Li with an electron capture. For this mixing to happen one expects the star to be quite evolved (an AGB star), and we see more Lithium-rich giants than this can explain.

Homework

- Calculate the Sun's mass loss rate due to the nuclear burning of 4 protons into helium
- Using `MESA-web`, which contains tabulated data for nuclear reactions from experiments, calculate the evolution until the end of iron core burning of a massive ($M>15M_\odot$) star and use the `trimmed_history.data` output to:

  1. determine the typical composition of the core at various evolutionary phases and explain it in terms of the experimentally derived properties of nuclei (**hint**: use the variables `center_*` to plot the mass fraction of abundances as a function of time)
  2. estimate the duration of each core burning phase.

  N.B.: You can also use the "movie" of the evolution of the star produced by `MESA-web` for you.

  N.B.: The values you will find may differ from the table above: that's ok! The simulation output depends on many physics assumptions which are probably not the same in the models `MESA-web` runs and those I quoted, this actually gives you an idea of *systematic* errors in the numerical computations

  N.B.: Computing the evolution of massive stars through *all* the burning phases is numerically challenging (and the stiffness of the nuclear reactions rate dependence on T is a big part of the challenge), therefore it is possible that your `MESA-web` model may not finish the run. The file `input.txt` in the `*.zip` file you download from `MESA-web` has a string at the end telling you if the model was successful or not. If your model is not and you can't get a successful model by the deadline by fiddling with the input parameters, mention which input you used (listed in that file) and list the composition and typical duration up to where your model evolved.

  Hint: I have tried with the following settings and got a model to form an iron core, that is successfully burn through all the viable nuclear fuel:

  - Initial Mass: $20M_\odot$ (solar mass)
  - Burning Modifiers: `none`
  - Nuclear Reaction Network: `approx21`

- Sampled Nuclear Reaction: `none`
- Initial Metallicity: 0.02
- Mixing Length Alpha: 2.0
- Mixing Length Theory Implementation: Cox (this is an algorithmic formulation of MLT)
- Convective Overshoot f: 0.000
- Convective Overshoot f0: 0
- Semi-Convection Alpha: 0
- Thermohaline Alpha: 0
- Thermohaline Mixing Implementation: Kippenhahn
- Boundary Mixing: `pred` = `.false.` / `predms` = `.true.` / `cpm` = `.false.` (the default)
- Red Giant Branch Wind Scheme: Dutch (this specifies stellar winds)
- RGB Wind Scaling Factor: 0.8
- Asymptotic Giant Branch Wind Scheme: Dutch
- AGB Wind Scaling Factor: 0.8
- Initial Rotational Value: 0.0 (initial rotation)
- Variance Control Target: 1e-2
- Mesh Delta Coefficient: 2.0
- $\mathrm{dX}_{\mathrm{nucdropminXlimit}}$: 1e-2
- MESA Release: `r12778`