400A - Radiative transfer

Mathieu Renzo

March 24, 2025

Materials: Mihalas *Stellar atmospheres* book, Chapter 3. de Koter's lecture notes on radiative transfer and stellar atmospheres, Shore's *Astrophysical Hydrodynamics* book, Lamers & Cassinelli 1994 book.

Stellar atmospheres

So far in this course we have mostly dealt with the *interior* of stars and derived equation to determine the stellar *structure* and what drives their *evolution* (which is typically "slow", $\tau_{\mathrm{nuc}} \gg \tau_{\mathrm{KH}} \gg \tau_{\mathrm{ff}}$).

The picture we have derived is that gravity drives a contraction of the (hot) stellar gas, which, because it has a finite temperature, shines. This loss of internal energy translates into a loss of gravitational potential because of the virial theorem relating the two, and consequently the temperature in the center increases. This continues until nuclear burning ignites to compensate for the energy losses: *stars don't shine because they burn, stars burn because they shine.*

The nuclear burning can be seen just as an attempt by the star to *delay* its gravitational collapse: as nuclear fuel is depleted the star is forced to evolve (until either gravity "wins" or the EOS deviates from the classical ideal gas, and quantum mechanical effects provide sufficient pressure that win over gravity, we will see these in more detail in the coming lectures).

However, many of the assumptions we have made to derive these equations and the picture of stellar evolution described above *do not apply to the stellar atmosphere*, which is the layer that produces the *detectable* photons from stars! In this lecture we will consider in more details the physics of the stellar atmosphere which are crucial to obtain empirical evidence to test the picture of stellar evolution we now can build, and to provide the outer boundary conditions needed to calculate a stellar model.

Definition

The *stellar atmosphere* is by definition the layer where the radiation field is *not* isotropic, but the photon flux has a net *radial* component.

N.B.: This does *not* mean that all the photons move radially! Just that on average there are more photons moving in the positive radial direction than the negative radial direction, but photons can still move in all direction!

This means that *by definition* the stellar atmosphere is *not* a black body, and it is *partially* transparent to photons (resulting in a positive radial component of the photon flux). We *cannot* use the diffusion approximation to treat radiative transport here, since the mean free path of photons ℓ_γ are becoming comparable to the thickness of the atmosphere (ρ decreases moving outward!).

To make models of stellar *spectra* and determine the outer boundary conditions we need to consider how radiation from the bottom flows through the atmospheric layer and consequently how this layer stratifies, determining the outer boundary pressure and temperature.

N.B.: real stars are even more complicated: in the photosphere radiative and (magneto)-hydrodynamical processes can launch stellar winds that remove mass from the star and connect smoothly the stellar material with the interstellar material!

By definition, the bottom layer of the atmosphere is the *photosphere*, that is this fictional surface that has T_{eff} such that a black body of this temperature would emits the same energy per unit time as the star does. This is from where the bulk of the stellar radiation comes from *by definition* (recall the discussion on spectral types!).

N.B.: $\mathrm{dL/dm} = \varepsilon_{\mathrm{nuc}} - \varepsilon_{\nu} + \varepsilon_{\mathrm{grav}} \equiv 0$ in the outer layers of the star where T is not high enough for nuclear burning, ρ is not high enough for ν emission, and we assume gravothermal equilibrium, so $\varepsilon_{\mathrm{grav}} \propto \mathrm{ds/dt} = 0$, so L is constant.

Basics of radiative transfer

The problem we need to consider in the stellar atmosphere is how radiation coming from the *photosphere* flows through the overlaying gas, and how this impact the observable radiation itself, and the structure of this gas.

N.B.: a useful thing for this problem is to "think like a photon", and imagine the physics it will encounter during its trip towards the detector!

Since by definition the radiation coming out from the photosphere is a black body, it is isotropic, so if you think of the atmosphere as a slab of gas with a thickness dr, the radiation illuminating it from below may come at an angle θ w.r.t. the slab.

We can define the specific intensity (per unit frequency or wavelength) I_ν as the amount of energy flowing through a surface element dA in a time interval dt and coming within a solid angle $d\Omega$ around the direction $\mathbf{n}$ with frequency in the range between ν and ν+dν:

$$I_\nu \equiv I_\nu(\theta) = \frac{dI}{d\nu} = \frac{dE_\nu}{d\nu dt dA d\Omega}\mathbf{n} \quad , \tag{1}$$

which has the dimensions of $[\mathrm{E}]/([\mathrm{L}^2][\mathrm{t}][\nu][\text{solid angle}])$. This would be constant as photons propagate along a path of length ds along the direction $\mathbf{n}$, however there can be processes that add photons:

- scattering from another direction onto the direction of interest,
- emission processes,

and process that can remove photons:

- scattering from the direction of propagation onto another direction
- absorption processes.

Moreover, I_ν itself may in general be time dependent (although this is not the case for stellar atmosphere), so we can write down the equation of radiative transfer as

$$\frac{dI_\nu}{ds} = \frac{1}{c}\frac{\partial I_\nu}{\partial t} + \mathbf{n}\cdot\nabla I_\nu = -\kappa_\nu \rho I_\nu + j_\nu \rho \quad , \tag{2}$$

where the l.h.s. expresses the total change in specific intensity along the direction $\mathbf{n}$ due to the intrinsic time-dependence (∂_{t}) of I_ν and the spatial dependence along the direction we are considering ($\mathbf{n} \cdot \nabla$), and the r.h.s. expresses the loss of radiation intensity due to scattering *and* absorption processes, which depends on κ_ν (the specific opacity we have already encountered) and is proportional to I_ν itself (you can't lose photons you don't have!), and the addition of radiation intensity from emission processes and scattering along the line of sight which depends on the emission coefficient j_ν.

N.B.: dimensional analysis reveals that each side has the units of $[I_\nu]/[L]$, this equation describes how the intensity changes along its path. The fact that photons propagate at the speed of light c make the leftmost factor of 1/c appear: $d/ds = c\partial_t + \mathbf{n}\cdot\nabla$. The density ρ on the l.h.s. expresses that the more matter there is (per unit volume), the more likely there will be absorption and emission.

The specific intensity at the bottom of the atmosphere is related to the photospheric emission by:

$$F \equiv \int_0^{+\infty} d\nu F_\nu \equiv \sigma T_{\mathrm{eff}}^4 = \int_0^{+\infty} d\nu \int d\Omega \cos(\theta) I_\nu \quad , \tag{3}$$

that is the black body flux F is obtained by integrating the specific intensity over the solid angles. Note the factor $\cos(\theta)$ that arises because I_ν is a vector and we only want the component normal to the surface element dA.

This last expression is going to be useful to connect the physics in the atmosphere with the interior, since we *define* the photosphere to have a flux σ ${T_{\mathrm{eff}}}^4$.

Simple solutions of the steady state radiative transfer equation

Steady state without emission

In absence of an explicit time dependence (∂_t I_ν =0) and emission processes (j_ν=0), this equation is easily solved calling s the length element along the direction $\mathbf{n}$ so that $\mathbf{n}\cdot\nabla \equiv d/ds$, and the solution becomes:

$$I_\nu = I_{\nu,0} e^{-\kappa_\nu \rho s} = I_{\nu,0} e^{-\tau_\nu} \quad , \tag{4}$$

where we introduce the definition of specific optical depth $d\tau_\nu = \kappa_\nu \rho$ ds. This variable is useful because it gives the scale-length of the problem as depending on $\kappa_\nu \rho = 1/\ell_{\gamma,\nu}$ with $\ell_{\gamma,\nu}$ the mean free path for a photon of frequency between ν and ν+dν. Effectively, this allows us to use τ_ν as the independent coordinate for the propagation of photons of frequency between ν and ν+dν.

Steady state with emission and absorption canceling each other

With the definition of $d\tau_\nu$, we can re-write Eq. 2 (still assuming no explicit time dependence, $\partial_t I_\nu$ = 0) as:

$$\frac{dI_\nu}{d\tau_\nu} = \frac{j_\nu}{\kappa_\nu} - I_\nu \equiv S_\nu - I_\nu \quad , \tag{5}$$

where in the last step we define the source function S_ν. In thermal equilibrium and at high optical depth, for instance in the interior region of a star, $dI_\nu/d\tau_\nu$=0 and I_ν = B(ν,T) is the black body function for the intensity, and this equation states $S_\nu = I_\nu \equiv$ B(ν, T).

This effectively is a statement that at thermal equilibrium, the emission processes, the absorption processes, and scattering in and out of the direction of interest all cancel each other out.

Eddington atmosphere

The simplest stellar atmosphere model that allows to define non-trivial outer boundary conditions is the so called "Eddington gray atmosphere", which provides an analytic T(τ) relation in the atmosphere that can be smoothly attached to the stellar interior where T $\equiv$ T_{eff} and used to calculate the pressure needed at such boundary to have hydrostatic equilibrium. In other words, the Eddington gray atmosphere allows one to define (non-trivial) outer boundary conditions for the stellar interior problem.

Let's start with the assumption of a *plane parallel atmosphere*, that is we neglect the *curvature* of the stellar atmosphere, which is acceptable if its radius is much larger than the length scale of interest at any point in it. This assumption reduces the problem to a one-dimensional problem along the vertical direction, and ds = -dz/cos(θ) for the element of length along a generic photon path ds, and rewrite the steady state ($\partial_t = 0$) radiative transfer equation as:

$$\cos(\theta)\frac{dI_\nu}{d\tau_\nu} = -(S_\nu - I_\nu) \quad . \tag{6}$$

N.B.: we define ds and dz to be antiparallel (introducing a minus sign), because we want dτ to be positive moving inwards toward negative z.

The second approximation of the Eddington atmosphere is that we assume a "gray" radiative transfer, meaning the opacity is *independent of frequency* $\kappa_\nu \rightarrow \kappa$, thus $\tau_\nu \rightarrow \tau$. We also neglect the frequency dependence of the source term S_ν. With these hypotheses we can now integrate this in dν from 0 to $+\infty$ and obtain:

$$\cos(\theta)\frac{dI}{d\tau} = -(S - I) \quad , \tag{7}$$

which can be solved analytically (multiply by $e^{-\tau/\cos(\theta)}$, rewrite the l.h.s. as a total derivative and integrate in dτ) getting

$$I(\tau,\theta) = \frac{\exp(\tau/\cos(\theta))}{\cos(\theta)} \int_\tau^{+\infty} S \exp(-\tau/\cos(\theta)) d\tau \quad , \tag{8}$$

where the r.h.s. is integrated from a certain optical depth τ outwards. We can recover the ν dependence of S as an optical depth dependence in this integral.

We can also define the radiation energy density u, the total flux F, and the radiation pressure as moments of the intensity I(τ,θ) w.r.t. cos(θ) (since θ always appears in a cosine, it is usual to change variable to cos(θ)=μ in radiative transfer calculations):

$$u \equiv u(\tau) = \frac{2\pi}{c}\int_{-1}^{1} I(\tau,\theta) d\cos(\theta) \;, F \equiv F(\tau) = 2\pi \int_{-1}^{1} I(\tau,\theta)\cos(\theta) d\cos(\theta) \;, P \equiv P(\tau) = \frac{2\pi}{c}\int_{-1}^{1} I(\tau,\theta)\cos^2(\theta) d\,\mathrm{co} \tag{9}$$

We can also define the average specific intensity as J(τ) = $(4\pi)^{-1} \int$ I(τ)d$\Omega \equiv 0.5\int_{-1}^{+1}$ I(τ)dcos(θ), so that J=c u/4π. and dividing Eq. 7 by two and integrating between -1 and 1 in cos(θ) we have

$$\frac{1}{4\pi}\frac{dF}{d\tau} = J - S \quad . \tag{10}$$

Now the total radiative gray flux in the atmosphere has to be constant, dF/dτ = 0: there is radiative equilibrium and what goes in must come out! So this equations tells us J=S.

We can also take Eq. 7 and multiply it by $\cos^2(\theta)$ and integrate between -1 and 1 in $\cos(\theta)$ to obtain:

$$\frac{dP}{d\tau} = \frac{F}{c} \quad . \tag{11}$$

The r.h.s. is constant, so this can be integrated to give P = Fτ/c + constant. One more hypothesis of the Eddington approximation is to *assume* that the gas is radiation pressure dominated (this was to allow him to proceed further): then we also know from thermodynamics that P=u/3 $\equiv$ 4π J/3c (using the definition of J and its relation with the radiation energy density u). Putting all these findings together:

$$S = J = \frac{3Pc}{4\pi} = \frac{3F}{4\pi}\left(\tau + \text{constant}\right) \quad , \tag{12}$$

that is we have an expression for the source function! Substituting for S in the solution for I we get:

$$I(\tau, \cos(\theta)) = \frac{3F}{4\pi}\frac{\exp(\tau/\cos(\theta))}{\cos(\theta)}\int_{\tau}^{+\infty}(\tau + \text{constant})\exp\left(-\frac{\tau}{\cos(\theta)}\right)d\tau \Rightarrow I(0,\cos(\theta)) = \frac{3F}{4\pi}(\cos(\theta){+}\text{constant}) \quad . \tag{13}$$

To determine the constant of integration, we can use the second Eq. 9 which defines F using the solution for I(τ=0,cos(θ)) in the integral:

$$F = 2\pi\int_{-1}^{1} I\cos(\theta)d\cos(\theta) = \frac{3F}{2}\int_{-1}^{1}\left(\cos^2(\theta) + \text{constant}\cos(\theta)\right)d\cos(\theta) = \frac{3F}{2}\left(\frac{1}{3}+\frac{\text{constant}}{2}\right) \Rightarrow \text{constant} = \frac{2}{3} \quad . \tag{14}$$

With this specification of the constant that we obtained imposing the flux to come from τ=0 $\Rightarrow$ κ = 0, so from the layer after which there is nothing impeding the photons anymore (**N.B.:** the only other option is ρ=0, so there is nothing, or ds=0, so the photons have not moved!), we completely specified the source function S $\equiv$ S(τ) and we can obtain I$\equiv$ I(τ) and use it to calculate the pressure!

Outer boundary conditions of the stellar problem: T_{eff} and P

From Eq. 10 and 11 we now have:

$$J = S = \frac{3F}{4\pi}\left(\tau + \frac{2}{3}\right) \quad , \tag{15}$$

but also, assuming that the atmosphere is also in LTE (including radiation!), J=S=B(ν,T)=σ T^4/π, so using that F=σ $T_{eff}{}^4$ we obtain:

$$T^4 = \frac{3}{4}T_{\rm eff}^4(\tau + \frac{2}{3}) \quad , \tag{16}$$

which is the Eddington T(τ) relation which connects the effective temperature of the black body to the outer temperature T(τ) under the approximations for the atmosphere:

1. plane parallel

2. gray (i.e., independent on frequency ν)

3. radiation dominated

4. Local thermal equilibrium.

N.B.: In the stellar atmosphere, T is a steep function of τ in this approximation!

N.B.: in this approach the photosphere correspond to τ=2/3, this factor comes from imposing T=$T_{\rm eff}$ in the radiation dominated, gray, plane parallel atmosphere. This number is a direct consequence of these specific approximations, and it makes sense that it is of order ~1: the black body radiation from the stellar interior comes from the region where τ goes from $\leq$1 (where the optical depth is low and we cannot assume black body) to $\tau \gg$1 (where $I_\nu = S_\nu$ and we have a black body distribution for radiation). Once again, it is important to remember that the photosphere is an idealization, and nothing that special occurs at τ=2/3, it's just a convenient location where we can stitch the Eddington gray atmospheric model to the interior model.

Finally, to find the outer boundary pressure, we need to integrate downward from τ=0 to τ(T=$T_{\rm eff}$)$\simeq$2/3 the hydrostatic equilibrium equation. We typically assume that the atmosphere is in hydrostatic equilibrium, however *can* be a big assumption, depending on the star and whether it loses mass and whether the interaction between radiation and the gas drives non-trivial dynamics. Furthermore, we usually assume that the gravity is constant, or in other words, we neglect the atmosphere's "self-gravity" since the bulk of the mass is inside its inner boundary. One can just assume that κ is constant throughout the atmosphere, an oversimplification that allows for an analytic calculation which yields:

$$P(\tau) = \frac{GM}{R^2\kappa}\tau \Rightarrow P(\tau = 2/3) = \frac{2}{3}\frac{GM}{R^2\kappa} \quad , \tag{17}$$

where M is the total mass of the star, R is the radius such that $L/(4\pi R^2) = \sigma T_{\rm eff}$, κ is the opacity assumed constant in the atmosphere, and we *define* the bottom of the atmosphere at τ=2/3 because of the Eddington T(τ) relation. Alternatively, one could use tabulated values of κ and a T(τ) to perform the integral.

Together with T=$T_{\rm eff}$, we now have specified the outer boundary conditions fixing T and P at τ=2/3 and completely determined the mathematical problem of the structure and evolution of a single, non-rotating, non-magnetic star of known total (initial) mass M and composition.

N.B.: While Eddington atmosphere are the simplest non-trivial case, it is still on approximations which can (and sometimes are) relaxed in stellar evolution modeling: this can move the outer boundary in τ location too!

N.B.: A "classic" generalization of this atmospheric model is the generic class of gray atmospheres where the constant of integration is *not* a constant, but a function of τ itself.

Saha equation

Let's also assume that LTE holds in the stellar atmosphere, therefore, the rate at which atoms are ionized I matches the rate at which there are recombinations R (principle of detailed balance). Therefore:

$$n_e n_+ R = n_0 I \Rightarrow \frac{n_e n_+}{n_0} = \frac{I}{R} \quad , \tag{18}$$

where n_e, n_+, and n_0 are the number densities of electrons, positive ions, and neutral atoms respectively (so we are imposing a balance per unit volume). But that must also be equal to the ratio of available states to all these particles, which in the limit of ideal gas we can calculate using Maxwell-Boltzmann statistics! The momentum terms of the ions and neutral atoms cancel each other in the ratio (neglecting the small mass difference between these 2), and we are left with

$$\frac{n_e n_+}{n_0} = 2\frac{(2\pi m_e k_B T)^{3/2}}{h^3} \exp\left(-\frac{\chi}{k_B T}\right) \quad , \tag{19}$$

where the first term comes from the momentum phase space of the electron (with 2 factor for its spin) and the exponential depends on the ionization potential χ. This is the so called Saha equation named after Megnhad Saha, which under the assumption of LTE (sometimes questionable in stellar atmospheres) allows to calculate the free electron and ion densities.

N.B.: The exponential factor comes from the Maxwell-Boltzmann statistical distribution of $\mathrm{dn_0}$ and $\mathrm{dn_e}$ $\mathrm{dn_+}$!

For any ion/atom for which we can calculate (or empirically determine in a lab) the ionization potential χ, or more in general the difference in their energy levels, we can write a similar equation! Thus once the temperature T of a gas is specified this allows us to predict what the photons filtering through the atmosphere will encounter, and thus what we expect will be "removed" from the distribution of photons coming out of the photosphere and the resulting spectrum of the stars.

N.B.: This equation also allows us to determine the number of free electrons and thus the chemical potential in the partial ionization zones of the stars!

Spectral line formation

Lines form because the black body spectrum coming from the photosphere (by definition) filter through the overlaying *atmospheres* where atomic radiative processes (mainly bound-bound and bound-free transitions) can *remove* some photons from the spectrum.

To predict the spectrum of a star, one needs to know the temperature, density, and velocity structure of the atmosphere (to be able to calculate the Doppler shifts!), whether it is in LTE (so electron populations are described by the Saha equation above) or non-LTE effects need to be accounted for (e.g., for maser lines), and solve the radiative transfer equation.

In some cases, the velocity structure depends on the radiation itself making this process extremely complicated, or more precisely, in the momentum equation of the gas, a radiative acceleration term dependent on the velocity (because of the Doppler-dependence of κ_ν) appears, making the dynamics of the radiation+gas highly non-linear. This is, for example, the case of radiatively driven stellar winds from massive stars (see for instance the book by Lamers & Cassinelli 1994 or the review Smith 2014).

Broadening mechanisms

While treating in detail all these processes would require an entire course on its own, we can give a brief qualitative description of some key effects here.

While considering these remember that for virtually all stars (except the Sun), the projected disk on the sky is *unresolved* (the size of the point-spread function of the telescope is bigger than the size of the stellar disk projected on the sky): in an observed spectrum you see all the surface at the same time!

Intrinsic width of lines

Because of the uncertainty principle, an electron in an ion allowing for a bound-bound or bound-free transition is not perfectly localized. A consequence of this is that the spectral lines formed by one particular ion in a particular energy state is not an infinitely sharp delta function $\delta(\nu_0)$ centered at $\nu_0 = \Delta$ E/h, but instead it is a Lorentzian profile with an intrinsic width.

Rotational broadening

If the star is rotating, some parts of the disk will be moving away from the observer (at a velocity $v_{rot} \times \sin(i)$ with i inclination angle to the line of sight), and some parts will be moving towards the observer (unless i=0, i.e. the star is seen rotation pole on, as seems to be the case for the North Star!).

This will introduce a Doppler shift from each part of the disk: this *rotational broadening* is usually described by a Gaussian, that needs to be convolved in frequency space with the intrinsic Lorentzian distribution coming from the QM of the transition.

The *convolution* of a Lorentzian and a Gaussian gives a Voigt profile after Woldermar Voigt.

Pressure broadening

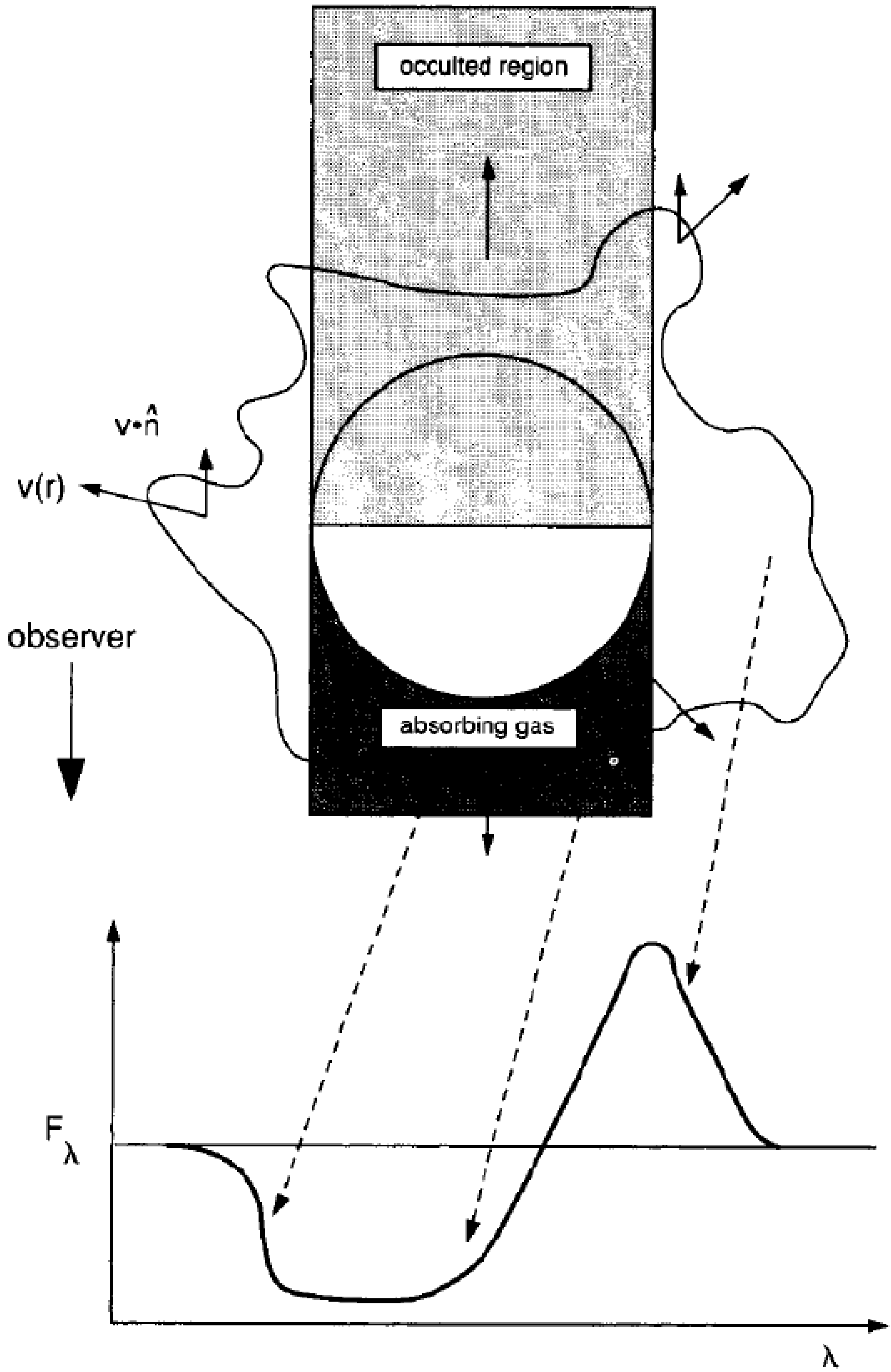
In a star, even in the relatively low ρ atmosphere, ions/atoms interacting with radiation are *not* in isolation! The presence of external forces (due to other ions/atoms, or global magnetic field, etc.) can modify the energy levels of each atom's Hamiltonian, and thus the central frequency ν_0 *and* the width of specific bound-bound transition. Collectively this is referred to as "pressure broadening".

As a concrete example, Zeeman splitting of the degenerate (in absence of magnetic field) ℓ=1, m=0,$\pm$1 triplet can result in small (non resolved) shift in frequency that are observed as a broadened line.

P Cygni profiles

If the atmosphere is "moving", for example because there is a wind outflow, a particular shape of the spectral lines will form. This is called after the first star in which this was detected a "P Cygni" line.

The wind moving toward the observer will absorb radiation like any atmosphere, but because of its motion the absorption will be moved to shorter wavelengths (blue-shifted). Viceversa, the wind moving in directions away from the observer will have electrons de-exciting and thus photon *emission* (if the de-excitation is radiative and not collisional), which will be redshifted to longer wavelengths, causing a specific shape of the line:



occulted region
v•n̂
v(r)
observer
absorbing gas
F_λ
λ

Emission lines

Some stars not only show *absorption* lines (i.e., "lack" of photons at certain wavelengths compared to the underlying black body spectrum produced at the photosphere), but also *emission lines*.

The P Cygni profiles mentioned above are in a sense an "intermediate" behavior between these two regimes.

Be stars

These are stars of spectral class B (recall the lecture on CMD/HRD), so fairly hot and massive, which show *emission* lines, typically Hα. A star is classified as Be if it is a B-type star that ever showed Hα in emission, even though these can be intermittent and disappear: long term spectroscopic followup, including the crucial contribution of amateur observers is important to understand the spectral behavior and thus the nature of these objects (see BeSS catalog containing professionally taken and amateur spectra of many bright Be stars!).

These stars are interpreted as being *fast rotating* ($\omega \geq 0.7\omega_{\mathrm{crit}}$) which shed a "decretion disk": the emission lines are not from the star directly, but from the disk of the star! A clear indication of the presence of the disk is the "double peaked" morphology of the Hα emission:

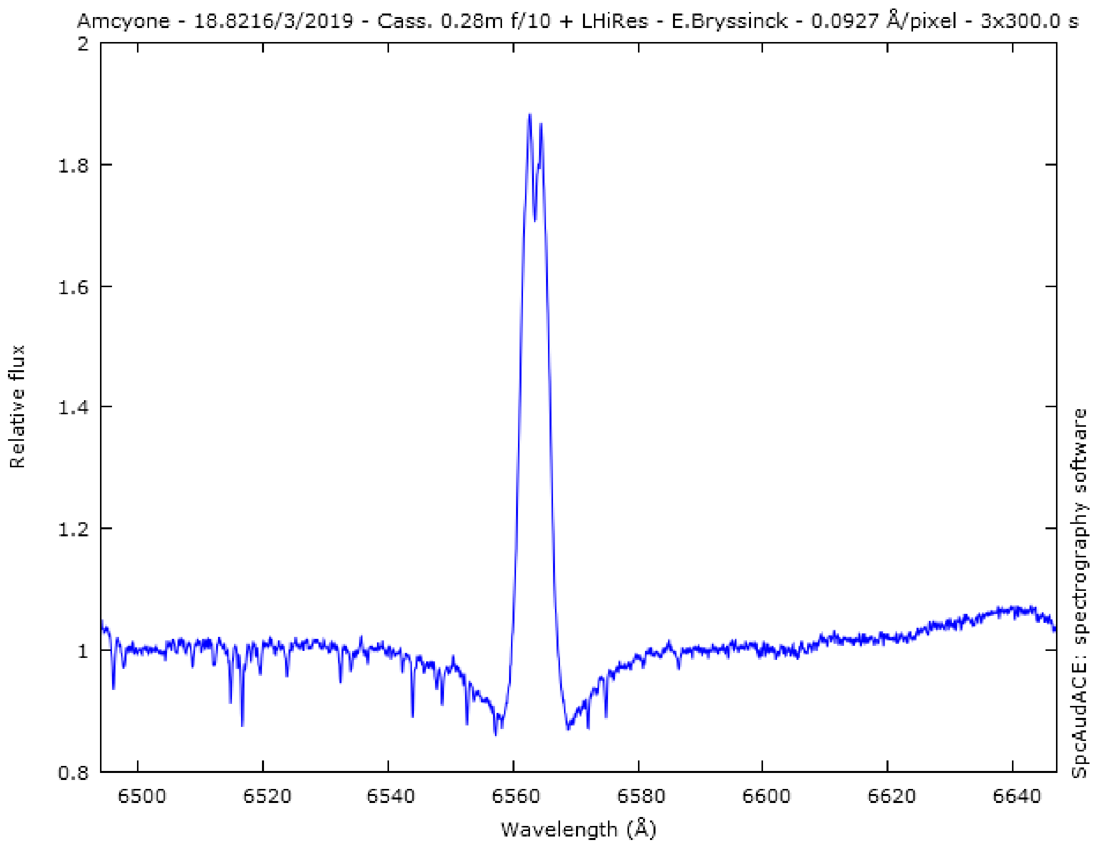


Figure 1: Spectrum of Alcyone (η Tau) on March 18$^{\mathrm{th}}$ 2019 centered around Hα ($\lambda \sim$6562 Angstrom) showing the typical double peaked emission suggesting the presence of a disk, obtained by the amateur astronomer E. Bryssinck.

- **Q**: can you infer why the double peaked morphology suggests a disk?

The formation path of these stars is still being actively investigated, but the fact that none are found with main sequence binary companions and many are found instead with a neutron star companion (periodically plunging through the disk producing X-rays making the system a Be-X-ray binary!) suggest that these may be accretor stars spun up by binary interactions (cf. Pols & Marinus 1994, Bodensteiner et al. 2020, Vinciguerra et al. 2020), although single star evolutionary pathways also exist (e.g., Langer 1998), see also the review by Rivinus et al. 2013.

B[e] stars

These are also B-type stars showing emission lines, but *forbidden* emission lines, that is radiative transitions where the angular momentum of the electron changes which are exponentially disfavored. These can only occur in low-density environments: if the density was high, the atoms/ions would much rather de-excite collisionally than with a radiative transition with $\Delta\ \ell > 0$.

Thus, the presence of a forbidden line (indicated by the squared brackets) suggests a very low density environment surrounding these stars. They tend to be brighter than Be stars (presumably, more massive!), and whether there is an evolutionary relation between the two classes is presently unclear.

Wolf-Rayet stars

Wolf-Rayet stars are a spectroscopic class *defined* by the presence of emission lines and the deficiency (but not necessarily total lack) of hydrogen (see also review by Shenar 2024).

They are further subdivided in classes based on the dominant lines visible (WNh if there is still significant amount of hydrogen, WN is it's nitorgen, WC for carbon, WO for oxygen). These are massive stars which have somehow shed a large portion of their H-rich envelopes (either because of winds of binary interactions) and are bright enough to drive strong outflows that are so dense that they are optically thick (remember $\tau(\mathrm{r})=\int_0^{\mathrm{r}} \kappa\rho\ \mathrm{dr'}$). In these dense winds collisional excitation of atoms/ions is possible followed by radiative de-excitation producing an *excess* of photons at the specific frequency of the atom/ion and transition considered, resulting in the emission line.