400A - opacity

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Materials: Chapter 5 of Onno Pols' lecture notes, Sec. 5.1.3 of Kippenhahn's book.

Opacity

We have seen in the previous lecture that in the equation for the energy transport by diffusion (either of photons or electrons, that is either radiative or conductive energy transport) there is a parameter κ that determines the "resistance" of the stellar gas to the passage of energy by diffusion. This was the analogy that comes out from the combination of the radiative opacity and conductive opacity

$$\frac{1}{\kappa} = \frac{1}{\kappa_{\mathrm{rad}}} + \frac{1}{\kappa_{\mathrm{cond}}} \quad . \tag{1}$$

Since we also saw that $dT/dr \propto \kappa F$, it is clear that the opacity κ is an extremely important parameter for the structure of the star. Effectively, what that proportionality means is that given an opacity profile $\kappa \equiv \kappa(r(m))$, the stellar structure will adjust in such a way that the temperature gradient dT/dr is sufficient to carry out the flux F that is necessary to maintain energy conservation layer-by-layer (accounting for the energy losses at the surface and throughout because of e.g., neutrinos)!

N.B.: The structure of each layer of the star is determined by (i) the microscopic properties of matter (e.g., κ_{rad}) and, more importantly, (ii) the energy losses by the outward flux F. The latter is the way the star compensates for the energy losses (its luminosity L).

The purpose of this lecture is to focus on κ_{rad} and understand what physically determines it, and understand how $\kappa_{rad} \equiv \kappa_{rad}(T, \rho, X_i)$ can be expressed as a function of the thermodynamics and composition of the stellar gas.

Rosseland mean opacity

But even before we discuss the microphysics, you know that the "resistance" of a material to the passage of light is dependent on the frequency of the radiation! Optical radiation clearly penetrates the Earth atmosphere, while X-rays don't; the small fraction of UV radiation that does penetrate the atmosphere and cause Sun burns does not penetrate the windshield of a car, but the optical wavelengths do, etc. So clearly in general $\kappa_{\mathrm{rad}} \equiv \kappa_{\mathrm{rad}}(\nu)$ is a function of the frequency of the light we are considering with $\nu = \mathrm{c}/\lambda$. What happened to that dependence in our stellar structure equations?

In the previous lecture we have written for the radiative energy flux

$$F_{\mathrm{rad}} = -\frac{1}{3}\frac{c}{\kappa_{\mathrm{rad}}\rho}\frac{du}{dr} \equiv -\frac{4ac}{3c\rho T^3}\frac{1}{\kappa_{\mathrm{rad}}}\frac{dT}{dr} \quad . \tag{2}$$

Here κ_{rad} should be interpreted as an "appropriate mean value". In reality, we should have written an equation like this for the specific flux (i.e., the radiative flux F_ν carried by radiation with frequency between ν and ν+dν) and integrate over frequencies to get the total radiative flux $\mathrm{F}_{\mathrm{rad}} = \int_0^{+\infty} \mathrm{F}_\nu \, \mathrm{d}\nu$. For the specific flux, we also should put in the equation the specific opacity κ_ν and consider only the energy density of radiation u_ν between ν and ν+dν:

$$F_\nu = -\frac{1}{3}\frac{c}{\kappa_\nu\rho}\frac{du_\nu}{dr} \quad . \tag{3}$$

Recall that $\mathrm{u}_\nu = \mathrm{h}\nu \ \mathrm{n}(\nu)$, with $\mathrm{n}(\nu)$ number density of photons ad a function of frequency determined by the condition of LTE (which is *approximately* correct in the stellar interior). The LTE approximation effectively means the photons are distributed according to a black body distribution for the intensity in the stellar interior:

$$u_\nu = \frac{4\pi}{c}B(\nu,T) = \frac{8\pi h}{c^3}\frac{\nu^3}{e^{h\nu/k_BT}-1} \quad . \tag{4}$$

Thus $\mathrm{du}_\nu/\mathrm{dr} = 4\pi/\mathrm{c} \times \partial \ \mathrm{B}(\nu, \mathrm{T})/\partial \ \mathrm{T} \times \mathrm{dT}/\mathrm{dr}$, and performing the integral over the frequencies:

$$F_{\mathrm{rad}} = \int_0^{+\infty} F_\nu d\nu = -\frac{1}{3}\frac{c}{\rho}\int_0^{+\infty}\frac{1}{\kappa_\nu}\frac{du_\nu}{dr} = -\frac{4\pi}{3c\rho}\frac{dT}{dr}\int_0^{+\infty}\frac{1}{\kappa_\nu}\frac{\partial B(\nu,T)}{\partial T}d\nu \quad . \tag{5}$$

Comparing this with Eq. 2 from the previous lecture we obtain that the $\kappa_{\rm rad}$ appearing there must be:

$$\frac{1}{\kappa_{\rm rad}} = \frac{\pi}{acT^3}\int_0^{+\infty} d\nu \frac{1}{\kappa_\nu}\frac{\partial B(\nu,T)}{\partial T} \quad , \tag{6}$$

which we can rewrite as

$$\frac{1}{\kappa_{\rm rad}} = \frac{\int_0^{+\infty} d\nu \frac{1}{\kappa_\nu}\frac{\partial B(\nu,T)}{\partial T}}{\int_0^{+\infty} d\nu \frac{\partial B(\nu,T)}{\partial T}} \quad , \tag{7}$$

which is the *harmonic mean of specific opacities* κ_ν *weighted with the temperature derivative of the Black Body distribution.* This is usually referred to as the Rosseland mean opacity after Svein Rosseland.

Physical interpretation

This may seem like a bunch of math that did not advance us much: we still have κ_ν (which in practice comes from atomic physics experimental and theoretical work), and the Rosseland mean opacity may seem to not have a straightforward physical interpretation.

However, consider that to a good approximation we can consider the stellar interior to be in LTE with a single temperature and a photon gas described by a black body. We can rewrite Eq. 3 (using Eq. 5) as:

$$F_\nu = -\frac{4\pi}{3\rho}\frac{dT}{dr}\frac{1}{\kappa_\nu}\frac{\partial B(\nu,T)}{\partial T} \quad , \tag{8}$$

Therefore at a location in the star of given ρ and dT/dr, the integral in the numerator of the Rosseland mean opacity is proportional to the flux F_ν: the Rosseland mean opacity weights the specific opacity (per unit frequency ν) with the available flux F_ν according to a black body distribution.

The frequency ν at which most of the photon energy is found by solving $\partial\ \mathrm{u}_\nu\ /\ \partial\ \mathrm{T} = 0$, which yields a maximum at $\mathrm{h}\nu = 4\mathrm{k_B T}$. This means that the Rosseland mean tends to "upweight" where most of the radiation energy is, and "down weight" the very low and very high frequencies.

In other words, $1/\kappa_{\rm rad}$ is smallest where most of the radiation energy can get through (which makes sense since we are developing an understanding for how energy is transported, and not for how energy is trapped!).

An unfortunate consequence of the harmonic nature of the Rosseland mean opacity is that it is not trivial to combine opacity of different gasses, the specific κ_ν have to be summed *before* taking the average.

Sources of radiative opacity

Now, let's consider the radiation-matter interactions that can be source of opacity (i.e., determine the κ_ν that we put in the Rosseland mean opacity to obtain κ_{rad}).

Bound-bound

Photons (orange wiggly line) can interact with the electrons in an atom/ion (especially if they have the "right" frequency close to $\nu \simeq \Delta$ E/h with Δ E the energy difference between the two levels for the electron). In this case the photon is absorbed by the ion and its energy goes into the energy level of the electron, which was bound to the nucleus before and after the interaction with the photon (hence the bound-bound name).

Because every atom/ion has specific energy levels, this opacity source may have a very complex frequency (i.e. photon energy) dependency. The transition energies must be determined solving the Hamiltonian for the electrons in the potential for the specific atom/ion, which can be extremely complicated and/or computationally unfeasible: for this reason, laboratory experiments are often used to determine opacities.

Note that ions of heavy elements with many electrons (e.g., iron) will tend to have *the most* lines (i.e., the largest number of possible bound-bound transitions), and dominate the opacity in the regime where they are not fully ionized.

This opacity source matters only until there are bound electrons to their respective ions in the stellar gas, which at very high T becomes more and more rare (since collisions between atoms would strip away the electrons). However, this term starts playing a role for T$\leq 10^6$ K, so still quite deep in the stars.

Bound-free

An incoming photon may have sufficient energy to photoionize an atom/ion. That is the absorption of the photon makes an electron jump from a bound energy level to an unbound energy level.

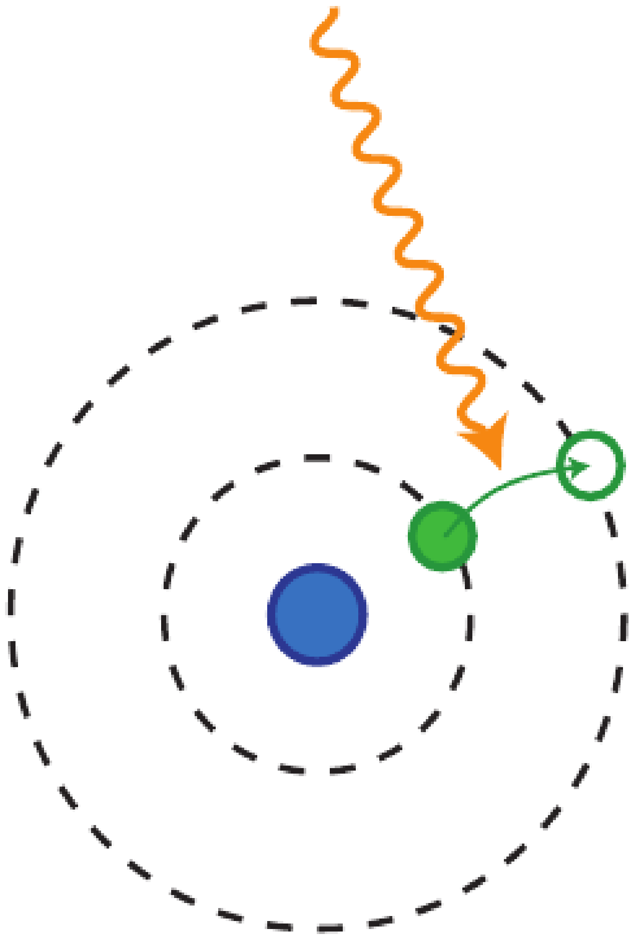

Figure 1: Cartoon of a bound-bound transition. The photon (orange wiggly line) is absorbed by the ion (nucleus in blue, electron in green) where an electron jumps to a higher energy level, represented by the dashed black line. Credits: R. Townsend. **N.B.:** the orbit of the electron is not a little circle like this, which would be unstable! It is instead an orbital which describes the spatial *probability distribution* of finding the electron there in accordance to quantum-mechanics.

As for bound-bound transition, bound-free photoionization requires the existence of electrons bound to nuclei, so its contribution to the opacity decreases at very high temperatures, when bound electrons are absent.

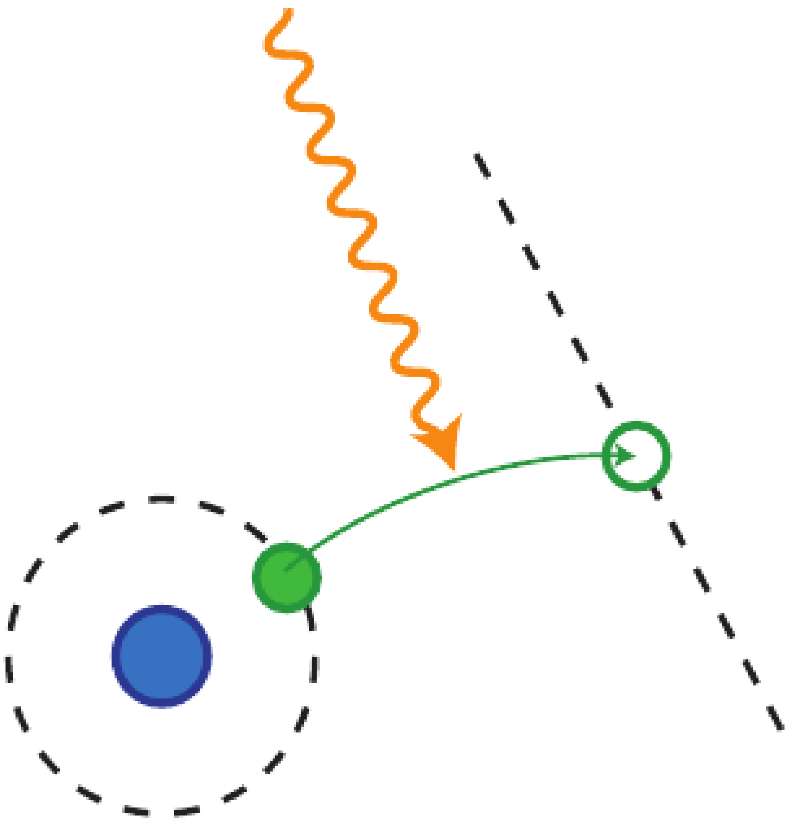

Figure 2: Cartoon of a bound-free transition. Credits: R. Townsend.

Free-free

Even unbound electrons can absorb a photon transitioning between two unbound energy levels of the continuum. This is effectively the opposite of bremstrahlung radiation, where the acceleration of an unbound electron results in the production of photons (or neutrinos!).

This process cannot occur if there are no free electrons, for example at very low temperatures.

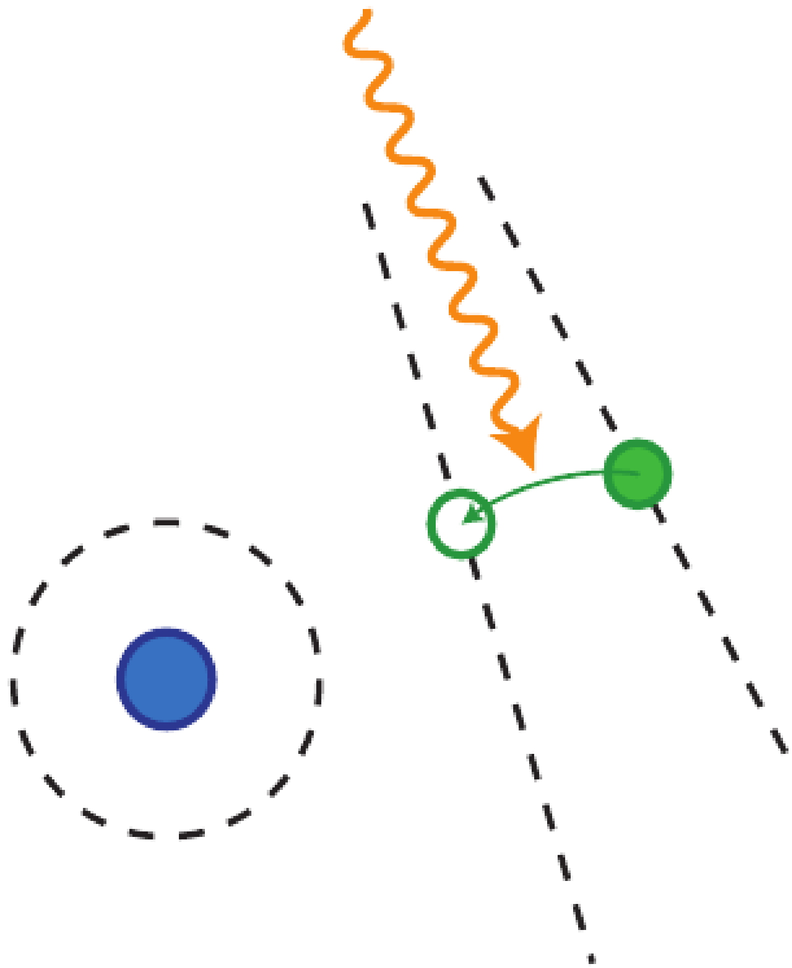

Figure 3: Cartoon of a free-free transition. Credits: R. Townsend.

Note that in the cartoon an ion/atom is still represented. The process of absorption of a photon by a single electron (γ+e $\rightarrow$ e) would violate conservation of the four-momentum, and it is not possible, but it is possible for an electron in the electromagnetic field of an ion.

Scattering

Another source of opacity is scattering, which unlike the processes above does not lead to the "disappearance" of a photon, but can still change its energy (and direction of propagation), thus affecting its ability to carry flux.

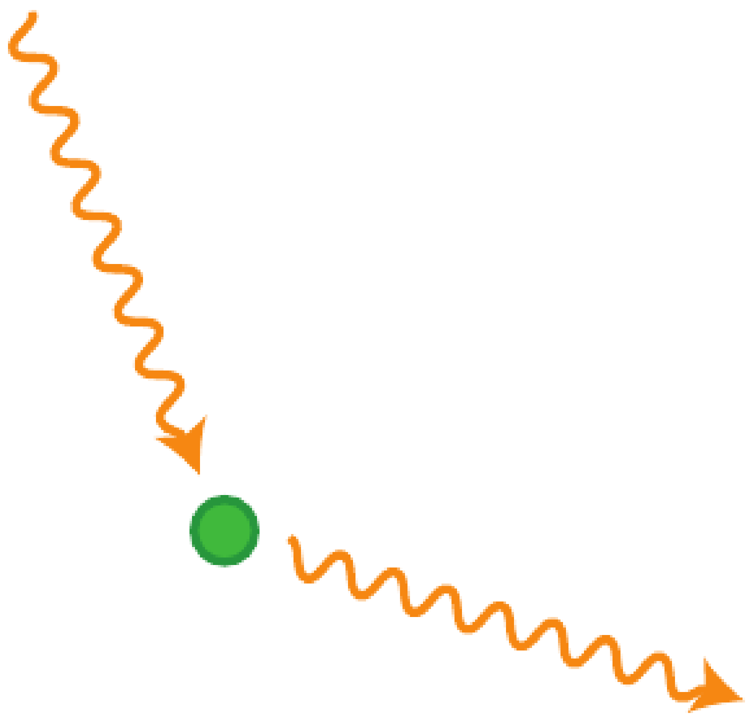

Figure 4: Cartoon of the scattering of a photon on an electron. Credits: R. Townsend.

At very high temperatures, scattering off free electrons is the main source of opacity (no bound-bound and bound-free processes without bound electrons), which greatly simplifies the $\kappa_{\rm rad}$(T,ρ) dependence.

The scattering of a classical electromagnetic wave off-an electron can be described by the Thomson scattering cross section, which divided by the $\mu_e m_u$ gives the corresponding opacity. Therefore, for T$\geq 10^6$ K, $\kappa_{rad} \equiv \kappa_{es}$:

$$\kappa_{es} = 0.2(1 + X) \ \ \mathrm{cm}^2 \ \mathrm{g} \ \ , \tag{9}$$

which does *not* depend on T or ρ, but only on the mass fraction of Hydrogen X (recall that $\mu_e = 2/(1{+}\mathrm{X})$ for fully ionized gas). If the gas is not fully ionized the expression here does not old.

Note that this opacity does *not* depend on the electromagnetic wave/photon frequency ν, so in the Rosseland mean, it comes out of the integral!

For very high energy, one needs to account also for the momentum exchange between radiation and the electron (Thomson $\rightarrow$ Compton scattering), which decreases the opacity. At even higher energies of the photons, one may need to use the Klein-Nishina formula.

N.B.: Ultimately in stellar evolution we use tabulated $\kappa_{rad} \equiv \kappa_{rad}(\rho, \mathrm{T})$ that (try to) account for all these effects without needing to calculate them on the fly while dealing with the star.

Molecules and dust

At T$\leq$4000K, atoms may bound together and form molecules, and even lower (T$\leq$1500K) dust grains may form. These are not the same dust you find on Earth (mostly small crystals, dead skin, etc.) but large agglomeration of molecules. These structures cause a very large increase in the opacity: the electrons in them can have many degrees of freedom that can be used to absorb and scatter photons (e.g., roto-vibrational molecular bands).

N.B.: molecular opacity is a field of research in *laboratory* astrophysics, when the relevant molecules can be synthesized and kept at the relevant T and ρ one can experimentally measure their κ_{rad} which is extremely complicated to calculate from first principles.

H^-

At low temperature hydrogen may capture an extra electron forming an H^- ion (i.e., a proton with 2 bound electrons). This is a fragile state, and in a pure hydrogen gas, it would not resist much, but if there are metals with one electron only (the first column of the periodic table, e.g., Na, K, Ca), they can provide extra electrons, allowing for the formation of this ion in stellar atmospheres.

N.B.: This negative ion can then provide most of the opacity in the envelope of non-metal-free cool stars, e.g., red giants or the Sun itself! An approximate relation for its opacity is

$$\kappa_{\mathrm{H}^-} = 2.5 \times 10^{-31} \frac{Z}{Z_\odot} \rho^{1/2} T^9 \ \mathrm{cm}^2 \ \mathrm{g} \quad . \tag{10}$$

- **Q**: since H$^-$ is the dominant source of opacity in cool stars, such as the Sun, red giants and supergiants, but for this ion to form metals able to lose an electron are required, do we expect red giants and supergiants for pop III stars? (The question is maybe less interesting for Sun-like stars since they are less luminous and thus even harder to detect, but still holds theoretically).

Conductive opacity

For an ideal gas, $\kappa_{\mathrm{cond}} \gg \kappa_{\mathrm{rad}}$ making conduction irrelevant in the combined opacity. This is because the Coulomb scattering cross section among charged particles in a plasma is larger than the cross section for interactions with photons.

Only for degenerate gas (at least partially), diffusion of energy through the thermal motion of particles (electrons, because of their lower mass) is important.

At very high densities, the electron mean-free path are very long (since collisions are forbidden by not having any level available below the Fermi energy), making conduction very efficient and allowing high density degenerate cores to become effectively isothermal (T=constant, dT/dr = 0).

Combining all these sources together

Ultimately, the metallicity (Z) or more specifically the detailed composition $\{\mathrm{X_i}\}$ can have a large impact on κ_{rad} and κ, together with the thermodynamic state of the gas (T,ρ), which determines which process dominates the blocking of photons

As you can see from the plot above, at fixed Z, there is maybe more structure as a function of T (because T determines the ionization levels, and thus the bound-bound and bound-free). The solid black line represents the T(ρ) profile of a stellar model.

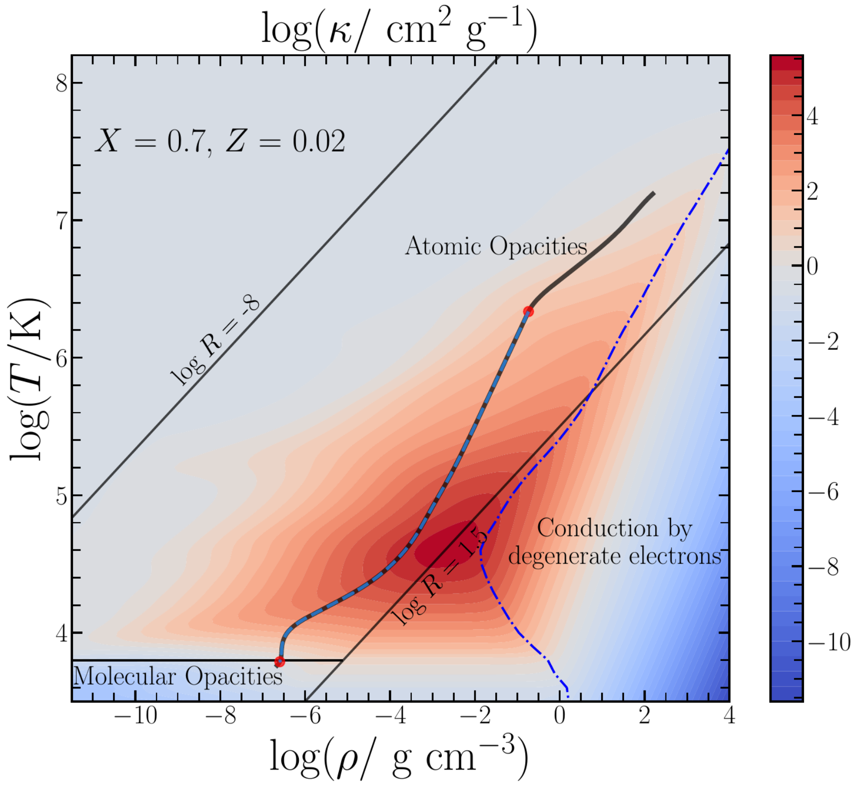

Figure 5: $\kappa \equiv \kappa(\mathrm{T},\rho)$ combining all the sources of opacities we discussed (and more) from Farag et al. 2024. This plot combines the atomic and molecular radiative opacities and the electron conduction opacities and is available in the `kap` module of the MESA code. See also Paxton et al. 2011.

Opacity "bumps" in the stellar interior and surface can lead to a steepening of the radiative gradient (recall dT/dr $\propto \kappa \times$ Flux), and cause the onset of other energy transport mechanisms and possibly stellar eruptions.

By "projecting" the plot above on either axes, one can obtain the κ(T) at fixed ρ (or $\kappa(\rho)$ at fixed T) and find that there are regimes where powerlaw approximations may be sufficient (e.g., the "Kramers" opacity law which gives $\kappa \propto T^{-7/2}\rho$, or the formula above for H^- opacity), but in practice to compute a stellar model one needs to use tabulated opacities from complex models and/or experiments carried out at LANL, Livermore, and other big, often military funded, laboratories.

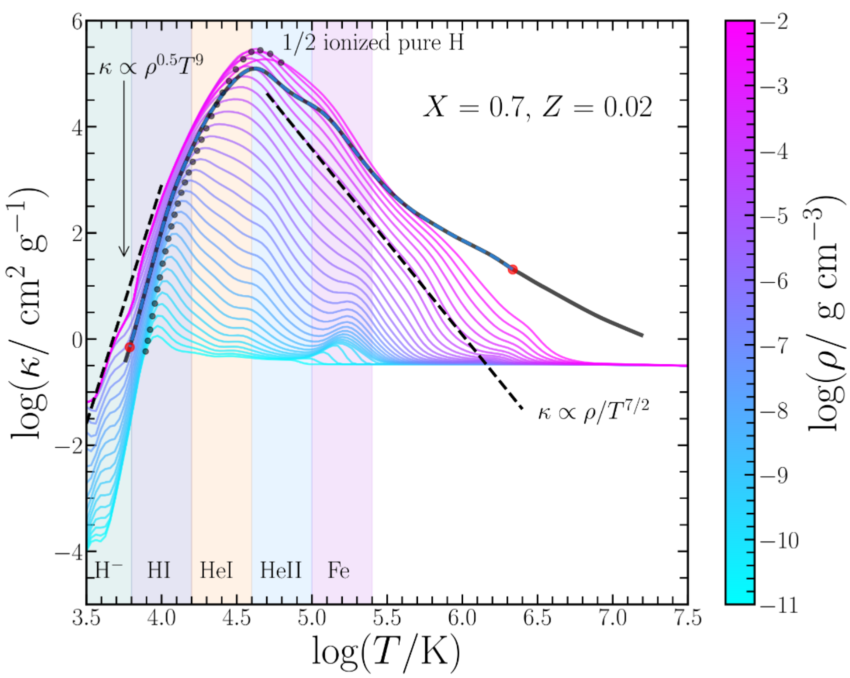

Figure 6: $\kappa_{\rm rad} \equiv \kappa_{\rm rad}$(T) for various fixed densities ρ (as indicated by the colorbar). This plot effectively shows various "slices" of the $\kappa_{\rm rad} \equiv \kappa_{\rm rad}(T,\rho)$ and allows one to see how powerlaw approximations can be used in certain regimes, but do not capture the full picture. Note the shaded background indicating ionization levels of important elements. Also from Farag et al. 2024

Homeworks

The Eddington Luminosity again

Using `MESA-web`, compute a $\geq$30 $M_\odot$ star until the maximum central temperature reaches above $\geq 10^{8.5}$ K. Make sure to save multiple profile files. Plot a series of κ(m) and/or κ(T) for the outer layers, and identify peaks that occur (**Hint**: this may be more easily done looking at κ(T)). Plot also L(m) and L_{edd}(m, κ) and, using your model, try to identify what happens in layers where L exceeds L_{edd}. We will discuss this in more detail in the next lecture.