400A - Qualitative evolution

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Materials: Onno Pols' lecture notes Chapter 9, 10, 11, Kippenhahn's book Chapters 22-24, 26-33, Hansen, Kawaler, Trimble book, Chapter 2, Farrell et al. 2022.

Pre-main sequence and Hayashi track

We have developed a set of 4 non-linear, coupled ODEs to describe the *structure* and *evolution* of stars (with physical approximations built-in), with an EOS for closure conditions plus a set of equations that describe the changes in composition due to nuclear processing and mixing (see for example this summary).

We have also worked out the ordering of timescales of the problem:

$$\tau_{\mathrm{nuc}} \gg \tau_{\mathrm{KH}} \gg \tau_{\mathrm{free\ fall}} \ , \tag{1}$$

which implies that for a star in hydrostatic equilibrium and LTE at every location throughout the star (i.e., global gravothermal equilibrium), the equations describing the (slow) *evolution* of the composition decouple from the 4 ODEs+EOS that describe the *structure* of the star.

The `MESA-web` stellar models at start at the *top right* of the Herzsprung-Russell diagram (so high L low T_{eff}) as a uniform sphere of gas with set composition that is in hydrostatic but not *necessarily* thermal equilibrium *yet*.

N.B.: there may also be short initial numerical transients lasting a few timesteps and irrelevantly short simulated physical time.

N.B.: the expression *top right* above is relative, in its future evolution the mode may evolve at even higher L later on (but not lower T_{eff}!)

These models thus evolve on a thermal timescale $\sim \tau_{\mathrm{KH}}$ losing energy (L goes down) and contracting – picture lines of constant $R=(L/(4\pi\sigma\ T_{\mathrm{eff}}^{4}))^{1/2}$! This *is* the gravothermal KH contraction which leads to the increase in average $\langle\ T\ \rangle$ by the virial theorem, and it goes on until either:

1. **degeneracy pressure stops this process by changing the EOS, tapping into quantum mechanical effects to provide extra pressure**: this should not happen to any of your models, but it is how you form a **Brown Dwarfs** (BD) and a planets. The distinction between these is whether some minor nuclear burning, energetically unimportant but still affecting the composition happens before the object sets on gravothermal equilibrium: BD experience deuterium burning, but deuterium is a loosely bound nucleus and its burning doesn't release a lot of energy.

2. **an energy source (namely nuclear burning) stops the collapse**. This is what happens to your stars, where H burning (as the lightest and most energy-releasing fuel) starts. The point where $L=L_{\mathrm{nuc}}$ because of H core burning is referred to as **Zero Age Main Sequence** (ZAMS), since the "main sequence" of the color magnitude diagram containing most stars is made of stars during their longest phase of evolution, hydrogen core burning.

From the ignition of H in the core onwards, for timescales $\tau \ll \tau_{\mathrm{nuc}}$, once the composition is decided (i.e., for fixed mass fractions $\{X_i\}$) the structure is determined by those 4 ODEs+EOS. This means that the *initial* structure of the star is *not* sensitive to the (complex, not-yet-fully understood) star formation process, and the ZAMS structure is fully determined by the composition!

Pre-main sequence stellar structure

If the gravothermal collapse of the mass of gas considered ends with the ignition of nuclear fuel (option 2. above), that is, if we make a star (recall $\langle T \rangle \propto \mu M/R$, so the mass determines whether we get here), then the phase from the beginning of the simulation until ZAMS is called *pre-main-sequence* (pre-MS).

During this phase the models start from the top-right of the diagram at high L, large R ($\Rightarrow$ low $\langle T \rangle$), and progressively contract and heat up. This is an idealization: often as stars are doing that, they still accrete mass, in the most massive stars (which have shorter KH timescales and pre-MS phases), accretion may not be over until even after ZAMS! See for instance the recent review by Offner et al. 2023.

N.B.: this accretion drives "outflows" in young stellar objects, which can be "dusty" and thus opaque, making the pre-main sequence evolution *embedded* and hard to directly observe.

The $\langle T \rangle$ is initially low (the gas that is collapsing is coming from the interstellar medium), and increasing. Since $\nabla = \partial \ln(T)/\partial \ln(P) \propto \kappa L$ and κ generally increases at low T, initially the stars have steep temperature gradient and are *fully convective*! This homogenizes their composition (but deuterium burning during the pre-MS can change this), and also determines that $\nabla \simeq \nabla_{\mathrm{ad}}$ to very good approximation.

Since almost arbitrarily high energy flux can be carried by (efficient) convection maintaining an adiabatic gradient, this means the luminosity of a fully convective star does not depend much on its structure (unlike a stably stratified *radiative* star where the temperature gradient of the *structure* is determined by the need of carrying the flux out).

This leads to the evolution of these stars along an almost vertical initial path on the HR diagram, the so-called *Hayashi track* (see also Sec. 9.1.1. of Onno Pols' lecture notes for an analytic approximation) after Chushiro Hayashi. The Hayashi track corresponds to solutions for fully convective stars. Wiggles around this vertical line are due to recombination and partial ionization zones leading to deviations of the temperature gradient ∇ from adiabatic and burning of light elements (^{2}H,^{7}Li) that releases some energy.

N.B.: The energy release per nucleon of the burning of light elements is very low and does not manage to alt the thermal-timescale quasi-static collapse.

The Hayashi line effectively determines a right, low-T_{eff} boundary on the HRD for stars in hydrostatic equilibrium: if a star were to be colder, it would have a steeper-than-adiabatic gradient somewhere, which would imply a higher convective flux (cf. convective energy flux)) and thus increase the luminosity of the star, moving the star upwards back onto the Hayashi track.

N.B.: for these cool temperatures, we already know that the opacity is dominated by H^-, molecules, and dust, and we have approximate powerlaw scalings with T_{eff} for analytic considerations, but `MESA-web` uses tabulated values (cf. opacity lecture and references therein).

The location in T_{eff} of the Hayashi track is dependent on the mass M of the star: more massive stars are hotter since the very beginning. This can be analytically derived imposing $\nabla = \nabla_{\mathrm{ad}}$ and solving the remaining 3 ODEs assuming some form for $\kappa \equiv \kappa(T,\rho)$ at the photosphere: effectively the outer boundary condition and atmospheric physics determines this.

Stars to the left, hotter side of the Hayashi track instead must *not be /fully convective* and have some radiative layers (recombination and light-elements burning chaging κ and μ)!

Main sequence

As the gravothermal collapse continues and $\langle$ T $\rangle$ increases, at some point, if we are making a star, by *definition* nuclear burning turns on (option 2. above). This is when the central temperature (which at this stage is the highest temperature in the star), is sufficient to obtain enough tunneling through the Coulomb barriers.

Because it is abundant, and its burning releases a lot of energy per nucleon ($\sim$ 6.5MeV/nucleon) because it produces the double-magic nucleus ^{4}He $\equiv \alpha$ (neutrons *and* protons fill their nuclear "shells", by analogy with electron shells in atomic physics), hydrogen is the first fuel to ignite, see also nuclear burning lecture.

Structure during the main sequence

As we discussed in the nuclear reaction cycles lecture, hydrogen burning can occur in two different ways: pp-cycle and CN-NO bi-cycle.

Looking at `MESA-web` models, we can see that the pp-cycle is sufficient to achieve the equilibrium condition $L_{nuc}=\int dm\ \varepsilon_{nuc} \equiv L$ in low mass stars (**N.B.:** $L \propto M^x$ with $x \geq 1$). This is because the pp-cycle has lower Coulomb barriers (shallower relation between ε_{nuc} and T) but a higher normalization (cf. pp $\rightarrow$ CNO transition).

- **Very low M main sequence $\Rightarrow$ fully convective**

For the lowest-mass stars, T_{eff} remains cold and the opacity remains high: they burn through the pp cycle, but remain *convective* throughout the main sequence. In this case, *all* of the stellar material is available to burn, there is no core/envelope structure at all! These stars however have (relatively speaking) very low L, thus they evolve very slowly. All these stars in the Universe are still on the main-sequence! This is the case of the $0.3M_\odot$ star you computed for a homework, which has an approximately polytropic EOS because it is fully convective, thus has $\nabla = \nabla_{ad} \Rightarrow P \propto \rho^{\Gamma_1}$.

- **Low M main sequence $\Rightarrow$ radiative core, convective envelope**

Moving slightly higher in mass, meaning also to higher T_{eff}, a radiative core appears. the burning is very concentrated in the innermost region, but they are cool enough to have high κ at the surface, and thus retain a convective *envelope*:

N.B.: we are seeing that the cooler T_{eff} is the deeper the convective envelope! Increasing T_{eff} the convective layer disappear in the deepest layers. This can be shown analytically (see Onno Pols' lecture notes sec. 7.2.3).

- **High M main sequence $\Rightarrow$ convective core, radiative envelope**

Increasing M $\Leftrightarrow$ T_{eff} further, the equilibrium condition $L=L_{nuc}$ cannot be satisfied anymore with the pp-chain, and the CN-NO bi-cycle kicks in. Because of its higher Coulomb barriers, it has a steeper temperature dependence: the energy release is even more concentrated, implying that ∇ in the core is very steep (recall $\nabla \propto \kappa\ L \propto \kappa\ L_{nuc}$), thus *the core becomes convective*. This means that convective mixing makes a larger mass of hydrogen available to the very central burning zone. At the same time, higher M $\Rightarrow$ higher T_{eff} and the envelope becomes radiative.

N.B.: The threshold initial masses dividing the three regimes above are somewhat uncertain and dependent on input physics and modeling assumptions.

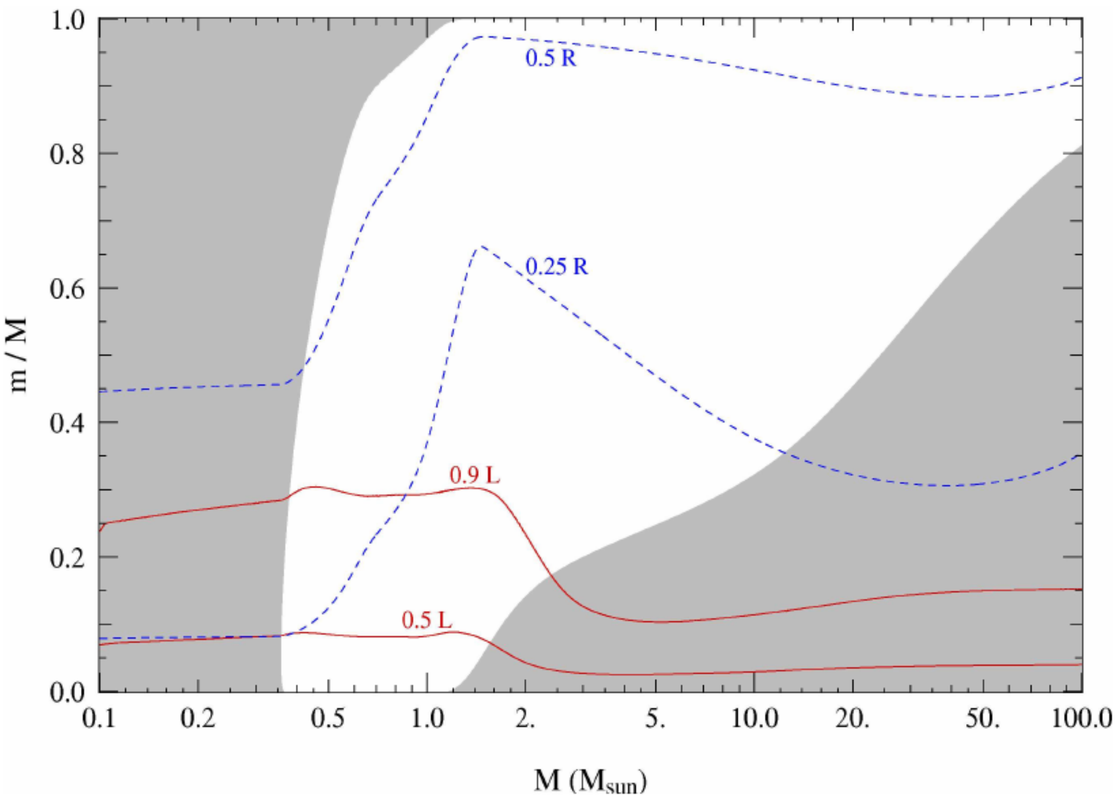

Figure 1: The "initial" gravothermal equilibrium structure of a star is determined only by mass M and composition. The figure (Fig. 9.8 in Onno Pols' notes, modified from Kippenhahn & Weigert) shows in gray the region in mass coordinate y=m/M that are convective as a function of the total mass M=$\int$ dm for Z=0.02 models. Red lines indicate where 50 and 90 % of the luminosity L is generated (the "burning region") and the blue dashed lines show r(m)=0.25M and r(m)=0.5M.

- **Q**: for your `MESA-web` models, what is the highest mass with a radiative main sequence core, and the lowest with convective main sequence core?

Evolution during the main sequence

During the main sequence L steadily increases on $\tau \sim \tau_{\rm nuc}$. This is because the conversion of hydrogen into helium decreases X (and increases Y), which enter in two key quantities, mean molecular weight and electron scattering opacity:

$$\mu \simeq \frac{1}{2X + \frac{3}{4}Y + \frac{Z}{2}} \quad , \kappa_{\rm es} = 0.2(1+X) \quad \mathrm{cm}^2\ \mathrm{g}^{-1} \quad . \tag{2}$$

Assuming a star to be in gravothermal equilibrium and assuming radiative energy transport (which we have just seen is not verified everywhere by `MESA-web` models!), we know that:

$$L \propto \frac{\mu^4 M^3}{\kappa} \quad , \tag{3}$$

This scaling relation is approximate and does not exactly hold if a star is not fully radiative (which we have already seen is not accurate!), but it tells that:

- the higher κ, that is, the harder it is for photons to get out, the lower the luminosity
- the higher the mass, the higher the luminosity ($\Rightarrow$ the higher the nuclear burning rate for a given fuel!), and since the mass exponent is larger than 1, this implies that *more massive stars have shorter lifetimes w.r.t. lower mass stars.* They do have more fuel available ($\propto$ M), but they burn through it at a higher rate ($\propto \mathrm{M}^3$)! In fact single-star lifetimes of stars that burn all the way to iron is only ∼10-50Myr ($\mathrm{M}_{\rm ZAMS} \geq 7.5\mathrm{M}_\odot$, with the exact lower limit depending on Z, rotation, binary interactions, cf. for example Doherty et al. 2017 and Poelarends et al. 2017), compared to $> 10^9$ years for $\mathrm{M}_{\rm ZAMS} \leq 2\mathrm{M}_\odot$.
- the higher the mean molecular weight μ (= number of particles per baryonic mass), the higher the luminosity.

Using Eq. 3 we can infer that the high power of μ drives the luminosity evolution of the stars during the main sequence: because hydrogen is converted into helium (X $\to$ Y), the **mass-weighted average $\langle\ \mu\ \rangle = \int$ dm μ(m)$/\int$ dm increases and thus L increases**.

N.B.: massive and low mass stars however have a very different morphology of the main sequence. For stars with radiative cores (burning through the pp-chain, $\mathrm{M} \leq 1.2\mathrm{M}_\odot$), L increases, R varies little, thus since $\mathrm{L}=4\pi\ \mathrm{R}^2 \sigma\ \mathrm{T_{eff}}^4$ in equilibrium, we also see a slight increase in temperature of the star during the main sequence. Conversely, massive stars with convective cores (burning through the CNO cycle, $\mathrm{M} \geq 1.2\mathrm{M}_\odot$) increase in radius and actually become *cooler* as they evolve during the main sequence. One can derive (see Onno Pols' notes chapter 7) analytic R(M) relations assuming a specific scaling for the energy generation to qualitatively explain this. In reality, the details of the core evolution (influenced by uncertain processes such as convective boundary mixing) and envelope (influenced by wind uncertainties) matter for the details.

N.B.: The relative role of μ and κ is slightly sensitive to metallicity too (because at lower Z the approximation $\kappa \simeq \kappa_{\rm es}$ is progressively better since fewer bound-bound and bound-free transitions are available, see also Xin et al. 2022). The opacity κ is dominant in determining the L and R at ZAMS for Z≃0.02, but the change in μ is determining their *evolution* along the main sequence.

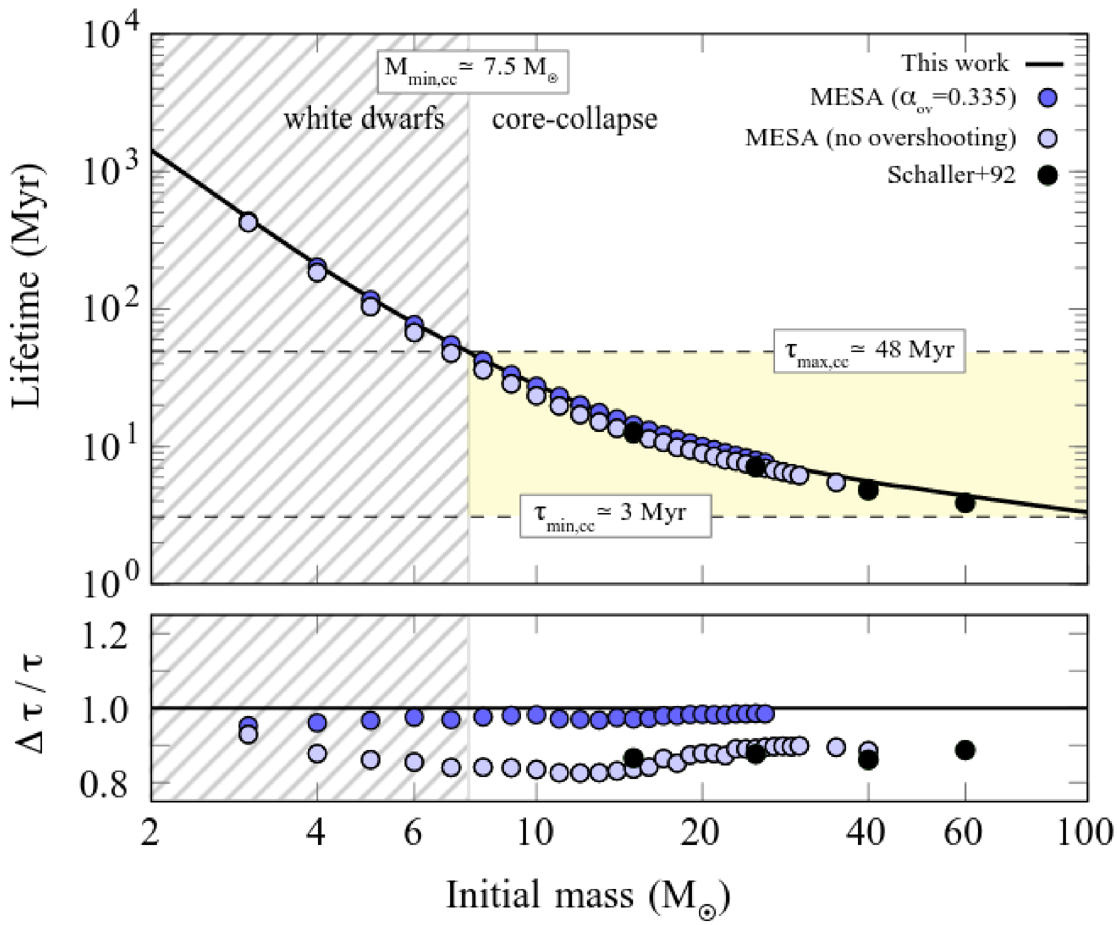

Figure 2: Stellar lifetime as a function of initial masses from Zapartas et al. 2017. MESA and GENEC models are shown, focusing on masses that result in a final core-collapse event. The bottom panel shows the deviations between the analytic fit and the numerical models.

- **Q**: based on the scaling in Eq. 3, how does the luminosity of two identical stars differing only in Z compare? Which star has the highest L? (**Hint**: you can compute more `MESA-web` models of your mass varying Z to check your answer!)

Looking at the Kippenhahn diagrams and composition diagrams from `MESA-web` we can also see what the model does in the core (something not *directly* accessible to observations - if not through neutrinos). For low mass stars with radiative cores and high $\rho_{\rm center}$ (something you can derive from the virial theorem + hydrostatic equilibrium + EOS), partial degeneracy already plays a role in sustaining the structure during the main sequence, and as the central burning region converts hydrogen into helium, the helium core becomes hot and degenerate - thus sustaining itself against gravitational collapse with the quantum effects due to the Fermi-Dirac statistics of electrons.

Conversely, high mass stars have a convective core: convective mixing connects the innermost burning region with a larger fuel reservoir. The progressive burning of hydrogen changes the center opacity (well approximated by electron scattering only in the hot, fully ionized interior) $\kappa \simeq \kappa_{\rm es}$=0.2(1+X) cm^2 g^{-1}. Specifically, as X decreases, so does κ, and since $\nabla = \partial \ln(T)/\partial \ln(\rho) \propto \kappa$ L, the temperature gradient becomes "less steep", meaning there is less need for convection: *during the main sequence of massive stars, the convective core receeds in mass coordinate.*

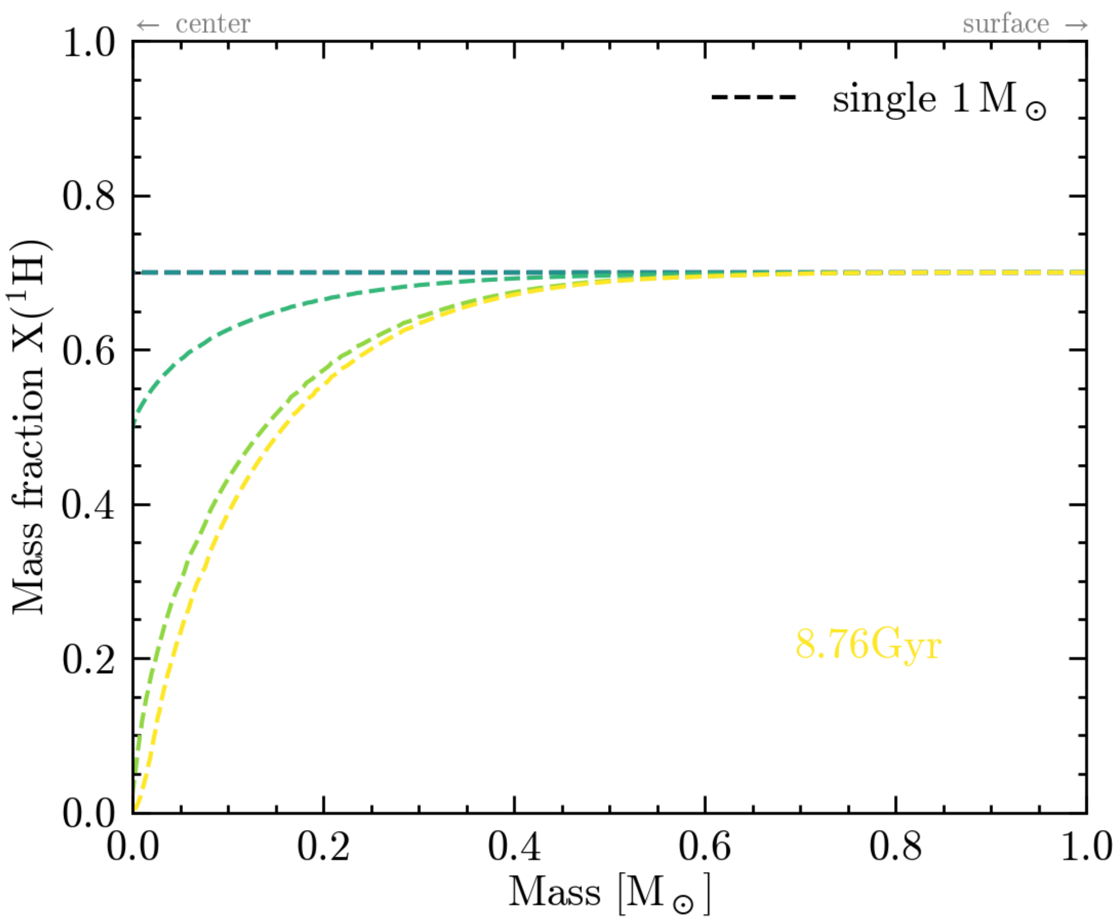

Figure 3: Hydrogen mass fraction X as a function of mass coordinate m for a single, non-rotating, 1M$_\odot$, Z=0.02 `MESA` model across its main sequence evolution. The color go from dark ($\sim$ ZAMS) to light ($\sim$ TAMS).

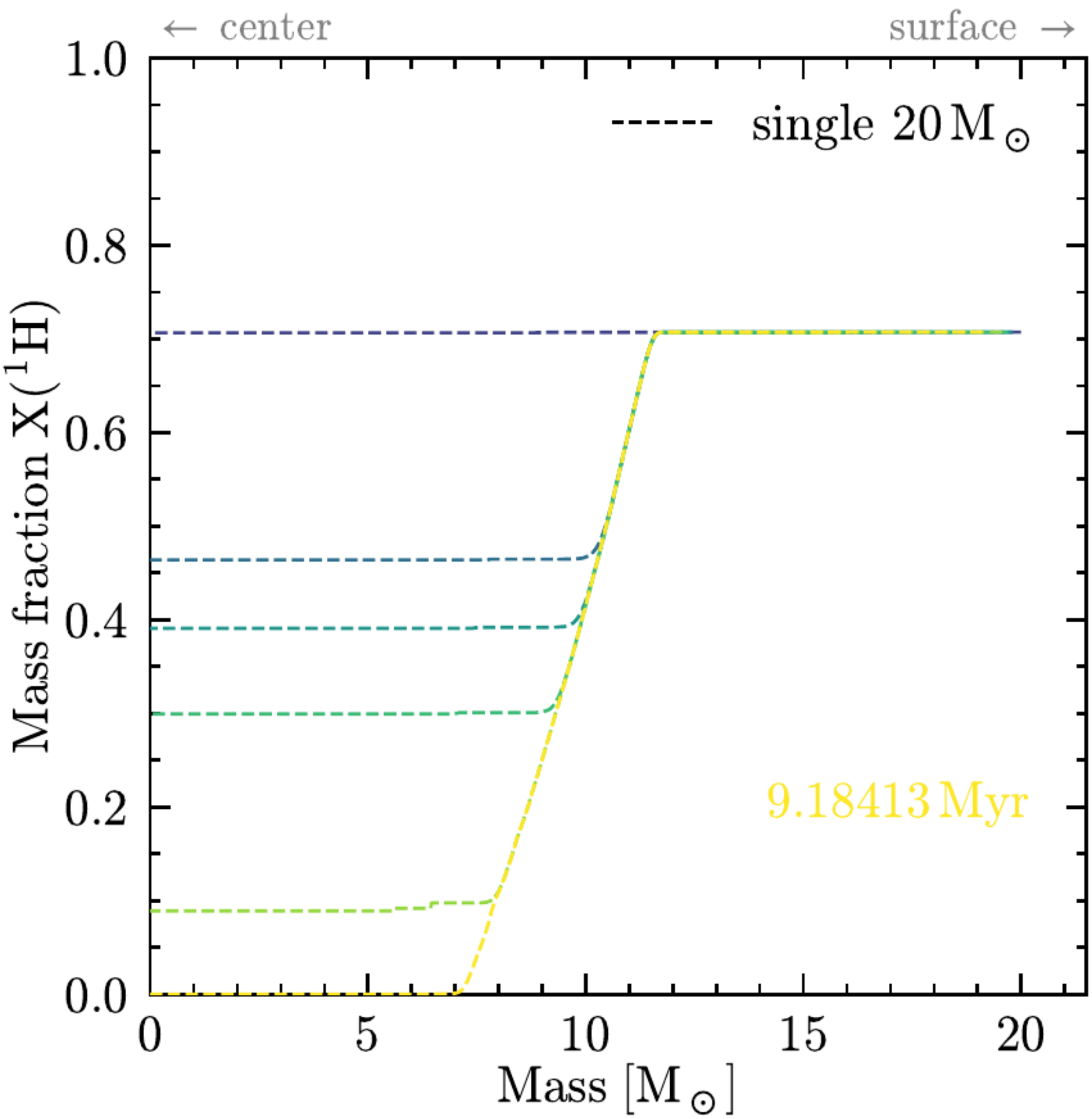

Figure 4: Hydrogen mass fraction X as a function of mass coordinate m for a single, non-rotating, $20M_{\odot}$, Z=0.001 `MESA` model across its main sequence evolution. The color go from dark ($\sim$ ZAMS) to light ($\sim$ TAMS), and as time passes the core recedes because of the change in κ.

End of the main sequence

"Low" mass stars with radiative cores

Very low mass stars smoothly evolve off the main sequence: if you look at the T(ρ) diagram in the movie produced by `MESA-web`, from the outlines of the track you can see where the nuclear burning moves.

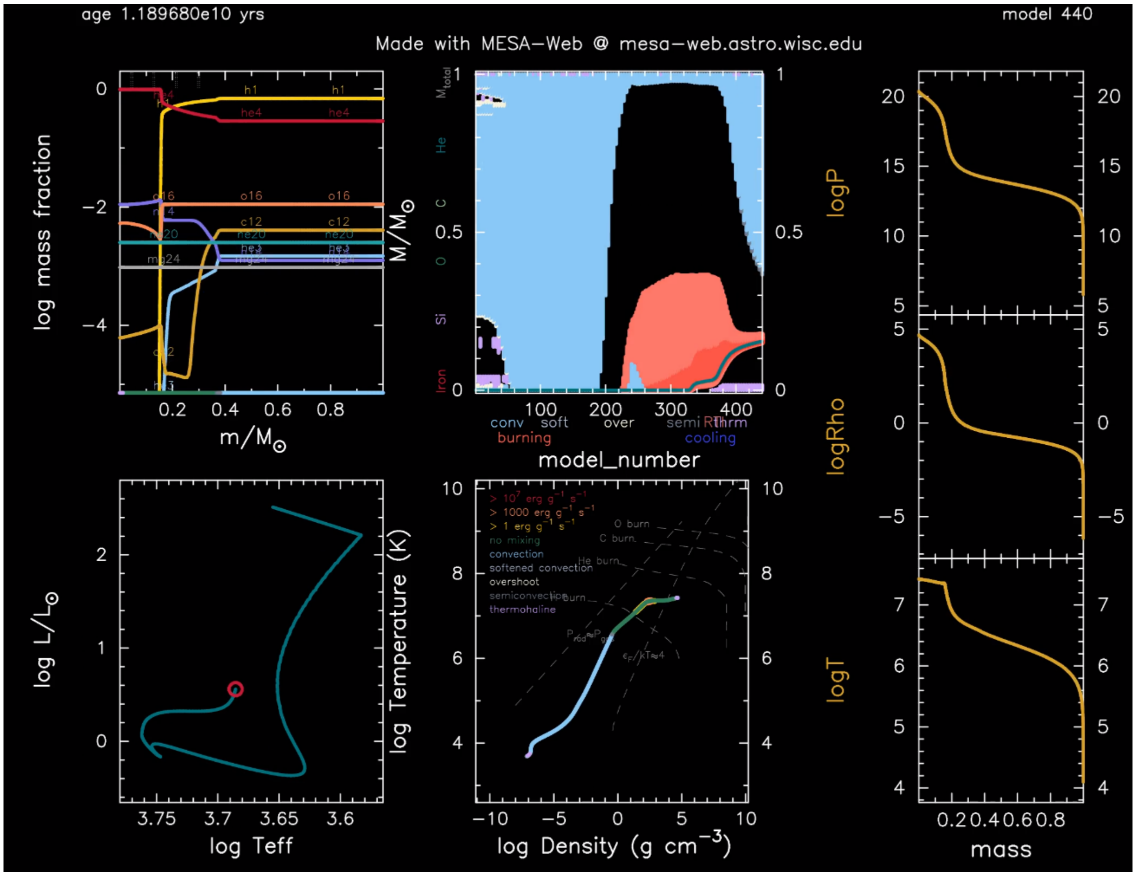

Figure 5: Screenshot of a `MESA-web` calculation of a $1M_\odot$ star shortly after the main sequence. The HRD (bottom left) shows a smooth end of the main sequence, and the Kippenhahn diagram and T(ρ) tracks (middle) show that all the burning is in a shell surrouding the inert He core. The bottom right panel shows that the inner region as a flattening T profile because of conduction efficiently transporting energy and erasing the dT/dr.

Since these are stars that were burning radiatively (the fully convective ones have not yet finished their main sequence even if they had been burning since the birth of the Universe!), they have just outside the region hot enough for hydrogen burning fresh fuel available that has not been mixed in the burning region. Therefore, **hydrogen ignites in a shell** around the now H-depleted, He-rich core.

Because of the gap in T to bridge the Coulomb barriers for hydrogen-burning and 3α, Helium core burning does *not* ignite immediately: the Helium core sits inert, contracts, degeneracy pressure starts to matter and conduction becomes important, leading to an almost *isothermal* He core sitting below the H shell.

The morphology of the end of the main sequence for low mass stars with radiative cores is *smooth*: the core contracts, the shell above it contracts and it is immediately hot enough to burn. The temperature of the shell is determined by the *contraction* of the inert He core, rather than by the energy generation by nuclear physics. Therefore, the shell is typically becoming hot enough to burn through the CNO cycle even for a low mass star.

"High" mass stars with convective cores

Increasing the mass above the threshold for activating the CN-NO bi-cycle (somewhere ∼1.1-1.3$M_\odot$ depending on assumptions), the morphology of the end of the main sequence changes.

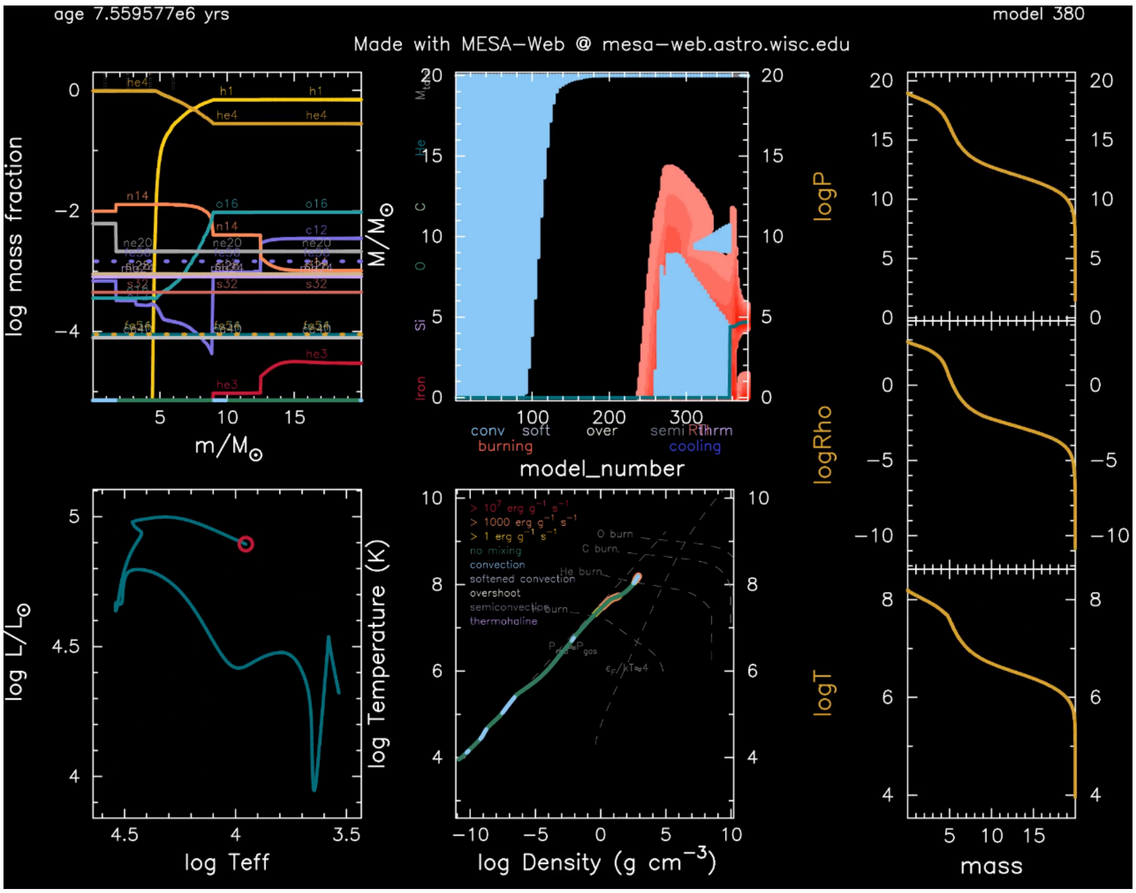

Figure 6: Screenshot of a `MESA-web` calculation of a 30$M_\odot$ star shortly after the main sequence. The HRD (bottom left) shows the "Henyey hook" feature, the Kippenhahn diagran and T(ρ) track shows that there is an off-center H-burning shell but the He in the core ignites promptly too. The core is not degenerate, but convective again, and mantains a nearly adiabatic temperature gradient.

In this case, during the main sequence the *burning* is even more centralized in mass and radius coordinate than for lower-mass pp-chain-sustained stars, but that drives *convection*. Therefore, convective mixing refuels the burning region from a larger reservoir, and when the fuel runs out, it means that there is a gap in the star between where T is hot enough for nuclear reactions and where viable fuel is. This causes an "overall contraction phase", also known as "Henyey hook", where the star, out of energy sources resumes its gravothermal collapse and shrinks in radius.

This process increases the temperature profile until the H-rich fuel left at the edge of the convective core ignites in a shell. However, the He core below, whose mass is set by the extent of convection (+convective boundary mixing) during the main sequence, is too big to be sustained by electron degeneracy pressure and too hot to be degenerate (recall that $\langle T \rangle \propto \mu M/R$): below the shell the contraction continues until He also promptly ignites through the 3α reaction, driving core convection!

H-shell and He burning

"[The post main sequence acts as a] *sort of magnifying glass, also revealing relentlessly the faults of calculations of earlier phases*" - Kippenhahn.

Low mass star "flashes"

For low mass stars the He core is sufficiently small to be electron-degeneracy supported, and there is H-rich fuel available right outside the region that was burning during the main sequence: after exhausting H in their core, they smoothly transition to a H-shell burning/He core degenerate phase. During this phase the core contracts and the envelope expands dramatically: the star appears as a red giant (RG)!

N.B.: during this phase the He core is degenerate and *conduction* by electrons efficiently transports energy making the whole core approximately isothermal. This leads to the Schonberg-Chandrasekhar maximum mass that it can have.

The microphysical reason for this expansion is not perfectly understood (and roughly once per decade a new tentative partial explanation is put forward). Nevertheless, we are confident that this does occur as we can see it happening across stellar populations. One partial explanation often invoked is the so called "mirror principle": when there is a shell source of energy, as the inner region contracts the outer regions expand (and viceversa). This "mirror principle" can be understood in terms of the virial theorem in its most complete form (including the $\ddot{I}$ term dependent on the moment of inertia): since the core contracts (decreasing the moment of inertia), the envelope needs to expand to compensate (increasing the moment of inertia). Another way to justify this semi-empirical "mirror principle" is to keep the shell energy generation constant (see Onno Pols' lecture notes, chapter 10).

The H-shell ignites wherever there is available fuel, its lower boundary temperature thus is determined by the structure of the contracting core, which typically exceeds the T threshold for the CNO cycle: even stars that burn through the pp-chain on the main sequence will do the CNO cycle later! The shell energy release also determines the structure of the envelope above: once the star is *not homogeneous* anymore, the simple gravothermal collapse due to the virial theorem complicates!

This also implies that it is the core structure which determines the properties of the shell, which determines the envelope properties (namely the luminosity): in fact we observe tight correlations between the core mass and the luminosity of the star.

As the evolution proceeds, the shell "climbs up in mass coordinate" (though its radius may stay constant or decrease even as the underlying inert He core contracts). The $T_{\rm eff}$ decreases and the convective envelope deepens ($T_{\rm eff}$ drops, $T_{\rm shell}$ is set by the core contraction and locked by nuclear reactions, thus ∇ steepens), this can reach the inner most layers (partially enriched in He, especially ^{3}He, and possibly ^{14}N if the star experienced some CN cycle), leading to the "first dredge up": material from the inner layers above the H-shell is mixed outwards by convection and becomes visible in the stellar atmosphere.

As the shell moves upwards by consuming H fuel (and dumping He ashes onto the core), it will encounter a layer mixed by convection in the first dredge up. The outward mixing of nuclearly processed material also corresponds to inward mixing of H-rich envelope material: the shell thus reaches a region that is *more fuel rich* than before! This makes the shell briefly exceed the $L_{nuc} = L$ condition, the overproduction of energy pushes the envelope to higher L, lower T_{eff}, and lowers the ρ in the shell, causing a decrease of L_{nuc}. This process ultimately results in stars crossing a certain luminosity threshold 3 times: observationally this produces a cumulation of stars at a certain luminosity or in other words a "bump" in the luminosity distribution.

N.B.: for massive stars, discussed below, the "first dredge up" may not occur as described here, but the H-shell will also move outwards towards more H-rich fuel causing a $3\times$ crossing of a certain luminosity.

He flash

Above is a `pgstar` movie of the He flash(es) in a $1M_\odot$ star computed with `MESA` by M. Cantiello. Note the panels are *different* than in the `MESA-web` configuration, and the HRD does *not* show the pre-main sequence.

As the H burning shell adds nuclear ashes to the underlying inert He core, until it reaches a mass that cannot be sustained by degeneracy pressure anymore, and He ignites. This typically occurs for $M_{He} \simeq 0.45 M_\odot$.

This ignition however happens in a degenerate environment where P does *not* depend on T! Therefore the energy released by the burning of He initially does not increase dP/dr and does not cause an expansion of the core, instead it all remains as internal energy, raising the temperature and increasing the nuclear burning rate: this situation (which presents itself any time there is a nuclear ignition in a degenerate environment) is clearly unstable and leads to the so called "Helium flash". Burning rises T until P transitions from being mostly due to electron degeneracy to being ideal gas again: this causes an abrupt change in pressure and a temporarily *dynamical* phase of the evolution!

Because this requires a specific He core mass, and the He core mass before the flashes is determining the total luminosity of the red giant, this means that pre-flash there is a "standardizable" maximum luminosity of red giants, the so called "tip of the red giant branch", which is nowadays used as an alternative method to measure distances for cosmological applications.

The occurrence of neutrino cooling in the core can cause the burning during the He flash to be initially off-center. Moreover, the star can react to the flash by (finally) expanding the core and decreasing the burning rate, and on a span of a few thermal timescale, minor secondary flash can occur as the core re-collapses, until He core burning finally stabilizes, lifting degeneracy and causing core convection.

Red clump and Horizontal branch

During He core burning, low mass stars have a convective core burning thought the 3α (and later $^{12}C(\alpha,\gamma)^{16}O$), surrounded by an inert He layer, and a H-burning shell wherever H becomes available. Above the H-burning shell, if there is a substantial H-rich envelope, it will be convective: these stars are close to the Hayashi track (by radius they are mostly convective), but on the hotter side (because of the existing radiative layers).

Since the He flash occurs as soon as the He core mass reaches a sufficient mass, all these stars have similar luminosities, and form the so-called "red clump" on the HR diagram, a noticeable feature in cluster and galaxy populations that can also be used for distance and age estimates (see also for example Girardi 2016). Since the mass of the He core at ignition for low mass stars is set by the He flash at $\sim 0.45 M_\odot$, the lower mass stars will have less envelope at this point (more has been processed into He to reach the threshold mass for the flash): from the red clump a there is a continuous almost horizontal line (they all have roughly the same luminosity set by the core mass) of stars in the HR diagram for low mass core-He burning stars whose coolest end is the red clump.

(continuing reading about the evolution of low mass here)

High mass stars and "Hertzsprung gap"

Stars with masses sufficiently high for the core to be convective during the hydrogen core burning main sequence ($M \geq 1.2 M_\odot$ roughly, depending on assumptions) will *not* have a phase of evolution with an inert, isothermal He core: the core is too big for degeneracy pressure to sustain it and after the main sequence it continues contracting until the 3α reaction activates and He burns. The prompt post "Henyey hook" appearance of two nuclear energy sources (He core and H shell) drives the star towards the cool side of the HR diagram very quickly ($\sim \tau_{\mathrm{KH}}$), becoming red supergiants (RSG)

Thus, in the HRD of a coeval stellar population, there will be many stars on the main sequence ($\tau \sim \tau_{\mathrm{nuc,H}}$) and close to the Hayashi track as RSG ($\tau \sim \tau_{\mathrm{nuc,He}}$), but very few in between: this is often referred to as the "Hertzsprung gap". **N.B.:** the scarcity of stars in the gap is only due to the timescales of evolution, it is not a forbidden region of the HRD.

Some stars may experience "blue loops" as their H-shell climbs upward in mass coordinate and encounters layers with more H (see for example Walmswell et al. 2015). The occurrence of these is very sensitive to numerical approximations and make solid predictions hard, but their physical nature in some cases is supported by observations. Depending on metallicity, some stars may even spend most of their He core burning time in a blue loop appearing hotter than a typical RSG.

Mass loss and single-star evolution path to Wolf-Rayet

As M increases (and consequently even more so L), mass loss becomes a progressively more important ingredient for the evolution of stars.

Stars can lose mass through:

- stellar winds (pressure driven for low mass stars, radiation driven for high mass stars)
- eruptive events (e.g., "luminous blue variable eruptions")
- binary interactions

All of these can directly or indirectly impact the internal structure of the star, and its appearance. Very massive stars may have such high mass loss rates that they lose their entire H-rich envelope already during the main sequence (becoming WNh stars). Moving to lower masses, they may evolve red-ward on the HR diagram (which increases the opacity κ and thus presumably the wind mass-loss rate, although this is highly debated presently, see Smith 2014, Renzo et al. 2017, Beasor et al. 2020, Decin et al. 2024), and then shed their H-rich envelope.

A star which has lost its envelope will "reveal" its He core, and if luminous enough, this will drive a thick wind that can enshroud the star and hide it below a "pseudo-photosphere". These winds can be so dense that collisional excitation produces emission lines, making the stars appear as WR (see e.g., Shenar 2024).

N.B.: stripped low and intermediate mass stars not luminous enough to drive WR-like outflows that produce emission lines are predicted and observed and require binary interactions to form, see Drout et al. 2023.

Late evolution

Low mass stars: AGB thermal pulses and WD cooling

After the end of He core burning, the *vast* majority of stars ($\sim$ 98% of all stars integrating over the birth-mass distribution for $M_{ZAMS} \leq 7.5 M_\odot$) is left with a carbon/oxygen rich degenerate core which is not massive enough to ignite further nuclear burning, and electron degeneracy sustains it. These stars however still need to lose their H-rich extended envelope and He-rich shell (which remain temporarily sustained by nuclear burning in shells) before they can finally rest as white dwarfs entirely sustained by degeneracy pressure. This process is relatively fast and involves copious episodic stellar outflows which are still an active topic of research.

A star in this phase is referred to as an "Asymptotic Giant Branch" (AGB) star: for most of its life the He layer is inert (no nuclear burning) and (partially) degenerate too, and it grows in mass because of the ashes of the overlaying H-burning shell, which sustains the H-rich envelope above it.

As the He layer grows in mass, it temporarily ignites: this energy release causes a *flash* (similar to He ignition in low mass stars in the first place), and expands the He layer, pushing outward the inner boundary of the H-shell, often until its density becomes too low for H-burning. Thus, as a consequence of the He shell flash, matter is pushed out and cools (possibly forming dust and increasing κ and thus the mass loss from the star), the H shell shuts off, but the He shell too does. This is because the flash was not hydrostatic self-regulated burning! The outer layers then re-collapse on a thermal timescale and as they contract, the H-burning shell ignites first (it's easier to burn H than He!), returning to the initial situation, but with a little less mass. This process of "thermal AGB pulses" ultimately will lead to the loss of all the H and He, leaving a "bare" CO core exposed, with only a very thin H/He atmosphere.

N.B.: ignition in a (partially) degenerate environment causes an abrupt increase in T and thus P from the ideal gas EOS, but the environment was supported by a T-independent degeneracy pressure: this leads to a discontinuity in time of the pressure and thus a *dynamical* event, referred to as a Flash. This can also occur in the core of massive stars!

N.B.: because of ν cooling, in AGB stars the center cools faster than the layers above it: this can lead to a "temperature inversion". At the boundary between this evolutionary end and the end of massive stars, the so-called "super-AGB" stars will ignite C off-center, but the ignition of carbon will then move inwards (in a $\sim$ meter thin shell) until it reaches the center, lifting the electron degeneracy by releasing nuclear energy, and allowing the star to evolve past C core burning.

After losing their envelopes to thermal pulses (possibly accompanied by a late enhancement of their stellar winds), low mass stars rapidly move from the top right (high L low T_{eff}) corner of the HR diagram to the lower left (low L high T_{eff}) corner becoming white dwarfs (WD): this process of contraction occurs on a thermal timescale. In WDs the gravothermal collapse stops because of the electron degeneracy pressure: the degeneracy decoupled their structure (which can be approximated assuming $k_B T \ll \varepsilon_{\mathrm{Fermi}} \Rightarrow T \simeq 0$) and their radiative properties.

These sit in the bottom left corner of the HR diagram: they actually have *hotter* surface temperatures compared to a main sequence star! This high T means they do radiate and lose energy, but because of the small radius ($R \simeq 0.01 R_\odot \simeq$ 1000km) they have a low luminosity $L = 4\pi\, R^2\, \sigma\, T_{\mathrm{eff}}$: their radiative cooling is very slow (timescale of billions of years). The WD will just "slide down slowly" on a cooling track. The WD cooling sequence provides a *clock* for stellar populations!

N.B.: Because of the M(R) relation for non-relativistic electron degeneracy gas in hydrostatic equilibrium, the *lower mass WDs have larger R*, thus for a given T_{eff}, they also have *higher L.*

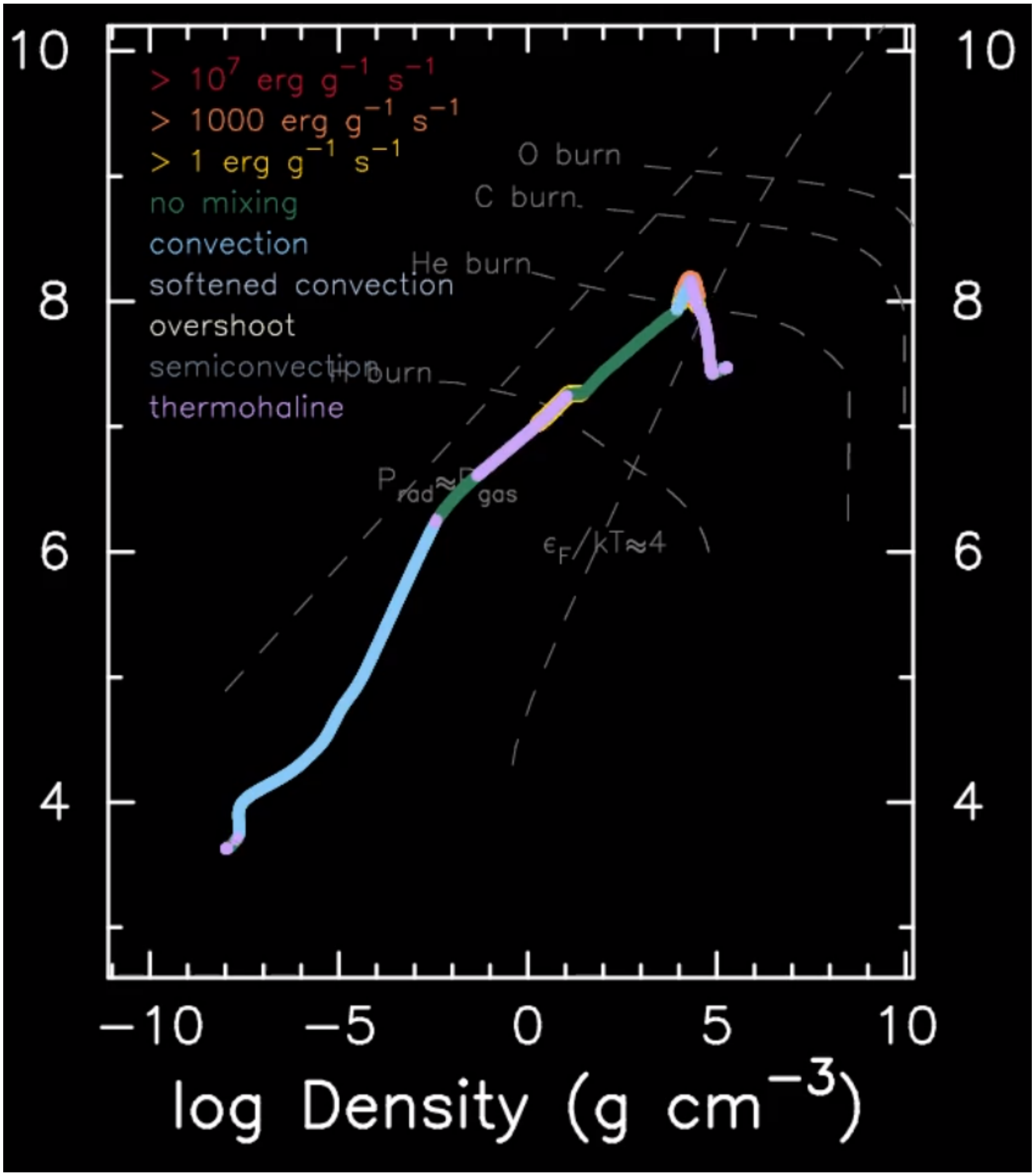

Figure 7: $T(\rho)$ diagram of a $1M_{\odot}$ `MESA-web` model during an AGB thermal pulse. Note the temperature inversion in the core, the presence of 2 burning shells in this snapshot (the He shell marked by the orange outline and the H shell marked by the yellow outline).

The *Gaia* spur: observational evidence for crystallization

As the WD cools, its core density increases, and its degenerate plasma will at some point crystallize. This phase transition releases latent heat thus slows down the cooling: in isochrones of WD populations we should expect an overabundance of stars in the region where we expect crystallization to occur, and this was tentatively observed thanks to the *Gaia* DR2 dataset for WDs within 100pc from Earth (Tremblay et al. 2019):

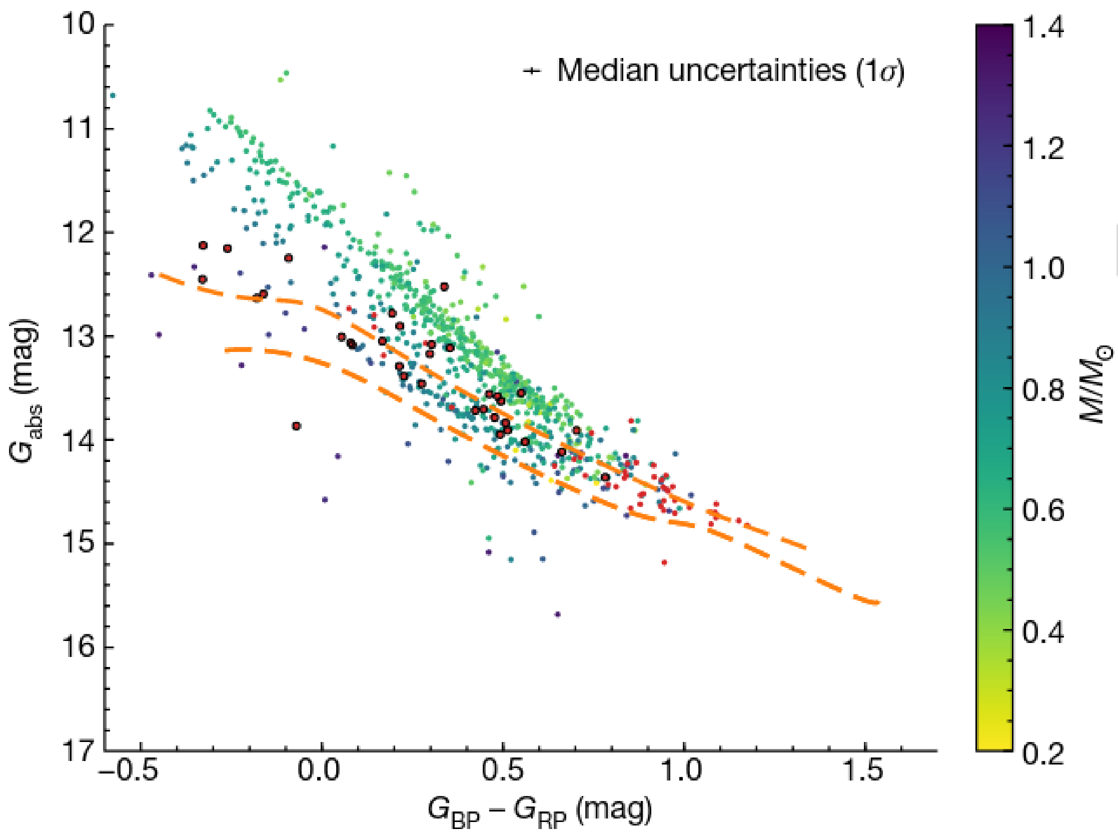

Figure 8: HR diagram for a WD sample within 100pc from *Gaia* DR2. The color of points indicates spectroscopically determined masses based on SDSS, red dots mark magnetized WD (again from their spectra), and dotted orange lines enclose the predicted region where crystallization of the WD covers between 20% (top) and 80% of the total mass, and contains an overabundance of stars as predicted by the release of latent heat. This is Fig. 2 from Tremblay et al. 2019

N.B.: Crystallization of a C-rich WD makes a stellar-mass, Earth-size diamond!

Other physical phenomena that can influence the evolution of WDs is the gravitational sedimentation of the composition, with heavier elements sinking and lighter elements rising, and for sufficiently high L (young WDs) there can also be radiative levitation, where the most opaque elements (typically the primordial iron present) will be pushed upwards by radiation.

- **Q**: Consider the formation of Helium WDs. These are observed, however, to form from a single star not-massive-enough to ignite He burning, it would take longer than the current age of the Universe (because low M $\Rightarrow$ much lower L $\Rightarrow$ $L_{nuc} = L$ drives a very slow evolution). Therefore, apparently, the existence of He WDs is paradoxical! Can you think of any solution to this apparent paradox?

High mass stars: ν speedup the evolution

As we discussed in the nuclear burning and neutrino lectures, for initially sufficiently massive stars ($M \geq 7$-$8 M_{\odot}$) the electron degeneracy pressure never suffice to stop the gravothermal collapse. As gravity drives their cores to higher and higher densities, $L_{\nu} \gg L_{\gamma}$ decoupling the neutrino-cooled core from the envelope: the nuclear timescale of the core becomes shorter than the thermal timescale of the envelope.

N.B.: partial electron degeneracy may play a role, depending on the mass, and at late burning phases "flashes" similar to the He flash in low mass stars can occur deep in the core.

Thus these stars proceed through burning all the way to iron, and each new fuel ignites in a more centralized, hotter mass range, surrounded by an inert layer, and then a shell of the previous nuclear fuel above it, creating the "onion structure" we have already seen.

While in theory the envelope should be completely "frozen" at this point, early observations of supernova explosions suggest that some *dynamical* coupling between core and envelope must occur in the last final years and months of the star, a topic of great research interest presently (see for example the review by Dessart 2024).

super-AGB stars

In the transition regime between intermediate mass stars (burning H convectively in their main sequence core but forming a WD at the end of their evolution) and massive stars we can define super-AGB stars: these burn partially carbon into a mixture of oxygen, neon, magnesium, after which they can experience thermal pulses (like lower mass AGB stars).

Typically, as the ONeMg core cools and contracts, it reaches densities sufficient to start electron captures, which remove the electrons sustaining the core leading to a so-called "electron capture SN".