400A - Binaries and dynamical masses

Mathieu Renzo

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Materials: Verbunt lecture notes (part 1), Sections 2.1-2.4 and 3.2 in Tauris & Van den Heuvel 2023 book, Chapter 7 of Carroll & Ostie's book, Chapter 3 in Prsa's book.

Measuring masses

This is typically a very hard problem: as we have seen, on the HRD/CMD there is a tight correlation between a star's magnitude and color. This holds for the majority ($>$90%) of stars, and thus for the longest phase of evolution. The linear relation between color and magnitude (that is effective temperature and luminosity) suggests there is one single parameter that determines the position on a star on this diagram. However, how do we determine what this parameter is, having only these observational data? It is generally not possible without relying on models. However, binaries can help!

Binaries are common

The Sun is clearly not in a binary system. However, it is **not** uncommon for stars to have gravitationally bound companion(s) orbiting around them, which can be of great help to determine their properties.

In fact, in this case we have two stars presumably born at roughly the same time, presumably with the same initial composition, and we can derive physical properties comparing them to each other.

Let's review some types of binaries.

Visual binaries

These are binaries that can be seen by naked eye. Typically, this means astrophysically very wide binaries that are unlikely to interact beyond the

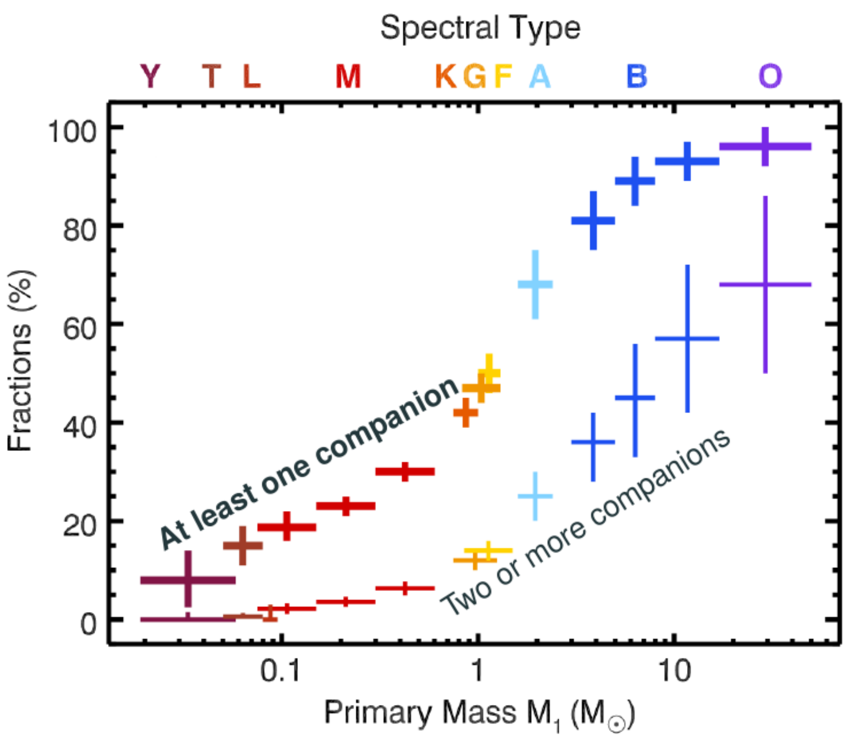

Figure 1: Fraction of binaries and higher multiplicity as a function of mass (bottom x-axis) or spectral type (top x-axis), modified from Offner et al. 2023.

fact that they are gravitationally bound and they orbit each other. Nevertheless, they have an important historical role, since Hershel observation of their motion of the sky over decades proved that Kepler's law are valid beyond the Solar system.

Mitchell 1767 also used statistical argument that all the visual binaries visible at the time could not be just chance alignment, and instead had to be somehow physically related to each other.

A classic example are Mizar & Alcor in the Big Dipper: the ability of finding the "double star" in this asterism was used as an eyesight test to become an astronomer in Hellenistic times. The name Alcor comes from the arabic for "hard to see"! Nowadays with modern telescopes we can see that Mizar is itself a quadruple system made of two binaries orbiting each other, and Alcor may itself be a binary, for a total of maybe 6 stars (e.g., Mamjek et al. 2010)!

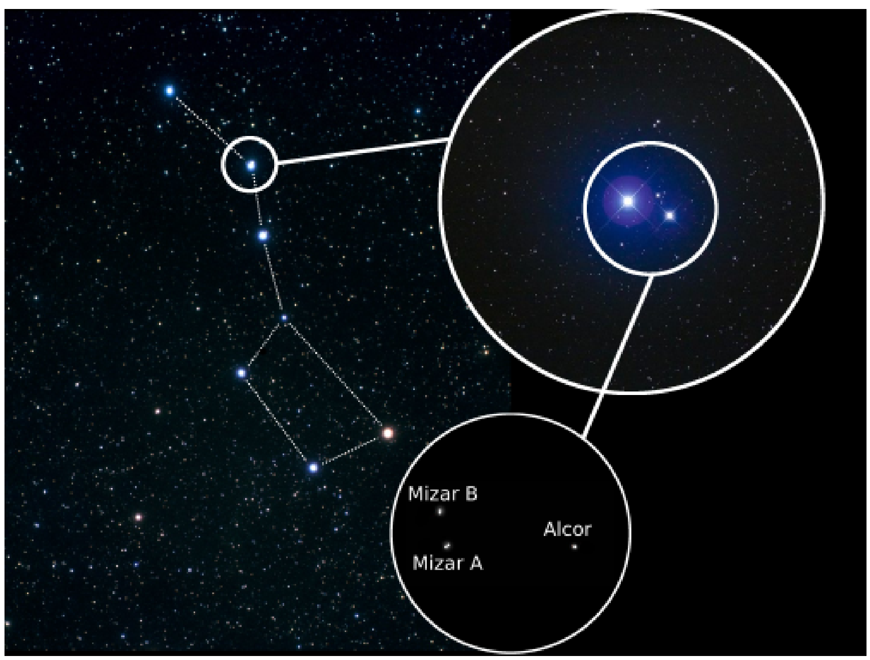

Figure 2: Zoom in on Mizar & Alcor from Renzo 2019

Astrometric binaries

Wide binaries where one of the two stars is too faint to be seen can still be found by looking at the peculiar motion of the star that can be seen on the sky. What matters is its *proper motion* (not the apparent motion due to the Earth rotation) with respect to background stars too far to be moving. For a single star, this is usually a straight line (because the timescale for it to turn around the Galactic center is typically $\sim$ hundreds of millions of years). However, if the star is in a binary, what moves along a straight line is the center of mass of the binary! The *photocenter* (i.e. the point from where we can detect light) will "wiggle" around this straight line due to orbital motion (and on top of the parallax periodic wiggle).

This was first detected for the binary Sirius A and Sirius B by Bessel in 1844 (the detection of photons from Sirius B followed in 1862 by Clark). Nowadays, this is a very timely because of the precise astrometric measurements by *Gaia*:

- this has been used to find *Gaia* BH1 (El Badry et al. 2023a), BH2 (El Badry 2023b), and BH3 (Gaia Collaboration, Panuzzo et al. 2024): these systems have stars orbiting around an otherwise undetectable Black Hole!
- it can also be used to find stars in wide orbits around NS or other faint stars, and possibly even massive planets!

Spectroscopic binaries

SB1 In a spectrum, one star of the binary dominates, but we can see its orbital motion as Doppler shifting of the spectral lines. Even if we never see the less bright star, we can infer its presence from the orbital motion of the brighter star.

N.B.: Radial velocity surveys that find massive and close-by planet are effectively finding SB1 binaries made of a star and a planet!

SB2 These are spectroscopic binaries where both stars are contributing light to the spectrum, meaning there will be two sets of spectral lines that are periodically Doppler shifted in couter-phase.

Eclipsing binaries

These are binaries when at some point in the orbit one star passes in front of the other. This blocks some of the light from one of the two stars causing an eclipse and a dip in the light curve (i.e., the plot of the luminosity of the system as a function of time).

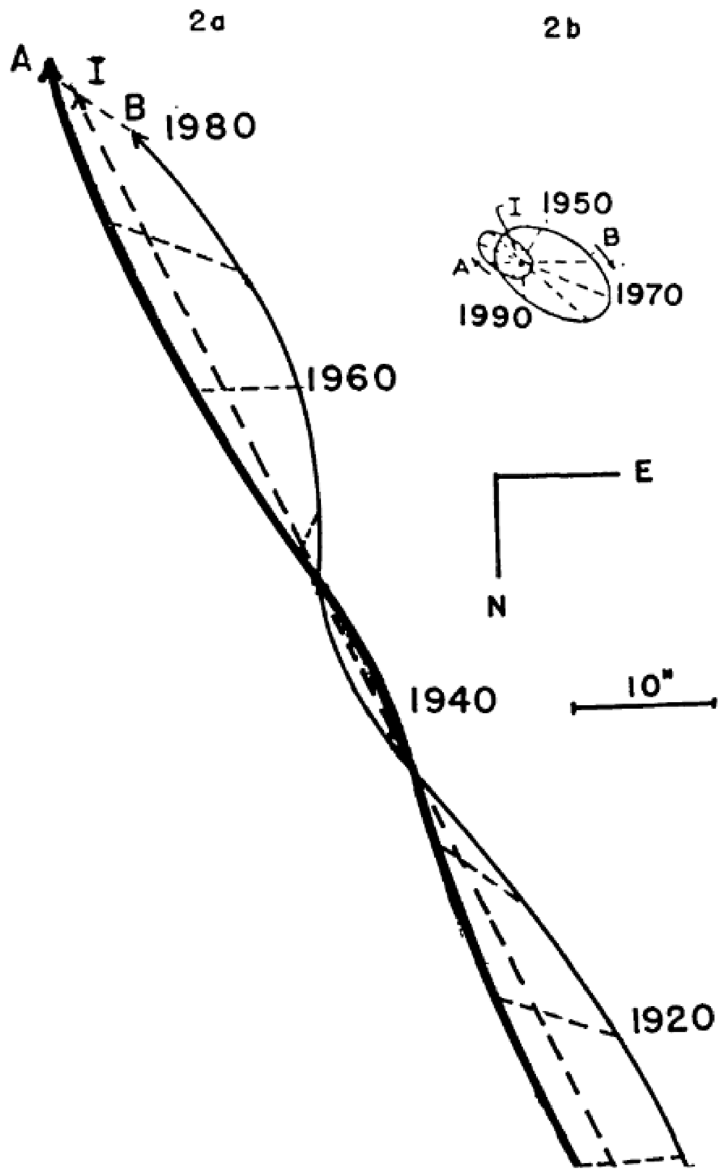

Figure 3: Path on the sky for Sirius A (thick line), Sirius B (thin line), and the center of mass (dashed line). Top right: absolute orbits around the center of mass. From Lippincott 1961.

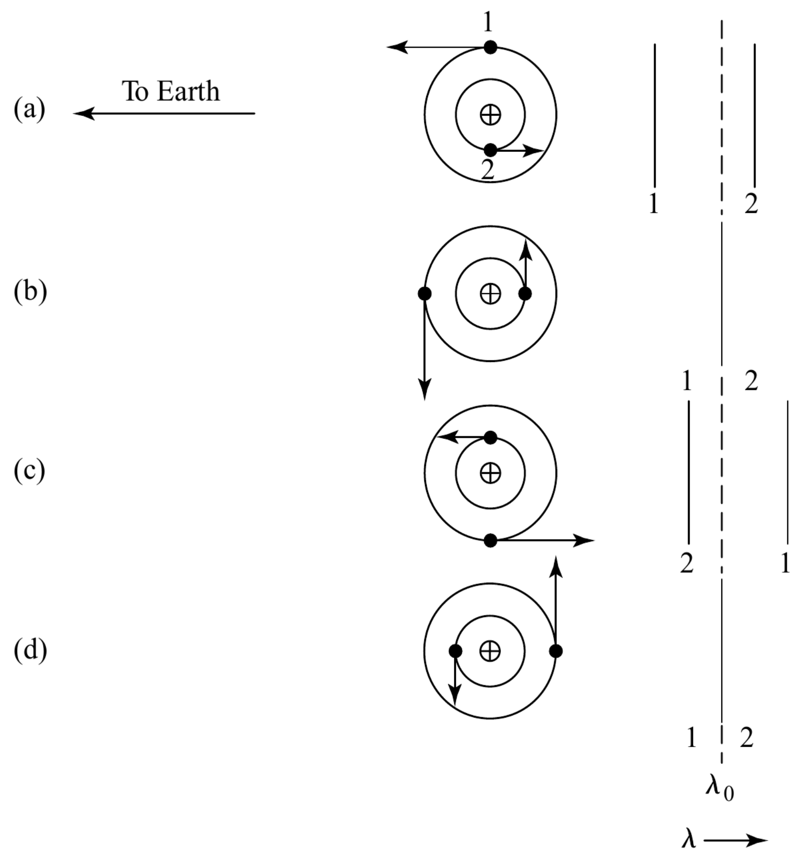

Figure 4: Schematic representation of the Doppler shift of spectral lines in binaries from which one can calculate radial velocities. Fig. 7.3 in Carrol & Ostie 2007 or 2.3 in Tauris & van den Heuvel 2023 books.

This of course can only happen if the inclination of the orbital plane of the binary w.r.t. the line of sight is not too large. Typically, astronomers define the inclination of the orbital plane i w.r.t. the plane of the sky, therefore the line-of-sight (which is perpendicular to the plane of the sky by definition) has an angle $\varphi = \pi/2$ - i w.r.t. the orbital plane. To have an eclipse, at least partial,

$$\sin(\varphi) \leq \frac{R_1 + R_2}{a} \quad , \tag{1}$$

where we use that $\sin(\varphi)=\cos(i)$, and a $\times$ cos(i) is the projected binary separation a on the plane of the sky, which is smaller than R_1+R_2 with R_i stellar radii, leads to an eclipse. The eclipse will be total if

$$\sin(\varphi) \leq \frac{R_1 - R_2}{a} \tag{2}$$

The time duration of an eclipse can be used to determine the radii of the two stars. Consider the following figure:

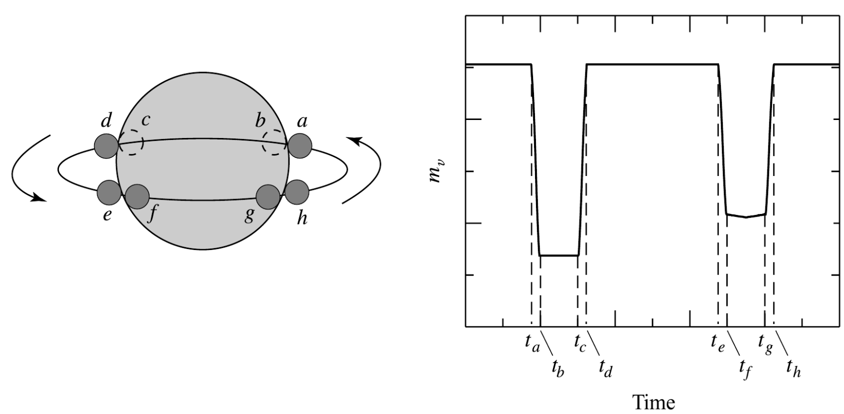

Figure 5: Light cuve (visual magnitude as function of time) for an eclipsing binary with i=$\pi/2$, figure 7.9 in Carrol & Ostie's book

one can see that, labeling with 2 the geometrically smaller star (darker gray) and assuming the orbit to be circular and the semi-major axis to be a and the period P:

$$\frac{t_d - t_a}{P} = \frac{2(R_1 + R_2)}{2\pi a} \quad , \tag{3}$$

and

$$\frac{t_c - t_b}{P} = \frac{2(R_1 - R_2)}{2\pi a} \quad . \tag{4}$$

These equations allow to calculate R_j/a and if also the masses are known also the individual radii R_j using Kepler's laws to get rid of the orbital semi-major axis a.

- **Q**: In the figure above, which star is assumed to be the hotter one?
- **Q**: Can you see what happens to the light curve minima if we have an inclination of the binary orbit w.r.t. the plane of the sky?

Dynamical mass determination

Visual binaries

Using Kepler's laws, we can use binaries to measure observationally masses without relying on hard, expensive, and uncertain stellar models.

In particular the third law can be written as

$$G(M_1 + M_2) = n^2 a^3 \quad , \tag{5}$$

where M_i are the masses of the stars, n$= 2\pi/$P is the angular velocity (a.k.a. "true anomaly") and a is the semimajor axis of the orbit (of the orbit of the reduced mass point $\mu = M_1M_2/(M_1+M_2)$ around the center of mass).

For a visual binary (see above) where we can measure the semi-major axes a_i of both stars around the center of mass, we also have another equation that follows from the definition of center of mass:

$$M_1 a_1 = M_2 a_2 \quad , \tag{6}$$

where $a_1 + a_2 =$ a. If we know the distance d to the binary (d $\gg$ a, so we can consider both stars at the same distance), then the angular size of the ellipse that each stars traces on the orbit of the sky is $\alpha_j = a_j/d$ for j=1,2. In reality, the orbit is typically not on the plane of the sky, but it has an inclination! This means what we can see is not a_j but the projection of it on the plane of the sky a_j sin(i). We can then solve these two equations for the individual stellar masses (just having observations long enough to trace the semi-major axes of wide visual binaries and knowing their distances, for example because we have parallax measurements).

N.B.: even without knowing the distance d, using Eq. 6 one can still determine the mass ratio $q=M_2/M_1$!

Spectroscopic (and eclipsing) binaries

In this case, we have access to more information thanks to the variable "radial velocity/ies" (RV) of the stars, that is the measurable Doppler shift of their spectral lines due to the orbital motion.

However, what we can see though this is only the *projected* orbital motion along the line of sight (because we only get Doppler shift along the line of sight), this is why the terminology is "radial velocities": it's velocities along the radii of the celestial sphere centered on the observer!

Following standard nomenclature, let's call z the direction of the line of sight, using xy for the plane of the sky. By definition the radial velocity of each star is dz/dt. ν here is the true anomaly (such that $d\nu/dt = n$), ω is the argument of periastron - that is the angle between the line of nodes (line where the plane of the sky, assumed to contain the focus of the ellipse, and the plane of the orbit intersect) and the direction of periastron.

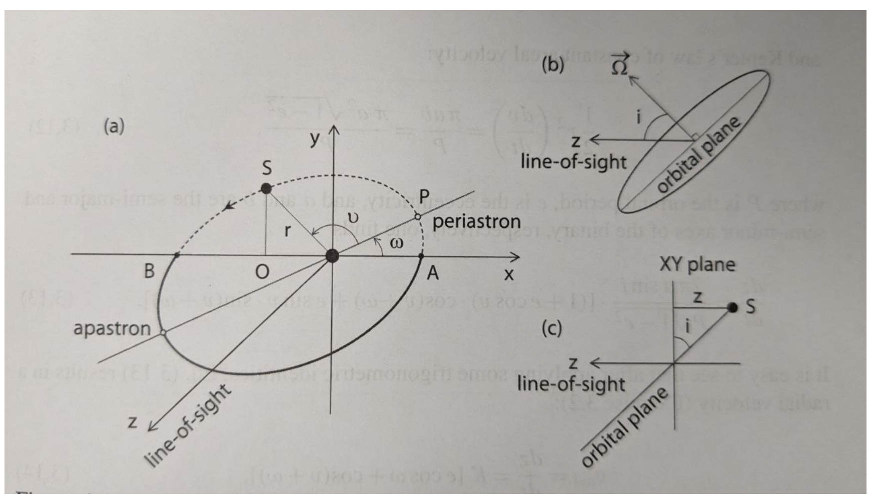

Figure 6: Geometry of a binary orbit. This is Fig. 3.3 in Tauris & van den Heuvel 2023 book.

The one can calculate: $z = r\sin(i)\,\sin(\nu+\omega)$ with r radius of the orbit (which for an elliptical orbit is a function of time), the sin(i) factor projects it on the plane of the sky and the sin(ν+ω) projects on the line of sight. The radial velocity is therefore:

$$\frac{dz}{dt} = \left(r\cos(\nu+\omega)\frac{d\nu}{dt} + \frac{dr}{dt}\sin(\nu+\omega) \right)\sin(i) \tag{7}$$

which using Kepler's second law

$$\frac{1}{2}r^2\frac{d\nu}{dt} = \frac{\pi ab}{P} = \frac{\pi a^2\sqrt{1-e^2}}{P} \quad , \tag{8}$$

where P is the orbital period, a and b are semi-major and semi-minor axes, and

$$e = \sqrt{1-b^2/a^2} \quad , \tag{9}$$

is the eccentricity, and the relation between r$\equiv$ r(ν) for an ellipse:

$$r = \frac{a(1-e^2)}{1+e\cos(\nu)} \quad , \tag{10}$$

one can finally rewrite the radial velocity from Eq. 7 as

$$v_{\rm rad} = \frac{dz}{dt} = K(ecos(\omega) + cos(\omega+\nu)) \tag{11}$$

where

$$K = \frac{2\pi a\sin(i)}{P\sqrt{1-e^2}} \tag{12}$$

is the so-called *semi-amplitude* of the radial velocity, which apart from the sin(i) factor depends only on observable quantities if one can get spectra of many epochs (i.e., return to take spectra of the same binary at many times, typically at least 7 spectra are required for a decent fit to all these parameters):

- P $\rightarrow$ from the time interval between repeating identical spectra
- e $\rightarrow$ from the shape of the measured radial velocities as a function of time (i.e., radial velocity curve)
- ω $\rightarrow$ similarly as e, it influences the shape of the radial velocity curve and can be deduced from it

- asin(i) $\rightarrow$ if the binary is SB2, then we can measure $K_1 \equiv K_1(a_1)$ and $K_2 \equiv K_2\ (a_2)$ and use the relation between $a_1+a_2 = a$ to work out asin(i)

- ν $\rightarrow$ this is less directly obtained by performing an orbital fit using other orbital parameters (mean anomaly and eccentric anomaly)

N.B.: In a real observation, one also needs to remove the component along the line of sight due to the motion of the Earth around the Sun, and the peculiar motion of the Sun (i.e., v_{rad} from Eq. 11 $\rightarrow$ v_{rad}-γ with γ peculiar velocity of the observer).

Therefore, for an SB2 binary we can observe $dz_j/dt = v_{rad,\,j}$ for each j=1,2 star, and we can determine K_1 and K_2. using Eq. 5 multiplied by $\sin^3$(i) to make the projection of the semi-major axis of the orbit on the line of sight, a $\times$ sin(i):

$$G(M_1 + M_2)\sin^3(i) = n^2 a^3 \sin^3(i) \quad , \tag{13}$$

and using 6 rewritten as

$$\frac{a_1 \sin(i)}{a_2 \sin(i)} = \frac{M_2}{M_1} \quad , \tag{14}$$

we can solve for $M_j \sin^3$(i), where except for the inclination angle of the orbit, we have the masses! In the case of an *eclipsing SB2 binary*, from the eclipses we can measure the inclination angle and obtain a direct measurement of the masses. These are the most precise mass measurements (so-called "dynamical masses", because they are based on orbital dynamics). They are only possible thanks to the occurrence in nature of stellar binaries!

In the case of an SB1 binary, when only one spectrum is visible, we can write

$$a = a_1 + a_2 = a_1(1 + \frac{a_2}{a_1}) = a_1(1 + \frac{M_1}{M_2}) = a_1\frac{(M_1 + M_2)}{M_2} \quad . \tag{15}$$

Thus, we can rewrite Eq. 5 $\times$ $\sin^3$(i) as

$$(M_1 + M_2)\sin^3(i)\frac{M_2^3}{(M_1 + M_2)^3} = \frac{n^2}{G} a_1^3 \sin^3(i) \tag{16}$$

where the r.h.s. depends only on observables, and thus this allow us to derive observationally the mass function

$$f(M_1, M_2) = \frac{M_2^3 \sin^3(i)}{(M_1 + M_2)^2} \quad , \tag{17}$$

which together with an estimate of the mass-ratio provides a limit on M_2.

Mass-luminosity relation

Using systems for which this mass and radius measurements are possible, we can derive empirical mass-luminosity and mass-radius relations:

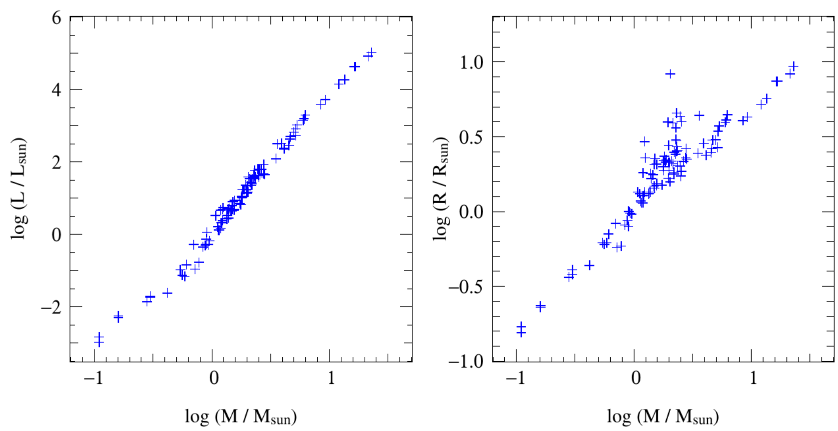

Figure 7: Mass-luminosity and Mass-radius empirical relations for a set of well measured SB2 eclipsing binaries. This is Fig. 1.3 in Onno Pols' lecture notes

In the best astronomical tradition, this was initially fit with a powerlaw, but as the range of stellar masses explored grew, this became a broken powerlaw:

$$L \propto M^x \tag{18}$$

with x=4 for $M \leq 0.8 M_\odot$, x=3 for $0.8 M_\odot < M < 50 M_\odot$, x=1 for $M \geq 50 M_\odot$. These thresholds are extremely approximated, probably metallicity dependent, as we will possibly see later in the course. Moreover, and importantly, these hold only for so-called "main sequence" stars, the one following also the $L(T_{eff})$ relatively tight relation on the color-magnitude diagram: evolved stars are a different problem!

Homework

Exercise 7.4 on Carroll & Ostie

Sirius is a visual binary with a period of 49.94 yr. Its measured trigonometric parallax is 0.37921" ± 0.00158" and, assuming that the plane of the orbit is in the plane of the sky, the true angular extent of the semimajor axis of the reduced mass is 7.61". The ratio of the distances of Sirius A and Sirius B from the center of mass is a $a_A / a_B = 0.466$.

1. Find the mass of each member of the system.

2. The absolute bolometric magnitude of Sirius A is 1.36, and Sirius B has an absolute bolometric magnitude of 8.79. Determine their luminosities. Express your answers in terms of the luminosity of the luminosity of the Sun

3. The effective temperature of Sirius B is approximately 24790 K ± 100 K. Estimate its radius, and compare your answer to the radii of the Sun and Earth. What kind of star is that hot with that radius?

Exercise 7.6 on Carroll & Ostie

From the light and velocity curves of an eclipsing, spectroscopic binary star system, it is determined that the orbital period is 6.31 yr, and the maximum radial velocities of Stars A and B are 5.4 km s^{-1} and 22.4 km s^{1}, respectively. Furthermore, the time period between first contact and minimum light (t_b t_a) is 0.58 d, the length of the primary minimum (t_c t_b) is 0.64 d, and the apparent bolometric magnitudes of maximum, primary minimum, and secondary minimum are 5.40 magnitudes, 9.20 magnitudes, and 5.44 magnitudes, respectively. From this information, and assuming circular orbits, find:

1. Ratio of stellar masses

2. Sum of the masses (assume $i = 90$ degrees)

3. Individual masses

4. Individual radii (assume that the orbits are circular)
5. Ratio of the effective temperatures of the two stars