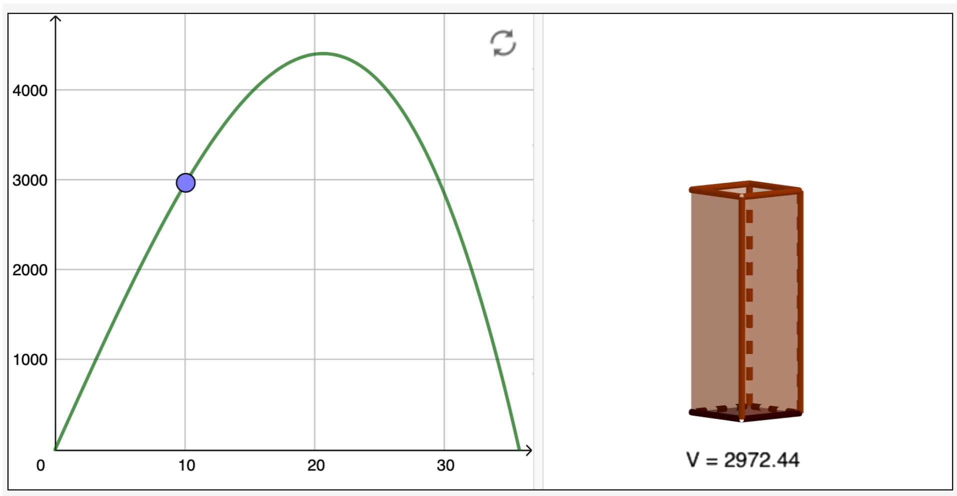
1. **(1 point)** `Riddle/BoxMaxVolumeScaffold.pg`



Note: This problem uses an interactive GeoGebra applet. It may take a few seconds for the figures to appear. **Instructions**

A square-bottomed box with no top must have a fixed surface area of 1056 cm^2. In this problem you will find the size of the box that will maximize its volume. The graph above shows a plot of the volume as a function of the base length. Move the indicated point along the graph to visualize how the volume changes for different lengths of the square base, and use this to estimate the optimal size of the bottom square.

Click on Part 1 to open the section. Enter your answers for the two questions in Part 1, then click on Submit Answers. Once your answers are both correct, you will be able to open Part 2. When your answers to Part 2 are both correct, continue to Part 3 to find the optimal width and height for the box with maximum volume.

Part 1: The Setup
Let x be the width of the square base and let h be the height of the box.

What is the surface area of the box in terms of x and h?

A = ____________

What is the volume of the box in terms of x and h?

V = ____________

Hint: *(Instructor hint preview: show the student hint after the following number of attempts: 0*
The surface area of an open box is the sum of the area of the square base and the area of the four sides of the box. The volume of the box is the area of the base times the height.

Solution: *(Instructor solution preview: show the student solution after due date.)*
The surface area of an open box is the sum of the area of the square base and the area of the four sides of the

box. The base has area x^2. Each side of the box has area xh. The surface area of an open box is therefore $A = x^2 + 4xh$. The volume of the box is $V = x^2 h$.

Part 2: The Calculus Part: Function and Derivative
Use the fact that $A = 1056$ to determine a function $V(x)$ that gives the volume of the box in terms of the width x of the base (this is the function plotted above).

$V(x)$ = ____________

What is the derivative of $V(x)$?

$V'(x)$ = ____________

Hint: *(Instructor hint preview: show the student hint after the following number of attempts: 0*
Solve the area equation for h and substitute into the volume formula to get a function for the surface area only in terms of x, then simplify as much as possible before taking the derivative.

Solution: *(Instructor solution preview: show the student solution after due date.)*
The area of the box is $x^2 + 4xh = 1056$. Solving this equation for h gives

$$\begin{aligned} x^2 + 4xh &= 1056 \\ 4xh &= 1056 - x^2 \\ h &= \frac{1056 - x^2}{4x} \end{aligned}$$

Now substitut into $V = x^2 h$ and simplify.

$$V(x) = x^2 \left(\frac{1056 - x^2}{4x} \right) = \frac{1}{4} \left(1056x - x^3\right)$$

The domain of this function is $0 \leq x \leq \sqrt{1056}$ since the you want $V(x) \geq 0$ with $x \geq 0$.

$$V'(x) = \frac{1}{4} \left(1056 - 3x^2\right)$$

Part 3: The Calculus Part: Maximizing the Volume
Use the derivative to find the **exact** width x that maximizes the volume.

x = ____________

What is the **exact** height of the box of maximum volume?

h = ____________

Solution: *(Instructor solution preview: show the student solution after due date.)*
To maximize V, take the derivative and set it equal to 0.

$$V'(x) = \frac{1}{4}\left(1056 - 3x^2\right) = 0$$

$$1056 - 3x^2 = 0$$

$$x^2 = \frac{1056}{3} = 352$$

$$x = \sqrt{352}$$

The graph shows that this is the global maximum on the domain $0 \le x \le \sqrt{A}$ of V. Calculus can verify this. Note that $V'' = -\frac{6}{4}x < 0$ and therefore the critical point is a local maximum by the second derivative test. But since V is a continuous function and there is only one critical point in the interval $0 \le x \le \sqrt{A}$, this must actually be the global maximum on this interval.

The optimal height is

$$h = \frac{A - x^2}{4x} = \frac{1056 - 352}{4\sqrt{352}}$$

$$= \frac{704}{4\sqrt{352}} = \frac{2(352)}{4\sqrt{352}}$$

$$= \frac{\sqrt{352}}{2}$$

Therefore the optimal height is half the optimal base length. Can you show that this is always the case for any value of the fixed surface area A?

Correct Answers:

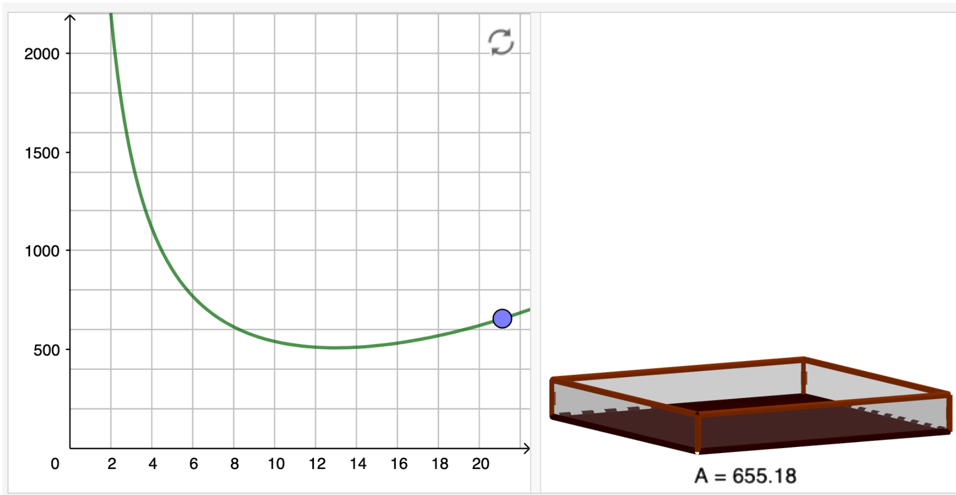
- `x^2+4*x*h`
- `x^2*h`
- `(1056*x-x^3)/4`
- `(1056-3*x^2)/4`
- `sqrt(352)`
- `[sqrt(352)]/2`

---

2. **(1 point)** `Riddle/BoxMinAreaScafford.pg`

Note: This problem uses an interactive GeoGebra applet. It may take a few seconds for the figures to appear. **Instructions**

A square-bottomed box with no top must contain a fixed volume of 1056 cm^3. In this problem you will find the size of the box that will minimize its surface area. The graph above shows a plot of the surface area as a function of the base length. Move the indicated point along the graph to visualize how the surface area



changes for different lengths of the square base of the box, and use this to estimate the optimal size of the bottom square.

Click on Part 1 to open the section. Enter your answers for the two questions in Part 1, then click on Submit Answers. Once your answers are both correct, you will be able to open Part 2. When your answers to Part 2 are both correct, continue to Part 3 to find the optimal width and height for the box with minimum surface area.

Part 1: The Setup
Let x be the width of the square base and let h be the height of the box.

What is the surface area of the box in terms of x and h?

$A =$ ____________

What is the volume of the box in terms of x and h?

$V =$ ____________

Hint: *(Instructor hint preview: show the student hint after the following number of attempts: 0*
The surface area of an open box is the sum of the area of the square base and the area of the four sides of the box. The volume of the box is the area of the base times the height.

Solution: *(Instructor solution preview: show the student solution after due date.)*
The surface area of an open box is the sum of the area of the square base and the area of the four sides of the box. The base has area x^2. Each side of the box has area xh. The surface area of an open box is therefore $A = x^2 + 4xh$. The volume of the box is $V = x^2h$.

Part 2: The Calculus Part: Function and Derivative
Use the fact that $V = 1056$ to determine a function $A(x)$ that gives the surface area of the box in terms of the width x of the base (this is the function plotted above).

$A(x) =$ ____________

What is the derivative of $A(x)$?

$A'(x)$ = ____________

Hint: *(Instructor hint preview: show the student hint after the following number of attempts: 0*
Solve the volume equation for h and substitute into the surface area formula to get a function for the surface area only in terms of x, then simplify as much as possible before taking the derivative.

Solution: *(Instructor solution preview: show the student solution after due date.)*
The volume of the box is $x^2h = 1056$. Solving this equation for h and substituting into A gives

$$A(x) = x^2 + 4x\left(\frac{1056}{x^2}\right) = x^2 + \frac{4224}{x}$$

The domain of this function is $x > 0$.

$$A'(x) = 2x - \frac{4224}{x^2}$$

Part 3: The Calculus Part: Minimizing the Surface Area
Use the derivative to find the **exact** width x that minimizes the surface area.

x = ____________

What is the **exact** height of the box with minimum surface area?

h = ____________

Solution: *(Instructor solution preview: show the student solution after due date.)*
To minimize $A(x)$, take the derivative and set it equal to 0.

$$A'(x) = 2x - \frac{4224}{x^2} = 0$$

$$2x^3 - 4224 = 0$$

$$x^3 = \frac{4224}{2} = 2112$$

$$x = (2112)^{1/3}$$

The graph suggests that this is the global minimum. Calculus can verify this. Note that

$$A''(x) = 2 + \frac{2(4224)}{x^3} > 0$$

and therefore the critical point is a local minimum by the second derivative test. But since A is a continuous function for $x > 0$ and there is only one critical point, this must actually be the global minimum for $x > 0$.

The optimal height is

$$h = \frac{V}{x^2} = \frac{1056}{x^2} = \frac{1056}{(2112)^{2/3}} = \frac{2112}{2(2112)^{2/3}} = \frac{(2112)^{1/3}}{2}$$

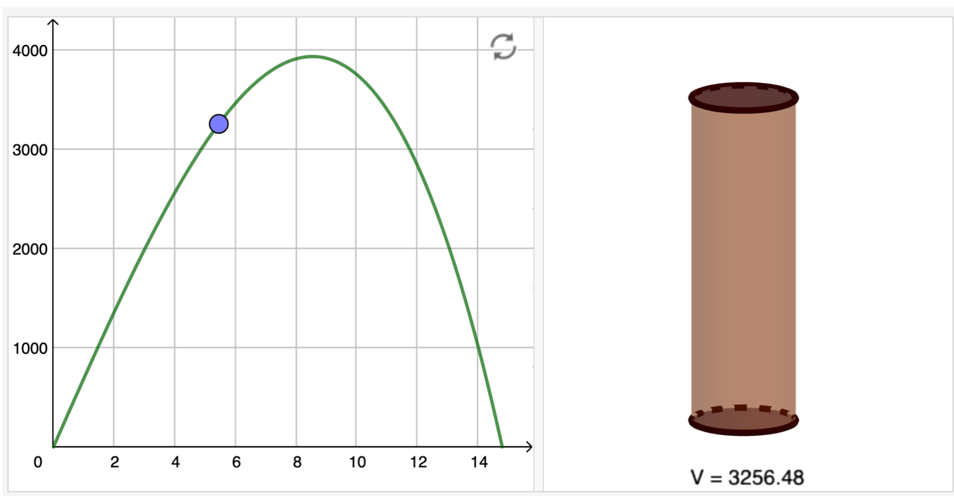
Therefore the optimal height is half the optimal base length. Can you show that this is always the case for any value of the fixed volume V?

Correct Answers:

- `x^2+4*x*h`
- `x^2*h`
- `x^2+4224/x`
- `2*x-4224/(x^2)`
- `(2*1056)^(1/3)`
- `(1056/4)^(1/3)`

---

3. (1 point) `Riddle/CanMaxVolumeScaffold.pg`



Note: This problem uses an interactive GeoGebra applet. It may take a few seconds for the figures to appear. **Instructions**

A closed cylindrical can must have a fixed surface area of 1554 cm^2. In this problem you will find the radius of the can that will maximize its volume. The graph above shows a plot of the volume as a function of the radius of the base. Use the slider to move the point on the graph to visualize how the volume changes for different values of the radius, and use this to estimate the optimal radius.

Click on Part 1 to open the section. Enter your answers for the two questions in Part 1, then click on Submit Answers. Once your answers are both correct, you will be able to open Part 2. When your answers to Part 2 are both correct, continue to Part 3 to find the optimal radius and height for the can of maximum volume.

Part 1: The Setup
Let r be the radius of the base and let h be the height of the cylinder.

What is the surface area of the cylinder in terms of r and h?

$A =$ ____________

What is the volume of the cylinder in terms of r and h?

$V =$ ____________

Hint: *(Instructor hint preview: show the student hint after the following number of attempts: 0*
The surface area of a closed cylinder is the sum of the areas of the top and bottom circles and the area of the side of the cylinder. If you could cut and unwrap the side of the cylinder, it would become a rectangle whose width is the circumference of the base circle and whose height is the height of the cylinder. The volume of the cylinder is the area of the base times the height.

Solution: *(Instructor solution preview: show the student solution after due date.)*
The surface area of a closed cylinder is the sum of the areas of the top and bottom circles and the area of the side of the cylinder. The top and bottom circles have combined area $2\pi r^2$. If you could cut and unwrap the side of the cylinder, it would become a rectangle whose width is the circumference of the base circle, $2\pi r$, and whose height is the height h of the cylinder. The surface area of a closed cylinder is therefore $A = 2(\pi r^2) + 2\pi rh$. The volume of the can is $V = \pi r^2 h$.

Part 2: The Calculus Part: Function and Derivative
Use the fact that $A = 1554$ to determine a function $V(r)$ that gives the volume of the cylinder in terms of the radius r of the base (this is the function plotted above).

$V(r) =$ ____________

What is the derivative of $V(r)$?

$V'(r) =$ ____________

Hint: *(Instructor hint preview: show the student hint after the following number of attempts: 0*
Solve the surface area equation for h and substitute into the volume formula to get a function for the volume only in terms of r, then simplify as much as possible before taking the derivative.

Solution: *(Instructor solution preview: show the student solution after due date.)*
Solving the surface area equation $2(\pi r^2) + 2\pi rh = 1554$ for h gives

$$\begin{aligned} 2(\pi r^2) + 2\pi rh &= 1554 \\ 2\pi rh &= 1554 - 2\pi r^2 \\ h &= \frac{1554 - 2\pi r^2}{2\pi r} \end{aligned}$$

Now substitute into $V = \pi r^2 h$ and simplify.

$$V(r) = \pi r^2 \left(\frac{1554 - 2\pi r^2}{2\pi r} \right) = \frac{1}{2} \left(1554r - 2\pi r^3 \right)$$

The domain of this function is $0 \le r \le \sqrt{\frac{1554}{2\pi}}$ since you want $V(r) \ge 0$ for $r \ge 0$.

$$V'(r) = \frac{1}{2}\left(1554 - 6\pi r^2\right)$$

Part 3: The Calculus Part: Maximizing the Volume
Use the derivative to find the **exact** radius r that maximizes the volume.

r = ___________

What is the **exact** height of the cylinder of maximum volume?

h = ___________

Solution: *(Instructor solution preview: show the student solution after due date.)*
To minimize V, take the derivative and set it equal to 0.

$$V'(r) = \frac{1}{2}\left(1554 - 6\pi r^2\right) = 0$$

$$1554 - 6\pi r^2 = 0$$

$$r^2 = \frac{1554}{6\pi} = \frac{259}{\pi}$$

$$r = \sqrt{\frac{259}{\pi}}$$

The graph shows that this is the global maximum of V on the domain $0 \le r \le \sqrt{\frac{1554}{2\pi}}$. Calculus can also verify this without referring to the graph. Note that $V'' = -6\pi r < 0$ and therefore the critical point is a local maximum by the second derivative test. But since V is a continuous function and there is only one critical point in the interval $0 \le r \le \sqrt{\frac{1554}{2\pi}}$, this must actually be the global maximum on this interval.

The optimal height is

$$h = \frac{A - 2\pi r^2}{2\pi r} = \frac{1554 - 2\pi\left(\frac{259}{\pi}\right)}{2\pi\sqrt{\frac{259}{\pi}}}$$

$$= \frac{1036}{2\pi\sqrt{\frac{259}{\pi}}} = \frac{\frac{518}{\pi}}{\sqrt{\frac{259}{\pi}}} = \frac{2 \cdot \frac{259}{\pi}}{\sqrt{\frac{259}{\pi}}}$$

$$= 2\sqrt{\frac{259}{\pi}}$$

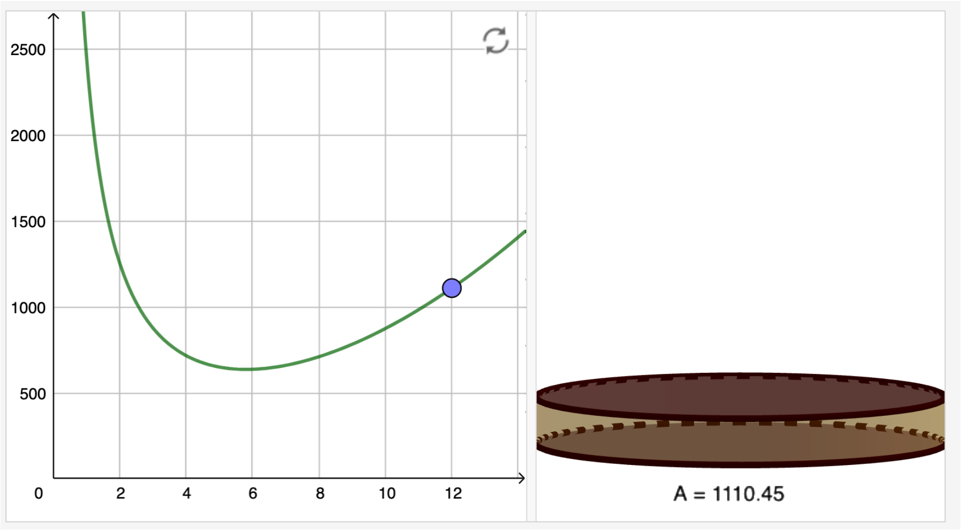
Therefore the optimal height is twice the optimal radius, i.e. in the optimal can the height and the diameter are the same. Can you show that this is always the case for any value of the fixed area A?

Correct Answers:

- `2*pi*r^2+2*pi*r*h`
- `pi*r^2*h`
- `(1554*r-2*pi*r^3)/2`
- `(1554-6*pi*r^2)/2`
- `sqrt(1554/(6*pi))`
- `2*sqrt(1554/(6*pi))`

---

4. (1 point) `Riddle/CanMinAreaScafford.pg`



Note: This problem uses an interactive GeoGebra applet. It may take a few seconds for the figures to appear. **Instructions**

A closed cylindrical can must have a fixed volume of 1102 cm^3. In this problem you will find the radius of the can that will minimize its surface area. The graph above shows a plot of the surface area as a function of the radius of the base. Use the slider to move the point on the graph to visualize how the surface area changes for different values of the radius, and use this to estimate the optimal radius.

Click on Part 1 to open the section. Enter your answers for the two questions in Part 1, then click on Submit Answers. Once your answers are both correct, you will be able to open Part 2. When your answers to Part 2 are both correct, continue to Part 3 to find the optimal radius and height for the can with minimum surface area.

Part 1: The Setup
Let r be the radius of the base and let h be the height of the cylinder.

What is the surface area of the cylinder in terms of r and h?

$A =$ ____________

What is the volume of the cylinder in terms of r and h?

$V =$ ____________

Hint: *(Instructor hint preview: show the student hint after the following number of attempts: 0*
The surface area of a closed cylinder is the sum of the areas of the top and bottom circles and the area of the side of the cylinder. If you could cut and unwrap the side of the cylinder, it would become a rectangle whose width is the circumferance of the base circle and whose height is the height of the cylinder. The volume of the cylinder is the area of the base times the height.

Solution: *(Instructor solution preview: show the student solution after due date.)*
The surface area of a closed cylinder is the sum of the areas of the top and bottom circles and the area of the side of the cylinder. The top and bottom circles have combined area $2\pi r^2$. If you could cut and unwrap the side of the cylinder, it would become a rectangle whose width is the circumferance of the base circle, $2\pi r$, and whose height is the height h of the cylinder. The surface area of a closed cylinder is therefore $A = 2(\pi r^2) + 2\pi rh$. The volume of the can is $V = \pi r^2 h$.

Part 2: The Calculus Part: Function and Derivative
Use the fact that $V = 1102$ to determine a function $A(r)$ that gives the surface area of the cylinder in terms of the radius r of the base (this is the function plotted above).

$A(r) =$ ____________

What is the derivative of $A(r)$?

$A'(r) =$ ____________

Hint: *(Instructor hint preview: show the student hint after the following number of attempts: 0*
Solve the volume equation for h and substitute into the surface formula to get a function for the surface area only in terms of r, then simplify as much as possible before taking the derivative.

Solution: *(Instructor solution preview: show the student solution after due date.)*
The volume of the can is $\pi r^2 h = 1102$. Solving this last equation for h and substituting into A gives

$$A(r) = 2\pi r^2 + 2\pi r\left(\frac{1102}{\pi r^2}\right) = 2\pi r^2 + \frac{2204}{r}$$

The domain of this function is $r > 0$.

$$A'(r) = 4\pi r - \frac{2204}{r^2}$$

Part 3: The Calculus Part: Minimizing the Surface Area
Use the derivative to find the **exact** radius r that minimizes the surface area.

$r =$ ____________

What is the **exact** height of the can of minimum surface area?

$h =$ ____________

Solution: *(Instructor solution preview: show the student solution after due date.)*
To minimize A, take the derivative and set it equal to 0.

$$A'(r) = 4\pi r - \frac{2204}{r^2} = 0$$

$$4\pi r^3 - 2204 = 0$$

$$r^3 = \frac{2204}{4\pi} = \frac{551}{\pi}$$

$$r = \left(\frac{551}{\pi}\right)^{1/3}$$

The graph suggests that this is the global minimum. Calculus can verify this. Note that

$$A''(r) = 4\pi + \frac{2(2204)}{r^3} > 0$$

and therefore the critical point is a local minimum by the second derivative test. But since A is a continuous function for $r > 0$ and there is only one critical point, this must actually be the global minimum for $r > 0$.

The optimal height is

$$h = \frac{V}{\pi r^2} = \frac{1102}{\pi r^2} = \frac{1102}{\pi\left(\frac{551}{\pi}\right)^{2/3}} = \frac{2\left(\frac{551}{\pi}\right)}{\left(\frac{551}{\pi}\right)^{2/3}} = 2\left(\frac{551}{\pi}\right)^{1/3}$$

Therefore the optimal height is twice the optimal radius, i.e. in the optimal can, the height and the diameter are the same. Can you show that this is always the case for any value of the fixed volume V?

Correct Answers:

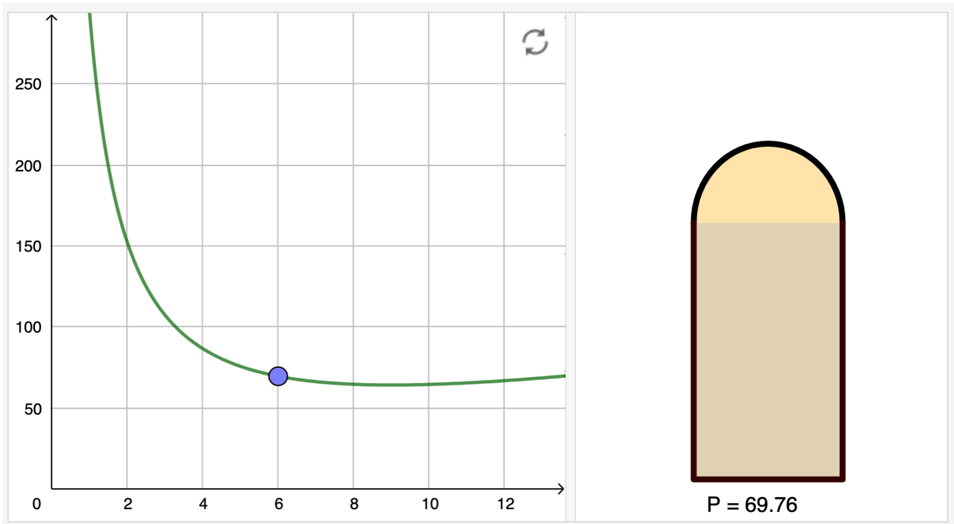
- `2*pi*r^2+2*pi*r*h`
- `pi*r^2*h`
- `2*pi*r^2+2204/r`
- `4*pi*r-2204/(r^2)`
- `[1102/(2*pi)]^(1/3)`
- `2*[1102/(2*pi)]^(1/3)`

---

5. **(1 point)** `Riddle/NormanWindowMinPerimeterScaffold.pg`

Note: This problem uses an interactive GeoGebra applet. It may take a few seconds for the figures to appear. **Instructions**

A window is in the shape of a rectangle with a semicircle at the top. The window must have a fixed area of 233. In this problem you will find the dimensions of the window that minimizes the total length of its perimeter. The graph above shows a plot of the perimeter as a function of the radius of the semicircle. Move



the indicated point along the graph to visualize how the length of the perimeter changes for different values of the radius of the semicircle, and use this to estimate the optimal radius.

Click on Part 1 to open the section. Enter your answers for the two questions in Part 1, then click on Submit Answers. Once your answers are both correct, you will be able to open Part 2. When your answers to Part 2 are both correct, continue to Part 3 to find the optimal radius and height for the window with minimal perimeter.

Part 1: The Setup
Let r be the radius of the semicircle and let h be the height of the rectangle. Express the area A of the window and the perimeter P of the window in terms of r and h.

$A =$ ________________

$P =$ ________________

Hint: *(Instructor hint preview: show the student hint after the following number of attempts: 0*
The perimeter consists of three sides of the rectangle and the semicircle. If r is the radius of the semicircle, what is the width of the rectangle in terms of r?

Solution: *(Instructor solution preview: show the student solution after due date.)*
The perimeter consists of three sides of the rectangle and the semicircle. Each side of the rectangle has height h. The base of the rectangle has width $2r$ since it is the same as the diameter of the semicircle.

The area of the rectangle is $2rh$ and the area of the semicircle is $\frac{1}{2}\pi r^2$. The area of the window is therefore $A = 2rh + \frac{1}{2}\pi r^2$.

The perimeter of the three sides of the rectangle is $2h + 2r$ and the perimeter of the semicircle is πr, half the circumference of the circle. The perimeter of the window is therefore $P = 2h + 2r + \pi r$.

Part 2: The Calculus Part: Function and Derivative

Use the fact that the area is required to be $A = 233$ to find a function $P(r)$ that gives the perimeter of the window in terms of the semicircle radius r (this is the function plotted above).

$P(r)$ = ____________

What is the derivative of $P(r)$?

$P'(r)$ = ____________

Hint: *(Instructor hint preview: show the student hint after the following number of attempts: 0*
Solve the area equation for h and substitute into the perimeter formula to get a function for the perimeter only in terms of r, then simplify as much as possible before taking the derivative.

Solution: *(Instructor solution preview: show the student solution after due date.)*
Solving the equation $2rh + \frac{1}{2}\pi r^2 = 233$ for h gives

$$\begin{aligned} 2rh + \frac{1}{2}\pi r^2 &= 233 \\ 2rh &= 233 - \frac{1}{2}\pi r^2 \\ h &= \frac{233 - \frac{1}{2}\pi r^2}{2r} \end{aligned}$$

Now substituting into P and simplify.

$$\begin{aligned} P(r) &= 2\left(\frac{233 - \frac{1}{2}\pi r^2}{2r}\right) + 2r + \pi r \\ &= \frac{233}{r} - \frac{1}{2}\pi r + 2r + \pi r \\ &= \frac{233}{r} + 2r + \frac{1}{2}\pi r \end{aligned}$$

The domain of this function is $r > 0$.

$$P'(r) = -\frac{233}{r^2} + 2 + \frac{1}{2}\pi$$

Part 3: The Calculus Part: Minimizing the Perimeter
Give your answers rounded to 3 decimal places.

Use the derivative to find the radius r that minimizes the perimeter.

r = ____________

What is the height of the window with minimum perimeter?

h = ____________

Solution: *(Instructor solution preview: show the student solution after due date.)*
To minimize $P(r)$, take the derivative and set it equal to 0.

$$P'(r) = -\frac{233}{r^2} + 2 + \frac{1}{2}\pi = 0$$

$$\frac{233}{r^2} = 2 + \frac{1}{2}\pi$$

$$r^2 = \frac{233}{2+\frac{1}{2}\pi} = \frac{466}{4+\pi}$$

$$r = \sqrt{\frac{466}{4+\pi}} \approx 8.077843$$

To three decimal places rounded, $r \approx 8.078$.

The graph suggests that this is the global minimum. Calculus can verify this. Note that $P'' = \frac{466}{r^3} > 0$ and therefore the critical point is a local minimum by the second derivative test. But since P is a continuous function for $r > 0$ and there is only one critical point, this must actually be the global minimum for $r > 0$.

The optimal height is $h = \frac{A - \frac{1}{2}\pi r^2}{2r} \approx 8.077843$ using the optimal radius found above. To three decimal places rounded, $h \approx 8.078$. It looks like the optimal radius and optimal height are the same! This is because

$$h = \frac{A - \frac{1}{2}\pi r^2}{2r} = \frac{233 - \frac{1}{2}\pi\left(\frac{466}{4+\pi}\right)}{2\sqrt{\frac{466}{4+\pi}}} = \frac{\frac{580(4+\pi)-580\pi}{2(4+\pi)}}{2\sqrt{\frac{466}{4+\pi}}}$$

$$= \frac{\frac{580(4)}{2(4+\pi)}}{2\sqrt{\frac{466}{4+\pi}}} = \frac{\frac{466}{4+\pi}}{\sqrt{\frac{466}{4+\pi}}} = \sqrt{\frac{466}{4+\pi}}$$

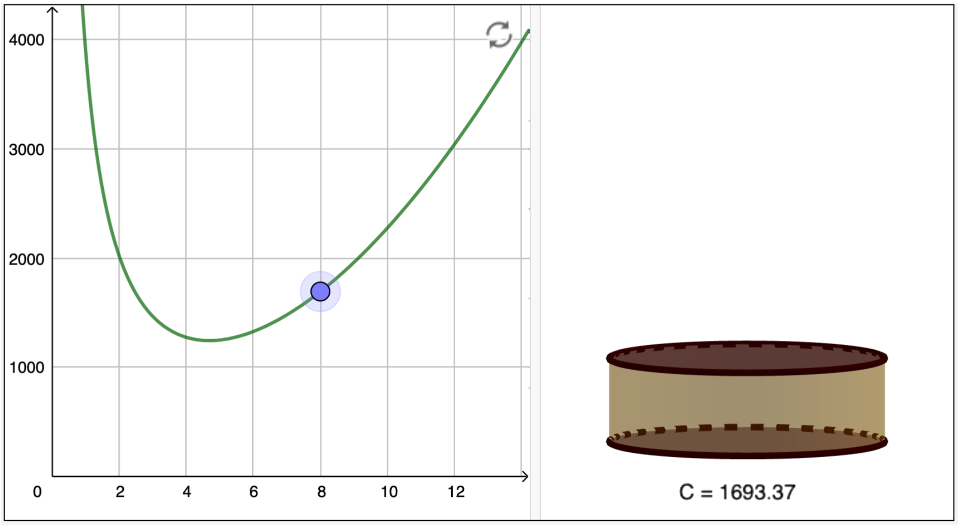
Therefore the optimal height is equal to the optimal radius which means that the height of the optimal rectangle is half the width of the rectangle. Can you show that this is always the case for any value of the fixed area A?

Correct Answers:

- `2*r*h+pi*r^2/2`
- `2*h+2*r+pi*r`
- `233/r+pi*r/2+2*r`
- `-233/(r^2)+pi/2+2`
- `sqrt(2*233/(4+pi))`
- `sqrt(2*233/(4+pi))`

---

6. (1 point) `Riddle/CanMinCostScafford.pg`



Note: This problem uses an interactive GeoGebra applet. It may take a few seconds for the figures to appear. **Instructions**

A closed cylindrical can must have a fixed volume of 974 cm^3.
The cost of materials for the top and bottom of the can is 3 cents per cm^2.
The cost of materials for the side of the can is 2 cents per cm^2.

In this problem you will find the radius of the can that will minimize the total cost. The graph above shows a plot of the total cost as a function of the radius of the base. Use the slider to move the point on the graph to visualize how the cost changes for different values of the radius, and use this to estimate the optimal radius.

Click on Part 1 to open the section. Enter your answers for the two questions in Part 1, then click on Submit Answers. Once your answers are both correct, you will be able to open Part 2. When your answers to Part 2 are both correct, continue to Part 3 to find the optimal radius and height for the can with minimum cost.

Part 1: The Setup
Let r be the radius of the base and let h be the height of the cylindrical can.

What is the cost of the can in cents in terms of r and h?

$C =$ ____________

What is the volume of the can in terms of r and h?

$V =$ ____________

Hint: *(Instructor hint preview: show the student hint after the following number of attempts: 0*
The total cost of the closed cylinder is the sum of the cost of the top and bottom circles and the cost of the side of the cylinder. The cost of each section is the cost per cm^2 multiplied by the area of the section. If you could cut and unwrap the side of the cylinder, it would become a rectangle whose width is the circumference of the base circle and whose height is the height of the cylinder. The volume of the cylinder is the area of the base times the height.

Solution: *(Instructor solution preview: show the student solution after due date.)*
The total cost of the closed cylinder is the sum of the cost of the top and bottom circles and the cost of the side of the cylinder. The top and bottom circles have combined area $2\pi r^2$, so the cost for the top an bottom is $(2\pi r^2)(3)$. If you could cut and unwrap the side of the cylinder, it would become a rectangle whose width is the circumference of the base circle, $2\pi r$, and whose height is the height h of the cylinder. The cost of the side is therefore $(2\pi rh)(2)$. The total cost of the closed cylinder is therefore $C = (2\pi r^2)(3) + (2\pi rh)(2) = 6\pi r^2 + 4\pi rh$. The volume of the can is $V = \pi r^2 h$.

Part 2: The Calculus Part: Function and Derivative
Use the fact that $V = 974$ to determine a function $C(r)$ that gives the total cost of the cylindrical can in terms of the radius r of the base (this is the function plotted above).

$C(r)$ = ____________

What is the derivative of $C(r)$?

$C'(r)$ = ____________

Hint: *(Instructor hint preview: show the student hint after the following number of attempts: 0*
Solve the volume equation for h and substitute into the cost formula to get a function for the total cost only in terms of r, then simplify as much as possible before taking the derivative.

Solution: *(Instructor solution preview: show the student solution after due date.)*
The volume of the can is $\pi r^2 h = 974$. Solving this equation for h and substituting into C gives

$$C(r) = 6\pi r^2 + 4\pi r\left(\frac{974}{\pi r^2}\right) = 6\pi r^2 + \frac{3896}{r}$$

The domain of this function is $r > 0$.

$$C'(r) = 12\pi r - \frac{3896}{r^2}$$

Part 3: The Calculus Part: Minimizing the Total Cost
Note: For r and h, round your answers to three decimal places. You may need to use more decimal places in the calculation so that the final answer is correct to three places. For C, round your answer in **dollars** to two decimal places.

Use the derivative to find the radius r that minimizes the total cost of the can.

r = ____ cm

What is the height of the can of minimum cost?

h = ____ cm

What is the minimum cost in dollars?

C =____

Solution: *(Instructor solution preview: show the student solution after due date.)*
To minimize C, take the derivative and set it equal to 0.

$$C'(r) = 12\pi r - \frac{3896}{r^2} = 0$$

$$12\pi r^3 = 3896$$

$$r^3 = \frac{3896}{12\pi}$$

$$r = \left(\frac{3896}{12\pi}\right)^{1/3} \approx 4.69277$$

To three decimal places rounded, $r \approx 4.693$.

The graph suggests that this is the global minimum. Calculus can verify this. Note that

$$C''(r) = 12\pi + \frac{2(3896)}{r^3} > 0$$

and therefore the critical point is a local minimum by the second derivative test. But since C is a continuous function for $r > 0$ and there is only one critical point, this must actually be the global minimum for $r > 0$.

The optimal height is $h = \dfrac{V}{\pi r^2} \approx 14.07831$ using the value of r found above. To three decimal places rounded, $h \approx 14.078$.

The minimum cost can be found using your function $C(r) = 6\pi r^2 + \frac{3896}{r} \approx 1245.319908$, then converting to dollars to get \$12.45.

See if you can show that if C_1 is the side cost and C_2 is the top and bottom cost, then

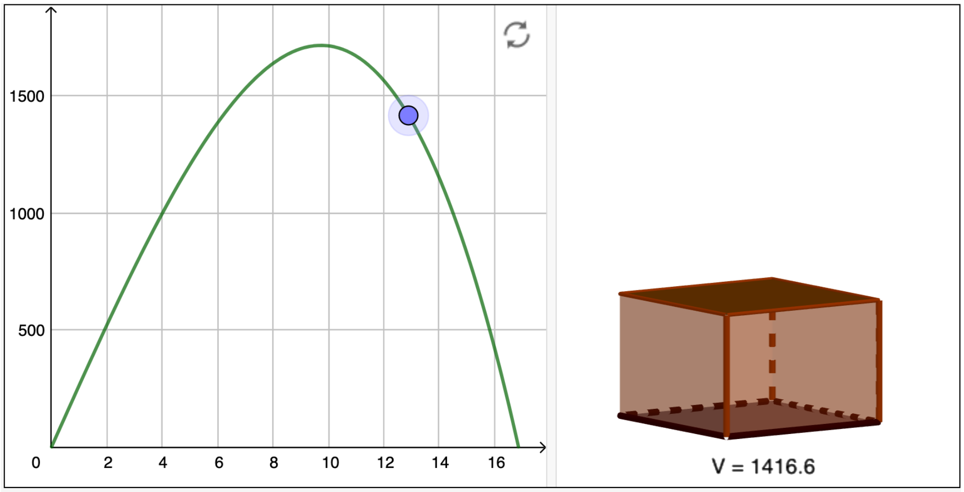
$$h = 2r\left(\frac{C_2}{C_1}\right) \text{ or } \frac{h}{2r} = \frac{C_2}{C_1}$$

no matter what the value of the fixed volume V is equal to. This means that the optimal height is in the same ratio to the optimal diameter as the top/bottom cost is to the side cost.

Correct Answers:

- `6*pi*r^2+4*pi*r*h`
- `pi*r^2*h`
- `6*pi*r^2+3896/r`
- `12*pi*r-3896/(r^2)`
- `4.69277`
- `14.07831`
- `12.453199`

7. (1 point) `Riddle/BoxMaxVolumeFixedCostScaffold.pg`



Note: This problem uses an interactive GeoGebra applet. It may take a few seconds for the figures to appear. **Instructions**

You have a budget of \$18.50 to construct a box with a square bottom and with a closed top.
The cost of the top of the box is 3 cents per square inch.
The cost of the bottom of the box is 3.5 cents per square inch.
The cost of the sides of the box is 1.75 cents per square inch.

In this problem you will find the size of the box that will maximize its volume when you use all of the budget of \$18.50. The graph above shows a plot of the volume as a function of the base length. Move the indicated point along the graph to visualize how the volume changes for different lengths of the square base, and use this to estimate the optimal size of the bottom square.

Click on Part 1 to open the section. Enter your answers for the two questions in Part 1, then click on Submit Answers. Once your answers are both correct, you will be able to open Part 2. When your answers to Part 2 are both correct, continue to Part 3 to find the optimal width and height for the box with maximum volume.

Part 1: The Setup
Let x be the width of the square base in inches and let h be the height of the box in inches.

What is the cost of the box **in cents** in terms of x and h?

$C =$ ____________ cents

What is the volume of the box in terms of x and h?

$V =$ ____________ cubic inches

Hint: *(Instructor hint preview: show the student hint after the following number of attempts: 0*
The cost of a closed box is the sum of the cost of the square top, the cost of the square bottom, and the cost

of the four sides of the box. The costs should be given in cents. The volume of the box is the area of the base times the height.

Solution: *(Instructor solution preview: show the student solution after due date.)*
The cost of a closed box is the sum of the cost of the square top, the cost of the square bottom, and the cost of the four sides of the box. The top and bottom each has area x^2. Each side of the box has area xh. The cost of a closed box in cents is therefore $C = 3x^2 + 3.5x^2 + (1.75)(4xh) = 10.5x^2 + 7xh$. The volume of the box is $V = x^2h$.

Part 2: The Calculus Part: Function and Derivative
Use the fact that the total cost is \$18.50 to determine a function $V(x)$ that gives the volume of the box in terms of the width x of the base (this is the function plotted above).

$V(x)$ = ____________

What is the derivative of $V(x)$?

$V'(x)$ = ____________

Hint: *(Instructor hint preview: show the student hint after the following number of attempts: 0*
Be careful of units! The cost expression in Part 1 is the cost in cents, but the budget amount is given in dollars.

Solve the cost equation for h and substitute into the volume formula to get a function for the surface area only in terms of x, then simplify as much as possible before taking the derivative.

Solution: *(Instructor solution preview: show the student solution after due date.)*
The cost of the box in cents is $3x^2 + 3.5x^2 + 4(1.75)xh = 10.5x^2 + 7xh = 1850$ where the budget \$18.50 has been coverted from dollars to cents. Solving this equation for h gives

$$10.5x^2 + 7xh = 1850$$

$$7xh = 1850 - 10.5x^2$$

$$h = \frac{1850 - 10.5x^2}{7x}$$

Now substitute into $V = x^2h$ to get a function just in terms of x.

$$V(x) = x^2\left(\frac{1850 - 6.5x^2}{7x}\right) = \frac{1}{7}\left(1850x - 6.5x^3\right)$$

The domain of this function is $0 \leq x \leq \sqrt{\frac{1850}{6.5}} \approx 16.87$ since the you want $V(x) \geq 0$ with $x \geq 0$.

$$V'(x) = \frac{1}{7}\left(1850 - 19.5x^2\right)$$

Part 3: The Calculus Part: Maximizing the Volume
Give your answers rounded to 3 decimal places.

Use the derivative to find the width x that maximizes the volume.

$x =$ _____

What is the height of the box of maximum volume?

$h =$ _____

What is the volume of the box?

$V =$ _____

Hint: *(Instructor hint preview: show the student hint after the following number of attempts: 0*
While you may enter your answer to x rounded to 3 decimal places, you might need to use **more** than 3 decimal places in x when computing the value of h to have it correctly rounded to 3 decimal places. And you may need to use even more decimal places in x and h when computing the value of V correctly rounded to 3 decimal places.
Solution: *(Instructor solution preview: show the student solution after due date.)*
To maximize V, take the derivative and set it equal to 0.

$$V'(x) = \frac{1}{7}\left(1850 - 19.5x^2\right) = 0$$

$$1850 - 19.5x^2 = 0$$

$$x^2 = \frac{1850}{19.5}$$

$$x = \sqrt{\frac{1850}{19.5}} \approx 9.740215$$

To three decimal places rounded, $x \approx 9.740$.

The graph shows that this is the global maximum on the domain $0 \leq x \leq 16.87$ of V. Calculus can verify this. Note that $V'' = -\frac{39}{7}x < 0$ and therefore the critical point is a local maximum by the second derivative test. But since V is a continuous function and there is only one critical point in the interval $0 \leq x \leq 16.87$, this must actually be the global maximum on this interval.

The optimal height is $h = \dfrac{1850 - 6.5x^2}{7x} \approx 18.088971$ with the value of x found above. To three decimal places rounded, $h \approx 18.089$.

The volume can be found using $V = x^2h \approx 1716.133179$ with the optimal values of x and h. To three decimal places rounded, $V \approx 1716.133$.

Note that to get h and V accurate to 3 decimal places, it may be necessary to use more decimal places for the intermediate calculations.

Correct Answers:

- `3*x^2+3.5*x^2+7*x*h`
- `x^2*h`
- `(1850*x-6.5*x^3)/7`
- `(1850-19.5*x^2)/7`
- `9.740215`
- `18.088971`
- `1716.133179`