

⇒ To find determinant  $D_n$  of  $A_n$ , where

$$A_n = \begin{vmatrix} 3 & 1 & 0 & \dots & 0 & 0 \\ 2 & 3 & 1 & \dots & 0 & 0 \\ 0 & 2 & 3 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 3 & 1 \\ 0 & 0 & 0 & \dots & 2 & 3 \end{vmatrix} n \times n$$

$$D_n = \begin{vmatrix} 3 & 1 & 0 & \dots & 0 & 0 \\ 2 & 3 & 1 & \dots & 0 & 0 \\ 0 & 2 & 3 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 3 & 1 \\ 0 & 0 & 0 & \dots & 2 & 3 \end{vmatrix} n \times n$$

Expanding along Row 1,

$$D_n = 3 \times \begin{vmatrix} 3 & 1 & 0 & \dots & 0 & 0 \\ 2 & 3 & 1 & \dots & 0 & 0 \\ 0 & 2 & 3 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 3 & 1 \\ 0 & 0 & 0 & \dots & 2 & 3 \end{vmatrix} (n-1) \times (n-1)$$

$$-1 \times \begin{vmatrix} 2 & 1 & 0 & \dots & 0 & 0 \\ 0 & 3 & 1 & \dots & 0 & 0 \\ 0 & 2 & 3 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 3 & 1 \\ 0 & 0 & 0 & \dots & 2 & 3 \end{vmatrix} (n-1) \times (n-1)$$

X

$$\text{Taking } X = \begin{vmatrix} 2 & 1 & 0 & \dots & 0 & 0 \\ 0 & 3 & 1 & \dots & 0 & 0 \\ 0 & 2 & 3 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 3 & 1 \\ 0 & 0 & 0 & \dots & 2 & 3 \end{vmatrix}_{(n-1) \times (n-1)}$$

Expanding along Column 1,

$$X = 2 \times \begin{vmatrix} 3 & 1 & 0 & \dots & 0 & 0 \\ 2 & 3 & 1 & \dots & 0 & 0 \\ 0 & 2 & 3 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 3 & 1 \\ 0 & 0 & 0 & \dots & 2 & 3 \end{vmatrix}_{(n-2) \times (n-2)} - 0$$

$$\Rightarrow X = 2 \times D_{n-2}$$

$$\therefore D_n = 3D_{n-1} - 2D_{n-2} \quad \text{--- (1)}$$

$$D_n - D_{n-1} = 2(D_{n-1} - D_{n-2})$$

Putting  $n = 3$  through  $n$ ,

$$D_n - D_{n-1} = 2(D_{n-1} - D_{n-2})$$

$$D_{n-1} - D_{n-2} = 2(D_{n-2} - D_{n-3})$$

$$D_{n-2} - D_{n-3} = 2(D_{n-3} - D_{n-4})$$

$$\vdots \qquad \qquad \qquad \vdots \qquad \qquad \qquad \vdots$$

$$D_4 - D_3 = 2(D_3 - D_2)$$

$$D_3 - D_2 = 2(D_2 - D_1)$$

Summing all the above equations,

$$\Rightarrow D_n - D_2 = 2D_{n-1} - 2D_1$$

$$\Rightarrow D_n = 2D_{n-1} + D_2 - 2P_1$$

$$D_1 = \begin{vmatrix} 3 \end{vmatrix} = 3$$

$$D_2 = \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} = 7.$$

$$D_n = 2D_{n-1} + 1 \quad - (2)$$

$$D_{n-1} = 2D_{n-2} + 1$$

$$\Rightarrow D_n = 2(2D_{n-2} + 1) + 1$$

$$D_n = 4D_{n-2} + (1+2)$$

$$D_n = 4(2D_{n-3} + 1) + (1+2)$$

$$D_n = 8D_{n-3} + (1+2+4)$$

$$D_n = 8(2D_{n-4} + 1) + (1+2+4)$$

$$D_n = 16D_{n-4} + (1+2+4+8)$$

likewise continuing,

$$D_n = 2^k D_{n-k} + (1+2+2^2+\dots+2^{k-1})$$

for  $k = (n-1)$ .

$$\Rightarrow D_n = 2^{n-1} D_{n-(n-1)} + (1+2+2^2+\dots+2^{n-2})$$

$$D_n = 2^{n-1} D_1 + \frac{(1-2^{n-1})}{1-2}$$

$$\therefore D_n = 3 \cdot 2^{n-1} + (2^{n-1} - 1)$$

$$D_n = 4 \cdot 2^{n-1} - 1$$

$$\Rightarrow \boxed{D_n = 2^{n+1} - 1}.$$