Indian Institute of Space Science and Technology

Department of Mathematics

Maths Club — Weekly Challenge #1

Date: October 13, 2025

Challenge Question

Consider the matrix

$$M = \begin{bmatrix} t+1 & t \\ 1 & 1 \end{bmatrix}$$

whose entries are polynomials in the variable t. Let $a(t)$ denote the polynomial in the first row and first column of M^{2025} and let $b(t)$ denote the polynomial in the first row and second column of M^{2025}.

Prove that $a(t)$ has 2025 distinct real roots $\alpha_1, \alpha_2, \ldots, \alpha_{2025}$ and that $b(t)$ has 2025 distinct real roots $\beta_1, \beta_2, \ldots, \beta_{2025}$ satisfying

$$\alpha_1 < \beta_1 < \alpha_2 < \beta_2 < \cdots < \alpha_{2025} < \beta_{2025}.$$

Instructions:

- Submit your detailed solution (typed or handwritten) by **25 Oct, 2025**.
- Use the link below for submission:

Submit Here: Maths Club – Weekly Challenge Submission

- Selected solutions will be featured in the **Maths Club GitHub repository**.

Source

- *A*dapted from: SMMC 2025