Form A

1. $\vec{\nabla} f(x,y) = \langle 2x+2y,\ 2x-6y-8 \rangle$

$z = f(2,1) + \frac{\partial f}{\partial x}(2,1)(x-2) + \frac{\partial f}{\partial y}(2,1)(y-1)$

$$\boxed{z = -3 + 6(x-2) - 10(y-1) \qquad \text{or} \qquad 6x - 10y - z = 5}$$

2. $\vec{u} = \dfrac{\langle -1-2,\ 5-1 \rangle}{\sqrt{3^2+4^2}} = \langle -\frac{3}{5}, \frac{4}{5} \rangle$

$D_{\vec{u}} f(2,1) = \vec{\nabla} f(2,1) \circ \vec{u} = \langle 6, -10 \rangle \circ \langle -\frac{3}{5}, \frac{4}{5} \rangle$

$= -\frac{18}{5} - \frac{40}{5} = \boxed{\frac{-58}{5}} = -11.6$

3. Solve $\vec{\nabla} f(x,y) = 0 \iff \left. \begin{aligned} \frac{\partial f}{\partial x} &= 2x+2y = 0 \\ \frac{\partial f}{\partial y} &= 2x-6y-8 = 0 \end{aligned} \right\} \Rightarrow \underbrace{x=1,\ y=-1}_{\text{inside } R}$

$f_{xx} = 2$, $f_{xy} = 2$, $f_{yy} = -6$, so at every point (x,y),

$D(x,y) = \begin{vmatrix} 2 & 2 \\ 2 & -6 \end{vmatrix} = -16 < 0$, indicating the critical point is

the location of a <u>saddle point</u>.