Solutions

1. (a)

$$\int_C \vec{F}\circ d\vec{r} = \int_0^{2\pi} \left\langle \frac{-3\sin t}{\sqrt{9\cos^2 t+9\sin^2 t}}, \frac{3\cos t}{\sqrt{9\cos^2 t+9\sin^2 t}} \right\rangle \circ \langle -3\sin t, 3\cos t\rangle \, dt$$

$$= 3\int_0^{2\pi} (\sin^2 t + \cos^2 t)\, dt = 3\int_0^{2\pi} dt = 3\big[t\big]_0^{2\pi} = \boxed{6\pi}$$

(b) There are many correct answers. Here are two

$$\vec{r}(t) = \langle -3\cos t, 3\sin t\rangle, \qquad 0 \le t \le 2\pi$$

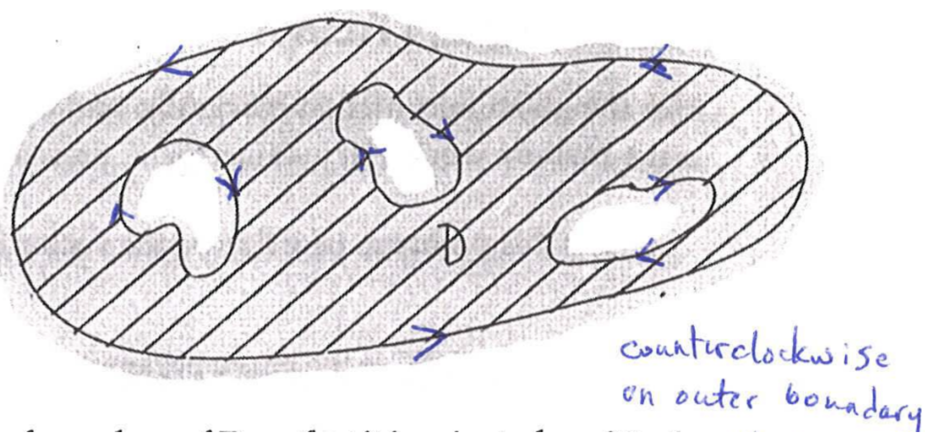
$$\vec{r}(t) = \langle 3\sin t, 3\cos t\rangle, \qquad -\pi \le t \le \pi$$

(c) Opposite of answer to part (a): -6π

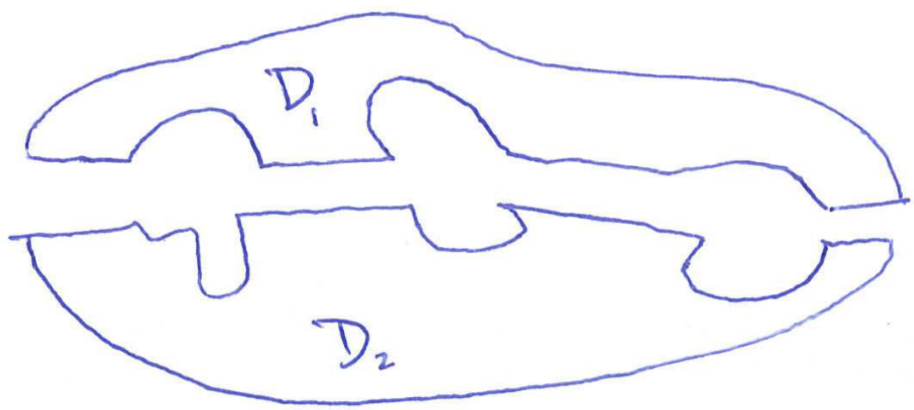
(d) No, because we did a line integral around a closed curve $\oint_C F\circ dr$ and the result wasn't zero. Another good reason, for those willing to calculate these derivatives:

$$\frac{\partial}{\partial y}\frac{-y}{\sqrt{x^2+y^2}} \text{ is different from } \frac{\partial}{\partial x}\frac{x}{\sqrt{x^2+y^2}}$$

2. (a)



(b) You must slice up D so the new regions, D_1, D_2 are without holes:



(c) $\int_{\partial D_1} \vec{F}\cdot\vec{T}\,ds$, $\int_{\partial D_1} \vec{F}\cdot d\vec{r}$, or $\int_{\partial D_1} P\,dx + Q\,dy$

(d) $= \iint_D (P_x + Q_y)\,dA$