Test 2: Friday, in-class, similar format to last time

Chs.

3: 3.1-ish, 3.2, 3.3

5: 5.1, 5.2, <u>5.3</u>

↑ H.W. today

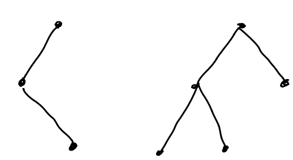
More: details and <u>problems</u> found on 11/2 on calendar.

Recursive definitions

1. $\mathcal{E}$ = extended binary trees

Base step: let the empty tree $\lambda \in \mathcal{E}$

Recursion: For input trees $T_1, T_2 \in \mathcal{E}$, obtain $T_1 \cdot T_2$ by attaching a new node to the roots of T_1, T_2.

Added to $\mathcal{E}$ after	$\mathcal{E}'$	A
Base step:	λ	λ
1^{st} recursion:		10,1
2^{nd} recursion:		10 $\underline{101}$ 1
3^{rd} recursion:	 . . .	

2. Basic: Let λ be an empty string in A.

Recursion: For each $x \in A$, $\underline{10x1} \in A$.

Friday 8.1 - modeling using recurrences

Did 5 examples

6. Come up w/ a recurrence relation for no. of ways to feed a vending machine \$n using

- \$1 coins (C)
- \$1 bills (O)
- \$5 bills (F)

Here a_n = # of ways, order mattering, to feed \$n into machine.

$$\boxed{\begin{aligned} a_0 &= 1 \\ a_1 &= 2 \\ a_2 &= 2^2 \\ a_3 &= 2^3 \\ a_4 &= 2^4 \end{aligned}}$$

counts ways to feed \$(n-1)

$$\underline{a_n} = 2a_{n-1} + a_{n-5}$$

5 degree recurrence requires 5 ICs.

counts ways to feed \$(n-5)

$$a_5 = 2a_4 + a_0 = 2 \cdot 2^4 + 1 = 33$$

$$a_6 = 2a_5 + a_1 = 2(33) + 2 = \underline{68}$$

$2^6 = \underline{64}$ using only $\underline{C, O}$s

$$2^6 \left\{ \begin{array}{l} CCCCCC \\ CCCCCO \\ CCCCOC \\ \vdots \\ OOOOOO \end{array} \right.$$

$$\boxed{\begin{array}{l} FC \\ FO \\ OF \\ CF \end{array}}$$

These additional four use F's.

8.2 Solving recurrences

Solve the recurrence

$$f_n = f_{n-1} + f_{n-2} \quad \text{w/ ICs.} \quad \begin{array}{l} f_0 = 0 \\ f_1 = 1 \end{array}$$

Key idea:

Assume $f_n = r^n$ for some number r.

Insert that formula into recurrence

$$r^n = r^{n-1} + r^{n-2}$$

Subtract and factor

$$r^{n-2}\left(r^2 - r - 1\right) = 0$$

Either $r^{n-2} = 0$ (not very interesting)

or $r^2 - r - 1 = 0$ (characteristic equation)

Quadratic formula: zeros of $ax^2 + bx + c$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In our char. eqn. $a = 1$, $b = -1$, $c = -1$

$\Rightarrow$ roots $r = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)} = \frac{1 \pm \sqrt{5}}{2}$

Two roots $r_1 = \frac{1-\sqrt{5}}{2}$, $r_2 = \frac{1+\sqrt{5}}{2}$.

At outset, we assumed $f_n = r^n$.

What is true:

$r_1^0, r_1, r_1^2, r_1^3, \ldots$ is a sequence which solves our recurrence

and $r_2^0, r_2, r_2^2, r_2^3, \ldots$

but they do not satisfy the ICs.

However, there is a way to mix them and get what we want

$$f_n = \alpha_1 \left(\frac{1-\sqrt{5}}{2} \right)^n + \alpha_2 \left(\frac{1+\sqrt{5}}{2} \right)^n$$