

-----  
Friday, September 04th 2020  
-----

Wrap-up of set theory (Appendix B)

- geometric series

$$a + ra + r^2a + r^3a + \dots + r^{n-1}a + \dots$$
$$= a(1 + r + r^2 + r^3 + \dots) = a \cdot \frac{1}{1-r}$$

converges if  $|r| < 1$

- Cartesian products of sets  $\underline{A \times B} = \{(a, b) \mid a \in A, b \in B\}$   
tuples

$$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$$

$$A = \{a, b, c, d, \dots, z\}, B = \{0, 1, 2, \dots, 9\}$$

$$A \times A \times A \times B \times B \times B \times B$$

cardinality of  $A \times B$  given cardinalities of  $A, B$

$$|A \times B| = |A| |B|$$

application: counting

How many license plates?

$$|A \times A \times A \times B \times B \times B \times B| = |A|^3 |B|^4 = 26^3 10^4$$

## Probability

Context:

- random process
- sample space  $S$ 
  - $s \in S$  is called an **outcome**
  - $E \subset S$  is called an **event**
- random variable  $X: S \rightarrow \mathbb{R}$ 
  - discrete vs. continuous random variables

Example

Random process: roll two dice

outcomes:  $(1, 1)$   
 $(1, 2)$   
 $(1, 3)$   
 $\vdots$   
 $(6, 6)$

$X(\text{outcome}) = \text{sum of pips}$

- in most situations, an event  $E \subset S$  is tied to a subset  $A$  of  $\mathbb{R}$ . That is,  $s \in E$  if and only if  $X(s) \in A$ .

The probability axioms:

$E: \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$   
 $= \{\text{get a 7}\}$

1.  $P(\emptyset) = 0$
2.  $P(S) = 1$

Disjoint  
Additivity

$$P(A \cup B) = P(A) + P(B) \quad \text{when } A \cap B = \emptyset$$

$$P(A_1 \cup A_2 \cup \dots \cup A_n) = \sum_{j=1}^n P(A_j) \quad \text{when mutually disjoint}$$

$$P(A_1 \cup A_2 \cup \dots) = \sum_{j=1}^{\infty} P(A_j) \quad \text{when " " " "}$$

Theoretical probabilities

- are assigned
- must follow the axioms

- are most useful if they mirror reality

**Example:** sum of pips from two dice

From this example, notice

- We can appeal to an equally-likely principle
- Counting outcomes is elementary

## Bijections

A function  $f: A \rightarrow B$  is

- injective (one-to-one) if  $f(x_1) = f(x_2)$  only when  $x_1 = x_2$ .
- surjective (onto) if for each  $y \in B$  there is  $x \in A$  with  $f(x) = y$ .
- bijective if both injective and surjective.

Two dice, take sum

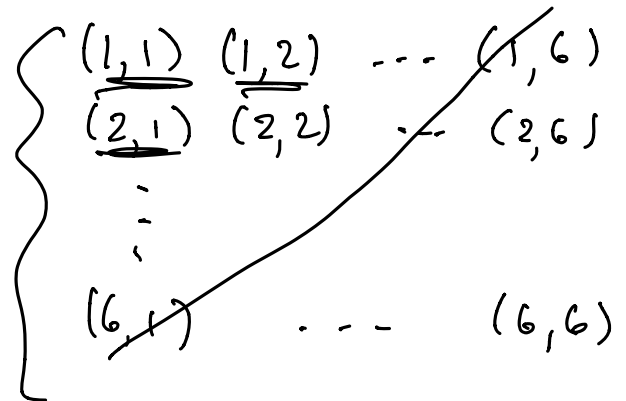
2, 3, 4, ..., 12 (values of random variable)

$$P(2) = \frac{1}{36}$$

$$P(3) = \frac{2}{36}$$

$$P(7) = \frac{6}{36}$$

$$P(12) = \frac{\quad}{\quad}$$



6 ways to get 7  
 36 outcomes

If it "feels like" outcomes are equally likely can assign probability

$$P(E) = \frac{|E|}{|S|}$$

"counting things"  
 is an important skill.

$$\sum_{x \in S} (x - m)$$

$$|S| = 10$$