E4.2

(b) $\Pr(X \le 1)$ corresponds to the area under the density curve in Figure 1 to the left of $x = 1$. That region is a triangle whose base length is 1 and whose height is 1/2. The area/probability we want is, thus, $(1/2)(1)(1/2) = 1/4$.

(c) The equation of the downslope in the density curve is $y = -\frac{1}{6}x + \frac{2}{3}$. This means the triangular region represented by $\Pr(X > 2)$ has a height of $(-1/6)(2) + 2/3 = 1/3$. Its base has length 2, so $\Pr(X > 2) = (1/2)(1/3)(2) = 1/3$. Hence, $\Pr(X \le 2) = 1 - 1/3 = 2/3$.

(d) The median m must satisfy $\Pr(X \ge m) = 1/2$. This means m must solve

$$\frac{1}{2} = \frac{1}{2}\left(\frac{2}{3} - \frac{1}{6}m\right)(4 - m).$$

After some algebraic manipulation, this equation becomes

$$m^2 - 8m + 10 = 0.$$

The quadratic formula yields two real roots, but only one of them lies in the interval $[0, 4]$, and that is $m = 4 - \sqrt{6} \doteq 1.551$.

E4.3

- Since $\displaystyle\int_{-1}^{1} (1 - x^2)\, dx = x - \frac{1}{3}x^3\Big|_{-1}^{1} = \frac{4}{3}$, the pdf is $f(x) = \frac{3}{4}(1 - x^2) \cdot 1_{[-1,1]}$.
- We compute $E(X)$ and $E(X^2)$, where X is the r.v. that corresponds to this pdf:

$$\begin{aligned}
\mu_X = E(X) &= \int_{-1}^{1} \frac{3}{4}\, x(1 - x^2)\, dx = \frac{3}{8}\, x^2 - \frac{1}{4}\, x^3\Big|_{-1}^{1} = 0 \\
E(X^2) &= \int_{-1}^{1} \frac{3}{4}\, x^2(1 - x^2)\, dx = \frac{1}{4}\, x^3 - \frac{3}{20}\, x^5\Big|_{-1}^{1} = \frac{1}{5}, \qquad \text{and so} \\
\sigma_X^2 &= E(X^2) - [E(X)]^2 = \frac{1}{5}.
\end{aligned}$$

E4.7

(a)

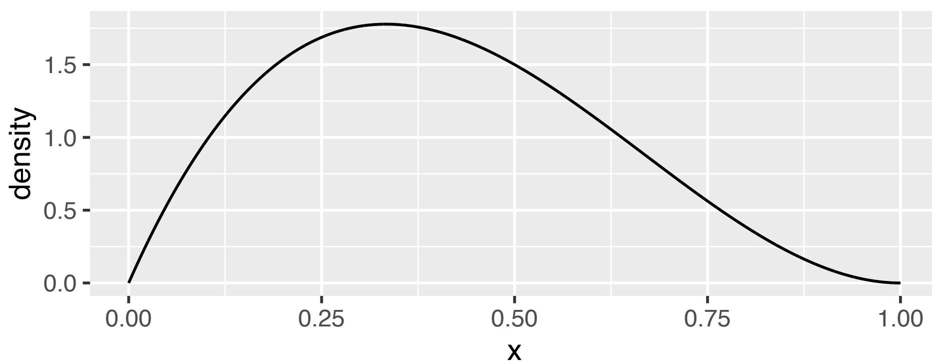
```
f = makeFun( dbeta(x, 2, 3) ~ x )
xf = makeFun( x*f(x) ~ x )
xxf = makeFun( x^2*f(x) ~ x )
mu = value(integrate(xf, 0, Inf)); mu
```

```
[1] 0.4
```

```
variance = value(integrate(xxf, 0, Inf)) - mu^2; variance
```

```
[1] 0.04
```

```
gf_dist("beta", params=c(2,3))
```



The mean is 0.4, and the variance is 0.04.

(b)

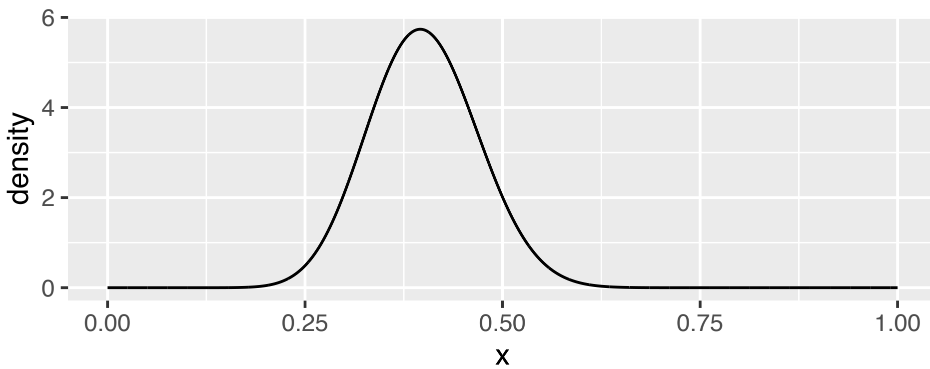
```
f = makeFun( dbeta(x, 20, 30) ~ x )
xf = makeFun( x*f(x) ~ x )
xxf = makeFun( x^2*f(x) ~ x )
mu = value(integrate(xf, 0, Inf)); mu
```

```
[1] 0.4
```

```
variance = value(integrate(xxf, 0, Inf)) - mu^2; variance
```

```
[1] 0.004705882
```

```
gf_dist("beta", params=c(20,30))
```



The mean is 0.4, and the variance is 0.00471.

(c)

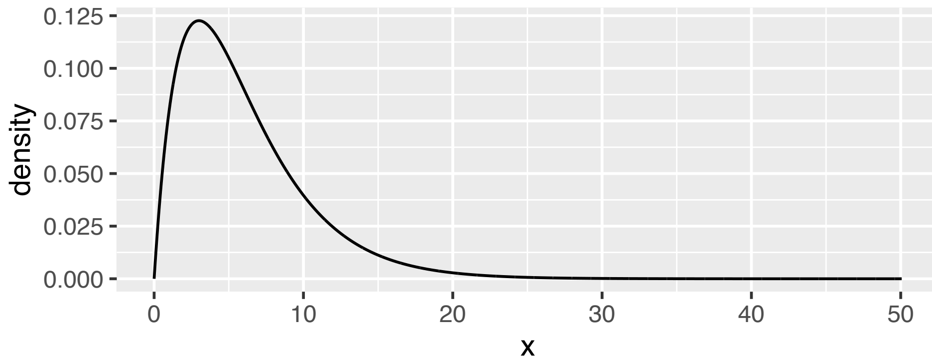
```
f = makeFun( dgamma(x, shape=2, scale=3) ~ x )
xf = makeFun( x*f(x) ~ x )
xxf = makeFun( x^2*f(x) ~ x )
mu = value(integrate(xf, 0, Inf)); mu
```

```
[1] 6
```

```
variance = value(integrate(xxf, 0, Inf)) - mu^2; variance
```

```
[1] 18
```

```
gf_dist("gamma", params=c(shape=2,scale=3))
```



The mean is 6, and the variance is 18.

(d)

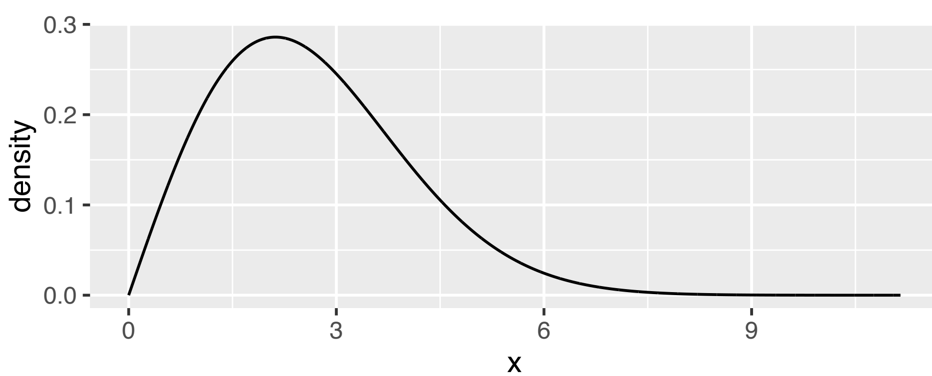
```
f = makeFun( dweibull(x, shape=2, scale=3) ~ x )
xf = makeFun( x*f(x) ~ x )
xxf = makeFun( x^2*f(x) ~ x )
mu = value(integrate(xf, 0, Inf)); mu
```

```
[1] 2.658681
```

```
variance = value(integrate(xxf, 0, Inf)) - mu^2; variance
```

```
[1] 1.931417
```

```
gf_dist("weibull", params=c(shape=2,scale=3))
```



The mean is 2.659, and the variance is 1.931.

E4.16

(a) This woman's z-score is $\frac{68-64.3}{2.6} = 1.423$.
(b) This man's z-score is $\frac{74-70}{2.8} = 1.429$.
(c) The z-scores from parts (a) and (b) are so similar, these heights are arguably equal on any *unusual* scale. With a slightly higher z-score, perhaps the man's height of 74 in is a bit more unusual.
(d) The requirement is equivalent to saying a person must be at or above the 97.5th percentile. For a woman, this is

```
qnorm(0.975, 64.3, 2.6)
```

```
[1] 69.39591
```

That is, she must be at least 69.396 in tall.

(e) For a man, the calculation is

```
qnorm(0.975, 70, 2.8)
```

```
[1] 75.4879
```

He must be 75.488 in tall.