$\mathbb{N}$	Odds
0	1
1	3
2	5
⋮	⋮

are in 1-1 corresp. Write $|\mathbb{N}| = |\{\text{odd pos. integers}\}|$

2. $\mathbb{N}$ and { positive odd integers }

$f : \mathbb{N} \xrightarrow{\text{bijective}} \{\text{pos. odds}\}$

$f(n) = 2n+1$

3. $\mathbb{N}$ and $\mathbb{Z}$

$|\mathbb{N}| = |\mathbb{Z}|$

$\mathbb{N}$	$\mathbb{Z}$
0	0
1	-1
2	1
3	-2
4	2
5	-3

odd $\to -(\text{odd}+1)/2$

even $\to \text{even}/2$

4. $\mathbb{N}$ and $\mathbb{Q}$

Georg Cantor

$$\frac{9}{10} + \frac{9}{10^2} + \frac{9}{10^3} + \frac{9}{10^4} + \cdots = \frac{9}{10} \cdot \frac{1}{1 - 1/10} = 1.$$

$0.49999\ldots = 0.5$

$0.99999\ldots = 1$

5. $\mathbb{N}$ and $[0,1] = \{x \in \mathbb{R} \mid 0 \leqslant x \leqslant 1\}$

cardinality is different

Suppose there is a bijection

$0.a_{11}a_{12}a_{13}a_{14}\ldots$

"Create" a number in $[0,1]$ that dodges the highlighted digits

$\mathbb{N}$		$[0,1]$
0	$\longleftrightarrow$	$0.a_{01}a_{02}a_{03}a_{04}\ldots$
1	$\longleftrightarrow$	$0.a_{11}a_{12}a_{13}a_{14}\ldots$
2	$\longleftrightarrow$	$0.a_{21}a_{22}a_{23}a_{24}\ldots$
⋮		
n	$\longleftrightarrow$	$0.a_{n1}a_{n2}a_{n3}\ldots$

a_{21} is the 1st digit to right of decimal of number from $[0,1]$ partnered w/ 2.

$b = 0.b_1b_2b_3b_4\ldots$

If $a_{01} = 2$, set $b_1 = 1$

$a_{01} \neq 2$, set $b_1 = 2$

So $\mathbb{N}$ and $[0,1$ are infinite sets, but $|\mathbb{N}| \neq |[0,1]|$. Write $|\mathbb{N}| = \aleph_0$, and $|\mathbb{R}| = \mathfrak{c}$.

- Question: If f, g are bijections, is $f \circ g$?
- When $f : A \to B$ is a bijection, $f^{-1} : B \to A$ exists (as a function)

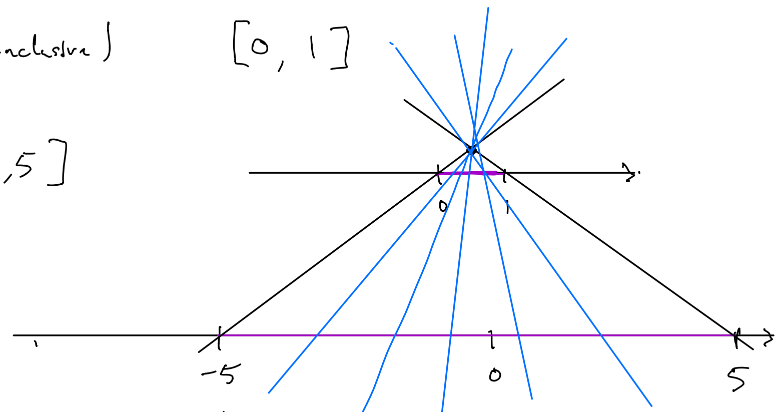
Sets that are countable (countably infinite) — ones w/ same cardinality as $\mathbb{N}$

Reals between 0, 1 (inclusive)

compare w/

$$[-5, 5]$$

$[0, 1]$

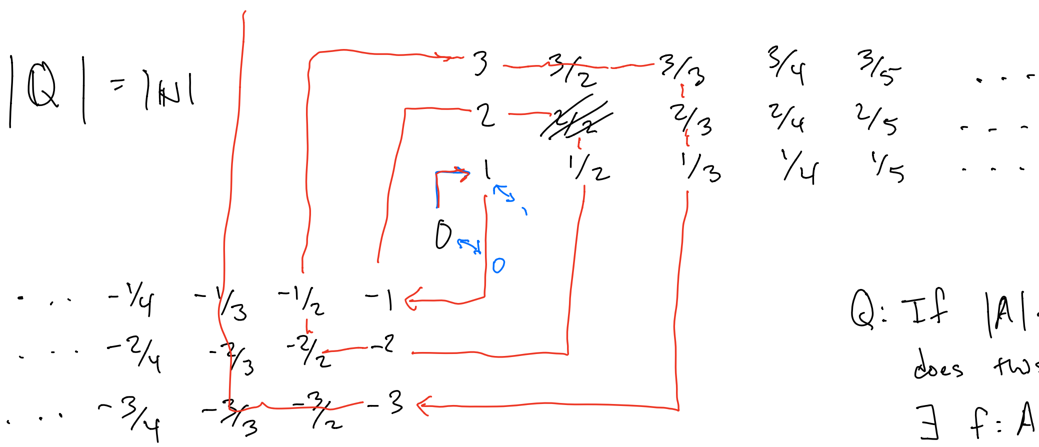


$f(x) = 10x - 5$

mapping $[0,1] \xrightarrow{\text{bij.}} [-5, 5]$

$$|[0,1]| = |[-5,5]|$$

$$|\mathbb{Q}| = |\mathbb{N}|$$



Q: If $|A| = |B|$ does this mean $\exists\, f: A \xrightarrow{\text{bij.}} B$?

True.

Fact: $|A| < |\mathcal{P}(A)|$

$|\mathbb{N}| = \aleph_0$ $\qquad$ $|[0,1]| = c$