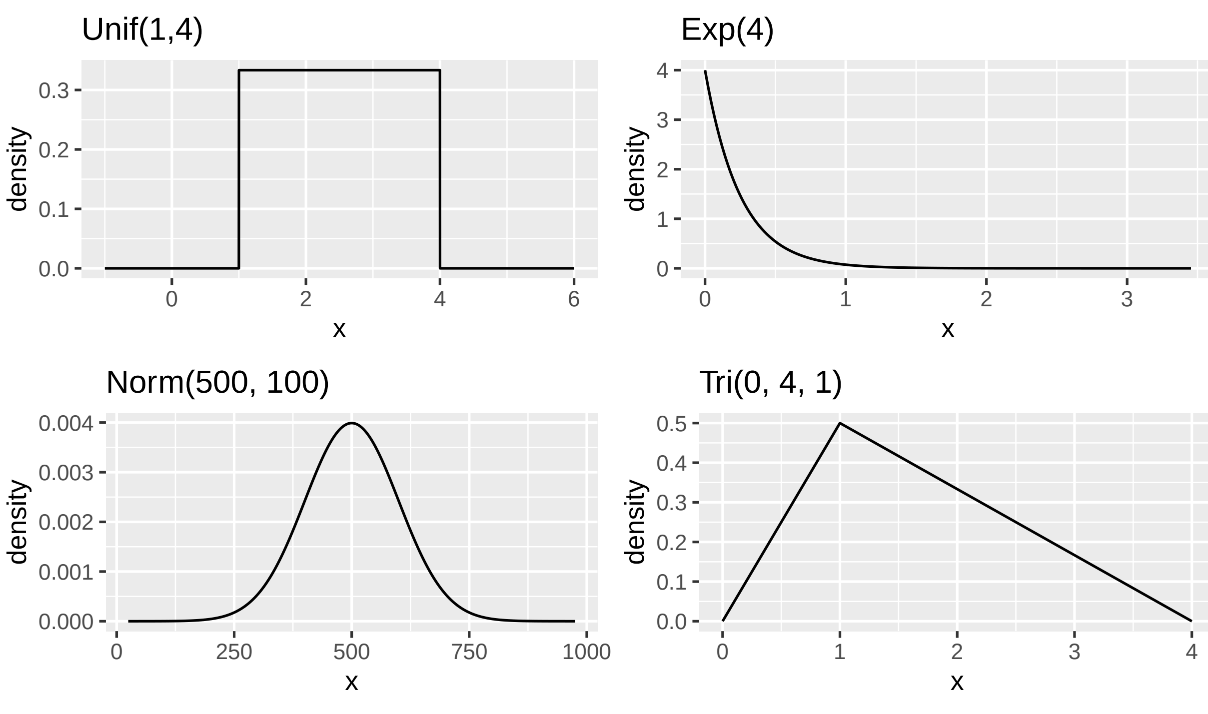
Some Important Distributional Families

Some Familiar(?) Ones

```
# triangle distributions are made available loading the triangle package
p1 <- gf_dist("unif", params = list(min = 1, max = 4), xlim = c(-1, 6),title = "Unif(1,4)")
p2 <- gf_dist("exp", rate = 4, title = "Exp(4)")
p3 <- gf_dist("norm", params = list(mean=500, sd=100), title = "Norm(500, 100)")
p4 <- gf_dist("triangle", params = list(a=0, b=4, c=1), title = "Tri(0, 4, 1)")

# grid.arrange() requires you load the gridExtra package
grid.arrange(p1,p2,p3,p4,ncol=2)
```



For $X \sim \text{Unif}(a, b)$, the pdf, mgf, mean and variance are, respectively,

$$
\begin{aligned}
f_X(x; a, b) &= \frac{1}{b-a}, \qquad a \leq x \leq b \\
M_X(t) &= \begin{cases} \dfrac{e^{bt} - e^{at}}{t(b-a)}, & t \neq 0 \\ 1, & t = 0 \end{cases} \\
\text{E}(X) &= \frac{1}{2}(a+b) \\
\text{Var}(X) &= \frac{1}{12}(b-a)^2
\end{aligned}
$$

For $X \sim \text{Exp}(\lambda)$, the pdf, mgf, mean and variance are

$$
\begin{aligned}
f_X(x; \alpha, \lambda) &= \lambda e^{-\lambda x}, \qquad x > 0 \\
M_X(t) &= \frac{\lambda}{\lambda - t} \\
\text{E}(X) &= \frac{1}{\lambda} \\
\text{Var}(X) &= \frac{1}{\lambda^2}
\end{aligned}
$$

For $X \sim \text{Norm}(\mu, \sigma)$, the pdf, mgf, mean and variance are

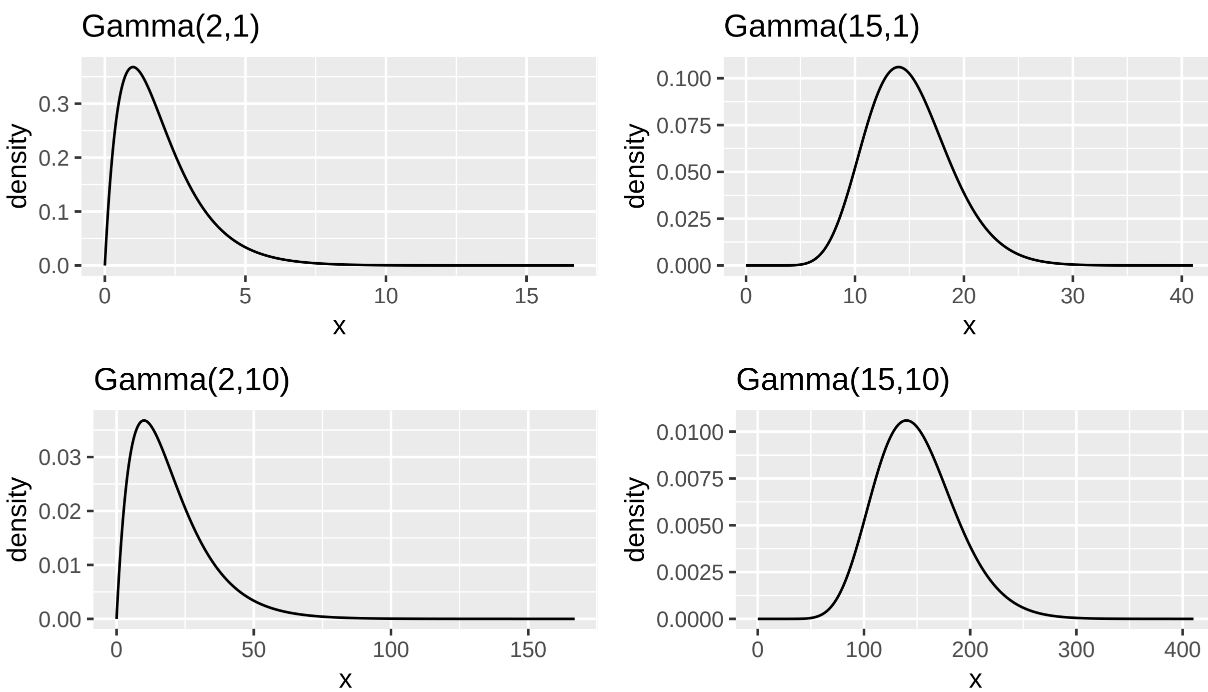
$$
\begin{aligned}
f_X(x; \alpha, \lambda) &= \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) \\
M_X(t) &= \exp\left(\mu t + \frac{\sigma^2 t^2}{2}\right) \\
\text{E}(X) &= \mu \\
\text{Var}(X) &= \sigma^2
\end{aligned}
$$

For $X \sim \text{Tri}(a, b, c)$, the pdf, mean and variance are

$$
\begin{aligned}
f_X(x; a, b, c) &= \begin{cases} \dfrac{2(x-a)}{(b-a)(c-a)}, & a \leq x \leq c \\ \dfrac{2(b-x)}{(b-a)(c-a)}, & c < x \leq b \\ 0, & \text{otherwise} \end{cases} \\
\text{E}(X) &= \frac{1}{3}(a+b+c) \\
\text{Var}(X) &= \frac{1}{18}(a^2 + b^2 + c^2 - ab - ac - bc)
\end{aligned}
$$

Gamma distributions

```
p1 <- gf_dist("gamma", params = list(shape = 2, rate = 1), title = "Gamma(2,1)")
p2 <- gf_dist("gamma", params = list(shape = 15, rate = 1), title = "Gamma(15,1)")
p3 <- gf_dist("gamma", params = list(shape = 2, scale = 10), title = "Gamma(2,10)")
p4 <- gf_dist("gamma", params = list(shape = 15, scale = 10), title = "Gamma(15,10)")
grid.arrange(p1,p2,p3,p4,ncol=2)
```

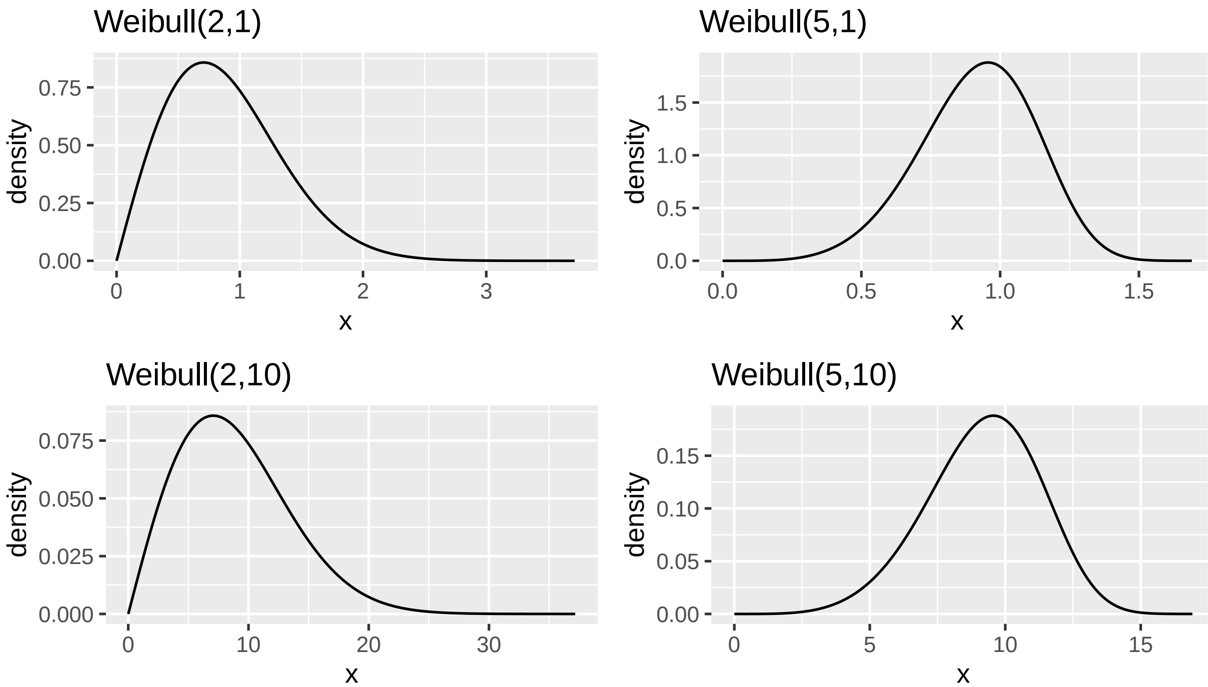


For $X \sim \text{Gamma}(\alpha, \lambda)$, the pdf, mgf, mean and variance are

$$
\begin{aligned}
f_X(x;\alpha,\lambda) &= \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}, \qquad x > 0 \\
M_X(t) &= \left(\frac{\lambda}{\lambda - t}\right)^\alpha \\
\text{E}(X) &= \frac{\alpha}{\lambda} \\
\text{Var}(X) &= \frac{\alpha}{\lambda^2}
\end{aligned}
$$

Weibull distributions

```
p1 <- gf_dist("weibull", params = list(shape = 2, scale = 1),title = "Weibull(2,1)")
p2 <- gf_dist("weibull", params = list(shape = 5, scale = 1),title = "Weibull(5,1)")
p3 <- gf_dist("weibull", params = list(shape = 2, scale = 10),title = "Weibull(2,10)")
p4 <- gf_dist("weibull", params = list(shape = 5, scale = 10),title = "Weibull(5,10)")
grid.arrange(p1,p2,p3,p4,ncol=2)
```

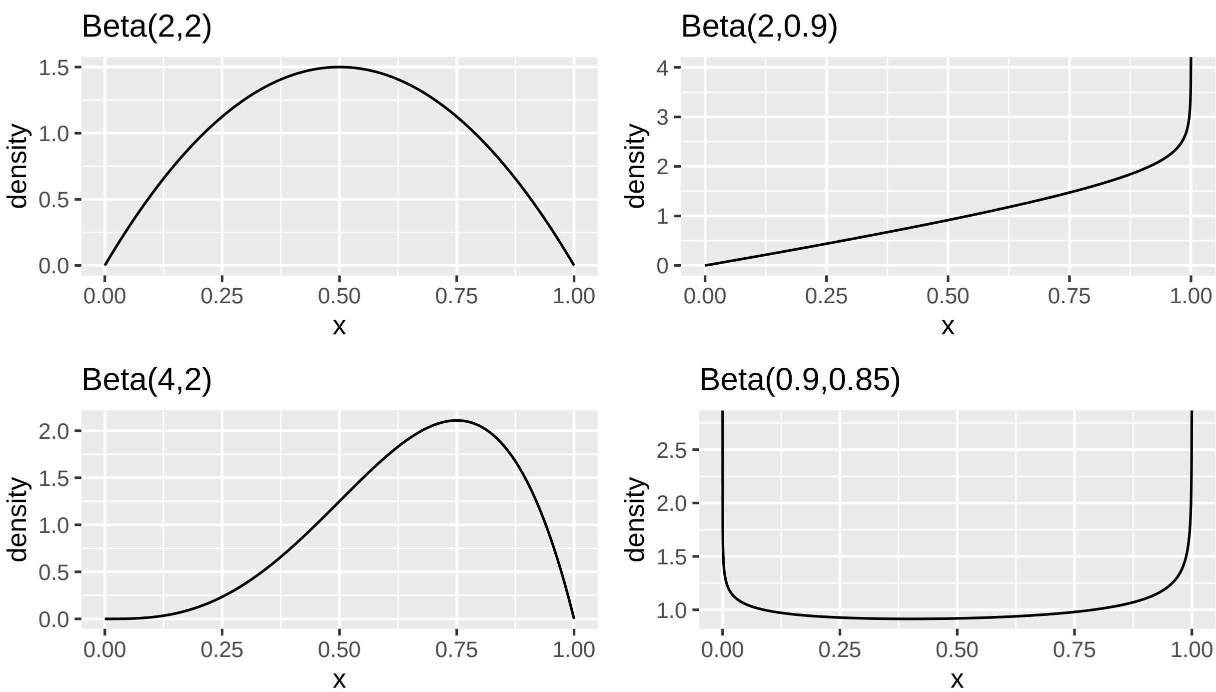


For $X \sim \text{Weibull}(\alpha, \lambda)$, the pdf, mean and variance are

$$\begin{aligned} f_X(x;\alpha,\beta) &= \frac{\alpha}{\beta^\alpha}\, x^{\alpha-1} \exp\left(-\left(\frac{x}{\beta}\right)^\alpha\right), \qquad x > 0 \\ \mathrm{E}(X) &= \beta\,\Gamma\left(1+\frac{1}{\alpha}\right) \\ \mathrm{Var}(X) &= \beta^2\left[\Gamma\left(1+\frac{2}{\alpha}\right) - \left[\Gamma\left(1+\frac{1}{\alpha}\right)\right]^2\right] \end{aligned}$$

Beta distributions

```
p1 <- gf_dist("beta", params = list(shape1 = 2, shape2 = 2), title = "Beta(2,2)")
p2 <- gf_dist("beta", params = list(shape1 = 2, shape2 = 0.9), title = "Beta(2,0.9)")
p3 <- gf_dist("beta", params = list(shape1 = 4, shape2 = 2), title = "Beta(4,2)")
p4 <- gf_dist("beta", params = list(shape1 = 0.9, shape2 = 0.85), title = "Beta(0.9,0.85)")
grid.arrange(p1,p2,p3,p4,ncol=2)
```

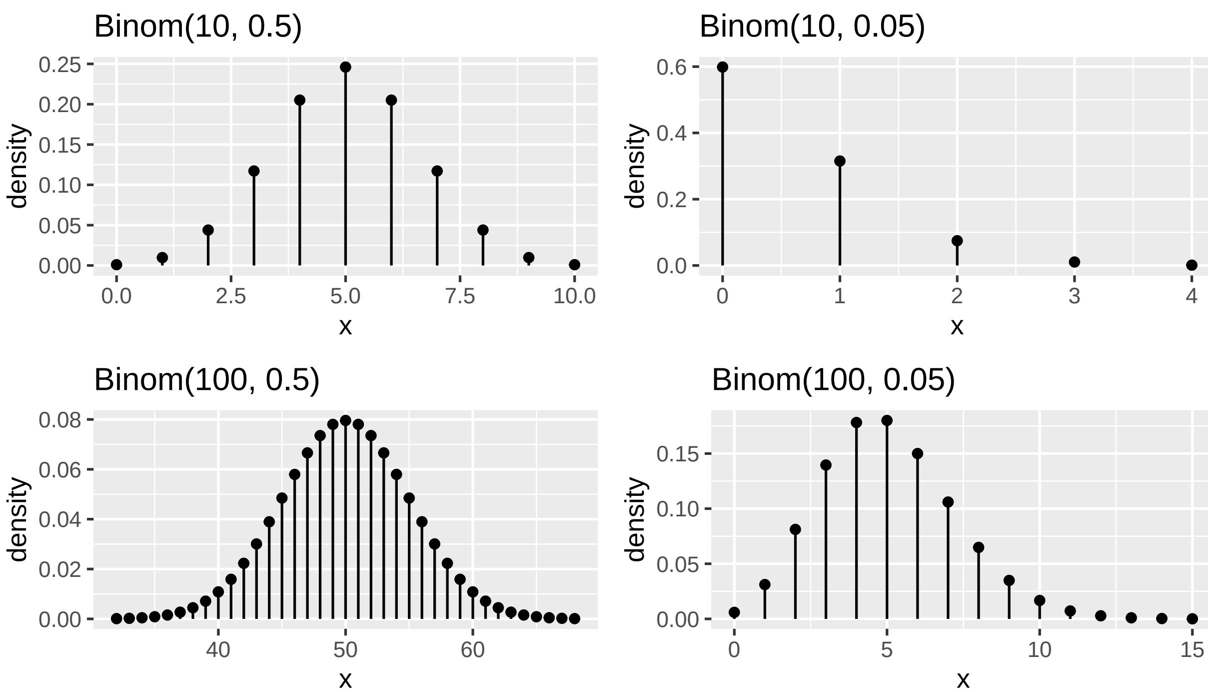


For $X \sim \text{Beta}(\alpha, \beta)$, the pdf, mgf, mean and variance are

$$
\begin{aligned}
f_X(x; \alpha, \beta) &= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \qquad 0 \le x \le 1 \\
M_X(t) &= 1 + \sum_{k=1}^{\infty} \left(\prod_{r=0}^{k-1} \frac{\alpha+r}{\alpha+\beta+r} \right) \frac{t^k}{k!} \\
\mathrm{E}(X) &= \frac{\alpha}{\alpha+\beta} \\
\mathrm{Var}(X) &= \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}
\end{aligned}
$$

Binomial distributions

```
p1 <- gf_dist("binom", size = 10, prob = 0.5, title = "Binom(10, 0.5)")
p2 <- gf_dist("binom", size = 10, prob = 0.05, title = "Binom(10, 0.05)")
p3 <- gf_dist("binom", size = 100, prob = 0.5, title = "Binom(100, 0.5)")
p4 <- gf_dist("binom", size = 100, prob = 0.05, title = "Binom(100, 0.05)")
grid.arrange(p1,p2,p3,p4,ncol=2)
```

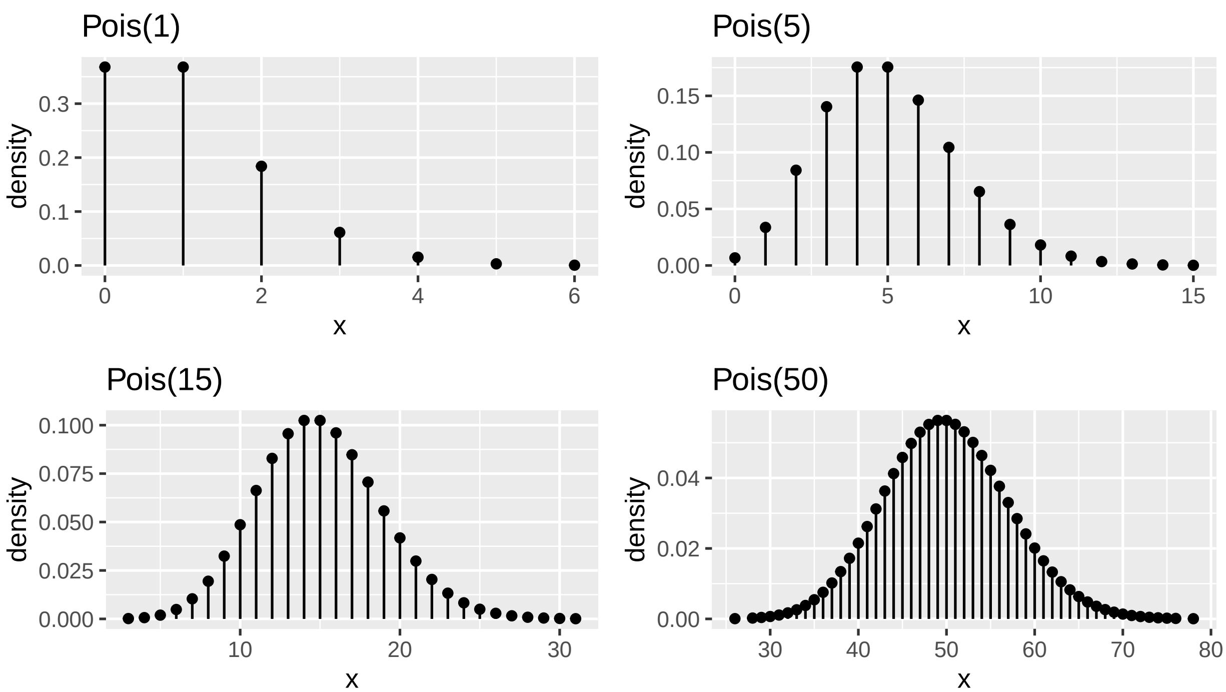


For $X \sim \text{Binom}(n, \pi)$, the pmf, mgf, mean and variance are

$$
\begin{aligned}
f_X(x; n, \pi) &= \frac{n!}{x!(n-x)!}\, \pi^x (1-\pi)^{n-x}, \qquad x = 0, 1, 2, \ldots, n \\
M_X(t) &= (\pi e^t + 1 - \pi)^n \\
\mathrm{E}(X) &= n\pi \\
\mathrm{Var}(X) &= n\pi(1-\pi)
\end{aligned}
$$

Poisson distributions

```
p1 <- gf_dist("pois", lambda = 1, title = "Pois(1)")
p2 <- gf_dist("pois", lambda = 5, title = "Pois(5)")
p3 <- gf_dist("pois", lambda = 15, title = "Pois(15)")
p4 <- gf_dist("pois", lambda = 50, title = "Pois(50)")
grid.arrange(p1,p2,p3,p4,ncol=2)
```



For $X \sim \text{Pois}(\lambda)$, the pmf, mgf, mean and variance are

$$
\begin{aligned}
f_X(x;\lambda) &= e^{-x}\frac{\lambda^x}{x!}, \qquad x = 0, 1, 2, ... \\
M_X(t) &= e^{\lambda(e^t - 1)} \\
\mathrm{E}(X) &= \lambda \\
\mathrm{Var}(X) &= \lambda
\end{aligned}
$$