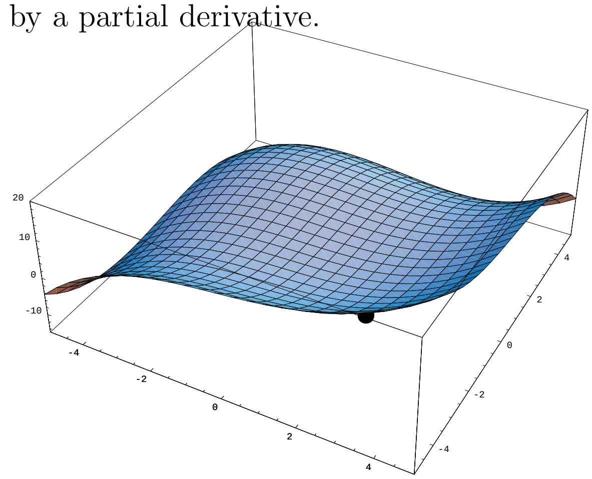
MATH 162: Calculus II

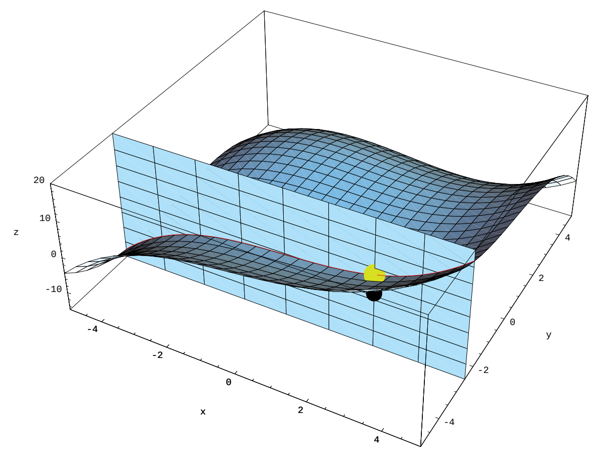
Framework for Thurs., Mar. 1

Partial Derivatives

Today's Goal: To understand what is meant by a partial derivative.



In the framework that introduced multivariate functions, we indicated that one can always turn a function of n variables (like the one depicted at top right) into a function of one particular variable by holding the others constant. Suppose (x_0, y_0) is an interior point to the dom(f) (the black dot, for example). Holding $y = y_0$ fixed and letting x take on values near x_0 traces out a curve on this surface. This curve is the intersection of our surface $z = f(x, y)$ and the plane $y = y_0$. (See the bottom figure.)



One might ask what the slope of this curve (the one where the plane and surface intersect) is above the point (x_0, y_0)—that is, at the location of the point $(x_0, y_0, f(x_0, y_0))$ (the yellow dot). The answer would be found by taking a derivative of the function of x that results by holding $y = y_0$ fixed:

$$\left.\frac{\partial f}{\partial x}\right|_{(x_0,y_0)} := \lim_{h\to 0} \frac{f(x_0+h, y_0) - f(x_0, y_0)}{h},$$

called the *partial derivative of f with respect to x at* (x_0, y_0). This partial derivative has various other notations:

$$\frac{\partial f}{\partial x}(x_0, y_0), \quad f_x(x_0, y_0), \quad \left.\frac{\partial z}{\partial x}\right|_{(x_0,y_0)}.$$

Of course, the partial derivative is a function of x and y in its own right. When we think of it that way, we write

$$\frac{\partial f}{\partial x} \quad \text{or} \quad f_x.$$

We can also take partial derivatives with respect to y (or other variables, if f is a function of more than 2 variables). The resulting partial derivatives may be differentiated again:

$\frac{\partial^2 f}{\partial y^2}$, or f_{yy} get this by differentiating f_y with respect to y.

$\frac{\partial^2 f}{\partial x \partial y}$, or f_{yx} get this by differentiating f_y with respect to x.

$\frac{\partial^2 f}{\partial y^2 \partial x}$, or f_{xyy} get this by differentiating f_x twice with respect to y.

The usual theorems that provide shortcuts to taking derivative may be applied, keeping in mind which variable(s) is being held constant.

Examples:

$$\frac{2x^2 - y}{3x - xy^2}$$

$$\exp(x/y^2)$$

$$\ln(xy^2)$$