Vector Functions

The position vector

One might take functions $x(t), y(t), z(t)$ as the components of a *vector function*:

$$\mathbf{r}(t) \;=\; x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}.$$

This can be thought of as a **position vector** describing the locations of a moving particle. That is, when drawn in *standard position* (i.e., with its initial point at the origin), the terminal point of $\mathbf{r}(t)$ moves so as to trace out a curve.

Limits and continuity of vector functions

While the following is not identical to the definition given in the text for the limit of a vector function, Theorem 3.1 shows the two are logically equivalent.

Definition: Let $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$ (so the component functions of $\mathbf{r}(t)$ are $x(t)$, $y(t)$ and $z(t)$). We say that

$$\lim_{t\to t_0} \mathbf{r}(t) \;=\; \mathbf{L} \;=\; L_1\mathbf{i} + L_2\mathbf{j} + L_3\mathbf{k}$$

precisely when each corresponding limit of the component functions

$$\lim_{t\to t_0} x(t) \;=\; L_1, \qquad \lim_{t\to t_0} y(t) \;=\; L_2, \qquad \text{and} \qquad \lim_{t\to t_0} z(t) \;=\; L_3$$

holds.

Not repeating the definition, here, for "the function $\mathbf{r}(t)$ is **continuous at** $t = t_0$," we note that it is a provable fact (or theorem) that

$$\mathbf{r}(t) \text{ is continuous at } t = t_0 \quad \text{iff} \quad \text{each of } x(t), y(t), z(t) \text{ is continuous at } t = t_0.$$

Example: The vector function $\mathbf{r}(t) = t/(t-1)^2\mathbf{i} + (\ln t)\mathbf{j}$ is continuous at all points t where its component functions $x(t) = t/(t-1)^2$ and $y(t) = \ln t$ are continuous—that is for $t > 0$. Thus, $\lim_{t\to t_0} \mathbf{r}(t)$ exists whenever $t_0 > 0$.