MATH 162: Calculus II
Framework for Mon., Apr. 16
Applications of Double Integrals

Today's Goal: To use double integrals meaningfully in solving problems.

Important Note: In conjunction with this framework, you should look over Section 13.3 of your text.

Area

If R is a bounded region of the plane, then $\iint_R dA$ gives the area of R. (This is because the volume under the curve $z = 1$ over the region R, while it has different units, is the same as the area of R.)

Average Value of a Function

For $y = f(x)$ (a function of one variable), we defined the average value of f over the interval $[a, b]$ to be

$$\frac{1}{b-a}\int_a^b f(x)\,dx \;=\; \frac{1}{\text{length of interval } [a,b]}\int_a^b f(x)\,dx.$$

Similarly,

Definition: If $f(x, y)$ is integrable over a region R of the plane, then the *average value* of f over R is defined to be

$$\frac{1}{\text{Area}(R)}\iint_R f(x,y)\,dA.$$

Integral of a Density

Some functions give the amount of something per unit of measurement, as in

- $f(x)$ is the number of grams per unit length in an (idealized) 1-dimensional string.
- $f(x, y)$ is the number of grams per unit area in an (idealized) 2-dimensional plate.
- $f(x, y, z)$ is the number of molecules per unit volume of a certain gas.

Such functions are collectively known as *densities*. The examples above are 1, 2 and 3-dimensional densities respectively.

If $f(x, y)$ gives the density (2-dimensional) of something at each point (x, y) in the region R, then $\iint_R f(x, y)\,dA$ is the total of that substance found in R.