Stat 145, Fri 4-Sep-2020 -- Fri 4-Sep-2020

Biostatistics

Spring 2020

Friday, September 04th 2020

Wrap-up of set theory (Appendix B)

- geometric series

$$a + ra + r^2 a + r^3 a + \dots + r^{n-1} a + \dots$$

$$= a\underbrace{(1 + r + r^2 + r^3 + \dots)}_{\text{converges if } |r|<1} = a \cdot \frac{1}{1-r}$$

- Cartesian products of sets A x B $= \{(a, b) \mid a \in A,\ b \in B\}$

  tuples

$$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$$

$$A = \{a, b, c, d, \dots, z\}, \quad B = \{0, 1, 2, \dots, 9\} \qquad A \times A \times A \times B \times B \times B \times B$$

  cardinality of A x B given cardinalities of A, B

$$|A \times B| = |A||B|$$

  application: counting

How many license plates?

$$|A \times A \times A \times B \times B \times B \times B| = |A|^3 |B|^4 = 26^3 10^4$$

Probability

Context:

Example
Random process: roll two dice

outcomes: (1, 1)
(1, 2)
(1, 3),
⋮
(6, 6)

$X(\text{outcome}) = \text{sum of pips}$

- random process
- sample space S
  - $s \in S$ is called an **outcome**
  - $E \subset S$ is called an **event**
- random variable $X\colon S \to \mathbb{R}$
  - discrete vs. continuous random variables
  - in most situations, an event $E \subset S$ is tied to a subset A of $\mathbb{R}$. That is, $s \in E$ if and only if $X(s) \in A$.

$E: \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$
$= \{\text{get a } 7\}$

The probability axioms:

1. $P(\emptyset) = 0$
2. $P(S) = 1$
3. $P(A_1 \cup A_2 \cup \cdots) = \sum_{j=1}^{\infty} P(A_j)$ when " "

Disjoint Additivity

$P(A \cup B) = P(A) + P(B)$ when $A \cap B = \emptyset$

$P(A_1 \cup A_2 \cup \cdots \cup A_n) = \sum_{j=1}^{n} P(A_j)$ when mutually disjoint

Theoretical probabilities

- are assigned
- must follow the axioms

- are most useful if they mirror reality

Example: sum of pips from two dice

From this example, notice

- We can appeal to an equally-likely principle
- Counting outcomes is elementary

Bijections

A function $f\colon A \to B$ is

- injective (one-to-one) if $f(x_1) = f(x_2)$ only when $x_1 = x_2$.
- surjective (onto) if for each $y \in B$ there is $x \in A$ with $f(x) = y$.
- bijective if both injective and surjective.

Two dice, take sum

$2, 3, 4, \ldots, 12$ (values of random variable)

$P(2) = 1/36$

$P(3) = 2/36$

$P(7) = \frac{6}{36}$

$P(12) = \frac{\quad}{\quad}$

$$\left\{\begin{array}{cccc} (1,1) & (1,2) & \cdots & (1,6) \\ (2,1) & (2,2) & \cdots & (2,6) \\ \vdots & & & \\ (6,1) & & \cdots & (6,6) \end{array}\right.$$

6 ways to get 7

36 outcomes

If it "feels like" outcomes are equally likely can assign probability

$$P(E) = \frac{|E|}{|S|}$$

"counting things" is an important skill.

$$\sum_{x \in S} (x - m)$$

$|S| = 10$