Parametric Equations and Polar Coordinates

1. Find parametric equations for the line segment joining the points $(3, 1)$ and $(5, 4)$.

2. Eliminate the parameter t in order to express the parametric equations in the alternate form $y = f(x)$:
$$x = \sqrt{t} + 3, \quad y = t - 49.$$

3. Find the points on the parametric curve $c(t) = (3t^2 - 4t, t^3 - 12t)$ where the tangent line has slope 3.

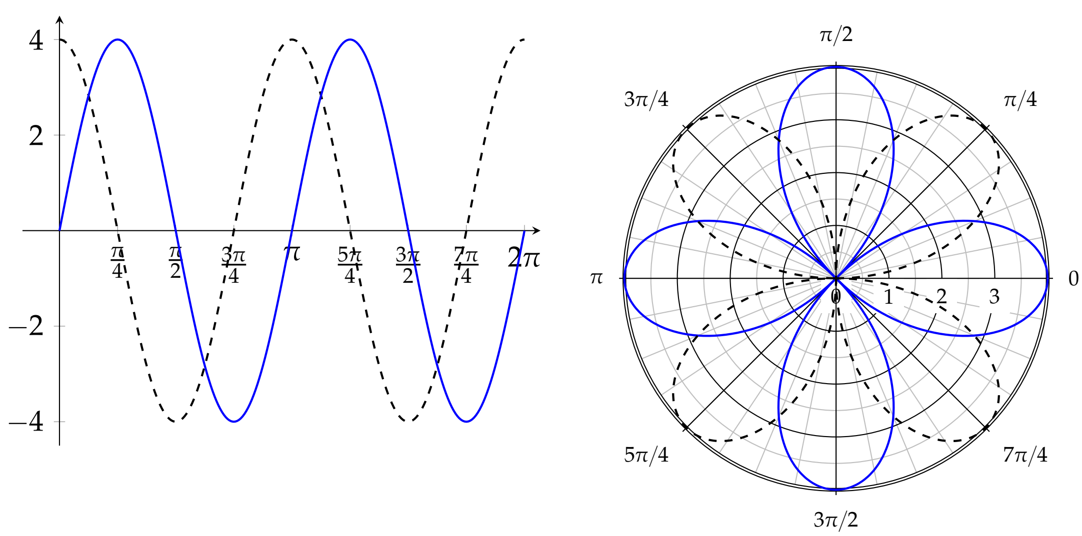
4. (a) Determine the speed ds/dt along the curve $(5\sin(6t), 8\cos(6t))$ at time $t = \pi/4$.
   (b) Determine the speed along the curve $(\ln(5t^2 + 5), 3t^3)$ at time $t = 1$.

5. How much area lies below the curve $c(t) = (3t - 2, t^2/2 + 1)$ over the interval $0 \leqslant t \leqslant 3$?

6. (a) Convert the equation $x = 4$ to an equation in polar coordinates.
   (b) Convert the equation $r = 3\sin\theta$ to an equation in rectangular coordinates.

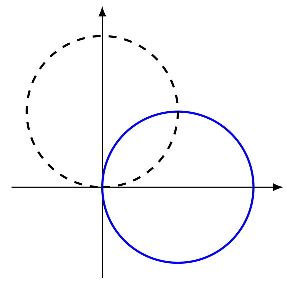
7. Describe the graph of the following polar equations:
   (a) $r = 6$
   (b) $\theta = 8$
   (c) $r = 7\sec\theta$
   (d) $r = 4\csc\theta$

8. Below is displayed the graphs of $r = 4\cos(2\theta)$ (solid curve) and of $r = 4\sin(2\theta)$ (dashed curve). On the left, the graphs are displayed in the θr-plane (i.e, as if r and θ were standard rectangular coordinates). On the right, they are displayed on the xy-plane, with the usual understanding of r as distance from the origin and θ as bearing. The interval $0 \leqslant \theta < 2\pi$ is sufficient to draw the four-petaled leaves without retracing any part of a leaf. On the right, mark the four intersections which correspond to intersections on the left. Why are some intersections on the right "absent" on the left?



9. Find the length of the polar arc $r = \sin^2(\theta/2),\ 0 \leqslant \theta \leqslant \pi$.

10. Find the area of region simultaneously inside both polar curves $r = 5\sin\theta$ and $r = 5\cos\theta$.



11. Use the graph of $r = 2(1 - \cos\theta)$ on the left, plotted in the θr-plane (like rectangular coordinates) to sketch its graph on the xy-plane.