Nested quantifiers

Translate the English sentence into sentences involving universal quantifiers and predicates with bound variables. For example,

> Every real number has a product with zero equalling zero

can be written as

> $\forall x(0 \cdot x = 0)$

with a domain of discourse the *reals*. The sentence

> There is a supervisor who oversees every process in this factory

can be written as

> $\exists x \forall y P(x, y)$,

where x has domain "employees in this factory", y has domain "process in the factory", and $P(x, y)$ is the predicate "x oversees y".

- Every process in this factory is overseen by some person.
- There is a positive integer that is smallest (i.e., at least as small as any other).
- There is a real number that has no reciprocal (multiplicative inverse).
- There is no smallest real number. [Try to write this both with and without a negation symbol.]
- Any secret any person knows can be revealed to the right person.

Rewrite the statement in as simple an English statement as possible. Then write the negation of that statement.

- $\forall$ colors C, $\exists$ an animal A such that A is colored C.
- $\exists$ a book b such that $\forall$ people p, p has read b.
- $\forall$ odd integers n, $\exists$ an integer k such that $n = 2k + 1$.
- $\exists$ real x such that for all real y, $x + y = 0$.

Negating nested quantifiers

- $\neg \forall x \exists y P(x, y) \equiv \exists x \forall y \neg P(x, y)$
- $\neg \exists x \forall y P(x, y) \equiv \forall x \exists y \neg P(x, y)$
- $\neg \forall x \forall y P(x, y) \equiv \exists x \exists y \neg P(x, y)$
- $\neg \exists x \exists y P(x, y) \equiv ?$
- $\neg \forall x \exists y \forall z P(x, y, z) \equiv ?$