2.88 $E(XY) = E(X)$

x	-1	0	1
$\Pr(X=x)$	$1/4$	$1/2$	$1/4$
$\Pr(Y=x)$	0	$1/2$	$1/2$
$\Pr(XY=x)$	$1/4$	$1/2$	$1/4$

$$= (-1)(1/4) + 0 + (1)(1/4)$$
$$= 0$$

$$\Rightarrow \mathrm{Cov}(X, Y) = E(XY) - E(X)E(Y)$$
$$= 0.$$

However, X and Y are not independent, as $f_{X,Y}(x,y) \neq f_X(x) f_Y(y)$.
For example, $f_{X,Y}(1,1) = \frac{1}{4}$, but $f_X(1) f_Y(1) = (1/4)(1/2) = 1/8$.

2.97 phyper(2, 12, 18, 17) yields the same answer. So do
1 - phyper(14, 17, 13, 18) and phyper(3, 18, 12, 13).

2.104 (a) K and Q are not independent. For instance, neither $\Pr(K=3)$ nor $\Pr(Q=3)$ are zero, but $\Pr(K=3 \text{ and } Q=3)$ is zero.

(b) $$\Pr(K=2 \mid Q=2) = \frac{\Pr(K=2 \text{ and } Q=2)}{\Pr(Q=2)} = \frac{\binom{4}{2}^2\binom{44}{1} \Big/ \binom{52}{5}}{\binom{4}{2}\binom{48}{3} \Big/ \binom{52}{5}}$$

$$= \frac{\binom{4}{2}\binom{44}{1}}{\binom{48}{3}} \doteq 0.01526.$$

2.105 (a) K and H are not independent. For instance, $\Pr(H=5) \neq 0$, but $\Pr(H=5 \mid K=2) = 0$.

(b) $$\Pr(K=2 \mid H=2) = \frac{\Pr(K=2 \text{ and } H=2)}{\Pr(H=2)} = \frac{\binom{52}{5}}{\binom{13}{2}\binom{39}{3}} \cdot \Pr(K=2 \text{ and } H=2).$$

But $\Pr(K=2 \text{ and } H=2) = \Pr(\text{heart king and } K=2 \text{ and } H=2) + \Pr(\text{no heart king and } K=2 \text{ and } H=2)$

$$= \left(\binom{3}{1}\binom{12}{1}\binom{36}{2} + \binom{3}{2}\binom{12}{2}\binom{36}{1}\right) \Big/ \binom{52}{5}$$

So, $$\Pr(K=2 \mid H=2) = \left(\binom{3}{1}\binom{12}{1}\binom{36}{2} + \binom{3}{2}\binom{12}{2}\binom{36}{1}\right) \Big/ \left(\binom{13}{2}\binom{39}{3}\right) \doteq 0.0418.$$

3.4 Since f, g are pdfs, for each $x \in \mathbb{R}$,

$$\alpha f(x) + (1-\alpha) g(x) \geq 0,$$

and $$\int_{-\infty}^{\infty} \left[\alpha f(x) + (1-\alpha) g(x)\right] dx = \alpha \int_{-\infty}^{\infty} f(x) dx + (1-\alpha) \int_{-\infty}^{\infty} g(x) dx$$

$$= \alpha \cdot 1 + (1-\alpha) \cdot 1 = 1.$$

3.10 (c) The pdf $f_X(x) = \frac{d}{dx} F_X(x) = \begin{cases} x/2, & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$

(d) $\Pr(Y \leq 1) = \Pr(X^2/4 \leq 1) = \Pr(-2 \leq X \leq 2) = F_X(2) - F_X(-2) = 1$

(e) $\Pr(Y \leq \frac{1}{4}) = \Pr(X^2/4 \leq 1/4) = \Pr(-1 \leq X \leq 1) = F_X(1) - F_X(-1) = 1/4$

(f) For $0 < y < 1$,

$$F_Y(y) = \Pr(Y \leq y) = \Pr(X^2/4 \leq y) = \Pr(-2\sqrt{y} \leq X \leq 2\sqrt{y}) = \Pr(X \leq 2\sqrt{y}) = y$$

So, $$F_Y(y) = \begin{cases} 0, & \text{if } y \leq 0 \\ y, & \text{if } 0 < y < 1 \\ 1, & \text{if } y \geq 1 \end{cases}$$

(g) The pdf $f_Y(y) = \frac{d}{dy} F_Y(y) = \begin{cases} 1, & \text{if } 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$

(h) $Y \sim \text{Unif}(0,1)$.

3.15 (a) $\Pr(X \leq 1) = F_X(1) = 1/4$.

(b) $\Pr(0.5 \leq X \leq 1) = F_X(1) - F_X(0.5) = 0.25 - 0.0625 = 0.1875$.

(c) $\Pr(X > 1.5) = 1 - F_X(1.5) = 1 - 0.5625 = 0.4375$.

(d) Solve $F_X(x) = 0.5 \Rightarrow x^2 = 2 \Rightarrow x = \sqrt{2}$.

(e) The pdf $f_X(x) = \frac{d}{dx} F_X(x) = \begin{cases} x/2, & \text{if } 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$

(f) $$E(X) = \int_{-\infty}^{\infty} x f_X(x) dx = \int_0^2 x \cdot \frac{x}{2} dx = \frac{1}{6} x^3 \Big|_0^2 = \frac{4}{3}.$$

(g) $E(X^2) = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx = \int_0^2 x^2 \cdot \frac{x}{2}\,dx = \frac{1}{8}x^4 \Big|_0^2 = 2$

$$\Rightarrow \quad Var(X) = E(X^2) - E(X)^2 = 2 - \left(\frac{4}{3}\right)^2 = \frac{2}{9}.$$

3.21 For $X \sim DUnif(n)$,

$$M_X(t) = E(e^{tX}) = \sum_{x=1}^{n} e^{tx} \cdot \frac{1}{n} = \frac{e^t}{n}\left(1 + e^t + e^{2t} + \cdots + e^{(n-1)t}\right)$$

$$= \frac{e^t}{n} \cdot \frac{e^{nt}-1}{e^t-1} = \frac{e^t(e^{nt}-1)}{n(e^t-1)}$$

3.22 For $X \sim Geom(\pi)$, $f_X(x) = (1-\pi)^x \pi$

$$\Rightarrow \quad M_X(t) = E(e^{tX}) = \sum_{x=0}^{\infty} e^{tx}(1-\pi)^x \pi = \pi \sum_{x=0}^{\infty} \left[e^t(1-\pi)\right]^x$$

$$= \pi \cdot \frac{1}{1-e^t(1-\pi)}$$

C.3 Let $\vec{v}, \vec{x}, \vec{y} \in \mathbb{R}^n$. Writing $\vec{v} = \langle v_1, v_2, \ldots, v_n \rangle$, we have

$$\vec{v} \circ (\vec{x} + \vec{y}) = \vec{v} \circ \langle x_1 + y_1, x_2 + y_2, \ldots, x_n + y_n \rangle$$

$$= \sum_{j=1}^{n} v_j (x_j + y_j) = \sum_{j=1}^{n} v_j x_j + \sum_{j=1}^{n} v_j y_j = \vec{v} \circ \vec{x} + \vec{v} \circ \vec{y}.$$

The proof is similar for $\vec{v} \circ (\vec{x} - \vec{y})$.

C.17 If A is $m \times n$, then A^T is $n \times m$, and the product AA^T is $m \times m$ (square).