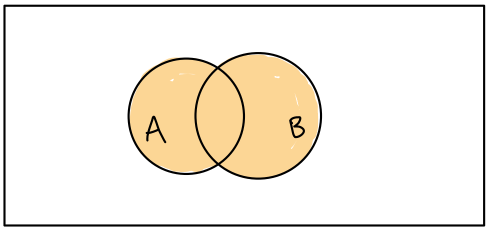
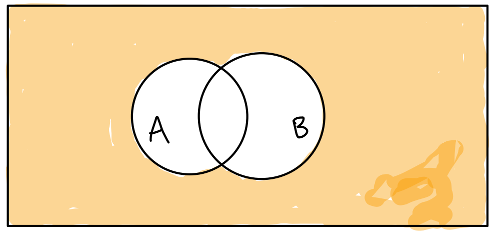
B.2 For $A \cup B$, defined as $\{x \mid x \in A \text{ or } x \in B\}$, to be the same as A, there can be nothing in B except elements which are also in A. That is, B must be a subset of A. A moment's reflection shows that $B \subset A$ is also a sufficient condition.

B.4 $A \cup B$ can be depicted as

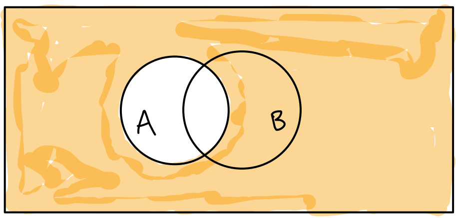


and, thus, $(A \cup B)^c$ is

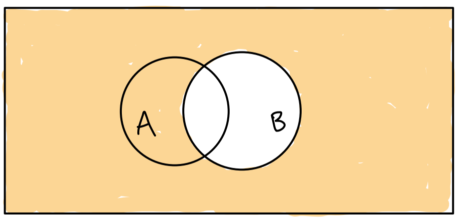


Whereas A^c, B^c are depicted below

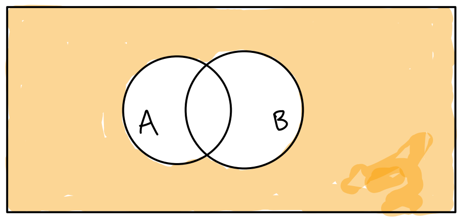
A^c



B^c



So, taken together, we obtain $A^c \cap B^c$



B.9 (a) $f(x) = 1$ precisely when $x \in A$ and $x \in B$; that is, when $x \in A \cap B$.

(b) $f(x) = 0$ whenever $x \notin A \cap B$; that is, precisely when x is absent from A or B.

(c) $f(x) = [\![x \in A \cap B]\!]$

B.11 (a) $\displaystyle\sum_{i=2}^{5} i^2 = 4 + 9 + 16 + 25 = 54.$

(b) $\displaystyle\sum_{n=1}^{4} n = 1 + 2 + 3 + 4 = 10.$

(c) $\displaystyle\sum_{x=1}^{5} (2x - 1) = 1 + 3 + 5 + 7 + 9 = 25.$

(d) $\displaystyle\prod_{n=2}^{4} n = (2)(3)(4) = 24.$

B.15 Let $S = \sum_{i=1}^{n} i = 1 + 2 + 3 + \cdots + n$. Then

$$S = 1 + 2 + 3 + \cdots + n,$$ and
$$S = n + (n-1) + (n-2) + \cdots + 1.$$

Adding these equations gives

$$2S = \underbrace{(n+1) + (n+1) + \cdots + (n+1)}_{n \text{ summands}} = n(n+1).$$

Dividing by 2 gives the result.

B.20 (a) $$\sum_{a=1}^{n} (2a-1)x = 2x \sum_{a=1}^{n} a - x \sum_{a=1}^{n} 1 = 2x \cdot \frac{n(n+1)}{2} - nx$$
$$= nx(n+1-1) = n^2 x.$$

(b) $$\sum_{y=1}^{n} xy = x \sum_{y=1}^{n} y = \frac{1}{2} nx(n+1)$$

(c) $$\sum_{x \in S} (x-m) = \sum_{x \in S} x - m \sum_{x \in S} 1 = \frac{|S|}{|S|} \sum_{x \in S} x - m|S|$$
$$= |S|(\bar{x} - m) = 10(\bar{x} - m).$$

(d) The assumption that
$$m = \sum_{x \in S} \frac{x}{10} = \frac{1}{|S|} \sum_{x \in S} x = \bar{x},$$

leads to

$$\sum_{x \in S} (x-m) = 10(\bar{x} - m) = 0.$$