Formulas Midterm 2

Coordinate changes:

$$\begin{aligned} x &= \rho \sin\phi \cos\theta \\ y &= \rho \sin\phi \sin\theta \\ z &= \rho \cos\phi \end{aligned}$$

$$\begin{aligned} \rho^2 &= x^2 + y^2 + z^2 \\ \tan\theta &= \frac{y}{x} \\ \phi &= \arccos\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right) \end{aligned}$$

$$\begin{aligned} r &= \rho \sin\phi \\ z &= \rho \cos\phi \end{aligned}$$

$$\begin{aligned} \rho &= \sqrt{r^2+z^2} \\ \phi &= \arccos\left(\frac{z}{\sqrt{r^2+z^2}}\right) \end{aligned}$$

Expansion factors:

$$\begin{aligned} dA &= r\,dr\,d\theta \\ dV &= r\,dr\,d\theta\,dz \\ &= \rho^2 \sin\phi\,d\rho\,d\phi\,d\theta \end{aligned}$$

Center of mass, 2D: $\left(\frac{M_x}{M}, \frac{M_x}{M}\right)$

$$\begin{aligned} M_y &= \iint_D x\rho(x,y)\,dA \\ M_x &= \iint_D y\rho(x,y)\,dA \end{aligned}$$

Moment of inertia, 2D:

$$\begin{aligned} I_y &= \iint_D x^2\rho(x,y)\,dA \\ I_x &= \iint_D y^2\rho(x,y)\,dA \\ I_0 &= \iint_D (x^2+y^2)\rho(x,y)\,dA = I_y + I_x \end{aligned}$$

Center of mass, 3D: $\left(\frac{M_{yz}}{M}, \frac{M_{xz}}{M}, \frac{M_{xy}}{M}\right)$

$$\begin{aligned} M_{yz} &= \iint_D x\rho(x,y,z)\,dV \\ M_{xz} &= \iint_D y\rho(x,y,z)\,dV \\ M_{xy} &= \iint_D z\rho(x,y,z)\,dV \end{aligned}$$