MATH 162: Calculus II

Framework for Wed., Feb. 7

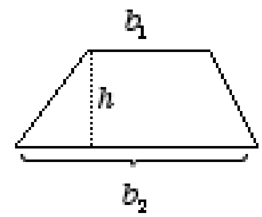
Numerical Integration

Numerical approximations to definite integrals

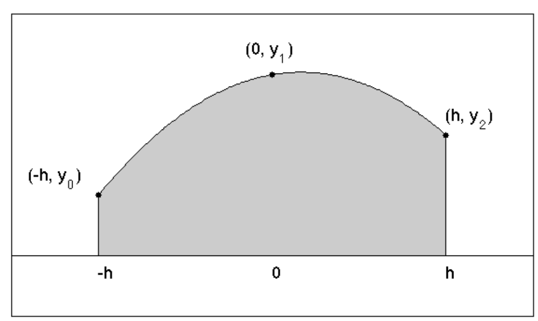
- Riemann (rectangle) sums already give us approximations

  Main types: left-hand, right-hand and midpoint rules

- Question: Why rectangles?
  - trapezoids
    - Area of a trapezoid with bases b_1, b_2, height h



    - Approximation to $\int_a^b f(x)\,dx$ using n steps all of width $\Delta x = (b-a)/n$ (Trapezoid Rule)
    - Remarkable fact: Trapezoid rule does not improve over midpoint rule.
  - parabolic arcs
    - $\int_{-h}^{h} g(x)\,dx$, when $g(x) = Ax^2 + Bx + C$ is chosen to pass through $(-h, y_0)$, $(0, y_1)$ and (h, y_2)



    - Approximation to $\int_a^b f(x)\,dx$ using n (even) steps all of width Δx (Simpson's Rule)

- Error bounds
  - No such thing available for a general integrand f
  - Formulas (available when f is sufficiently differentiable)
    * **Trapezoid Rule**. Suppose f'' is continuous throughout $[a, b]$, and $|f''(x)| \leq M$ for all $x \in [a, b]$. Then the error E_T in using the Trapezoid rule with n steps to approximate $\int_a^b f(x)\,dx$ satisfies

$$|E_T| \leq \frac{M(b-a)^3}{12n^2}.$$

    * **Simpson's Rule**. Suppose $f^{(4)}$ is continuous throughout $[a, b]$, and $|f^{(4)}(x)| \leq M$ for all $x \in [a, b]$. Then the error E_S in using Simpson's rule with n steps to approximate $\int_a^b f(x)\,dx$ satisfies

$$|E_S| \leq \frac{M(b-a)^5}{180n^4}.$$

  - Use
    * For a given n, gives an upper bound on your error
    * If a desired upper bound on error is sought, may be used to determine *a priori* how many steps to use