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Math 251, Wed 30-Sep-2020 -- Wed 30-Sep-2020
Discrete Mathematics
Fall 2020


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Wednesday, September 30th 2020
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Wk 5, We
Topic:: Sequences and sums
Read:: Rosen 2.4
HW::   WW SequencesAndSeries due Sat.
HW::   PS07 due Mon.
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Sequences

> **Definition 1:** A **sequence** is a function $a\colon A \to B$ for which $A \subseteq \mathbb{Z}$.

- Most commonly, $A = \mathbb{N}$ or $\mathbb{Z}^+$, the integers beginning with either 0 or 1.
- Since the domain of a includes only integers, you can talk about $a(2)$, $a(1000)$, etc., but not $a(2.3)$.
- Usually a subscript notation is adopted, a_n instead of $a(n)$, but both refer to the same thing, the value of the sequence for input n.
- The specification of inputs is somewhat arbitrary, less important than the outputs themselves. Different ways of naming the sequence

$$1,\ \frac{1}{2},\ \frac{1}{3},\ \frac{1}{4},\ \cdots$$

include

$a_0 \quad a_1 \quad a_2 \quad a_3 \quad \cdots$ valid

equally valid: $a_5 = 1$

$a_6 = 1/2$

$a_7 = 1/3$

$a_8 = 1/4$

$\vdots$

85, 81, 77, —, —, —

Patterns and formulas

Say our first term is subscripted w/ 0.

Can you recognize a pattern and add several terms?

arithmetic 1. 3, 10, 17, 24, 31, . . . 38, 45, 52 (pattern: add 7)

geometric 2. 5, 10, 20, 40, 80, . . . 160, 320, 640, . . . (pattern: mult. by 2)

3. 2, 7, 9, 16, 25, . . . 41, 66, 107, . . . (pattern: add previous two terms)

arithmetic 4. 18, 14, 10, 6, 2, . . . −2, −6, −10, . . . (subtract 4)

5. 11, 67, 15, 63, 19, 59, . . . 23, 55, 27

Sequences described with a recurrence relation. Write five terms for a sequence that follows this prescription. Answers can be checked for validity. The valid ones are called **solutions**.

1. $a_n = 2a_{n-1} - a_{n-2}$ 0, 1, 2, 3, 4, . . .

geometric 2. $a_n = \frac{1}{3}a_{n-1}$ 162, 54, 18, 6, 2, . . .

3. $a_n = 2 - a_{n-2}$ 1, 1, 1, 1, . . . 1, 3, 1, −1, 1, 3, . . .

Are solutions to these unique? No!

If not, could additional specifications make them unique? (initial conditions)

Yes, by giving initial conditions

#1: Need a_0, a_1 specified

#2: saying $a_0 = 162$

#3: Again need a_0, a_1 specified

Special sequences: arithmetic and geometric

common difference.

Arithmetic sequences: Need starting value a_0

Recurrence relation $a_n = a_{n-1} + d$ (some set d)

Ex.) $a_0 = 5$, $d = 11$ generates arithmetic seq. $5, 16, 27, 38, \ldots$

~~Recording~~ recurrence relation $a_n = a_{n-1} + 11$.

So $a_4 = a_3 + 11 = (a_2 + 11) + 11 = a_2 + 2(11) = a_1 + 3(11) = a_0 + 4(11)$

$= 5 + 4(11)$

More generally $a_n = 5 + 11n$ closed/explicit formula

Geometric sequence: Need starting value a_0, a multiplier r (common ratio)

recurrence relation: $a_n = r\, a_{n-1}$

Ex) $a_0 = 5$, $r = 2$ $\longrightarrow$ geometric seq: $5, 10, 20, 40, 80, \ldots$

$a_4 = 2 \cdot a_3 = 2 \cdot (2a_2) = 2^2 a_2 = 2^3 a_1 = 2^4 a_0$

Generally $a_n = 2^n a_0$

Series: summing terms in a sequence

- summation notation

$$\sum_{j=0}^{5} a_j = a_0 + a_1 + a_2 + a_3 + a_4 + a_5$$

- different forms but same sum

$k = j - 3$

$j = k + 3$

$$\sum_{k=0}^{4} \left[2(k+3) - 1\right]^2 = (2 \cdot 3 - 1)^2 + \cdots + (2 \cdot 7 - 1)^2$$

- examples

1. $\sum_{j=3}^{7} (2j-1)^2$ $= (2\cdot3-1)^2 + (2\cdot4-1)^2 + (2\cdot5-1)^2 + (2\cdot6-1)^2 + (2\cdot7-1)^2$

$a_j = (2j-1)^2$ $=$