Some Chapter 3 problems

1. Suppose a list of positive integers is already sorted in order of decreasing size. It is possible that numbers are repeated more than once.
   (a) Describe an algorithm that locates all instances of an integer in the list.
   (b) Give a worst-case estimate the number of comparisons used.
2. Suppose f, g are both functions from the set of positive integers into the reals.
   (a) State what it means for $f(n)$ to be $O(g(n))$.
   (b) Prove directly that $f(n) = 2n^2 + n + 5$ is $O(n^3)$.
   (c) Prove or disprove: $g(n) = n^3$ is $O(2n^2 + n + 5)$.
3. Give a big-O estimate for
   (a) $f(n) = (n! + 1)(2^n + 1) + (3^{n-2} + n^{n-3})(n^3 + 2^n)$.
   (b) $f(n) = \sum_{j=1}^{n} j(j+1)$ (You may use that $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$.)

For more, see

- Chapter 3 Review Questions: 2, 3, 4, 5
- Chapter 3 Supplemental Exercises: 4, 11-18, 21, 22

Some Chapter 5 problems

1. Find and prove a formula for the sum of the first n even integers

$$2 + 4 + \cdots + 2n.$$

   What role does mathematical induction play in the *finding* and/or the *proving*?
2. (a) What does it mean for a function to be well-defined?
   (b) Give a recursive definition for $f(n) = (n+1)!$. Is your function well-defined?

For more, see

- Chapter 5 Review Questions: 1, 4, 11
- Chapter 5 Supplemental Exercises: 3, 6, 7, 8