2. (a) This is the problem of having 10 slots you fill with letters C, T, V and D (D for divider) so that precisely two D's are used.

Counting ways to choose 2 slots for D's answers the question:

$$\binom{10}{2} = \frac{10!}{2!\ 8!} = \frac{(10)(9)}{2} = 45.$$

(b) $9!$ would reflect the number of permutations (strings) one can make out of 9 distinct letters. To use that as an answer would overcount strings, but predictably so: there are

$4! = 24$ equivalent arrangements of the 4 E's

$2! = 2$ " " " " 2 N's

$2! = 2$ " " " " 2 S's

So,
$$\frac{9!}{4!\ 4!\ 2!} = \frac{9\cdot8\cdot7\cdot6\cdot5}{2\cdot4\cdot3\cdot2} = (9)(7)(5) = 315 \text{ strings.}$$

3. Note: $\Pr(A \mid B) + \Pr(A^c \mid B) = 1$

$$\Rightarrow \Pr(A^c \mid B) = 1 - \Pr(A \mid B) = 1 - \Pr(A) \quad \text{(given information)}$$
$$= \Pr(A^c).$$

Since $\Pr(A^c \mid B) = \Pr(A^c)$, A^c and B are independent.

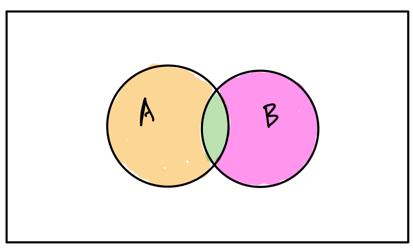
4. Let A denote the event that a person has symptom A.

B denote the event that a person has symptom B.

We know $\Pr(A \setminus B) = 0.31$ (orange shading)

We know $\Pr(B \setminus A) = 0.23$ (pink shading)

We know $\Pr(A \cap B) = 0.09$ (green shading)



(a) $\Pr(\text{neither symptom}) = 1 - 0.31 - 0.23 - 0.09 = 0.37.$

(b) $\Pr(A \cap B \mid B) = \Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)} = \dfrac{0.09}{0.23 + 0.09} = 0.28125$

5. (a) $E(2X) = 2E(X) = 38.$

(b) $E(X+Y) = E(X) + E(Y) = 19 - 8 = 11.$

(c) $E(X-Y) = E(X) - E(Y) = 19 + 8 = 27.$

(d) $Var(X+2Y) = Var(X) + Var(2Y)$ (by independence)

$= Var(X) + 4Var(Y) = 4 + 12 = 16.$

6. (a) We require

$$1 = \sum_{j=0}^{4} c \cdot \frac{j^2}{10} = \frac{c}{10}(0+1+4+9+16) = 3c$$

So, $c = \frac{1}{3}$.

(b) $Pr(X \le 3) = \frac{1}{3} \cdot \frac{1}{10}(0+1+4+9) = \frac{7}{15} \doteq 0.4667$

(c)

$$E(X) = \sum_{j=0}^{4} j f(j) = 0 + \frac{1}{30} + 2 \cdot \frac{4}{30} + 3 \cdot \frac{9}{30} + 4 \cdot \frac{16}{30}$$

$$= \frac{10}{3} \doteq 3.3333. \quad \text{(the mean)}$$

$$E(X^2) = \sum_{j=0}^{4} j^2 f(j) = 0 + \frac{1}{30} + 2^2 \cdot \frac{4}{30} + 3^2 \cdot \frac{9}{30} + 4^2 \cdot \frac{16}{30}$$

$$= \frac{354}{30} = 11.8.$$

So, $Var(X) = E(X^2) - [E(X)]^2 = \frac{354}{30} - \left(\frac{10}{3}\right)^2 = \frac{31}{45} \doteq 0.6889$

7. (a) For X counting defects on a 1 sq-ft piece, $X \sim Pois(0.5)$.

$$Pr(X \ge 1) = 1 - Pr(X=0) = 1 - e^{-0.5}\frac{(0.5)^0}{0!} = 1 - e^{-1/2} \doteq 0.3935$$

(b) For Y counting defects on a 4 sq-ft. piece, $Y \sim Pois(2)$.

$$Pr(Y \ge 2) = 1 - Pr(Y=0) - Pr(Y=1) = 1 - e^{-2}\left(\frac{2^0}{0!}\right) - e^{-2}\left(\frac{2}{1!}\right)$$

$$= 1 - 3e^{-2} \doteq 0.5940.$$

(c) Let Z count the number, out of 3, of 1 sq-ft. pieces w/ a defect. Then $Z \sim Binom(3, 0.3935)$. $Pr(Z \ge 1) = 1 - Pr(Z=0) = 1 - \binom{3}{0}(1-0.3935)^3 \doteq 0.7769.$

(d) 1 - pbinom(0, 3, 0.3935) works. So do others.