

## Answers

1. (a) and (d)
2. (b) only
3. (a) IV (b) III (c) I (d) II
4. Deduce the correct answers from how yours were marked/scored.
5. (a) . . . , choose a sample of size at least 30.  
(b) . . . , you should quadruple the sample size (i.e., obtain a sample which is 4 times as large).
6. (a) Does the new drug cause a greater reduction in blood pressure than the current "best" drug?  
(b) The variable *treatment group* is categorical (values are "old drug" or "new drug"), and serves as the explanatory variable. The *amount blood pressure is reduced* is quantitative, and is the response variable.  
(c) There is the mean amount that patients see their blood pressure reduced by under the "old" (current best) drug, and the mean amount of reduction under the "new" drug. These can be labeled  $\mu_1$  and  $\mu_2$ , though  $\mu_N$  and  $\mu_O$  are more descriptive. Using the latter labels, the hypotheses are
 
$$\mathbf{H}_0: \mu_N - \mu_O = 0, \quad \mathbf{H}_a: \mu_N - \mu_O > 0.$$
- (d) Change in blood pressure is quantitative, so each group yields an  $\bar{x}$  value; we might call them  $\bar{x}_1$  and  $\bar{x}_2$ , although  $\bar{x}_N$  and  $\bar{x}_O$  are more descriptive names. The sample statistic is the difference of these two:  $\bar{x}_N - \bar{x}_O$  (though they may be subtracted the other way, too).
- (e) A randomization sample in this situation typically does not reuse numbers from the same patient—so we will **not** sample with replacement. The important thing is that we take the null hypothesis into account—that the choice of one treatment or the other does not affect results. Thus, one way to produce a randomization statistic is to write the result (difference in blood pressure) of each patient on a slip of paper and place these 46 slips in a bag, mixing them up. Draw out (*without replacement*) 23 slips and call that the "new drug" group, using them to produce a mean  $\bar{x}_N$ . The remaining 23 slips make up the "old drug" group, from which you calculate  $\bar{x}_O$ . The difference  $\bar{x}_N - \bar{x}_O$  is your randomization statistic.
- (f) In our setting, a Type I error is that the new drug is actually equally effective as the old one, but their performances on this particular group of patients gave enough different results to be *statistically significant*, and we concluded the new drug is better.
- (g) Reducing the significance level  $\alpha$  always makes it harder to reject the null hypothesis, thereby decreasing the chances of a Type I error (i.e., less likely we might convict an innocent defendant). At the same time, this increases the chances of a Type II error (that is, it becomes more likely we fail to convince a guilty defendant).
- (h) A  $P$ -value measures the likelihood that we obtain, in similar random samples, a test statistic as extreme or more so as the one obtained in the given sample, in a world where the null hypothesis is true.
- (i) This  $P$ -value is not less than the required significance level 0.01, and so we fail to reject the null hypothesis. The researchers do not have compelling evidence the new drug is an improvement over the old one.
7. (a) A 98% CI would extend from the 1st percentile to the 99<sup>th</sup> percentile. The corresponding  $t^*$ -value, read from the RStudio output, is  $t^* \doteq 2.441$ . We, thus, have 98% CI

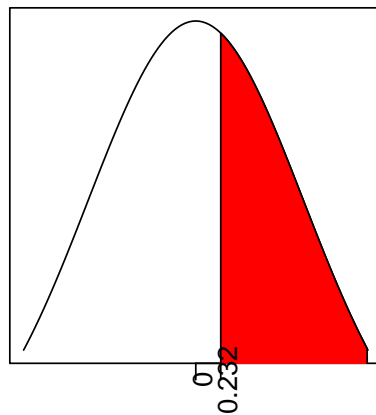
$$34.85 \pm (2.441) \left( \frac{17.9}{\sqrt{35}} \right), \quad \text{or} \quad [27.46, 42.24].$$

- (b) The procedure we have followed in part (a) presumes that the sampling distribution for  $\bar{x}$  is normal. The CLT guarantees this is so for large enough sample sizes, and the rule of thumb given by the Lock's is that a sample size of 30 is generally large enough. We have a sample size of 35, so we are reasonably assured that our resulting confidence interval can be relied on to behave as it should (either contain the true population mean, or be among the rarest 2% of instances when the true mean does not fall inside a 98% CI).
8. (a) Here,  $\hat{p} = 751/1493 \doteq 0.503$ , and the standard error (the one we would have in a world where the null hypothesis is true) is  $SE = \sqrt{(.5)^2/1493} \doteq 0.0129$ . Thus, our standardized z-statistic (since we are dealing with proportions, not means) is

$$z = \frac{0.503 - 0.5}{0.0129} \doteq 0.232.$$

- (b) Either of these plots suffices for the instructions:

**Norm(0,1)**



**Norm(0.5,0.013)**

