Fallacies made by male math students

① $\sqrt{x^2+4} = x+2$ False!!!

$\underline{5} = \sqrt{25} = \sqrt{9+16} = \underbrace{\sqrt{3^2+4^2}}_{\longleftarrow \text{ start } \longrightarrow} = \sqrt{3^2} + \sqrt{4^2} = 3+4 = \underline{7}$

② $\int (t^2+2)e^{-3t}\,dt = \underbrace{\left(\frac{1}{3}t^3+2t\right)}_{\text{antideriv. of } (t^2+2)}\underbrace{\left(-\frac{1}{3}e^{-3t}\right)}_{\text{antideriv. of } e^{-3t}} + C$ False!

<u>False</u> product rule

$$\frac{d}{dt}\left(f(t)\cdot g(t)\right) = f'(t)\cdot g'(t) \quad \text{False!}$$

Reverse of this False product rule, also false.

③ Pertinent to Ch. 5

$\mathcal{L}\{e^{3t}\cdot e^{-7t}\}$ "start"

At least one of these processes is invalid

One "process" (A)

$$\mathcal{L}\{e^{3t}\cdot e^{-7t}\} = \mathcal{L}\{e^{-4t}\} = \frac{1}{s-(-4)} = \frac{1}{s+4}$$

Another "process" (B)

different!

$$\mathcal{L}\{e^{3t}\cdot e^{-7t}\} = \frac{1}{s-3}\cdot\frac{1}{s+7} = \frac{1}{(s-3)(s+7)}$$

<u>Fallacy</u>: $\mathcal{L}\{f(t)\cdot g(t)\} = \mathcal{L}\{f(t)\}\,\mathcal{L}\{g(t)\}$

False!!!

So,

	On the t-side		on s-side
addition/subtraction	$f(t) \pm g(t)$	$\longrightarrow$	$\mathcal{L}\{f(t)\} \pm \mathcal{L}\{g(t)\}$
mult. by a const.	$c\,f(t)$	$\longrightarrow$	$c\,\mathcal{L}\{f(t)\}$

No $\quad f(t)\cdot g(t) \quad \not\longrightarrow \quad \mathcal{L}\{f(t)\}\cdot\mathcal{L}\{g(t)\}$

Yes $\quad f * g \quad \longrightarrow \quad \mathcal{L}\{f(t)\}\cdot\mathcal{L}\{g(t)\}$

Ways for combining fns. : $\quad +,\ -,\ \times,\ \div,\ $ compose $f(g(t))$

New way: Convolution

$$(f * g)(t) = \int_0^t f(t-w)\,g(w)\,dw$$

symbol for convolution

Theorem: $\quad \mathcal{L}\{(f*g)(t)\} = \mathcal{L}\{f(t)\}\cdot\mathcal{L}\{g(t)\}$

Often used to return from s-side (to take an inverse L.T.)

Ex. $\quad \mathcal{L}^{-1}\left\{\dfrac{1}{s^2}\cdot\dfrac{1}{s-3}\right\}$

One approach: partial fractions

$$\frac{1}{s^2(s-3)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-3}$$

Another approach afforded by the Convolution Theorem

$\frac{1}{s^2}$ comes (see table, p. 242) from t

$\frac{1}{s-3}$ " " e^{3t}

product (regular mult.) $\frac{1}{s^2} \cdot \frac{1}{s-3}$ comes from convolution of t, e^{3t} $\int_0^t (t-w)\, e^{3w}\, dw$

$$\mathcal{L}^{-1}\left\{ \frac{1}{s^2(s-3)} \right\} = \int_0^t (t-w)\, e^{3w}\, dw$$

$u = t - w$
$dv = e^{3w}\, dw$
$du = -dw$
$v = \frac{1}{3} e^{3w}$

$$= \frac{1}{3}(t-w)\, e^{3w} \Big|_0^t + \frac{1}{3}\int_0^t e^{3w}\, dw$$

$$= \underbrace{0}_{\substack{\text{when} \\ w=t}} - \underbrace{\frac{1}{3}(t-0)e^0}_{\substack{\text{when} \\ w=0}} + \frac{1}{9} e^{3w} \Big|_0^t$$

$$= -\frac{1}{3}t + \frac{1}{9}e^{3t} - \frac{1}{9}$$ Answer

Ex

$$\mathcal{L}\Big\{ \underbrace{\int_0^t \cos(3t-3w)\sin(2w)\, dw}_{\underline{\cos(3t)} \,*\, \underline{\sin(2t)}} \Big\} = \mathcal{L}\{\cos(3t)\} \cdot \mathcal{L}\{\sin(2t)\}$$

mult. on s side

$$= \boxed{\frac{s}{s^2+9} \cdot \frac{2}{s^2+4}}$$

convolving on t-side

Ex.) $ay'' + by' + cy = f(t), \quad y(0)=0, \; y'(0)=0$

Take L.T. both sides

$$a\,\mathcal{L}\{y''\} + b\,\mathcal{L}\{y'\} + c\,\mathcal{L}\{y\} = \mathcal{L}\{f\}$$

$$a\left[s^2 Y - sy(0) - y'(0)\right] + b\left[sY - y(0)\right] + cY = F$$

$$(as^2 + bs + c)Y = F$$

$$Y(s) = \underbrace{\left(\frac{1}{as^2+bs+c}\right)}_{\text{transfer fn.}} \cdot F(s) = H(s)\,F(s)$$

Soln.

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\{H(s) \cdot F(s)\}$$

mult. on s-side

"impulse response"

Call $\underline{h(t)} = \mathcal{L}^{-1}\{H(s)\}$. Already have $f(t) = \mathcal{L}^{-1}\{F\}$

By Convolution Thm,

$$y(t) = (h * f)(t)$$

Special Case: $f(t) = \delta(t)$

$$\boxed{ay'' + by' + cy = \delta(t), \text{ zero ICs}}$$

$$y(t) = h * \delta = \int_0^t \underbrace{\delta(t-w)}_{\substack{\text{activated} \\ \text{when } w=t}} h(w)\,dw = h(t)$$

sifting property of Dirac delta

Properties of convolution

Associativity $\quad (f * g) * h = f * (g * h)$

Distributivity $\quad (f + g) * h = (f * h) + (g * h)$

Commutativity $\quad (f * g)(t) = \int_0^t f(t-w)\, g(w)\, dw$

$$= \int_0^t f(w)\, g(t-w)\, dw$$

$$= (g * f)(t)$$

Convolution w/ 0

$$(\underbrace{f * 0}_{\uparrow})(t) = \int_0^t f(t-w) \cdot 0 \, dw = 0$$

zero fn