Exercise 4.36

(a) Task: Method of CI construction for a difference of means $\mu_1 - \mu_2$ when i.i.d. samples are taken from populations $Norm(\mu_1, \sigma_1)$, $Norm(\mu_2, \sigma_2)$.

Assume: σ_1, σ_2 are available / known.

CI: $$\underbrace{(\bar{X}_1 - \bar{X}_2)}_{\text{pt. est for } \mu_1 - \mu_2} \pm Z^* \cdot \underbrace{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}_{SE_{\bar{X}_1 - \bar{X}_2}}$$

Why valid:

Know: $\bar{X}_1 \sim Norm\left(\mu_1, \frac{\sigma_1}{\sqrt{n_1}}\right)$, $\bar{X}_2 \sim Norm\left(\mu_2, \frac{\sigma_2}{\sqrt{n_2}}\right)$

Given that $\bar{X}_1, \bar{X}_2$ are independent

$$Var(\bar{X}_1 - \bar{X}_2) = Var(\bar{X}_1) + Var(\bar{X}_2)$$

$$= \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$

$$\Rightarrow \quad \underset{\uparrow \atop SE_{\bar{X}_1 - \bar{X}_2}}{\sigma_{\bar{X}_1 - \bar{X}_2}} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

(b) Task: Look at coverage rate

For samples taken from Norm(8, 2)

True value $\mu_1 - \mu_2 = 8 - 8 = 0$

If sample 1 taken from Norm($\underline{8}$, 2)
2 " " Norm($\underline{6}$, 3)

Known populations leads to known $\mu_1 - \mu_2$

true value $\underline{\mu_1 - \mu_2} = 2$

If Sample 1 comes from Norm(5, 2) $\longrightarrow \mu_1 = 5$
2 " " Exp($\lambda = 1/3$) $\rightarrow \mu_2 = 3$

$$X \sim Exp(\lambda) \implies E(X) = \frac{1}{\lambda}$$

$$Var(X) = 1/\lambda^2$$