Some questions to explore

Use this class period to investigate, with a couple of neighbors, the following questions. You will be asked to write problems yourself which, in general, is a proven help in learning mathematical (likely other disciplines as well) concepts.

1. Draw a graph for functions $y = f(x)$ and $y = g(x)$, creating a region of interest in the plane.

   Is your region *vertically simple*? Is it horizontally simple? What features do you study on the graph in order to answer these questions?

   Write a generic integral whose value corresponds to the area of the region in the case it is vertically simple. Would this integral produce the area between f and g if the two functions cross in mid-region?

   How must you change/overhaul your integral in the case where the region is not vertically simple?

2. Draw again a region of interest in the plane between functions $y = f(x)$ and $y = g(x)$. We have two formulas,

   $$\int_a^b 2\pi x[f(x) - g(x)]\,dx \qquad \text{and} \qquad \int_a^b \pi\left([f(x)]^2 - [g(x)]^2\right)\,dx,$$

   for computing volumes when a region is rotated about an axis to produce a solid.

   In what instances would the first of these formulas apply? Write a specific problem, perhaps one you could find in a textbook, for which the answer would be obtained using this *cylindrical shell* formula.

   In what instances would the second formula apply? Again, write a specific problem for which the answer could be obtained using this *washer* formula.

   Now write a *volume* problem for which neither formula quite works on its own, but can be adapted in order to your problem. Trade your problem with another student or group, having them write an integral expression for yours while you write one for hers/his/theirs. (You will need to know a correct answer to yours so that you can critique the other person's/group's work.)

3. What is special about these volumes of rotation? What makes them different (easier?) than some of the volume problems of Section 6.2?