2.24 Use S to be the event "is smoker"

W to be the event "is a women"

M to be the event "is a man"

C to be the event "has cancer"

We have been told

$Pr[C \mid M \cap S] = 23 \cdot Pr[C \mid M \cap S^c]$,

$Pr[C \mid W \cap S] = 13 \cdot Pr[C \mid W \cap S^c]$,

$Pr[S \mid M] = 0.231$, $Pr[S \mid W] = 0.183 \Rightarrow Pr[S^c \mid M] = 0.769$, $Pr[S^c \mid W] = 0.817$.

(a) $$Pr[W \mid S] = \frac{\#\text{ of women who smoke}}{\#\text{ of smokers}} = \frac{21.1}{21.1 + 24.8} \doteq 0.46$$

(b) $$Pr[S \mid W \cap C] = \frac{Pr[S \cap W \cap C]}{Pr[W \cap C]} = \frac{Pr[C \mid S \cap W] \cdot Pr[S \cap W]}{Pr[C \cap W \cap S] + Pr[C \cap W \cap S^c]}$$

$$= \frac{Pr[C \mid S \cap W]\, Pr[S \cap W]}{Pr[C \mid S \cap W] \cdot Pr[S \cap W] + Pr[C \mid S^c \cap W] \cdot Pr[S^c \cap W]}$$

$$= \frac{13 \cdot Pr[C \mid S^c \cap W]\, Pr[S \cap W]}{13 \cdot Pr[C \mid S^c \cap W] \cdot Pr[S \cap W] + Pr[C \mid S^c \cap W](1 - Pr[S \cap W])}$$

$$= \frac{13\, Pr[S \cap W]}{13\, Pr[S \cap W] + Pr[S^c \cap W]} = \frac{13\, Pr[S \mid W] Pr[W]}{13\, Pr[S \mid W] Pr[W] + Pr[S^c \mid W] Pr[W]}$$

$$= \frac{13\, Pr[S \mid W]}{13\, Pr[S \mid W] + Pr[S^c \mid W]} = \frac{13(0.183)}{13(0.183) + 0.817} \approx 0.744$$

(c) Similarly to part (b),

$$Pr[S \mid M \cap C] = \frac{23(0.231)}{23(0.231) + 0.769} = \boxed{0.874}$$

2.51 $X \sim Geom(\pi) \Rightarrow Pr(X = x) = (1-\pi)^x \pi$

(a) $$Pr(X \geq k) = \left[(1-\pi)^k + (1-\pi)^{k+1} + \dots\right]\pi$$

$$= (1-\pi)^k \pi \left[1 + (1-\pi) + (1-\pi)^2 + \dots\right] = \frac{(1-\pi)^k \pi}{1-(1-\pi)} = (1-\pi)^k$$

(b)

$$\Pr(X=x \mid X \geq k) = \frac{\Pr(X \geq k \text{ and } X=x)}{\Pr(X \geq k)}$$

$$= \begin{Bmatrix} 0, & x<k \\ \dfrac{\pi(1-\pi)^x}{(1-\pi)^k}, & x \geq k \end{Bmatrix} = \begin{Bmatrix} 0, & x<k \\ \pi(1-\pi)^{x-k}, & x \geq k \end{Bmatrix} = \Pr(X=x-k).$$

(c) Saying $X \geq k$ is like starting over.

2.62 (a) $X=2$: $\binom{4}{2}\left[\binom{26}{5} - 2\binom{13}{5}\right] \Big/ \binom{52}{5} = 0.1459$

$X=4$: $\binom{4}{1}\binom{13}{2}\binom{13}{1}^3 \Big/ \binom{52}{5} = 0.2637$

$X=3$: $1 - \left(\Pr(X=1) + \Pr(X=2) + P(X=4)\right) = 0.5884$

x	$\Pr(X=x)$
1	0.00198
2	0.1459
3	0.5884
4	0.2637

(b) $E(X) = (0.00198) + (2)(0.1459) + (3)(0.5884) + (4)(0.2637) = 3.114$

2.97 phyper(2, 12, 18, 17) yields the same answer. So do

1 - phyper(14, 17, 13, 18) and phyper(3, 18, 12, 13).

3.5 For $X \sim \exp(\lambda)$, we have cdf $F_X(x) = \begin{cases} 0, & x<0 \\ 1-e^{-\lambda x}, & x \geq 0 \end{cases}$

For the median, we solve $0.5 = 1 - e^{-\lambda x} \Rightarrow x = \frac{1}{\lambda}\ln 2$.

The first quartile x satisfies $0.25 = 1 - e^{-\lambda x} \Rightarrow x = -\frac{1}{\lambda}\ln(3/4)$.

The third quartile x satisfies $0.75 = 1 - e^{-\lambda x} \Rightarrow x = \frac{2}{\lambda}\ln 2$.

3.22 For $X \sim Geom(\pi)$, $f_X(x) = (1-\pi)^x \pi$

$$\Rightarrow M_X(t) = E(e^{tX}) = \sum_{x=0}^{\infty} e^{tx}(1-\pi)^x \pi = \pi \sum_{x=0}^{\infty} \left[e^t(1-\pi)\right]^x$$

$$= \pi \cdot \frac{1}{1-e^t(1-\pi)}$$

3.23 $$M_Y(t) = \int_{-\infty}^{\infty} e^{ty} f(y)\,dy = \int_0^1 y e^{ty}\,dy + \int_1^2 (2-y)e^{ty}\,dy$$

$$= \frac{1}{t} y e^{ty}\Big|_0^1 - \frac{1}{t}\int_0^1 e^{ty}\,dy + \frac{1}{t}(2-y)e^{ty}\Big|_1^2 + \frac{1}{t}\int_1^2 e^{ty}\,dy$$

$$= \frac{1}{t}e^t - \frac{1}{t^2}\Big[e^{ty}\Big]_0^1 - \frac{1}{t}e^t + \frac{1}{t^2}\Big[e^{ty}\Big]_1^2$$

$$= \frac{1}{t^2}\left(1-e^y\right) + \frac{1}{t^2}\left(e^{2y}-e^y\right) = \frac{1}{t^2}\left(1-2e^y+e^{2y}\right) = \frac{1}{t^2}\left(1-e^y\right)^2.$$

3.31 $$M_X'(t) = 2e^{2t}(1-t^2)^{-1} + 2te^{2t}(1-t^2)^{-2} \longrightarrow E(X) = M_X'(0) = 2$$

$$M_X''(t) = 4e^{2t}(1-t^2)^{-1} + 8te^{2t}(1-t^2)^{-2} + 2e^{2t}(1-t^2)^{-2} + 8t^2e^{2t}(1-t^2)^{-3}$$

$$\longrightarrow E(X^2) = M_X''(0) = 6$$

Thus, $\mathrm{Var}(X) = 6 - 2^2 = 2.$

3.33 $$M_X'(t) = \frac{18}{(3-t)^3} \longrightarrow E(X) = M_X'(0) = 2/3$$

$$M_X''(t) = \frac{54}{(3-t)^4} \longrightarrow E(X^2) = M_X''(0) = 2/3$$

So, $\mathrm{Var}(X) = \frac{2}{3} - \left(\frac{2}{3}\right)^2 = \frac{2}{9}.$

3.37 (a) Since $X \sim \mathrm{Binom}(n, \pi)$ has MGF $M_X(t) = \left(1-\pi+\pi e^t\right)^n$,
when $M_X(t) = \left(\frac{1}{2}(e^t+1)\right)^{10}$, $X \sim \mathrm{Binom}(10, 1/2)$.

(b) Since $X \sim \mathrm{Norm}(\mu, \sigma)$ has MGF $M_X(t) = e^{\mu t + \sigma^2 t^2/2}$,
when $M_X(t) = e^{t + t^2/2}$, $X \sim \mathrm{Norm}(1, 1)$.

(c) Since $X \sim \mathrm{Exp}(\lambda)$ has MGF $M_X(t) = \frac{1}{1 - t/\lambda}$,
when $M_X(t) = \frac{1}{1-2t}$, $X \sim \mathrm{Exp}(1/2)$.

(d) Since $X \sim \mathrm{Gamma}(\alpha, \lambda)$ has MGF $M_X(t) = \frac{1}{(1-t/\lambda)^\alpha}$
when $M_X(t) = (1-2t)^{-3}$, $X \sim \mathrm{Gamma}(\alpha = 3, \lambda = 1/2)$, or $\mathrm{Gamma}(\alpha=3, \beta=2)$.

3.38 $X \sim \text{Gamma}(\alpha, \lambda)$, so $M_X(t) = \dfrac{1}{(1 - t/\lambda)^\alpha}$. Setting $Y = 3X$, we have

$$M_Y(t) = E(e^{tY}) = E(e^{t(3X)}) = E(e^{(3t)X}) = M_X(3t) = \frac{1}{(1 - 3t/\lambda)^\alpha}$$

$\Rightarrow Y \sim \text{Gamma}(\alpha, \lambda/3)$.

3.39 (a) pexp(2) − pexp(0) = 0.865

(b) pexp(1,2) − pexp(0,2) = 0.865

(c) $\dfrac{2}{2\sqrt{3}} \cdot (b-a) \cdot \dfrac{1}{b-a} = \dfrac{1}{\sqrt{3}} \doteq 0.5774$.

(d) $E(X) = 1/3$, $Var(X) = 2/63$

diff(pbeta(1/3 + c(-1,1) * sqrt(2/63), 2, 4))

= 0.6522.

3.62 (a) Because $R \sim \text{Norm}(100, 20)$, his obtaining 150 would correspond to a Z-score

$$Z_R = \frac{150 - 100}{20} = 2.5$$

For $C \sim \text{Norm}(110, 15)$, $Z_c = \dfrac{150 - 110}{15} = 2.667$

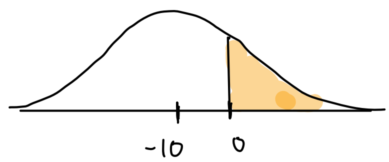
A higher Z-score corresponds to a rarer event. Thus, Ralph should reach scores of 150 (or higher) more often than Claudia.

(b) By normality and independence,

$$R - C \sim \text{Norm}(-10, \sqrt{15^2 + 20^2}) = \text{Norm}(-10, 25)$$

$Pr(R > C) = 1 - \text{pnorm}(0, -10, 25)$

$\doteq 0.345$



(c) Let $\bar{R}, \bar{C}$ be their averages over three games.

$\bar{R} \sim \text{Norm}(100, 20/\sqrt{3})$, $\bar{C} \sim \text{Norm}(110, 15/\sqrt{3})$ and $\bar{R} - \bar{C} \sim \text{Norm}(-10, 25/\sqrt{3})$.

$Pr(\bar{R} > \bar{C}) = 1 - \text{pnorm}(0, -10, 25/\sqrt{3}) \doteq 0.244$.

(d) Let X = # of games won by Ralph. Assuming independence, $X \sim \text{Binom}(3, 0.345)$.

$Pr(X \geq 2) = 1 - \text{pbinom}(1, 3, 0.345) = 0.275$.

C.4 (a) $\text{proj}(\vec{u} \to \vec{v}) = \dfrac{\langle 1,0\rangle \cdot \langle 1,1\rangle}{\langle 1,1\rangle \cdot \langle 1,1\rangle} \langle 1,1\rangle = \dfrac{1}{2}\langle 1,1\rangle = \langle \tfrac{1}{2}, \tfrac{1}{2}\rangle$.

(b) $\text{proj}(\vec{u} \to \vec{v}) = \dfrac{\langle 1,0\rangle \cdot \langle 1,-1\rangle}{\langle 1,-1\rangle \cdot \langle 1,-1\rangle} \langle 1,-1\rangle = \dfrac{1}{2}\langle 1,-1\rangle = \langle \tfrac{1}{2}, -\tfrac{1}{2}\rangle$.

(c) $\text{proj}(\vec{u} \to \vec{v}) = \dfrac{\langle 1,0\rangle \cdot \langle 1,2\rangle}{\langle 1,2\rangle \cdot \langle 1,2\rangle} \langle 1,2\rangle = \dfrac{1}{5}\langle 1,2\rangle = \langle \tfrac{1}{5}, \tfrac{2}{5}\rangle$

(d) $\text{proj}(\vec{u} \to \vec{v}) = \dfrac{\langle 1,2,3\rangle \circ \langle 1,1,1\rangle}{\langle 1,1,1\rangle \circ \langle 1,1,1\rangle} \langle 1,1,1\rangle = \dfrac{6}{3}\langle 1,1,1\rangle = \langle 2,2,2\rangle.$

(e) $\text{proj}(\vec{u} \to \vec{v}) = \dfrac{\langle 1,1,1\rangle \circ \langle 1,2,3\rangle}{\langle 1,2,3\rangle \circ \langle 1,2,3\rangle} \langle 1,2,3\rangle = \dfrac{6}{14}\langle 1,2,3\rangle = \left\langle \frac{3}{7}, \frac{6}{7}, \frac{9}{7}\right\rangle$

(f) $\text{proj}(\vec{u} \to \vec{v}) = \dfrac{\langle 1,2,3\rangle \circ \langle 1,-1,0\rangle}{\langle 1,-1,0\rangle \circ \langle 1,-1,0\rangle} \langle 1,-1,0\rangle = -\dfrac{1}{2}\langle 1,-1,0\rangle = \left\langle -\frac{1}{2}, \frac{1}{2}, 0\right\rangle$

C.17 If A is $m\times n$, then A^T is $n \times m$, and the product AA^T is $m \times m$ (square).

C.21 This statement is true. To demonstrate it, let $B = (A^T)^{-1}$. Then $I = BA^T$. Taking transposes of both sides and noting $I^T = I$, we have $I = (BA^T)^T = AB^T$, showing that $B^T = A^{-1}$. Transposing again gives $B = (A^{-1})^T$.

C.24 (a) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix} = (ad-bc)\,I.$

(b) It is evident that $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, when multiplied by the now-rescaled $\dfrac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, is I.

(c) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^{-1} = -\dfrac{1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}.$