B.21 (a) From the given pmf, we have

$$\sum_x f(x) = \frac{1}{6} + \frac{1}{3} + \frac{1}{4} + \frac{1}{6} + \frac{1}{12} = 1$$

(b) $$\sum_x x f(x) = (0)\left(\frac{1}{6}\right) + \frac{1}{3} + (2)\left(\frac{1}{4}\right) + (3)\left(\frac{1}{6}\right) + (4)\left(\frac{1}{12}\right) = \frac{5}{3}$$

(c) $$\sum_x x^2 f(x) = (0)^2\left(\frac{1}{6}\right) + \frac{1}{3} + (2)^2\left(\frac{1}{4}\right) + (3)^2\left(\frac{1}{6}\right) + (4)^2\left(\frac{1}{12}\right)$$

$$= \frac{1}{3} + 1 + \frac{3}{2} + \frac{4}{3} = \frac{25}{6}$$

2.1 (c) $A^c = \{HHH, HHT, HTH, THH\}$

$A \cap B = \{HTT, TTT\}$

$A \cup C = \{HTT, THT, TTH, TTT, THH\}$

2.2 (c) $A \cap B = \{(3,6),(4,5),(4,6),(5,6)\}$

$B \cup C = \{(1,2),(1,3),(1,4),(1,5),(1,6),(2,3),(2,4),$
$(2,5),(2,6),(3,4),(3,5),(3,6),(4,5),(4,6),$
$(5,6),(5,5),(6,5)\}$

$A \cap (B \cup C) = \{(3,6),(4,5),(4,6),(5,6),(5,5),(6,5)\}$

2.5 Using the language of positions and dividers, there are

$12 + 3 - 1 = 14$ positions,

and out of those a need to choose 2 of them for the dividers.
So, the number of orders is $\binom{14}{2} = \frac{14!}{2!\,12!} = 91.$

2.6 There are $\binom{52}{5}$ possible (and equally-likely) hands. Out of those,

$$\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2} = (13)(4)(12)(6) = 3744$$

are full houses. So, $\Pr(\text{full house}) = \frac{3744}{{}_{52}C_5} \doteq 0.001440576,$

or about 1 in 694.

2.7 There are $\binom{52}{5}$ possible (and equally-likely) hands. Out of those,

$$\binom{13}{2}\binom{4}{2}\binom{4}{2}\binom{11}{1}\binom{4}{1} = (78)(6^2)(11)(4) = 123552$$

contain two-pair (but escape being classified as full houses). So

$$\Pr(\text{two pair}) = \frac{123552}{{}_{52}C_5} \doteq 0.04753902 \quad \text{or about 1 in 21}$$

2.8 There are $\binom{52}{5}$ possible (and equally-likely) hands. Out of those,

$$\binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{1}\binom{4}{1} = (13)(4)(66)(4^2) = 54912$$

have 3-of-a-kind (but escape being classified as full houses). So

$$\Pr(\text{3-of-a-kind}) = \frac{54912}{{}_{52}C_5} \doteq 0.02112845,$$

or about 1 in 47.

2.9 A software solution might begin with a user-defined function like this one:

```
probNoRepeat = function(n) {
    if (n = 1) { return(1) } else {
        product = 1
        for (i in 1:(n-1)) { product = product * (365-i)/365 }
        return (product) }
}
```

(a) $\Pr(\text{at least two of 10 people share birthday}) = 1 - \text{probNoRepeat}(10) \doteq 0.1169482$

(b) $\text{probNoRepeat}(22) \doteq 0.524$ and $\text{probNoRepeat}(23) \doteq 0.493$. So, 23 people.

2.14 By the Inclusion-Exclusion Principle,

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B).$$

Coupled with the fact that $\Pr(A \cup B) \le 1$, we have

$$\Pr(A) + \Pr(B) - \Pr(A \cap B) \le 1 \quad \Rightarrow \quad \Pr(A \cap B) \ge \Pr(A) + \Pr(B) - 1.$$

2.17 (a) $P(\text{bad}) = \dfrac{2+1}{8+10} = \boxed{\dfrac{1}{6}}$

(b) $P(\text{bad} \mid \text{assembly line } 1) = \frac{2}{8} = \boxed{\frac{1}{4}}$

(c) $$P(\text{assembly line } 1 \mid \text{bad}) = \frac{P(\text{bad} \mid \text{assembly line } 1) \cdot P(\text{assembly line } 1)}{P(\text{bad})}$$

$$= \frac{(1/4)(8/18)}{1/6} = \boxed{\frac{2}{3}}$$

2.19 $$\Pr(B \mid A) = \frac{\Pr(A \cap B)}{\Pr(A)} = \frac{\Pr(A \mid B)\Pr(B)}{\Pr(A)} = \frac{\Pr(A)\Pr(B)}{\Pr(A)} = \Pr(B).$$

2.30 There are 10 letters in the word STATISTICS. If these letters were all distinct, there would be $10!$ permutations of them. To use that count here would overcount by a factor $3!$ because of the three S's, by another factor of $3!$ because of the three T's, and $2!$ because of two I's. Thus, there are $$\frac{10!}{(3!)^2\, 2!} = \frac{(10)(9)(8)(7)\cdot(5)(4)}{2} = 50400$$