2. (a) $\underline{III}$ (b) $\underline{IV}$ (c) I (d) $\underline{VII}$ (e) $\underline{V}$ (f) $\underline{VII}$ (g) I

3. (a)

$$\int_0^{\pi/2} \vec{r}(t)\, dt = \frac{1}{2}\sin(2t)\,\hat{i} - \frac{3}{2}t^2\,\hat{j} + \frac{1}{2}\cos(2t)\,\hat{k} \Big|_0^{\pi/2}$$

$$= \left(\frac{1}{2}\hat{i} - \frac{3\pi^2}{8}\hat{j} - \frac{1}{2}\hat{k} \right) - \frac{1}{2}\hat{k}$$

$$= \frac{1}{2}\hat{i} - \frac{3\pi^2}{8}\hat{j} - \hat{k} \qquad \text{or} \qquad \left\langle \frac{1}{2},\ \frac{-3\pi^2}{8},\ -1 \right\rangle$$

(b) $\vec{r}'(t) = \left\langle -2\sin(2t),\ -3,\ -2\cos(2t) \right\rangle$

$\Rightarrow \|\vec{r}'(t)\| = \sqrt{4\sin^2(2t) + 9 + 4\cos^2(2t)} = \sqrt{13}$.

$$\text{length} = \int_1^4 \|\vec{r}'(t)\|\, dt = \sqrt{13}\int_1^4 dt = 3\sqrt{13}.$$

(c) $\vec{T}(t) = \dfrac{\vec{r}'(t)}{\|\vec{r}(t)\|} = \dfrac{1}{\sqrt{13}}\vec{r}'(t) = \left\langle \dfrac{-2}{\sqrt{13}}\sin(2t),\ \dfrac{-3}{\sqrt{13}},\ \dfrac{-2}{\sqrt{13}}\cos(2t) \right\rangle$

4. (a)

$$\frac{\partial f}{\partial z} = \frac{-6xy\,e^{-3z}(z-x^2) - 2xy\,e^{-3z}}{(z-x^2)^2} = \frac{-2xy\,e^{-3z}(3z - 3x^2 + 1)}{(z-x^2)^2}$$

(b) The domain excludes those points (x,y,z) for which $z = x^2$. This is a sheet-like set of points, slicing through the xz-plane in the shape of a parabola, and splitting xyz-space into 2 distinct parts that contain only interior points. So, the domain is open.

(c) This, by definition, is $\dfrac{\partial f}{\partial x}$, so it equals

$$\frac{2ye^{-3z}(z-x^2) - (2x)2xye^{-3z}}{(z-x^2)^2} = \frac{2ye^{-3z}(1 - 3x^2)}{(z-x^2)^2}$$

5. (a) $\overrightarrow{PQ} = \langle -4, 7, 3 \rangle$, So the line can be written as

$$\vec{r}(t) = \langle 1, -3, 2 \rangle + t\langle -4, 7, 3 \rangle = \langle 1-4t, -3+7t, 2+3t \rangle.$$

(b) A normal vector to this plane is

$$\vec{n} = \vec{OP} \times \vec{OQ} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ -3 & 4 & 5 \end{vmatrix} = \begin{vmatrix} -3 & 2 \\ 4 & 5 \end{vmatrix} \hat{i} - \begin{vmatrix} 1 & 2 \\ -3 & 5 \end{vmatrix} \hat{j} + \begin{vmatrix} 1 & -3 \\ -3 & 4 \end{vmatrix} \hat{k}$$

$$= -23\hat{i} - 11\hat{j} - 5\hat{k}.$$

Using the origin as our point, the equation is

$$\vec{n} \cdot \langle x-0, y-0, z-0 \rangle = 0, \qquad \text{or} \qquad -23x - 11y - 5z = 0$$

(c) The angle θ is the acute angle between normal vectors:

$$\langle -23, -11, -5 \rangle \quad \text{and} \quad \langle 0, 0, 1 \rangle :$$

$$\cos\theta = \frac{|-5|}{\sqrt{23^2 + 11^2 + 5^2} \cdot \sqrt{1}} = \frac{5}{\sqrt{529 + 121 + 25}} = \frac{5}{\sqrt{675}} = \frac{1}{3\sqrt{3}}$$

Or, alternately (w/ the other plane)

$$\cos\theta = \frac{2}{\sqrt{25+1+4}} = \frac{2}{\sqrt{30}} .$$