Sample work problem, asked in class on Oct. 2

Problem: A bucket of water of mass 10 kg is pulled at constant velocity up to a platform 40 m above the ground. This takes 18 minutes, during which time 2 kg of water drips out at a steady rate through a hole in the bottom. Find the work needed to raise the bucket to the platform.

Solution: I will take the positive y-axis as moving in the upward direction, with $y = 0$ at the bottom (the position from which the bucket starts its journey). The time, 18 minutes, it takes for the entire trip, is irrelevant information. What is relevant, however, is that the bucket is lifted at constant velocity, so that when it loses 2 kg over 40 m, we can translate that into a constant rate-of-lost-mass-per-unit-of-lift of $(1/20)$ kg/m. Thus, at height $0 \leqslant y \leqslant 40$, the bucket will have lost $(1/20)y$ kg, giving it a

$$\text{mass at height } y = 10 - \frac{y}{20}, \qquad \text{and} \qquad \text{weight at height } y = 9.8\left(10 - \frac{y}{20}\right).$$

That weight is lifted an infinitesimal amount dy, giving

$$\text{work element } = \ 9.8\left(10 - \frac{y}{20}\right)\, dy.$$

The total work is obtained by integrating:

$$W \ = \ \int_0^{40} 9.8\left(10 - \frac{y}{20}\right)\, dy \ = \ 98y - \frac{49}{200}y^2\Big|_0^{40} \ = \ 3528 \text{ Joules.}$$