Benford's Law

See p. 103

Let X be the leading digit of some recorded number on a balance sheet, tax return, etc. Consider the pmf(?):

```
log(2:10, 10) - log(1:9, 10)

[1] 0.30103000 0.17609126 0.12493874 0.09691001 0.07918125 0.06694679 0.05799195
[8] 0.05115252 0.04575749
```

Multinomial

The setting is the same as binomial except for these alterations:

- We assume each of the n trials has $k \geq 2$ possible outcomes. In binomial, $k = 2$.
- In binomial settings, π is the probability of "success" and, necessarily, the probability of "failure" is $1 - \pi$. Now we have individual probabilities for each of the k outcomes: π_1 for outcome 1, π_2 for outcome 2, ..., π_k for outcome k. Naturally,

$$\pi_1 + \pi_2 + \cdots + \pi_k = 1.$$

  When convenient, we will denote this list of probabilities by a vector $\vec{\pi} = \langle \pi_1, \pi_2, \ldots, \pi_k \rangle$.

- In binomial settings, we counted successes, often denoting this count as X. If $X \sim \text{Binom}(n, \pi)$, then $n - X$ is the number of failures.

  Now, we count occurrences of each of the outcomes: X_1 is the number of times in n trials that outcome 1 occurs, X_2 is the number of times in n trials that outcome 2 occurs, ..., X_k is the number of times in n trials that outcome k occurs. We have

$$X_1 + X_2 + \cdots + X_k = n,$$

  and will sometimes refer to the full list in vector form $\mathbf{X} = \langle X_1, X_2, \ldots, X_k \rangle$.

random vector $\vec{X}$

The pmf for such a **random vector X** can be derived, yielding

$$P(\mathbf{X} = \mathbf{x}) = \binom{n}{\mathbf{x}} \pi_1^{x_1} \pi_2^{x_2} \cdots \pi_k^{x_k} = \binom{n}{x_1 x_2 \cdots x_k} \pi_1^{x_1} \pi_2^{x_2} \cdots \pi_k^{x_k}.$$

This involves new notation for a **multinomial coefficient**

$$\binom{n}{\mathbf{x}} = \binom{n}{x_1 x_2 \cdots x_k} := \binom{n}{x_1}\binom{n - x_1}{x_2}\binom{n - x_1 - x_2}{x_3} \cdots \binom{x_k}{x_k} = \frac{n!}{x_1! x_2! \cdots x_k!}.$$

$$\binom{5}{\langle 1,3,1 \rangle} = \frac{5!}{1!\,3!\,1!}$$

Ex.) Rock-paper-scissors: 3 outcomes

play 20 times $(n=20)$

$X_1 = \#$ of "rock"

$X_2 = \#$ of "paper"

$X_3 = \#$ of "scissors"

$\pi_1 = \pi_2 = \pi_3 = 1/3$ $\qquad\qquad \vec{\pi} = \langle \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \rangle$

$$P\big(\underbrace{\langle X_1, X_2, X_3 \rangle}_{\vec{X}} = \langle 8, 8, 4 \rangle \big)$$

Ch. 2, Section 8