

$$1. \quad \vec{\nabla} f(x, y) = \langle 4x + 3y - 1, 3x + 2y \rangle$$

$$z = f(1, -2) + \frac{\partial f}{\partial x}(1, -2)(x - 1) + \frac{\partial f}{\partial y}(1, -2)(y + 2)$$

$$\boxed{z = -1 - 3(x - 1) - 1(y + 2) \quad \text{or} \quad 3x + y + z = -1}$$

$$2. \quad \vec{u} = \frac{\langle 4, -1, 2, -2 \rangle}{\sqrt{3^2 + 4^2}} = \left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$$

$$D_{\vec{u}} f(1, -2) = \vec{\nabla} f(1, -2) \cdot \vec{u} = \langle -3, -1 \rangle \cdot \left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$$

$$= -\frac{9}{5} - \frac{4}{5} = \boxed{\frac{-13}{5}} = -2.6$$

$$3. \quad \text{Solve } \vec{\nabla} f(x, y) = 0 \quad \Leftrightarrow \quad \left. \begin{aligned} \frac{\partial f}{\partial x} &= 4x + 3y - 1 = 0 \\ \frac{\partial f}{\partial y} &= 3x + 2y = 0 \end{aligned} \right\} \Rightarrow \underbrace{x = -2, y = 3}_{\text{inside } R}$$

$$f_{xx} = 4, \quad f_{xy} = 3, \quad f_{yy} = 2, \quad \text{so at every point } (x, y),$$

$$D(x, y) = \begin{vmatrix} 4 & 3 \\ 3 & 2 \end{vmatrix} = -1 < 0, \quad \text{indicating the critical point is}$$

the location of a saddle point.