Form B

1. (a) $\overline{VI}$ (b) $\overline{II}$ (c) $\overline{III}$ (d) $\overline{III}$ (e) $\overline{III}$ (f) $\overline{VII}$ (g) $\overline{VII}$

3. (a) $$\int_0^{\pi/2} \vec{r}(t)\,dt = \frac{-1}{3}\cos(3t)\,\hat{i} - \frac{1}{3}\sin(3t)\,\hat{j} - \frac{5}{2}t^2\hat{k}\,\Big|_0^{\pi/2}$$
$$= \left(\frac{1}{3}\hat{j} - \frac{5\pi^2}{8}\hat{k}\right) - \left(-\frac{1}{3}\hat{i}\right)$$
$$= \frac{1}{3}\hat{i} + \frac{1}{3}\hat{j} - \frac{5\pi^2}{8}\hat{k} \quad \text{or} \quad \left\langle \frac{1}{3}, \frac{1}{3}, \frac{-5\pi^2}{8} \right\rangle$$

(b) $\vec{r}\,'(t) = \langle 3\cos(3t), 3\sin(3t), -5 \rangle$

$$\Rightarrow \|\vec{r}\,'(t)\| = \sqrt{9\cos^2(3t) + 9\sin^2(3t) + 25} = \sqrt{34}$$

$$\text{length} = \int_1^4 \|\vec{r}\,'(t)\|\,dt = \sqrt{34}\int_1^4 dt = 3\sqrt{34}.$$

(c) $$\vec{T}(t) = \frac{\vec{r}\,'(t)}{\|\vec{r}\,'(t)\|} = \frac{1}{\sqrt{34}}\vec{r}\,'(t) = \left\langle \frac{3}{\sqrt{34}}\cos(3t), \frac{3}{\sqrt{34}}\sin(3t), \frac{-5}{\sqrt{34}} \right\rangle$$

4. (a) $$\frac{\partial f}{\partial z} = \frac{-6xye^{-3z}(z-x^2) - 2xye^{-3z}}{(z-x^2)^2} = \frac{-2xye^{-3z}(3z-3x^2+1)}{(z-x^2)^2}$$

(b) The domain excludes those points (x,y,z) for which $z = x^2$. This is a sheet-like set of points, slicing through the xz-plane in the shape of a parabola, and splitting xyz-space into 2 distinct parts that contain only interior points. So, the domain is open.

(c) This, by definition, is $\frac{\partial f}{\partial x}$, so it equals

$$\frac{2ye^{-3z}(z-x^2) + (2x)2xye^{-3z}}{(z-x^2)^2} = \frac{2ye^{-3z}(z+x^2)}{(z-x^2)^2}$$

5. (a) $\overrightarrow{PQ} = \langle -2, 4, -4 \rangle$, so the line can be written as

$$\vec{r}(t) = \langle 3, -1, 2 \rangle + t\langle -2, 4, -4 \rangle = \langle 3-2t, -1+4t, 2-4t \rangle.$$

(b) A normal vector to this plane is

$$\vec{n} = \overrightarrow{OP} \times \overrightarrow{OQ} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -1 & 2 \\ 1 & 3 & -2 \end{vmatrix} = \begin{vmatrix} -1 & 2 \\ 3 & -2 \end{vmatrix} \hat{\imath} - \begin{vmatrix} 3 & 2 \\ 1 & -2 \end{vmatrix} \hat{\jmath} + \begin{vmatrix} 3 & -1 \\ 1 & 3 \end{vmatrix} \hat{k}$$

$$= -4\hat{\imath} + 8\hat{\jmath} + 10\hat{k}.$$

Using the origin as our point, the equation is

$$\vec{n} \circ \langle x-0, y-0, z-0 \rangle = 0, \quad \text{or} \quad -4x + 8y + 10z = 0$$

(c) The angle θ is the acute angle between normal vectors:

$\langle -4, 8, 10 \rangle$ and $\langle 0, 0, 1 \rangle$:

$$\cos\theta = \frac{10}{\sqrt{4^2+8^2+10^2} \cdot \sqrt{1}} = \frac{}{\sqrt{16+64+100}} = \frac{10}{\sqrt{180}} = \frac{\sqrt{5}}{3}$$

Or, alternately (w/ the other plane)

$$\cos\theta = \frac{|-4|}{\sqrt{9+4+16}} = \frac{4}{\sqrt{29}}.$$