Probability

- Concepts and terminology: random process, sample space, event, mutually exclusive, subset, union = "or", intersection = "and", independence, etc.

- Counting principles: Bijection Principle, Multiplication Principle, Inclusion-Exclusion Principle (See Table 2.3, p. 49)

- assigning probability

    - empirically

    - according to the assumption of equal likelihood

    - axioms of probability

- Further rules

    - complement rule

    - multiplication: $\Pr(A \text{ and } B) = \Pr(A \mid B)\Pr(B)$

    - addition: $\Pr(A \text{ or } B) = \Pr(A) + \Pr(B) - \Pr(A \text{ and } B)$

    - law of total probability:
      $\Pr(A) = \Pr(A \text{ and } B) + \Pr(A \text{ and } B^c) = \Pr(A \mid B)\Pr(B) + \Pr(A \mid B^c)\Pr(B^c)$

    - Bayes' Theorem

Discrete distributions

- Random variables

- Roles of pmf and cdf; the relationship between them

- mean and variance

    - ways they are calculated

    - relationships of mean and variance if $Y = aX + b$, or $Z = X + Y$

- Meaning of quantile/percentile/median

- Named families: Bernoulli, binomial, negative binomial, geometric, Poisson, hypergeometric

    - context and assumptions that make one applicable over another

    - ability to turn questions into a query of pmf or cdf

    - writing accurate R commands that return answers relevant to the scenario

Hypothesis testing

- Be able to write hypotheses relevant to a problem description

- Be able to define and identify Type I/II errors

- Be able to describe and write commands to calculate the P-value

- Understand how α and Type I error relate

- Understand how $\beta = \Pr(H_0 \text{ not rejected} \mid H_0 \text{ is false})$, Type II error, and power, are related, as well as the issues involved in calculating these

- Binomial testing

Joint distributions

- use and relations between joint pmfs, marginal distributions, and conditional distributions

- independence of variables: definition and implications on distributions

- meaning of covariance; how it appears in calculation of $\mathrm{Var}(X + Y)$

Continuous distributions

- Random variables
    - computing moments, expected value, variance, moment generating function, all from the definitions
    - using integration to evaluate probabilities, find cdf
    - using differentiation to find pdf
    - functions of r.v.s, and the effect on mean and variance

- Distributional families
    - Named ones: uniform, exponential, normal
    - working with tools provided in R

Along with certain proofs appearing in homework, study proofs of

- Lemma 2.2.12

- Theorems 2.5.8 and 3.2.3

- Lemma 2.5.9

- Theorem 2.6.8 E(X+Y)=E(X)+E(Y) etc.

- Lemma 2.6.12

- Lemma 2.6.13

- Lemma 3.2.6

- Theorem 3.3.6