# Form A Solutions

1. (a) Going through the list of elements in $A$, we have

$$f(-4)=-4,\quad f(-3)=-4,\quad f(-2)=-2,\quad f(-1)=-2,\quad f(7)=6,\quad f(8)=8,\quad f(9)=8.$$

   Thus, $f(A)=\{-4,-2,6,8\}$.

   (b) For each integer $x$, $f(x)$ is the largest even integer that does not exceed $x$. Since $f(12)=12$ and $f(13)=12$, and no other $x \in \mathbb{Z}$ satisfies $f(x)=12$, the desired preimage is $\{12,13\}$.

   (c) $f$ is not surjective, as every odd integer is in the codomain but not in the range.

2. Let us temporarily use propositional variables to rewrite $p$. Taking

   $q$: one owns a craw
   $r$: one earns a deno badge
   $s$: one is a grog

   then statement $p$ can be written in these equivalent forms:

$$(q \wedge r) \to s \;\equiv\; \neg(q \wedge r) \vee s.$$

   (a) The negation of $p$, in symbols, is

$$\neg(\neg(q \wedge r) \vee s) \;\equiv\; q \wedge r \wedge \neg s.$$

   Writing this in English, we have "One owns a craw and earns a deno badge, but is not a grog."

   (b) The contrapositive of $p$ is $\neg s \to \neg(q \wedge r) \equiv \neg s \to (\neg q \vee \neg r)$. In English, this is "If you are not a grog, then you do not own a craw or have not earned a deno badge."

3. $p \leftrightarrow q \;\equiv\; (p \to q) \wedge (q \to p) \;\equiv\; (\neg p \vee q) \wedge (\neg q \vee p)$.

4. (a) $p \oplus q \equiv (p \wedge \neg q) \vee (\neg p \wedge q)$

   (b) $p \wedge q$

   (c) $q \to p \equiv \neg q \vee p$

5. (a) Something like this: "Every student has watched *Casablanca*."

   (b) "There is exactly one movie that no student has watched."

   (c) $\exists m\,(R(m) \wedge \exists s_1 \exists s_2 (s_1 \neq s_2 \wedge W(s_1,m) \wedge W(s_2,m)))$

   (d) The statement you are out to negate can be written as $\exists m(R(m) \wedge \forall s\, W(s,m))$. Following our rules of negation,

$$\begin{aligned}\neg\exists m(R(m) \wedge \forall s\, W(s,m)) \;&\equiv\; \forall m\, \neg(R(m) \wedge \forall s\, W(s,m)) \;\equiv\; \forall m\,(\neg R(m) \vee \neg\forall s\, W(s,m))\\ &\equiv\; \forall m\,(\neg R(m) \vee \exists s\, \neg W(s,m)) \;\equiv\; \forall m\,(R(m) \to \exists s\, \neg W(s,m)).\end{aligned}$$

   This is option (ii).

6. (a) $B \subseteq A$

   (b) neither

7. (a) 6

(b) $2^5 = 32$

(d) $|A \times B| = |A| \cdot |B| = 30$

(e) This statement is False. For there to be a bijection $f$, each element in $A$ would be paired with just one in $B$, and likewise each element in $B$ would be paired with one in $A$. That cannot happen when $|A| \neq |B|$, as is the case here.

8. A membership table is one way to carry this out.

| $A$ | $B$ | $C$ | $B - C$ | $A - (B - C)$ | $A - B$ | $(A - B) \cup C$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 1 | 0 | 1 |

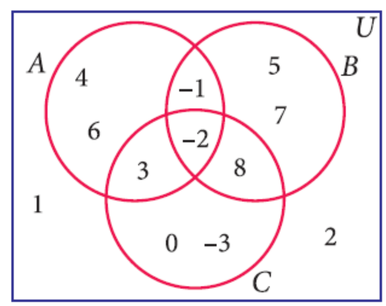
Comparing the $A \cup (B - C)$ column with the $(A \cup B) - C$ one, we see discrepancies in rows 6 and 8. Thus, these sets are not equal.

Another approach is to use specific sets, or a Venn diagram. We illustrate both, taking

$$U = \{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8\}, \quad A = \{-2, -1, 3, 4, 6\},$$

$$B = \{-2, -1, 5, 7, 8\}, \quad C = \{-3, -2, 0, 3, 8\}.$$

Drawing Venn diagrams with these elements inserted, we have $A - (B - C) = \{-2, 3, 4, 6\}$, while $(A - B) \cup C = \{-3, -2, 0, 3, 4, 6, 8\}$.




9. Many answers are correct. Here are several:

(a) Each of $f(x) = x^2$, $f(x) = |x|$, $f(x) = 0$, or $f(x) = \lfloor x \rfloor$ suffices, as each fails the horizontal line test as a function from $\mathbb{R}$ to $\mathbb{R}$.

(b) Each of $f(x) = 2x + 5$, $f(x) = 1 - 7x$, or $f(x) = x^3$ suffices, as each passes the horizontal line test and has range $\mathbb{R}$.

10. (a) $a_n = 8(7)^n$ (b) $a_n = 99 + 13n$

11. (i) This sum involves finitely many, $311 - 3 + 1 = 309$, to be exact, terms of an arithmetic series with first term $15 - 8(3) = -9$ and last term $15 - 8(311) = -2473$. The sum, then, is

$$\left(\frac{1}{2}\right)(309)(-9 + -2473) = \left(\frac{1}{2}\right)(309)(-2482) = -383469.$$

(ii) The sum involves infinitely many terms of a geometric series with $a_0 = 90/25$ and $r = 1/5$. Since $|r| < 1$, the series converges to

$$s = \frac{a_0}{1 - r} = \frac{90/25}{1 - 1/5} = \frac{90/25}{4/5} = \frac{90}{25} \cdot \frac{5}{4} = \frac{450}{100} = 4.5.$$