

$$1. \quad \vec{\nabla} f(x, y) = \langle 4x+3y-1, 3x+2y \rangle$$

$$Z = f(1, -2) + \frac{\partial f}{\partial x}(1, -2)(x-1) + \frac{\partial f}{\partial y}(1, -2)(y+2)$$

$$Z = -1 - 3(x-1) - 1(y+2) \quad \text{or} \quad 3x+y+Z = -1$$

$$2. \quad \vec{u} = \frac{\langle 4-1, 2-2 \rangle}{\sqrt{3^2 + 4^2}} = \left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$$

$$D_{\vec{u}} f(1, -2) = \vec{\nabla} f(1, -2) \circ \vec{u} = \langle -3, -1 \rangle \circ \left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$$

$$= -\frac{9}{5} - \frac{4}{5} = \boxed{-\frac{13}{5}} = -2.6$$

$$3. \quad \text{Solve } \vec{\nabla} f(x, y) = 0 \iff \begin{cases} \frac{\partial f}{\partial x} = 4x+3y-1 = 0 \\ \frac{\partial f}{\partial y} = 3x+2y = 0 \end{cases} \Rightarrow \underbrace{x=-2, y=3}_{\text{inside } R}$$

$f_{xx} = 4, f_{xy} = 3, f_{yy} = 2$ , so at every point  $(x, y)$ ,

$$D(x, y) = \begin{vmatrix} 4 & 3 \\ 3 & 2 \end{vmatrix} = -1 < 0, \text{ indicating the critical point is}$$

the location of a saddle point.