2.2/18. $(B - A) \cup (C - A) = (B \cup C) - A$

Math 251, Wed 7-Oct-2020 -- Wed 7-Oct-2020
Discrete Mathematics
Fall 2020

Wednesday, October 7th 2020

Topic:: Comparison of functions
[[notes/lect11Oct2019.pdf
Read:: Rosen 3.2
HW[[PS08 due Wed.

Highlights of homework
 - Exercise 18, Section 2.2 (p. 157 in preview)
 - Exercise 40, Section 2.3 (p. 175 in preview)
 - Exercise 42, Section 2.3 (p. 175 in preview)

 - Exercise 74, Section 2.3 (p. 176 in preview)
    show: ceil(ceil(x/2) / 2) = ceil(x / 4)
    proof: Let x be a real number. It is 4n - r, for integer n, 0 \le r < 4
      In the case 0 \le r < 2
         ceil(x / 2) = ceil(2n - r/2) = 2n
         ==> ceil(ceil(x/2)) = ceil(2n) = n

Can do approach this by

1) Showing LHS is a subset of RHS and similarly RHS $\subseteq$ LHS

Let $x \in (B - A) \cup (C - A)$

then $x \in B - A$ or $x \in C - A$

if in $B - A$ then $x \in (B \cup C) - A$

likewise, if in $C - A$ then $x \in (B \cup C) - A$.

So $(B-A) \cup (C-A) \subseteq (B \cup C) - A$

2) Use a membership table

A	B	C	$B-A$	$C-A$	LHS $(B-A) \cup (C-A)$
0	0	1			

$\checkmark$

2.3/#40

(a) $f : A \to B$

$f(S \cup T) = f(S) \cup f(T)$ set equality problem

LHS $\subseteq$ RHS ?

Let $y \in f(S \cup T) = \{f(x) \mid x \in S \vee x \in T\}$

So there is an $x \in S \cup T$ for which $f(x) = y$

Does this mean that $y \in f(S) \cup f(T)$
$= \{f(x) \mid x \in S\} \cup \{f(x) \mid x \in T\}$?

Yes, so LHS $\subseteq$ RHS.

still to do: Show RHS $\subseteq$ LHS

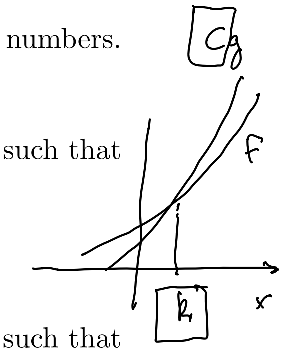
Comparing the Growth of Functions as Inputs (x or n) $\to \infty$

Section 3.2

Suppose f and g are real-valued functions on a domain that includes nonnegative real numbers. We say that

common

Definitions



- f **is of order at most** g, written $f(x)$ is $O(g(x))$, iff there exists $C > 0$ and $k \geqslant 0$ such that

$$|f(x)| \leqslant C|g(x)|, \qquad \text{for all real numbers } x > k.$$

  We call C, k **witnesses** to this **Big-**O relationship.
- f **is of order at least** g, written $f(x)$ is $\Omega(g(x))$, iff there exists $C > 0$ and $k \geqslant 0$ such that

$$|f(x)| \geqslant C|g(x)|, \qquad \text{for all real numbers } x > k.$$

- f **is of order** g, written $f(x)$ is $\Theta(g(x))$, iff f is simultaneously of order at most g and of order at least g.

Note: Similar definitions hold for sequences (functions from $\mathbb{N}$ to $\mathbb{R}$).

Examples:

1. Find witnesses that demonstrate $f(x) = 3x^3 + 2x + 7$ is $O(x^3)$.

Comparing f w/ $g(x) = x^3$.

To demonstrate, need k, C

Try $C = 5$ $\quad |3x^3 + 2x + 7| \leq |5x^3|$ for $x \geq$ ___

Note: $|3x^3 + 2x + 7| \leq \underbrace{|3x^3|}_{\leq 3x^3 \text{ if } x>0} + \underbrace{|2x|}_{\leq x^3 \text{ if } x>2} + \underbrace{|7|}_{\leq x^3 \text{ if } x \geq 2} \leq 5x^3$ if $x \geq 2$

Triangle Ineq $\quad |\text{sum}| \leq \Sigma |\ |$

$C = 5$, $k = 2$ witnesses

2. Show that $f(x) = \dfrac{15\sqrt{x}(2x+9)}{x+1}$ is $\Theta(x^{1/2})$.

Say f is $O(x^{1/2})$ and f is $\Omega(x^{1/2})$

To show f is $\Omega(x^{1/2})$, notice

$$\frac{2x+9}{x+1} \underset{\substack{\text{true} \\ \text{for } x>0}}{\geq} \frac{x+9}{x+1} \geq \frac{x+9}{x+9} = 1$$

So $15\sqrt{x} \cdot \dfrac{2x+9}{x+1} \geq 15\sqrt{x}$ when $x > 0$

Can use $C = 15$, $k = 0$ as witnesses to f being $\Omega(x^{1/2})$.