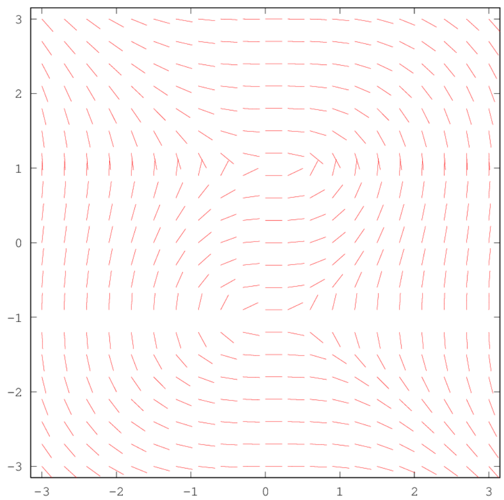
Example 1: One done in class this am

Find and plot solutions to the DE

$$\frac{dy}{dx} = \frac{x^2}{1-y^2}.$$

As this differential equation is in normal form, we may plot its direction field.

```
f = inline('x.^2 ./ (1-y.^2)', 'x','y')
dirfield(f, [-3 3], [-3 3])      % dirfield.m is home-grown
```



Noting the DE is separable, we worked out, in class, solutions $y(x)$ implicitly given as

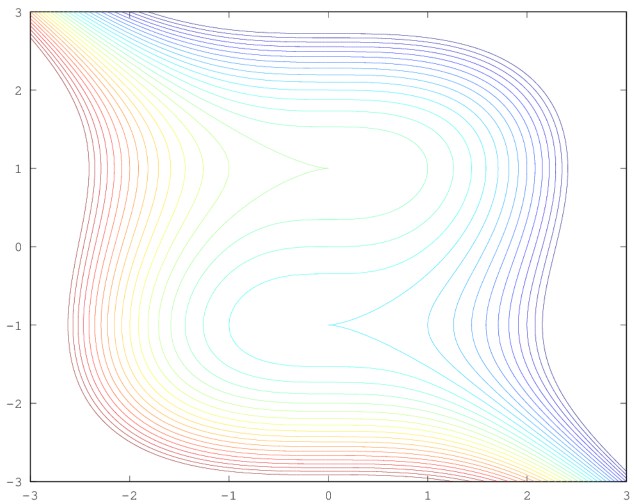
$$y - \frac{1}{3}y^3 = \frac{1}{3}x^3 + C.$$

One can rearrange this equation to put it in the form

$$-x^3 + 3y - y^3 = \tilde{C}.$$

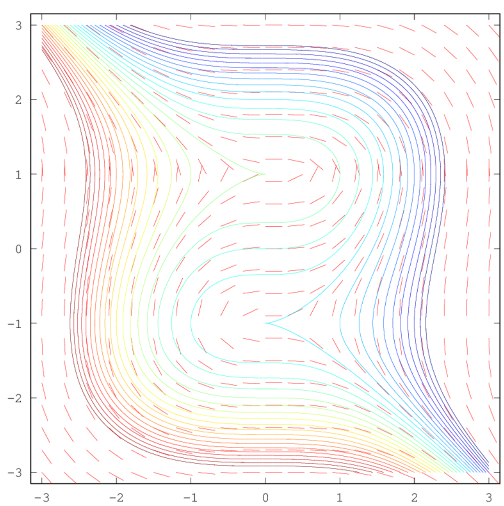
It is this form I consider as I plot level curves (i.e., graphs of this equation for different values of $\tilde{C}$).

```
g = inline('-x.^3+3*y-y.^3','x','y')
[X,Y] = meshgrid(-3:.01:3,-3:.01:3);
Z = g(X,Y);
contour(X,Y,Z,-12:16)
```



Putting the direction field and level curves together, we have this view:

```
f = inline('x.^2 ./ (1-y.^2)', 'x','y')
dirfield(f, [-3 3], [-3 3])      % dirfield.m is home-grown
g = inline('-x.^3+3*y-y.^3','x','y')
[X,Y] = meshgrid(-3:.01:3,-3:.01:3);
Z = g(X,Y);
hold on, contour(X,Y,Z,-12:16), hold off
```



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