1.13 We have

$$SS(c) = \sum (x_i - c)^2$$

so

$$SS'(c) = -2\sum (x_i - c) = 2nc - 2\sum x_i = 2n(c - \bar{x})$$

The only zero of SS' occurs when $c = \bar{x}$. By the 1st derivative test, $c = \bar{x}$ corresponds to a minimum of $SS(c)$.

2.62 (a) $X=2$: $\binom{4}{2}\left[\binom{26}{5} - 2\binom{13}{5}\right] / \binom{52}{5} = 0.1459$

$X=4$: $\binom{4}{1}\binom{13}{2}\binom{13}{1}^3 / \binom{52}{5} = 0.2637$

$X=3$: $1 - \left(Pr(X=1) + Pr(X=2) + P(X=4)\right) = 0.5825$

x	$Pr(X=x)$
1	0.00198
2	0.1459
3	0.5825
4	0.2637

(b) $E(X) = (0.00198) + (2)(0.1459) + (3)(0.5825) + (4)(0.2637) = 3.096$

2.70 (a) For $X \sim DUnif(10)$, $Pr(X=3) = 1/10$, $Pr(X=12) = 0$, $Pr(X \leq 3) = 0.3$

(b) $E(X) = (1)\left(\frac{1}{n}\right) + (2)\left(\frac{1}{n}\right) + \cdots + n\left(\frac{1}{n}\right) = \frac{1}{n}\sum_{i=1}^{n} i$

$$= \left(\frac{1}{n}\right)\frac{n(n+1)}{2} = \frac{n+1}{2}.$$

$$E(X^2) = \frac{1}{n}\sum_{i=1}^{n} i^2 = \frac{1}{n}\cdot\frac{n(n+1)(2n+1)}{6} = \frac{1}{6}(2n^2 + 3n + 1).$$

$$\Rightarrow Var(X) = \frac{1}{3}n^2 + \frac{1}{2}n + \frac{1}{6} - \frac{1}{4}(n^2 + 2n + 1)$$

$$= \frac{1}{12}n^2 - \frac{1}{12}.$$

2.85

$$\begin{aligned}
Cov(X,Y) &= E(XY) - E(X)E(Y) \\
&= E(XY) - \mu_X E(Y) - \mu_Y E(X) + \mu_X\mu_Y \\
&= E(XY) - E(\mu_X Y) - E(\mu_Y X) + \mu_X\mu_Y \\
&= E(XY - \mu_X Y - \mu_Y X + \mu_X\mu_Y) \\
&= E\left((X-\mu_X)(Y-\mu_Y)\right)
\end{aligned}$$

2.90 (a) $\text{dpois}(0, 2) = e^{-2} \cdot \frac{2^0}{0!} = e^{-2} \doteq 0.1353$

(b) $\text{dpois}(2, 2) = e^{-2} \cdot \frac{2^2}{2!} = e^{-2}(2) \doteq 0.2707$

(c) If X = # of customers in that hour,

$\Pr(X > 6) = 1 - \text{ppois}(6, 6) \doteq 0.3937$

$\Pr(X < 6) = \text{ppois}(5, 6) \doteq 0.4457$

$\Pr(X = 6) = \text{dpois}(6, 6) \doteq 0.1606$

(d) If X = # of customers during Sam's 4-hour shift, then

$\Pr(20 \le X \le 30) = \text{ppois}(30, 24) - \text{ppois}(19, 24) \doteq 0.7239$

(e) A Poisson model would be not-so-appropriate when

i. the rate doesn't stay constant across time periods, or

ii. when "arrivals" are not independent.

2.91 Pr(Z scores at least 44 goals in 89 games)

$= 1 - \text{ppois}\left(43, (89)\left(\frac{206}{506}\right)\right) \doteq 0.1157$

It is not terribly unlikely — not significant even at the 10% level — that he might score 44 goals during any 89-game stretch during the regular season.

2.93 A start might be to make a duplicate data frame but with a "total" column:

```
myFumbles <- mutate(Fumbles, total = week1 + week2 + week3)
```

Then compare

```
gf_dhistogram(~total, data = myFumbles, binwidth = 1)
```

with

```
gf_dist("pois", params = c(mean(~total, data = myFumbles)))
```

3.1 (a) $$\int_{-\infty}^{\infty} f(x)\,dx = k\int_{-2}^{2}(x^2-4)\,dx = k\left(\frac{1}{3}x^3 - 4x\Big|_{-2}^{2}\right) = k\left[\left(\frac{8}{3}-8\right)-\left(-\frac{8}{3}+8\right)\right]$$

$$= -\frac{32}{3}k$$

This quantity equals 1 iff $k = \frac{-3}{32}$.

(b) $\Pr(X \geq 0) = \frac{1}{2}$, by symmetry.

(c) $\Pr(X \geq 1) = \int_1^\infty f(x)\,dx = \frac{-3}{32}\left(\frac{1}{3}x^3 - 4x\Big|_1^2\right) = -\frac{3}{32}\left[\left(\frac{8}{3}-8\right)-\left(\frac{1}{3}-4\right)\right]$

$= \left(\frac{-3}{32}\right)\left(\frac{-5}{3}\right) = \frac{5}{32}$.

(d) $\Pr(-1 \leq X \leq 1) = 1 - 2\cdot\Pr(X \geq 1) = 1 - \frac{10}{32} = \frac{11}{16}$.

3.2 (a) $\int_{-\infty}^{\infty} g(x)\,dx = k\int_0^3 (x^2-3x)\,dx = k\left(\frac{1}{3}x^3 - \frac{3}{2}x^2\Big|_0^3\right) = \left(9 - \frac{27}{2}\right)k = -\frac{9}{2}k$.

This integral is 1 when $k = -\frac{2}{9}$.

(b) $\Pr(X \leq 1) = -\frac{2}{9}\int_0^1 (x^2-3x)\,dx = -\frac{2}{9}\left(\frac{1}{3}x^3 - \frac{3}{2}x^2\Big|_0^1\right) = \left(-\frac{2}{9}\right)\left(\frac{-7}{6}\right) = \frac{7}{27}$.

(c) $\Pr(X \leq 2) = -\frac{2}{9}\left(\frac{1}{3}x^3 - \frac{3}{2}x^2\Big|_0^2\right) = \left(-\frac{2}{9}\right)\left(\frac{-10}{3}\right) = \frac{20}{27}$

(d) $\Pr(1 \leq X \leq 2) = \Pr(X \leq 2) - \Pr(X \leq 1) = \frac{1}{27}(20-7) = \frac{13}{27}$.

3.5 For $X \sim \exp(\lambda)$, we have cdf $F_X(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-\lambda x}, & x \geq 0 \end{cases}$

For the median, we solve $0.5 = 1 - e^{-\lambda x} \Rightarrow x = \frac{1}{\lambda}\ln 2$.

The first quartile x satisfies $0.25 = 1 - e^{-\lambda x} \Rightarrow x = -\frac{1}{\lambda}\ln(3/4)$.

The third quartile x satisfies $0.75 = 1 - e^{-\lambda x} \Rightarrow x = \frac{2}{\lambda}\ln 2$.