Systems of linear equations

1. General knowledge
   (a) classifications
      i. as consistent/inconsistent
      ii. as homogeneous/nonhomogeneous (meaning of ”trivial solution”)
   (b) understanding equations as constraints, variables as freedoms
   (c) types of solution sets that can be encountered and how to interpret a solution set geometrically
   (d) how to use back substitution to determine solution sets, when in echelon form; how to write solutions in parametrized form
2. Solving using Gaussian elimination
   (a) forming a coefficient and augmented matrix corresponding to a linear system
   (b) knowledge of the elementary row operations (EROs)
   (c) ability to employ EROs, both to take a matrix to echelon form, and to RREF
   (d) identification of basic and free variables, along with the rank of a matrix
   (e) various conclusions you can draw knowing only the dimensions of the coefficient matrix $\mathbf{A}$, the rank of $\mathbf{A}$, and the rank of the augmented matrix
   (f) solving for multiple right-hand sides by attaching multiple augmented columns
   (g) solving for the inverse matrix, when it exists

Matrix operations

1. addition, scalar multiplication of matrices; matrix multipication
   (a) properties these have
   (b) Be able to carry out each by hand.
2. special matrices
   (a) for a given set of dimensions, the zero matrix
   (b) for a given matrix $\mathbf{A}$, its additive inverse
   (c) the identity matrices
   (d) for a given square matrix $\mathbf{A}$, its multiplicative inverse (when nonsingular)
   (e) for a given matrix $\mathbf{A}$, its transpose $\mathbf{A}^T$
3. given a system, the equivalent formulation as matrix equation $\mathbf{Ax} = \mathbf{b}$
4. Solving $\mathbf{Ax} = \mathbf{b}$ by using $\mathbf{A}^{-1}$

Matrix determinants

1. requirements on $\mathbf{A}$ for it to have a determinant, notation of determinants, and what a determinant indicates
2. computing determinants
   (a) using Laplace expansion (i.e., expansion in cofactors)
   (b) simplification when matrix is upper- or lower-triangular

   (c) using Gaussian elimination
3. properties determinants have
   (a) linearity in each row/column
   (b) $\det(\mathbf{AB}) = \det(\mathbf{A})\det(\mathbf{B})$
   (c) under EROs
   (d) $\det(\mathbf{A}) = \det(\mathbf{A}^T)$
4. Cramer's Rule