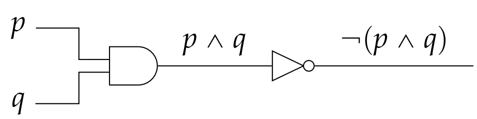
Disjunctive Normal Form

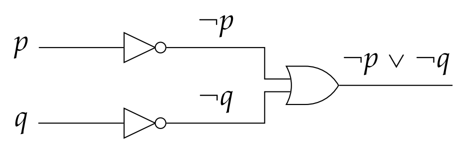
A compound proposition is in **disjunctive normal form** (DNF) if

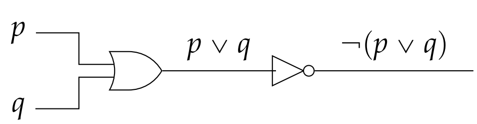
- negations occur only on the atomic propositions.
- conjunctions occur only on inputs containing no disjunctions.
- there are no operations besides negation, conjunction and disjunction.

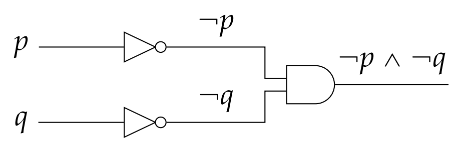
Several (tauto)logical equivalences can be used to re-express compound propositions in DNF.

- $p \to q \;\equiv\; \neg p \vee q$, to eliminate implications.
  Note how this provides direction for removing biconditionals, too.
- $p \oplus q \;\equiv\; (p \vee q) \wedge (\neg p \vee \neg q)$, to eliminate EXCLUSIVE ORs.
- $\neg(p \vee q) \;\equiv\; \neg p \wedge \neg q$, and $\neg(p \wedge q) \;\equiv\; \neg p \vee \neg q$, to move negation inside of conjunction/disjunction.

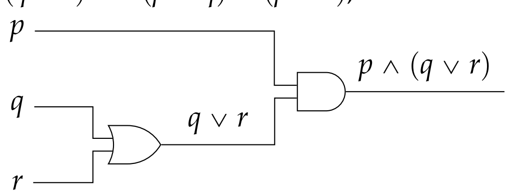


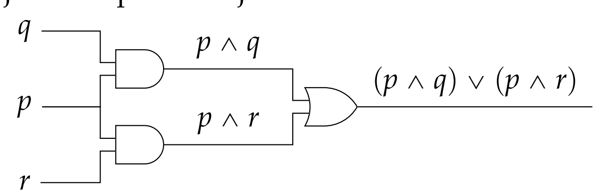






- $p \wedge (q \vee r) \;\equiv\; (p \wedge q) \vee (p \wedge r)$, to move a conjunction past a disjunction.





Exercise: Put the compound proposition $(p \to (q \wedge r)) \vee \neg(p \vee \neg(r \vee s))$ into DNF.
Answer: $(\neg p \vee (q \wedge r)) \vee ((\neg p \wedge r) \vee (\neg p \wedge s))$.

While reading this page, I hope you have become convinced that every compound proposition is logically equivalent to a logical statement written in terms of its atomic propositions and using only the three operators: $\neg$, $\wedge$, and $\vee$. We say the three operators are **functionally complete**. In fact, it is possible to show that $\neg$, $\wedge$, these two alone, are functionally complete, as is the pair $\neg$, $\vee$.