Integrals using various combinations of techniques

Write down a strategy for computing the following integrals:

1. $\displaystyle\int \frac{4x^2-1}{x(x^2-1)}\,dx$
2. $\displaystyle\int \frac{x}{(x^2-1)^{3/2}}\,dx$
3. $\displaystyle\int \frac{x^2}{(x^2-1)^{3/2}}\,dx$
4. $\displaystyle\int \frac{dx}{x^2(x-1)^2}\,dx$
5. $\displaystyle\int x\sec x\tan x\,dx$
6. $\displaystyle\int \frac{x+1}{(x^2+4x+8)^2}\,dx$ Note: $\sin(2\theta) = 2\sin\theta\cos\theta$
7. $\displaystyle\int \frac{dx}{5+e^x}$
8. $\displaystyle\int \frac{\sqrt{x}}{1+x^3}\,dx$
9. $\displaystyle\int \sqrt{x^2-6x+5}\,dx$
10. $\displaystyle\int \frac{dx}{x^2+2x+5}$

In carrying out Number 9 above, you likely encounter

$$\begin{aligned}
\int \sec\theta\tan^2\theta\,d\theta &= \int \sec\theta(1+\sec^2\theta)\,d\theta = \int \sec\theta\,d\theta + \int \sec^3\theta\,d\theta \\
&= \ln|\sec\theta+\tan\theta| + \int \sec^3\theta\,d\theta \\
&= \ln|\sec\theta+\tan\theta| + \sec\theta\tan\theta - \int \sec\theta\tan^2\theta\,d\theta \qquad (\text{with } u=\sec\theta \text{ and } dv=\sec^2\theta\,d\theta).
\end{aligned}$$

Now you add $\int \sec\theta\tan^2\theta\,d\theta$ to both sides and divide by 2.

For Number 6 above, you complete the square and substitute $2\tan\theta = x+2$:

$$\begin{aligned}
\int \frac{x+1}{[(x+2)^2+4]^2}\,dx &= \int \frac{2\tan\theta-1}{(4\tan^2\theta+4)^2}2\sec^2\theta\,d\theta = \frac{1}{8}\int \frac{2\tan\theta-1}{\sec^2\theta}\,d\theta \\
&= \frac{1}{4}\int \sin\theta\cos\theta\,d\theta - \frac{1}{8}\int \cos^2\theta\,d\theta = \frac{1}{8}\sin^2\theta - \frac{1}{16}\int [1+\cos(2\theta)]\,d\theta \\
&= \frac{1}{8}\sin^2\theta - \frac{1}{16}\theta - \frac{1}{32}\sin(2\theta) + C = \frac{1}{8}\sin^2\theta - \frac{1}{16}\theta - \frac{1}{16}\sin\theta\cos\theta + C \\
&= \frac{1}{8}\frac{(x+2)^2}{x^2+4x+8} - \frac{1}{16}\arctan\left(\frac{x+2}{2}\right) - \frac{1}{8}\frac{x+2}{x^2+4x+8} + C.
\end{aligned}$$