MATH 162: Calculus II

Framework for Fri., Apr. 13

Double Integrals, General Regions

Today's Goal: To understand the meaning of double integrals over more general bounded regions R of the plane, and to be able to evaluate such integrals.

Important Note: In conjunction with this framework, you should look over Section 13.2 of your text.

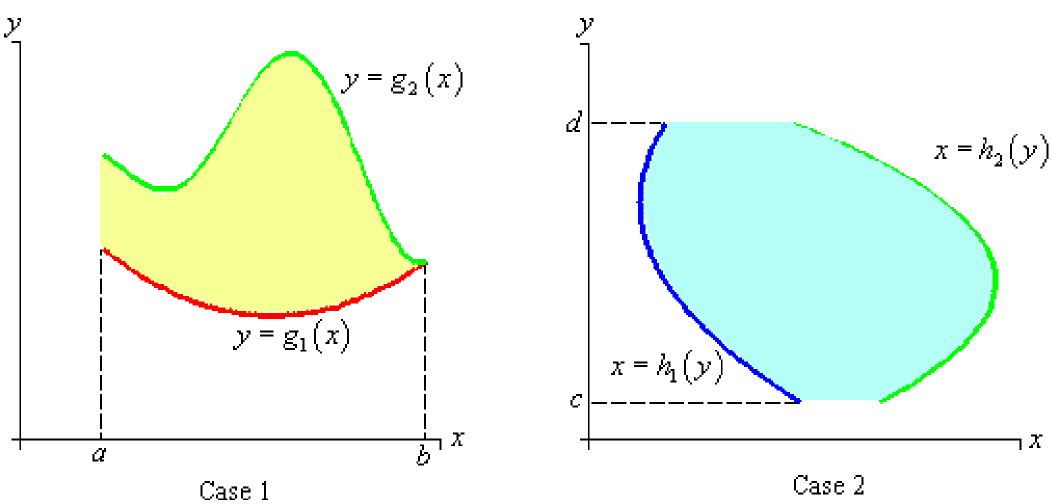
Double Integrals as Iterated Integrals: General Treatment

Q: What if we seek $\iint_R f(x,y)\,dA$ when R is not a rectangle whose sides are parallel to the coordinate axes?

A1: If R is a "nice enough" region (and, for our study, it will be), we can, once again, define $\iint_R f(x,y)\,dA$ in terms of Riemann sums. The twist here is that, for any given partition of R, the rectangles will only partially fill up R.

We will not pursue this train of thought further.

A2: Use a more general form of Fubini's theorem. You can see the formal statement on p. 792 of your text. It deals with 2 cases (pictured):



Case 1: the upper and lower boundaries of R each are functions of x on a common interval; that is, a region $R:\ a \le x \le b,\ g_1(x) \le y \le g_2(x)$. Then

$$\iint_R f(x,y)\,dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx.$$

Example: Evaluate $\iint_R (x + 2y)\, dA$ over the region R that lies between the parabolas $y = 2x^2$ and $y = 1 + x^2$.

Case 2: the left and right boundaries of R each are functions of y on a common interval; that is, a region $R:\ c \leq y \leq d,\ h_1(x) \leq x \leq h_2(x)$. Then

$$\iint\limits_R f(x, y)\, dA \ = \ \int_c^d \int_{h_1(y)}^{h_2(y)} f(x, y)\, dx\, dy.$$

Example: Set up an integral for a function $f(x, y)$ over the region bounded by the y-axis and the curve $x + y^2 = 1$.