2.48 (a) $\Pr(\text{No misses for Freddie}) = (0.8)^5 = 0.32768$.

(b) $\Pr(\text{exactly one miss for Freddie}) = (0.32768) \cdot \text{pgeom}(4, 0.2) \doteq 0.22031$

(c) $\Pr(\text{Freddie misses at least twice}) = 1 - \Pr(\text{no misses}) - \Pr(1 \text{ miss}) \doteq 0.45201$.

(d) In (a), we learned that Freddie's goal, 5 consecutive "made" shots, happens with probability 0.32768. A "failure" in this endeavor occurs everytime, shy of completing the task, a shot is missed. Thus, the X we are tracking to get its pmf is $X \sim \text{geom}(0.32768)$.

2.96 We may consider this a hypergeometric setting: Our "urn" contains 22 white balls (those who lost cash) and 23 black ones (those who lost a ticket). There are $k = 23$ selected as to be respondents who say "no, I would not attend," and $X = 9$ come from the "lost cash" group. Our P-value

$$\Pr(X \le 9) = \text{phyper}(9, 22, 23, 23) \doteq 0.149.$$

This is not a statistically significant result, and we fail to reject random chance as the source of the difference in proportions.

2.104 (a) K and Q are not independent. For instance, neither $\Pr(K=3)$ nor $\Pr(Q=3)$ are zero, but $\Pr(K=3 \text{ and } Q=3)$ is zero.

(b) $$\Pr(K=2 \mid Q=2) = \frac{\Pr(K=2 \text{ and } Q=2)}{\Pr(Q=2)} = \frac{\binom{4}{2}^2\binom{44}{1} \Big/ \binom{52}{5}}{\binom{4}{2}\binom{48}{3} \Big/ \binom{52}{5}}$$

$$= \frac{\binom{4}{2}\binom{44}{1}}{\binom{48}{3}} \doteq 0.01526.$$

3.1 (a) $$\int_{-\infty}^{\infty} f(x)\,dx = k\int_{-2}^{2}(x^2-4)\,dx = k\left(\frac{1}{3}x^3 - 4x\Big|_{-2}^{2}\right) = k\left[\left(\frac{8}{3}-8\right)-\left(-\frac{8}{3}+8\right)\right]$$

$$= -\frac{32}{3}k$$

This quantity equals 1 iff $k = \dfrac{-3}{32}$.

(b) $\Pr(X \ge 0) = \frac{1}{2}$, by symmetry.

(c) $\Pr(X \ge 1) = \int_1^\infty f(x)\,dx = -\frac{3}{32}\left(\frac{1}{3}x^3 - 4x\Big|_1^2\right) = -\frac{3}{32}\left[\left(\frac{8}{3}-8\right)-\left(\frac{1}{3}-4\right)\right]$

$= \left(\frac{-3}{32}\right)\left(\frac{-5}{3}\right) = \frac{5}{32}$.

(d) $\Pr(-1 \le X \le 1) = 1 - 2\cdot\Pr(X \ge 1) = 1 - \frac{10}{32} = \frac{11}{16}$.

3.4 Since f, g are pdfs, for each $x \in \mathbb{R}$,

$$\alpha f(x) + (1-\alpha)g(x) \ge 0,$$

and $\int_{-\infty}^{\infty}\left[\alpha f(x) + (1-\alpha)g(x)\right]dx = \alpha\int_{-\infty}^{\infty} f(x)dx + (1-\alpha)\int_{-\infty}^{\infty} g(x)dx$

$$= \alpha\cdot 1 + (1-\alpha)\cdot 1 = 1.$$

3.15 (d) Solve $F_X(x) = 0.5 \Rightarrow x^2 = 2 \Rightarrow x = \sqrt{2}$.

(e) The pdf $f_X(x) = \frac{d}{dx}F_X(x) = \begin{cases} x/2, & \text{if } 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$

(f) $E(X) = \int_{-\infty}^{\infty} x f_X(x)\,dx = \int_0^2 x\cdot\frac{x}{2}\,dx = \frac{1}{6}x^3\Big|_0^2 = \frac{4}{3}$.

(g) $E(X^2) = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx = \int_0^2 x^2\cdot\frac{x}{2}\,dx = \frac{1}{8}x^4\Big|_0^2 = 2$

$\Rightarrow \mathrm{Var}(X) = E(X^2) - E(X)^2 = 2 - \left(\frac{4}{3}\right)^2 = \frac{2}{9}$.

3.10 (a) $\Pr(X \le 1) = F_X(1) = 1/4$.

(b) $\Pr(X \le 1/4) = F_X(1/4) = \left(\frac{1}{4}\right)\left(\frac{1}{4}\right)^2 = 1/64 \doteq 0.0156$

(c) The pdf $f_X(x) = \frac{d}{dx}F_X(x) = \begin{cases} x/2, & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$

(d) $\Pr(Y \le 1) = \Pr(X^2/4 \le 1) = \Pr(-2 \le X \le 2) = F_X(2) - F_X(-2) = 1$

(e) $\Pr(Y \le \frac{1}{4}) = \Pr(X^2/4 \le 1/4) = \Pr(-1 \le X \le 1) = F_X(1) - F_X(-1) = 1/4$

(f) For $0 < y < 1$,

$$F_Y(y) = \Pr(Y \le y) = \Pr(X^2/4 \le y) = \Pr(-2\sqrt{y} \le X \le 2\sqrt{y}) = \Pr(X \le 2\sqrt{y}) = \frac{(2\sqrt{y})^2}{4} = y.$$

So,

$$F_Y(y) = \begin{cases} 0, & \text{if } y \le 0 \\ y, & \text{if } 0 < y < 1 \\ 1, & \text{if } y \ge 1 \end{cases}$$

(g) The pdf $f_Y(y) = \frac{d}{dy} F_Y(y) = \begin{cases} 1, & \text{if } 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$

(h) $Y \sim \text{Unif}(0, 1)$.

3.18 (1) Case: discrete

$$E(aX + b) = \sum_x (ax + b) f(x) = a \sum_x x f(x) + b \sum_x f(x) = a\,E(X) + b \cdot 1$$

or Case: cont.

$$= \int_{-\infty}^{\infty} (ax + b) f(x)\,dx = a \int_{-\infty}^{\infty} x f(x)\,dx + b \int_{-\infty}^{\infty} f(x)\,dx = a\,E(X) + b$$

(2)

$$\begin{aligned} Var(aX+b) &= E((aX+b)^2) - [E(aX+b)]^2 \\ &= E(a^2X^2 + 2abX + b^2) - (aE(X) + b)^2 \\ &= E(a^2X^2) + E(2abX + b^2) - a^2[E(X)]^2 - 2abE(X) - b^2 \end{aligned}$$

This step uses result (1).

$$\begin{aligned} &= a^2 E(X^2) + 2ab\,E(X) + b^2 - a^2[E(X)]^2 - 2abE(X) - b^2 \\ &= a^2\left(E(X^2) - [E(X)]^2\right) = a^2\,Var(X). \end{aligned}$$

3.21 For $X \sim \text{DUnif}(n)$,

$$M_X(t) = E(e^{tX}) = \sum_{x=1}^{n} e^{tx} \cdot \frac{1}{n} = \frac{e^t}{n}\left(1 + e^t + e^{2t} + \cdots + e^{(n-1)t}\right)$$

$$= \frac{e^t}{n} \cdot \frac{e^{nt} - 1}{e^t - 1} = \frac{e^t(e^{nt} - 1)}{n(e^t - 1)}$$

3.34 (a) $X \sim Geom(\pi) \Rightarrow f_X(x) = (1-\pi)^x \pi$. Thus,

$$M_X(t) = \sum_{x=0}^{\infty} e^{tx}(1-\pi)^x \pi = \pi \sum_{x=0}^{\infty} \left[e^t(1-\pi)\right]^x = \frac{\pi}{1-e^t(1-\pi)}$$

(c') $$M_X'(t) = \frac{\pi(1-\pi)e^t}{\left[1-e^t(1-\pi)\right]^2} \Rightarrow E(X) = M_X'(0) = \frac{\pi(1-\pi)}{\left[1-(1-\pi)\right]^2} = \boxed{\frac{1-\pi}{\pi}}$$

$$M_X''(t) = \frac{\pi(1-\pi)e^t\left[1+e^t(1-\pi)\right]}{\left[1-e^t(1-\pi)\right]^3} \Rightarrow E(X^2) = M_X''(0) = \frac{(1-\pi)(2-\pi)}{\pi^2}$$

Thus, $$Var(X) = E(X^2) - \left[E(X)\right]^2 = \frac{(1-\pi)(2-\pi)}{\pi^2} - \left(\frac{1-\pi}{\pi}\right)^2$$

$$= \frac{1-\pi}{\pi^2}\left[2-\pi-(1-\pi)\right] = \boxed{\frac{1-\pi}{\pi^2}}.$$