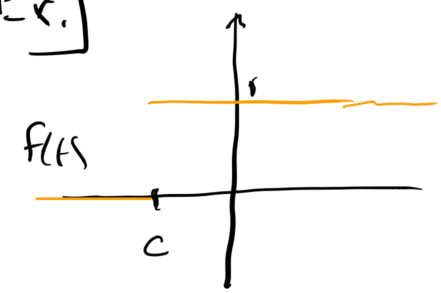
Goal: Modeling switches, and take Laplace transforms

Ex.



$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t) e^{-st}\,dt$$

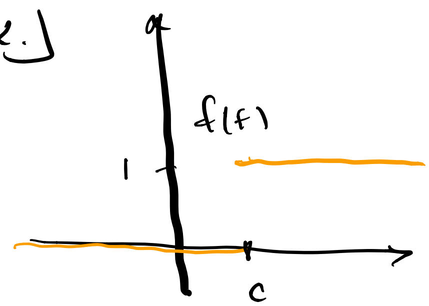
$$= \int_0^\infty 1 \cdot e^{-st}\,dt$$

$$= \text{from Monday} = \frac{1}{s}$$

has same Laplace transform as constant fn. 1

– i.e., L.T. sees no distinction.

Ex.



$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t) e^{-st}\,dt$$

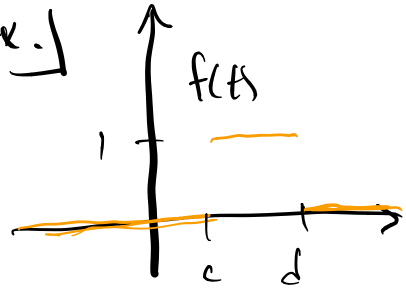
$$= \underbrace{\int_0^c 0 \cdot e^{-st}}_{=0} + \int_c^\infty 1 \cdot e^{-st}\,dt$$

$$= \int_c^\infty e^{-st}\,dt = -\frac{1}{s} e^{-st}\Big|_c^\infty$$ divergent if $s \le 0$

for $s>0$ $$= 0 - \left(-\frac{1}{s} e^{-sc}\right) = \frac{1}{s} e^{-cs}$$

Special case: $c = 0$ — same result as const. $f(t) = 1$.

Ex.



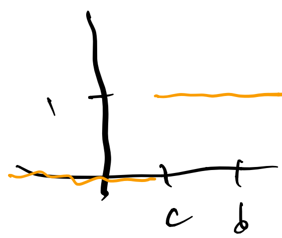
$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)\, e^{-st}\, dt$$

$$= \int_c^d 1 \cdot e^{-st}\, dt$$

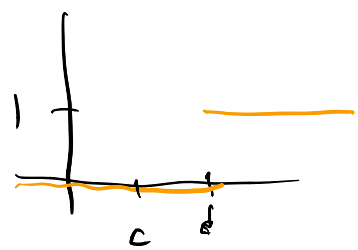
$$= -\frac{1}{s} e^{-st} \Big|_c^d = -\frac{1}{s} e^{-sd} - \left(-\frac{1}{s} e^{-sc}\right)$$

$$= \frac{1}{s} e^{-cs} - \frac{1}{s} e^{-ds}$$

L.T. of



L.T. of



Define (give name to) fns. like this

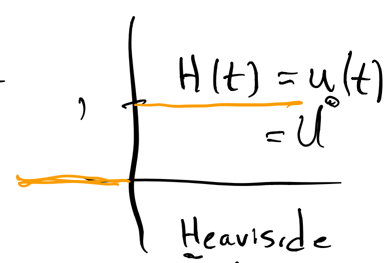
Alt. names

$\boxed{\begin{array}{l} \mathcal{U}(t-c) \\ u_c(t) \end{array}}$

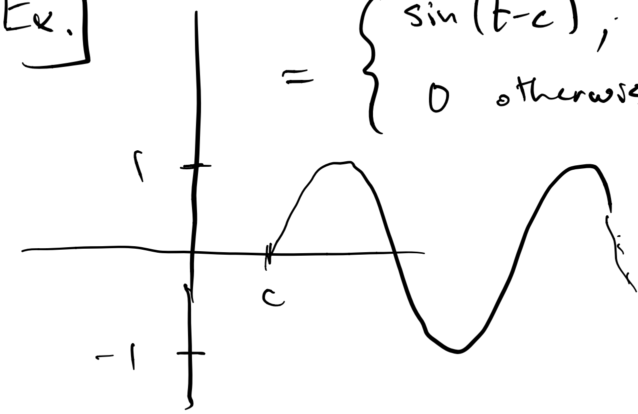
$u(t-c)$

$H(t-c)$

Note: Shifted, by c units, of this fn.

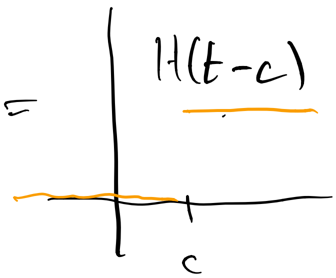


Ex.

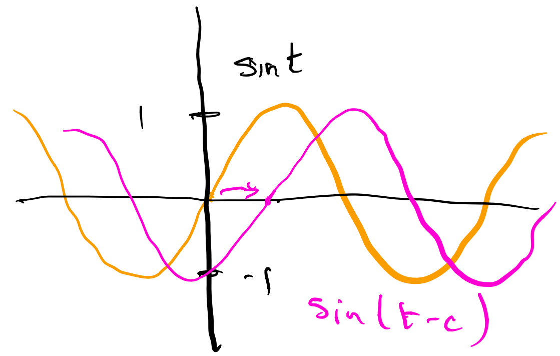


$$= \begin{cases} \sin(t-c), & t > c \\ 0 & \text{otherwise} \end{cases} = H(t-c)\sin(t-c)$$

$=$



$\times$



Q: What sort of expression for transform for

$$H(t-c)\,f(t-c) \ ?$$

$$\mathcal{L}\{H(t-c)f(t-c)\} = \int_0^\infty H(t-c) f(t-c) e^{-st}\,dt$$

when $c \geq 0$

$$= \int_0^c 0 \cdot f(t-c) e^{-st}\,dt + \int_c^\infty 1 \cdot f(t-c) e^{-st}\,dt$$

$$= 0 + \int_0^\infty f(z)\, e^{-s(z+c)}\,dz$$

under subst. $z = t-c \quad dz = dt$

$$= \int_0^\infty f(z)\, e^{-sz} \cdot \underbrace{e^{-sc}}_{\text{const. in the integral}}\, dz$$

$$= e^{-sc} \int_0^\infty f(z)\, e^{-sz}\, dz$$

z dummy variable of integration

rename z as t

$$= e^{-sc} \int_0^\infty f(t)\, e^{-st}\, dt$$

$$= e^{-sc} \cdot \mathcal{L}\{f(t)\}$$

Use for this result? Helps to <u>bring</u> <u>us</u> <u>back</u>

<u>time domain</u> $\longrightarrow$ <u>frequency domain s</u>

$g(t)$ $\qquad$ $\mathcal{L}\{g(t)\} = G(s)$

$\longrightarrow$ focus has been on this direction

$\longleftarrow$ will need to be able to come back

Ex.] Have a transformed fn. (on s-side)

$$e^{-2s} \cdot \frac{2}{s^2+4}$$

Want to know: What t-side fn. does this come from?

that is, what $f(t)$ has $\mathcal{L}\{f(t)\}$ = this?

From Monday, know

$$\mathcal{L}\{\sin(at)\} = \frac{a}{s^2+a^2}$$

So $$\mathcal{L}\{\sin(2t)\} = \frac{2}{s^2+4}$$

Above result says

$$\mathcal{L}\{H(t-c)\,f(t-c)\} = e^{-sc}\,\underline{\mathcal{L}\{f(t)\}}$$

in ours $\mathcal{L}\{\sin(2t)\}$

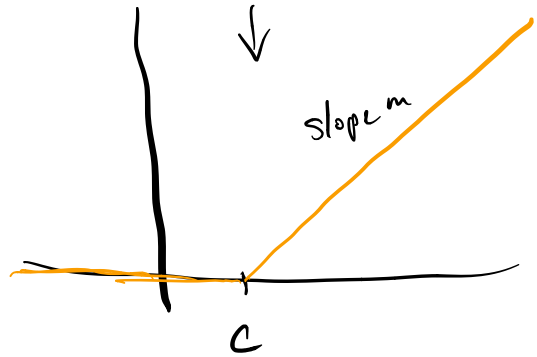
Ans.

$$H(t-2)\sin(2(t-2))$$

Unshifted sine $\sin(2t) \to \frac{2}{s^2+4}$

So $H(t-\underline{2})\sin(2(t-\underline{2}))$ = desired fn.

Ex.) Express the fn. using Heaviside step fn.



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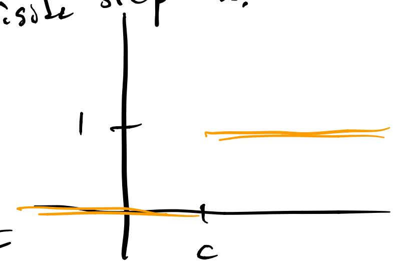
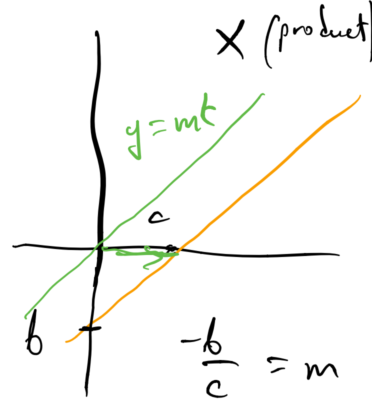


× (product)



$\frac{-b}{c} = m$

$\Rightarrow b = -mc$

On right

top: $H(t-c)$

bottom: line $f(t) = mt + \text{intcpt.}$

$= mt - mc$

Desired fn. is product of these

$$H(t-c) \cdot (mt - mc)$$

$$= H(t-c)\, \underbrace{m \cdot (t-c)}_{\text{shifted, by } c \text{ units of } y = mt}$$