MATH 231: Differential Equations with Linear Algebra

Hand-Checked Assignment #1, due date: Mon., Mar. 15, 2021

Write up, carefully and legibly, your solutions to the following problems. While you do not need to present one problem per page, please do not put problems side-by-side (i.e., no two-column format). To submit your work it must be

- scanned (all pages) to a single .pdf file (one multi-page file containing all graded (⋆1–⋆8) problems). A really excellent way to produce the .pdf file is using your phone, along with one of the apps suggested at this site

  https://help.gradescope.com/article/0chl25eed3-student-scan-mobile-device

  (**Scannable** by **Evernote**, if you use an ios device, and **Genius Scan**, if you use an Android one). Here is a video made from an iPhone demonstrating how you can scan pages using Evernote Scannable to a .pdf and then use AirDrop to get the file to a Mac.

- submitted to

  https://www.gradescope.com

  as **hc01**. The upload process involves more than simply sending it to Gradescope, and here is another video demonstrating how you do that.

⋆1 Consider the augmented matrix

$$\left[\begin{array}{cccc|c} 0 & 2 & 1 & 3 & 3 \\ 2 & 1 & 2 & -1 & 4 \\ 1 & -3 & 1 & 1 & 7 \\ 2 & 0 & 1 & -2 & 2 \end{array}\right].$$

(a) Write down the corresponding linear system of 4 (algebraic) equations in variables x_1, x_2, x_3 and x_4 that corresponds to this augmented matrix.

(b) Carry out the following sequence of **elementary row operations** (EROs) in the given order, writing the new form of the augmented matrix after each step.

i. ERO1: swap rows 1 and 3; i.e., $\mathbf{r}_1 \leftrightarrow \mathbf{r}_3$

ii. ERO3: add (-2) multiples of row 1 to row 2; that is, $(-2)\mathbf{r}_1 + \mathbf{r}_2 \to \mathbf{r}_2$

iii. ERO3: add (-2) multiples of row 1 to row 4; $(-2)\mathbf{r}_1 + \mathbf{r}_4 \to \mathbf{r}_4$

iv. ERO1: swap rows 2 and 3; $\mathbf{r}_2 \leftrightarrow \mathbf{r}_3$

v. ERO3: add $(-7/2)$ multiples of row 2 to row 3; $(-7/2)\mathbf{r}_2 + \mathbf{r}_3 \to \mathbf{r}_3$

vi. ERO3: add (-3) multiples of row 2 to row 4; $(-3)\mathbf{r}_2 + \mathbf{r}_4 \to \mathbf{r}_4$

vii. ERO3: add $(-8/7)$ multiples of row 3 to row 4; $(-8/7)\mathbf{r}_3 + \mathbf{r}_4 \to \mathbf{r}_4$

What you should have after the 7 steps is

$$\left[\begin{array}{cccc|c} 1 & -3 & 1 & 1 & 7 \\ 0 & 2 & 1 & 3 & 3 \\ 0 & 0 & -3.5 & -13.5 & -20.5 \\ 0 & 0 & 0 & 17/7 & 17/7 \end{array}\right].$$

[Note: While a given matrix has many echelon forms, you should get this particular one if you followed the sequence of EROs given above.]

(c) While part (b) yields an echelon form for the original augmented matrix, it is not in **reduced row echelon form** (RREF). Describe (using notation akin to the instructions given to you in part (b)) a sequence of EROs which, starting from the echelon form above, takes the matrix to RREF. Give both your sequence of EROs, and the contents of the matrix after each step.

(d) Write, in vector form, the solution of the system of equations in part (a).

★2 From Section 1.1, p. 11, do Exercise 1.1.2 parts (b), (c) and (d).

★3 From Section 1.1, p. 11, do Exercise 1.1.3 parts (a) and (c).

★4 From Section 1.1, p. 11, do Exercise 1.1.4.

★5 From Section 1.1, p. 12, do Exercise 1.1.10.

★6 From Section 1.2, p. 13, do Exercise 1.2.2.

★7 From Section 1.4, p. 23, do Exercise 1.4.1 parts (b) and (c).

★8 From Section 1.4, p. 23, do Exercise 1.4.5.

★9 [These are for practice only, not to be handed in.]

(a) From Section 1.2, p. 16, do Exercise 1.2.1.

(b) From Section 1.3, p. 20, do Exercise 1.3.1.

(c) From Section 1.3, p. 20, do Exercise 1.3.2.