

Math 251, Mon 7-Sep-2020 -- Mon 7-Sep-2020
 Discrete Mathematics
 Fall 2020

Monday, September 07th 2020

Due:: PS01

Monday, September 7th 2020

Wk 2, Mo

Topic:: Satisfiability

Read:: Rosen 1.1-1.2

HW[[PS02 due Fri.

Use DeMorgan's laws to write the negation of

(a) The food is good or the service is excellent.

p

q

$$\neg(p \vee q)$$

$$\equiv \neg p \wedge \neg q$$

(b) The food is good and the service is excellent.

p

q

$$\neg(p \wedge q) \equiv (\neg p) \vee (\neg q)$$

(c) The food is good and service excellent, or the price is high.

p

q

r

$$\neg((p \wedge q) \vee r)$$

$$\equiv \neg(p \wedge q) \wedge \neg r$$

$$\equiv (\neg p \vee \neg q) \wedge \neg r$$

Find a simpler proposition that is logically equivalent to

(a) not (p or (not p and q))

(b) not q and (p \rightarrow q) \rightarrow not p

tautology

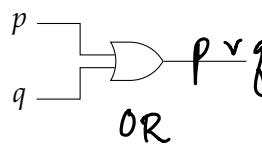
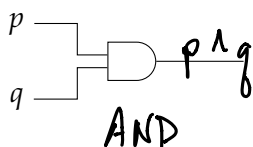
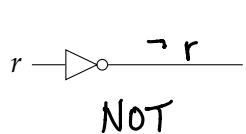
$$\neg q \wedge (p \rightarrow q) \rightarrow \neg p \equiv \neg(\neg q \wedge (p \rightarrow q)) \vee \neg p$$

$$\equiv \neg(\neg q) \vee \neg(p \rightarrow q) \vee \neg p$$

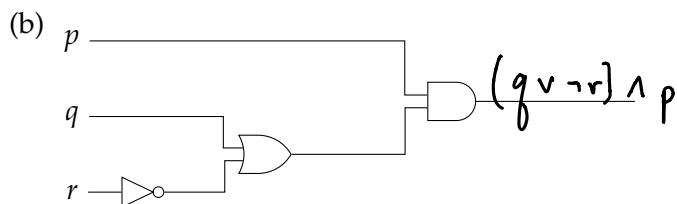
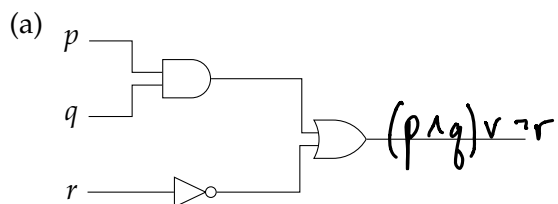
$$\equiv q \vee \neg(\neg p \vee q) \vee \neg p \equiv q \vee (p \wedge \neg q) \vee \neg p$$

$$\equiv \boxed{(q \vee p) \wedge (q \vee \neg q)} \vee \neg p \equiv \boxed{q \vee p \vee \neg p} \equiv \top$$

Circuit diagrams 1.2



Write out in symbols the appropriate compound proposition:



Are (a) and (b) logically equivalent?

p	q	r	$p \wedge q$	$q \vee \neg r$	$(p \wedge q) \vee \neg r$	$(q \vee \neg r) \wedge p$
F	F	F	F		T	F
F	F	T				
F	T	F				
F	T	T				
T	F	F				
T	F	T				
T	T	F				
T	T	T				

No.

Satisfiability

A proposition is **satisfiable** if there is some set of truth values assignable to its propositional variables (atoms) under which the proposition is true. In other words, the proposition is *not* a contradiction.

Examples:

1. $\neg p$ If $p = F$, then $\neg p$ is T

2. $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$ Set $p = T, q = T, r = T$

3. $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$ $p = T, q = F, r = F$

4. $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p) \wedge (p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$ Not satisfiable

5. Are these propositions **consistent** (simultaneously satisfiable)?

- You have a virus and you use the #CampusClear app. \wedge
- If you have a sore throat, the app says "don't come to class." $\wedge \quad r \rightarrow s$
- You do not come to class when the app says not to. \wedge
- You have a sore throat only if you have a virus. $\wedge \quad r \rightarrow p$
- You come to class.

p : You have a virus set T
 q : You use the app set T
 r : You have a sore Throat F
 s : app says don't go F
 t : You don't go to class F

You know, among A, B, and C,

there is one knight (always tells the truth)

there is one knave (every statement is a lie), and

there is one spy (can lie or tell the truth).

They make these statements.

A: "I am the knight."

B: "I am the knave."

C: "B is the knight."

Insights:

- A statement "I am ____" is self-referential
- A knave cannot say "I am a knave."
- A knight must finish the statement "I am the" with "knight."

A : knight

B : spy

C : knave