3.30 $M_X'(t) = \alpha k(1-\alpha t)^{-k-1} \longrightarrow E(X) = M_X'(0) = \alpha k$

$M_X''(t) = \alpha^2(k+1)k(1-\alpha t)^{-k-2} \longrightarrow E(X^2) = M_X''(0) = \alpha^2 k(k+1)$

So, $Var(X) = \alpha^2 k(k+1) - \alpha^2 k^2 = \alpha^2 k$.

3.41 (a) $E(X) = 5/7$, $Var(X) = (5)(7)/((5+7)^2(1+5+7)) = 35/1872 \doteq 0.01870$.

(b) qbeta(0.5, 5, 2) $\doteq$ 0.73555

(c) $Pr[X \le E(X)]$ = pbeta(5/7, 5, 7) $\doteq$ 0.4516

(d) $Pr(0.2 \le X \le 0.4)$ = pbeta(0.4, 5, 2) − pbeta(0.2, 5, 2) $\doteq$ 0.03936.

(e) $Pr\left[E(X) - \sqrt{Var(X)} \le X \le E(X) + \sqrt{Var(X)}\right] \doteq 0.08336$.

3.45 Taking into account that points-scored is always an integer, the normal-quantile plot is quite linear in its appearance, making a normal model appear appropriate. At the top end, there may be the suggestion that he should have had a higher personal-season-best.

3.54 (a) $\int_0^1\int_0^1 x^2y^3\,dy\,dx = \int_0^1 \frac{1}{4}x^2\left[y^4\right]_0^1 dx = \frac{1}{12}\left[x^3\right]_0^1 = \frac{1}{12}$.

So, the pdf
$$f_{X,Y}(x,y) = \begin{cases} 12x^2y^3, & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

(b)
$$Pr(X<Y) = \int_0^1\int_0^y 12x^2y^3\,dx\,dy = 4\int_0^1 y^3\left[x^3\right]_0^y dy = 4\int_0^1 y^6\,dy = \frac{4}{7}.$$

(c) $f_X(x) = 12x^2\int_0^1 y^3\,dy = 3x^2$, and $f_Y(y) = 12y^3\int_0^1 x^2\,dx = 4y^3$.

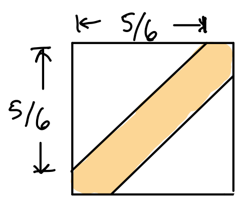
Note that $f_{X,Y}(x,y) = f_X(x)f_Y(y)$, so X and Y are independent.

3.55 (a) $f_{X,Y}(x,y) = 1$ for $(x,y) \in [5,6]\times[5,6]$.

(b) $Pr((X,Y) \in [5,5.5]\times[5,5.5]) = 1/4$.

(c) For them to arrive within 10 minutes of each other, (X,Y) must be a point in the orange shaded region, which has area (= probability)

$$1-(2)(\tfrac{1}{2})(5/6)^2 = \frac{11}{36}.$$



$[5,6]\times[5,6]$

3.63 $X \sim \text{Pois}(\lambda_1)$ and $Y \sim \text{Pois}(\lambda_2)$, so $M_X(t) = e^{e^t\lambda_1 - \lambda_1}$, and $M_Y(t) = e^{e^t\lambda_2 - \lambda_2}$.

By independence of X, Y,

$$M_{X+Y}(t) = M_X(t)M_Y(t) = e^{e^t\lambda_1} \cdot e^{-\lambda_1} \cdot e^{e^t\lambda_2} \cdot e^{-\lambda_2}$$

$$= e^{e^t\lambda_1 + e^t\lambda_2} \cdot e^{-(\lambda_1+\lambda_2)} = e^{e^t(\lambda_1+\lambda_2) - (\lambda_1+\lambda_2)}$$

This is the mgf for another Poisson r.v. with parameter $\lambda_1 + \lambda_2$. Thus

$$X + Y \sim \text{Pois}(\lambda_1 + \lambda_2).$$

3.64 $X \sim \text{Binom}(n, \pi_1)$ and has $M_X(t) = (\pi_1 e^t + 1 - \pi_1)^n$. As well,

$Y \sim \text{Binom}(n, \pi_2)$ and has $M_Y(t) = (\pi_2 e^t + 1 - \pi_2)^n$. By independence of X and Y,

$$M_{X+Y}(t) = M_X(t)M_Y(t) = \left[(\pi_1 e^t + 1 - \pi_1)(\pi_2 e^t + 1 - \pi_2)\right]^n$$

$$= \left[\pi_1\pi_2 e^{2t} + (\pi_1+\pi_2)e^t - 2\pi_1\pi_2 e^t + 1 - (\pi_1+\pi_2) + \pi_1\pi_2\right]^n$$

This is not the mgf of a binomial r.v.

4.1 Let $\bar{x} = \frac{1}{n}\sum X_i$ be the first sample moment about the origin (a.k.a. the sample mean). The population mean for $\text{Binom}(1, \pi)$ is $1 \cdot \pi = \pi$. Our estimate is $\hat{\pi} = \bar{x}$.

4.4 For $X \sim \text{NBinom}$, $E(X) = \frac{s}{\pi} - s$. So, we set

$$\frac{s}{\hat{\pi}} - s = \bar{x} \implies \hat{\pi} = \frac{s}{s + \bar{x}}.$$

4.7 favstats reveals $\bar{x} \doteq 0.6091$, $s \doteq 0.248$, $n = 134$ $\Rightarrow v = 0.06105$.

From the formulas:

$$\hat{\alpha} = \bar{x}\left(\frac{\bar{x}(1-\bar{x})}{v} - 1\right) \doteq 1.7665, \quad \hat{\beta} = (1-\bar{x})\left(\frac{\bar{x}(1-\bar{x})}{v} - 1\right) \doteq 1.1337$$

The beta distribution using these shape parameters gives a very poor fit to the data. By filtering out the players with FTPct = 0, the new parameter estimates from remaining players are

$$\hat{\alpha} = 4.824, \quad \hat{\beta} = 2.387,$$

and the fit is vastly improved.

4.11 For these examples, we estimate θ in Unif$(0, \theta)$ using $\hat{\theta} = 2\bar{x}$. Since

$$E(\hat{\theta}) = 2\,E(\bar{x}) = 2\mu = 2\left(\frac{\theta}{2}\right) = \theta,$$

this $\hat{\theta}$ is unbiased.