For Math 251, Sept. 21

From Rosen 2.2

DeMorgan's laws for two sets

→ ① $\overline{A \cup B} = \bar{A} \cap \bar{B}$ analog for propositions $\neg(p \vee q) \equiv \neg p \wedge \neg q$

② $\overline{A \cap B} = \bar{A} \cup \bar{B}$

For many sets

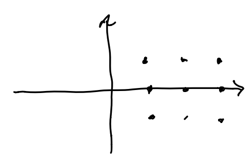
① $\overline{A_1 \cup A_2 \cup \cdots \cup A_n} = \bar{A_1} \cap \bar{A_2} \cap \cdots \cap \bar{A_n}$

② $\infty \; \overline{\bigcap_{i=1}^{\cancel{n}} A_i} = \infty \bigcup_{i=1}^{n} \bar{A_i}$

2.3 functions

A <u>relation</u> from a set A to a set B is a subset of $A \times B$.

Ex.) $A = \{1, 2, 3\}$, $B = \{-1, 0, 1\}$



$|A \times B| = |A||B| = 9$

Full $A \times B = \{(1,-1), (1,0), (1,1), (2,-1), (2,0), (2,1), (3,-1), (3,0), (3,1)\}$

Any subset of this is a relation from A to B.

Note: Number of possible relations from A to B

$=$ Number of subsets of $A \times B$, i.e. $|\mathcal{P}(A \times B)| = 2^9 = 512$.

A <u>function</u> $f: A \to B$ is a particular type of relation from A to B in which not element of B appears paired w/ 2 elements from A (passes a vertical line test).

Notation

$$\underset{\text{fn. name}}{f} : \overset{\text{domain}}{A} \to \overset{\text{codomain}}{B}$$

read as "f is a function from A to B".

To each $x \in A$, $f(x)$ is called "the image of x under f", the entry in B to which x is paired.

"range of f" $= \{ f(x) \mid x \in A \}$

Ex.) $f : \mathbb{R} \to \mathbb{R}$ is given by $f(x) = x^2$.

domain: $\mathbb{R}$

codomain: $\mathbb{R}$

image of f = range: $[0, \infty)$ (Always true: range $\subseteq$ codomain)

Inside $\mathbb{R}$, let $S = [-1, 1]$.

$f(S) = [0, 1]$ (called "image of S under f")

a set, not simply one real number

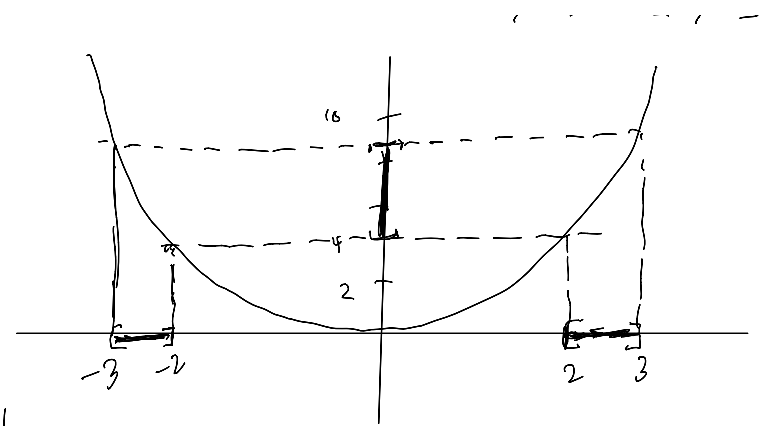
<u>preimage of some subset</u> of the codomain under f

Say $T \subseteq B$

Write $f^{-1}(T) = \{ x \in \text{domain of} \mid f(x) \in T \}$

Again, with $f(x) = x^2$, what is

$$f^{-1}([4, 9]) = [-3, 3] - (-2, 2)$$
$$= [-3, -2] \cup [2, 3]$$



pseudocode

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float absVal ( float x ) {

    return ______
                                   }
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Ex.) floor $\lfloor\ \rfloor : \mathbb{R} \to \mathbb{Z}$

$\lfloor x \rfloor$ = returns the largest integer k w/ $k \le x$.

range of $\lfloor\ \rfloor$: $\mathbb{Z}$

When the range = codomain, say that the function is surjective (onto is a synonym).

Ex.) Take $\lfloor\ \rfloor : \mathbb{R} \to \mathbb{R}$ is not surjective.

Two functions $f, g : A \to B$ are called equal precisely when

- they share domain / codomain
- $\forall x \in A,\ f(x) = g(x)$.

Ex.) $\dfrac{x^2 - 1}{x + 1} = \dfrac{\cancel{(x+1)}(x-1)}{\cancel{x+1}} = x - 1$

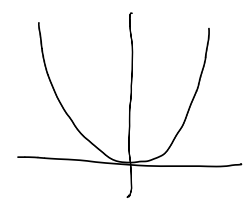
false : they don't have same domain.

A function $f : A \to B$ is injective one-to-one if (precisely when)

$$\forall x_1, x_2 \; \underbrace{\left(f(x_1) = f(x_2) \longrightarrow x_1 = x_2\right)}_{\text{means the graph passes a horizontal line test}} .$$

Say that $f : A \to B$ is bijective precisely when it is both surjective and injective.

Ex.) $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^2$



not surjective
not injective

Ex.) $f : [0, \infty) \to [0, \infty)$ given by $f(x) = x^2$
is bijective.