Math 251, Fri 17-Sep-2021 -- Fri 17-Sep-2021
Discrete Mathematics
Fall 2021

Friday, September 17th 2021

Wk 4, Fr
Topic:: Functions
Read:: Rosen 2.3

Loose ends from Section 2.2
- DeMorgan: Generalization to arbitrary unions/intersections
- cardinality of the power set of A
- inclusion-exclusion principle: |A \cup B| = |A| + |B| - |A \cap B|
  special case: A, B disjoint

2.3 Functions
- to each input there is exactly one output

- functions as
  sets of coordinate pairs
  other representations
    with arrows
    formulas
    graphs

- meaning of
  f: A -> B
  domain, codomain
  image, preimage
    of singletons
    of sets
  image under f of A is also called the range
    note range doesn't necessarily equal codomain

B
Braden
Adam
Oscar
David

Loose ends from 2.2

① De Morgan

$$\overline{A \cap B} = \bar{A} \cup \bar{B}$$

applied to intersection of 3 sets

$$\overline{A \cap B \cap C} = \bar{A} \cup \bar{B} \cup \bar{C}$$

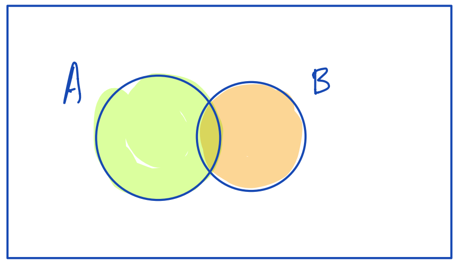
n sets

$$\overline{\bigcap_{i=1}^{n} A_i} = \bigcup_{i=1}^{n} \bar{A_i}$$

(Shorthand for $\overline{A_1 \cap A_2 \cap \dots \cap A_n} = \bar{A_1} \cup \bar{A_2} \cup \dots \cup \bar{A_n}$)

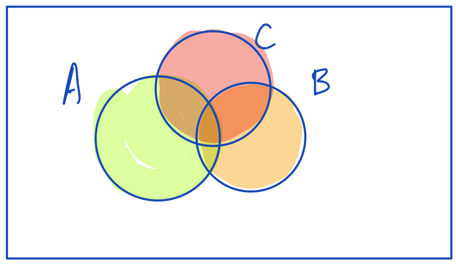
② Inclusion - Exclusion Principle

$$|A \cup B| = |A| + |B| - |A \cap B|$$



Special case: A, B disjoint

$$|A \cup B| = |A| + |B|$$



3 sets:

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap C| - |A \cap B| - |B \cap C| + |A \cap B \cap C|$$

③ $A = \{1, 2, 3\}$

subsets of A:

binary	subset	binary	subset
000	$\{\}$	110	$\{1, 2\}$
100	$\{1\}$	101	$\{1, 3\}$
010	$\{2\}$	011	$\{2, 3\}$
001	$\{3\}$	111	$\{1, 2, 3\}$

$P(A)$ is a set $\{\{\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

$$|P(A)| = 2^{3} = 8.$$

$|A|$

choice Y or N

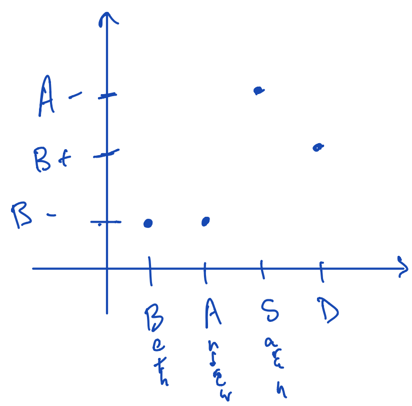
2.3 functions

- Representing them

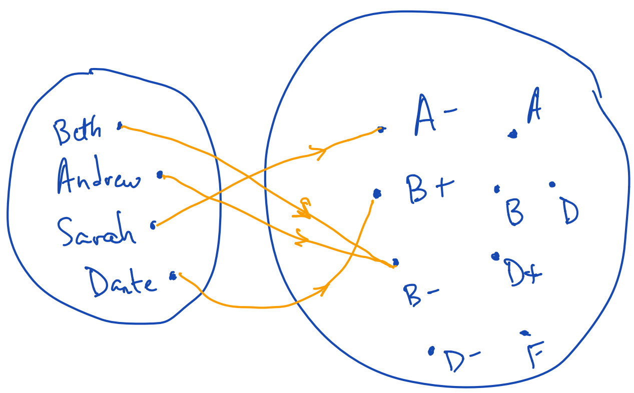
1. Coordinate Pairs

(Beth, "B-")
(Andrew, "B-")
(Sarah, "A-")
(Dante, "B+")

2. Graphs



3. Arrows



Write $f : A \to B$ means f is a function taking elements from A (a set) to elements of B (another set).

Here A is the <u>domain</u>

B is the <u>codomain</u>

We write $f(x)$ for the "image under f of x"

previous example: The image of Dante under grading fn. is B+.

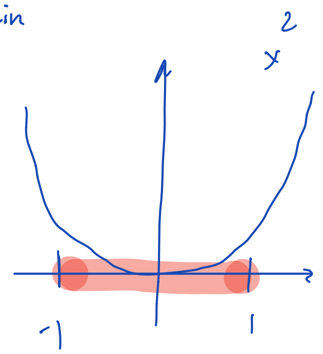
We say: Dante is a preimage under grading map of B+.

Say $f: \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^2$.

The image under f of $x = 2$ is 4 (since $f(2) = 4$).

New: idea of image under f of a subset of the domain

$$f([-1, 1]) = [0, 1]$$



Generally: $f: A \to B$, given subset $C \subseteq A$

then $f(C) = \{ f(x) \mid x \in C \}$ image of C under f

Related idea: Given subset $D \subseteq B$, the preimage of D under f,

$$f^{-1}(D) = \{ x \in A \mid f(x) \in D \}.$$

Ex.) $f: \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^2$

$$f^{-1}([4, 9]) = [2, 3] \cup [-3, -2]$$

Change anything (even domain or codomain) of some function, it is then considered a new function

Ex.) $f(x) = x^2$ is a different function when from domain $\mathbb{R}$ to codomain $\mathbb{R}$ than when from domain $\mathbb{R}$ to codomain $[0, \infty)$.

Definition: For $f: A \to B$ we call $f(A)$ the <u>range</u> of f.
The range of f is always a subset of the codomain.
When the range equals the codomain, f is said to be surjective (or onto).

Examples:

1. $f: \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^2$ is not surjective, as there are elements in the codomain $\mathbb{R}$ (-1, for instance), which are not in the range.

2. $f: \mathbb{R} \to [0,\infty)$ given by $f(x) = x$ is surjective, as the range of f is $[0,\infty)$.

3. The floor function $\lfloor\ \rfloor: \mathbb{R} \to \mathbb{R}$ gives the greatest integer $\leq x$ as output. So, $\lfloor -3.16 \rfloor = -4$, $\lfloor 9.65 \rfloor = 9$, and $\lfloor 2 \rfloor = 2$. This function is not surjective. But, when looked at as a function from $\mathbb{R} \to \mathbb{Z}$, it is surjective.

   The preimages

$$\lfloor\ \rfloor^{-1}(\{2\}) = [2,3)$$

$$\lfloor\ \rfloor^{-1}(\{-3,-2,-1,0\}) = [-3,1)$$

$$\lfloor\ \rfloor^{-1}([0.1,0.5]) = \emptyset.$$