

3.30 $M'_X(t) = \alpha k (1 - \alpha t)^{k-1} \rightarrow E(X) = M'_X(0) = \alpha k$

$$M_X''(t) = \alpha^2(k+1)k(1-\alpha t)^{-k-2} \rightarrow E(X^2) = M_X''(0) = \alpha^2 k(k+1)$$

$$\text{So, } \text{Var}(X) = \alpha^2 k(k+1) - \alpha^2 k^2 = \alpha^2 k.$$

3.41 (a) $E(X) = 5/7$, $Var(X) = (5)(2)/((5+2)^2(1+5+2)) = 5/196 \approx 0.0255$

$$(b) \text{ qbeta}(0.5, 5, 2) = 0.73555$$

$$(c) \Pr[X \leq E(X)] = \text{pbeta}(5/7, 5, 2) \doteq 0.4516$$

$$(d) \Pr(0.2 \leq X \leq 0.4) = \text{pbeta}(0.4, 5, 2) - \text{pbeta}(0.2, 5, 2) = 0.03936.$$

$$(e) \Pr[E(X) - \sqrt{\text{Var}(X)} \leq X \leq E(X) + \sqrt{\text{Var}(X)}] = \text{pbeta}\left(\frac{5}{7} + \sqrt{\frac{5}{196}}, 5, 2\right) - \text{pbeta}\left(\frac{5}{7} - \sqrt{\frac{5}{196}}, 5, 2\right) \doteq 0.662$$

3.45 Taking into account that points-scored is always an integer, the normal-quantile plot is quite linear in its appearance, making a normal model appear appropriate. At the top end, there may be the suggestion that he should have had a higher personal-season-best.

3.54 (a) $\int_0^1 \int_0^1 x^2 y^3 dy dx = \int_0^1 \frac{1}{4} x^2 [y^4]_0^1 dx = \frac{1}{12} [x^3]_0^1 = \frac{1}{12}.$

So, the pdf

$$f_{X,Y}(x,y) = \begin{cases} 12x^2y^3, & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

$$(b) \quad P_r(X < Y) = \int_0^1 \int_0^y 12x^2 y^3 dx dy = 4 \int_0^1 y^3 [x^3]_0^y dy = 4 \int_0^1 y^6 dy = \frac{4}{7}.$$

(c) $f_x(x) = 12x^2 \int_0^1 y^3 dy = 3x^2$, and $f_y(y) = 12y^3 \int_0^1 x^2 dx = 4y^3$.

Note that $f_{X,Y}(x,y) = f_X(x)f_Y(y)$, so X and Y are independent.

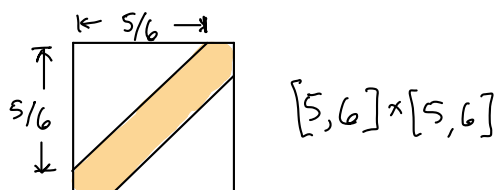
3.55 (a) $f_{X,Y}(x,y) = 1$ for $(x,y) \in [5,6] \times [5,6]$.

(b) $\Pr((X, Y) \in [5, 5.5] \times [5, 5.5]) = 1/4.$

(c) For them to arrive within 10 minutes of each other, (X, Y) must be a point in the orange shaded region, which has area (= probability)

$[5, 6] \times [5, 6]$

$$1 - (2)\left(\frac{1}{2}\right)\left(\frac{5}{6}\right)^2 = \frac{11}{36}.$$



3.63 $X \sim \text{Pois}(\lambda_1)$ and $Y \sim \text{Pois}(\lambda_2)$, so $M_X(t) = e^{e^t \lambda_1 - \lambda_1}$, and $M_Y(t) = e^{e^t \lambda_2 - \lambda_2}$.

By independence of X, Y ,

$$\begin{aligned} M_{X+Y}(t) &= M_X(t) M_Y(t) = e^{e^t \lambda_1 - \lambda_1} \cdot e^{e^t \lambda_2 - \lambda_2} \\ &= e^{e^t \lambda_1 + e^t \lambda_2 - (\lambda_1 + \lambda_2)} = e^{e^t (\lambda_1 + \lambda_2) - (\lambda_1 + \lambda_2)}. \end{aligned}$$

This is the mgf for another Poisson r.v. with parameter $\lambda_1 + \lambda_2$. Thus

$$X + Y \sim \text{Pois}(\lambda_1 + \lambda_2).$$

3.64 $X \sim \text{Binom}(n, \pi_1)$ and has $M_X(t) = (\pi_1 e^t + 1 - \pi_1)^n$. As well,

$Y \sim \text{Binom}(n, \pi_2)$ and has $M_Y(t) = (\pi_2 e^t + 1 - \pi_2)^n$. By independence of X and Y ,

$$\begin{aligned} M_{X+Y}(t) &= M_X(t) M_Y(t) = [(\pi_1 e^t + 1 - \pi_1)(\pi_2 e^t + 1 - \pi_2)]^n \\ &= [\pi_1 \pi_2 e^{2t} + (\pi_1 + \pi_2) e^t - 2\pi_1 \pi_2 e^t + 1 - (\pi_1 + \pi_2) + \pi_1 \pi_2]^n \end{aligned}$$

This is not the mgf of a binomial r.v.

4.1 Let $\bar{X} = \frac{1}{n} \sum X_i$ be the first sample moment about the origin (a.k.a. the sample mean). The population mean for $\text{Binom}(1, \pi)$ is $1 \cdot \pi = \pi$. Our estimate is $\hat{\pi} = \bar{X}$.

4.4 For $X \sim \text{NBinom}$, $E(X) = \frac{\Delta}{\pi} - \Delta$. So, we set

$$\frac{\Delta}{\hat{\pi}} - \Delta = \bar{X} \quad \Rightarrow \quad \hat{\pi} = \frac{\Delta}{\Delta + \bar{X}}.$$

4.7 farrstats reveals $\bar{X} \doteq 0.6091$, $s \doteq 0.248$, $n = 134 \Rightarrow v = 0.06105$.

From the formulas:

$$\hat{\alpha} = \bar{X} \left(\frac{\bar{X}(1-\bar{X})}{v} - 1 \right) \doteq 1.7665, \quad \hat{\beta} = (1-\bar{X}) \left(\frac{\bar{X}(1-\bar{X})}{v} - 1 \right) \doteq 1.1337$$

The beta distribution using these shape parameters gives a very poor fit to the data. By filtering out the players with $\text{FTPct} = 0$, the new parameter estimates from remaining players are

$$\hat{\alpha} = 4.824, \quad \hat{\beta} = 2.387,$$

and the fit is vastly improved.

4.9 Take $\bar{x} = \frac{1}{n} \sum x_i$ and $v = \frac{1}{n} \sum (x_i - \bar{x})^2$.

For an exponential distribution, $\hat{\lambda} = \frac{1}{\bar{x}} \doteq 0.00763$

For a gamma distribution,

$$\frac{\hat{\alpha}}{\hat{\lambda}} = \bar{x} \Rightarrow \hat{\alpha} = \hat{\lambda} \bar{x}$$

$$\frac{\hat{\alpha}}{\hat{\lambda}^2} = v \Rightarrow \frac{\hat{\lambda} \bar{x}}{\hat{\lambda}^2} = \frac{\bar{x}}{\hat{\lambda}} = v$$

$$\Rightarrow \hat{\lambda} = \frac{\bar{x}}{v} \doteq 1.858 \quad \text{and} \quad \hat{\alpha} = \hat{\lambda} \bar{x} = \frac{\bar{x}^2}{v} \doteq 0.0141.$$