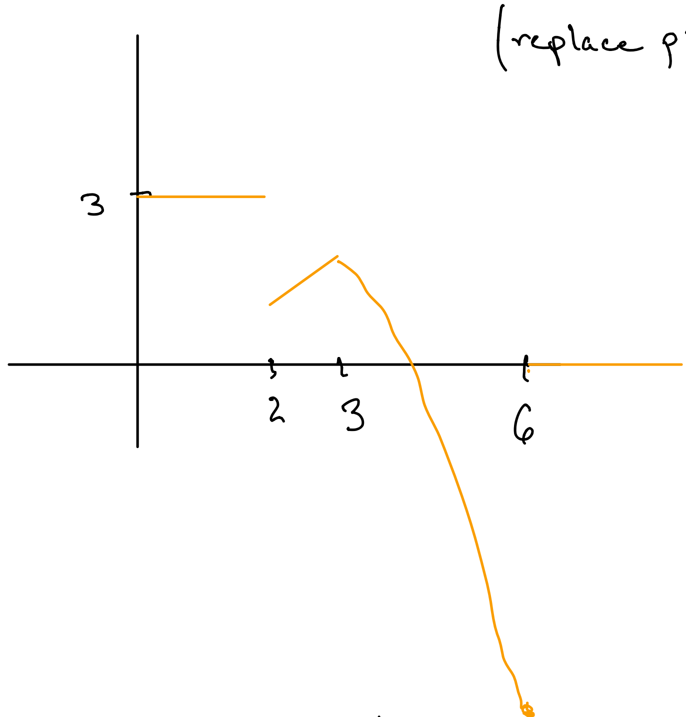
Follow up to yesterday

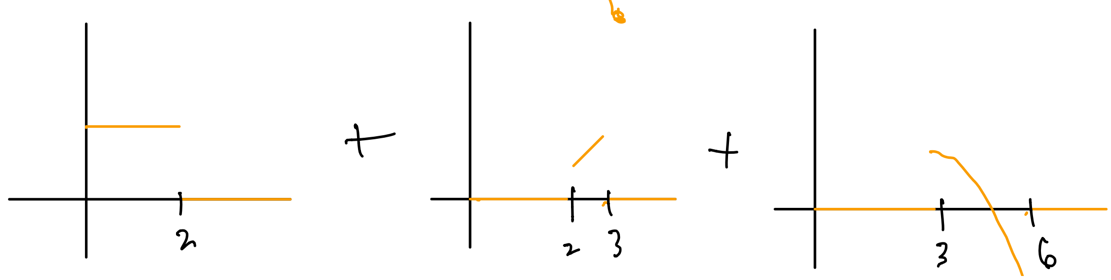
Write the function

$$f(t) = \begin{cases} 3, & 0 \le t < 2 \\ t-1, & 2 \le t \le 3 \\ 5 - \tfrac{1}{3}t^2, & 3 < t \le 6 \\ 0, & t > 6 \end{cases}$$

using Heaviside step in the expression.

(replace piecewise-defined version)



on at 2 off at 3

$3\cdot\left[H(t)-H(t-2)\right]$ $\left[H(t-2)-H(t-3)\right]\cdot$ $\left[H(t-3)-H(t-6)\right]$

on at 0

off at 2

$(t-1)$

$\cdot\left(5-\frac{1}{3}t^2\right)$

$$f(t) = 3\left[H(t)-H(t-2)\right] + (t-1)\cdot\left[H(t-2)-H(t-3)\right] + \left(5-\tfrac{1}{3}t^2\right)\cdot\left[H(t-3)-H(t-6)\right]$$

Inverse Laplace transforms

Have fn. of $s$ already, say $G(s)$; want the fn. of $t$, $g(t)$, for which $\mathcal{L}\{g(t)\} = G(s)$.

say $g(t) = \mathcal{L}^{-1}\{G(s)\}$.

Do using our catalog. See p. 242 in textbook

See ones we derived as well as

| $f(t)$ | $F(s) = \mathcal{L}\{f(t)\}$ |
|---|---|
| $t^n e^{at}$ | $\dfrac{n!}{(s-a)^{n+1}}$ |
| $e^{at}\cos(bt)$ | $\dfrac{s-a}{(s-a)^2+b^2}$ |

$e^{at}\sin(bt)$ $\qquad\qquad$ $\dfrac{b}{(s-a)^2+b^2}$

Ex. Find the inverse Laplace transform for

(a) $\dfrac{3}{2s+1}$ ⟵ In the middle column, nearest match

$$= \frac{3}{2}\cdot\frac{1}{s+1/2} = \frac{3}{2}\cdot\frac{1}{s-(-1/2)}$$

matches 3rd entry

$$\mathcal{L}^{-1}\left\{\frac{3}{2s+1}\right\} = \mathcal{L}^{-1}\left\{\frac{3}{2}\cdot\frac{1}{s-(-1/2)}\right\}$$

$$= \frac{3}{2}\mathcal{L}^{-1}\left\{\frac{1}{s-(-1/2)}\right\} = \underline{\frac{3}{2}e^{-\frac{1}{2}t}}.$$

(b) $\dfrac{3s+5}{s^2+9} = \dfrac{3s}{s^2+9} + \dfrac{5}{s^2+9}$

$$\mathcal{L}^{-1}\left\{\frac{3s}{s^2+9}\right\} + \mathcal{L}^{-1}\left\{\frac{5}{s^2+9}\right\} = 3\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\} + \frac{5}{3}\mathcal{L}^{-1}\left\{\frac{3}{s^2+9}\right\}$$

$$= \boxed{3\cos(3t) + \frac{5}{3}\sin(3t)}.$$

L.T. table
www.math.ucsd.edu/~helton/20Dweb2011/
files/laplace_table.pdf

(c) $\dfrac{2s-4}{s^2+6s+10}$ — sort of like $e^{at}\cos(bt)$ line
— complete the square

$$= \frac{2s-4}{s^2+6s+9+1} = \frac{2s-4}{(s+3)^2+1}$$

$$= \frac{2(s+3)-10}{(s+3)^2+1} = 2\cdot\frac{s+3}{(s+3)^2+1} - 10\cdot\frac{1}{(s+3)^2+1}$$

$$\mathcal{L}^{-1}\left\{2\cdot\frac{s+3}{(s+3)^2+1}\right\} - 10\,\mathcal{L}^{-1}\left\{\frac{1}{(s+3)^2+1}\right\}$$

$$= 2e^{-3t}\cos t - 10e^{-3t}\sin t$$

(d) $\dfrac{3}{(s-2)(s+5)}$ $\overset{\text{use partial fractions}}{=}$ $\dfrac{A}{s-2} + \dfrac{B}{s+5}$

mult. thru by CD

$$3 = A(s+5) + B(s-2)$$

$\uparrow$

/

Equality must hold for all choices of $s$

Fortuitous choices of $s$:

@ $s = -5$:

$$3 = A(-5+5) + B(-5-2)$$

$$\Longrightarrow B = -3/7$$

@ $s = 2$:

$$3 = A(2+5) + B(2-2)$$

$$\Longrightarrow A = 3/7$$

$$\mathcal{L}^{-1}\left\{\frac{3/7}{s-2}\right\} + \mathcal{L}^{-1}\left\{\frac{-3/7}{s+5}\right\} = \boxed{\frac{3}{7}e^{2t} - \frac{3}{7}e^{-5t}}$$