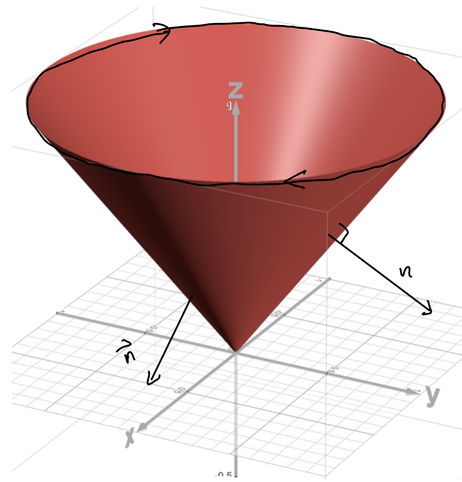

1. (a) One parametrization uses x, y as parameters:

$$\langle x,\ y,\ \sqrt{x^2+y^2}\ \rangle, \quad -1 \le x \le 1,\ -\sqrt{1-x^2} \le y \le \sqrt{1-x^2}$$

Another employs taking x, y over to polar coords.

$$\langle r\cos\theta,\ r\sin\theta,\ r \rangle, \qquad 0 \le r \le 1,\ 0 \le \theta \le 2\pi.$$

(b) Using the first parametrization above, we get a vector normal to the surface

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & \frac{x}{\sqrt{x^2+y^2}} \\ 0 & 1 & \frac{y}{\sqrt{x^2+y^2}} \end{vmatrix}$$

$$= \left\langle \frac{-x}{\sqrt{x^2+y^2}},\ \frac{-y}{\sqrt{x^2+y^2}},\ 1 \right\rangle.$$

This points away from the xy-plane, a matter easily fixed by multiplying it by (-1). To make it a unit vector, note that

$$\sqrt{\left(\frac{-x}{\sqrt{x^2+y^2}}\right)^2 + \left(\frac{-y}{\sqrt{x^2+y^2}}\right)^2 + 1} = \sqrt{\frac{x^2+y^2}{x^2+y^2} + 1} = \sqrt{2}.$$

So, the unit normal we want is $\left\langle \frac{x}{\sqrt{2(x^2+y^2)}},\ \frac{y}{\sqrt{2(x^2+y^2)}},\ \frac{-1}{\sqrt{2}} \right\rangle.$

(c) See the arrows added to the edge curve above.

(d) One answer $\langle \sin t,\ \cos t,\ 1 \rangle, \quad 0 \le t \le 2\pi$

(e) $\iint_S \operatorname{curl} \vec{F} \cdot \vec{N}\, dS = \oint_C \vec{F} \cdot d\vec{r}$. Orientation along C must be positive in relation to the orientation of the surface. If a wrongly-oriented parametrization were used for $\oint_C \vec{F} \cdot d\vec{r}$, its value would be the opposite sign of $\iint_S \operatorname{curl} \vec{F} \cdot d\vec{S}$.

2. By the Divergence Theorem,

$$\iint_{\partial E} \vec{F} \cdot d\vec{S} = \iiint_E \operatorname{div} F\, dV = \iiint (4x^3 + 4xy^2)\, dV = 4 \iiint x(x^2+y^2)\, dV$$

$$= 4 \int_0^{2\pi} \int_0^1 \int_0^{2+r\cos\theta} r^4 \cos\theta\, dz\, dr\, d\theta = 4 \int_0^{2\pi} \int_0^1 r^4 \left(2\cos\theta + r\cos^2\theta\right) dr\, d\theta$$

$$= 4\int_0^{2\pi}\left(\frac{2}{5}\cos\theta\, r^5 + \frac{1}{6}\cos^2\theta\, r^6 \Big|_0^1\right) d\theta = 4\int_0^{2\pi}\left(\frac{2}{5}\cos\theta + \frac{1}{6}\cos^2\theta\right) d\theta$$

$$= 4\int_0^{2\pi} \frac{2}{5}\cos\theta + \frac{1}{12}\left[1+\cos(2\theta)\right] d\theta$$

$$= 4\left[\frac{2}{5}\sin\theta + \frac{1}{12}\theta + \frac{1}{24}\sin(2\theta)\right]_0^{2\pi} = \frac{2}{3}\pi$$

3. Among the three vars. x, y, z, its easiest to solve for x

$$x = 4 + y^3 - z^2(y-1)$$

facilitating the use of y, z as parameters

$$\vec{r}(y,z) = \langle 4+y^3 - z^2(y-1),\ y,\ z\rangle \Rightarrow \vec{r}_y = \langle 3y^2 - z^2,\ 1,\ 0\rangle$$

$$\vec{r}_z = \langle -2z(y-1),\ 0,\ 1\rangle$$

So, a normal vector is found via the cross product

$$\vec{r}_y \times \vec{r}_z = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3y^2 - z^2 & 1 & 0 \\ -2z(y-1) & 0 & 1 \end{vmatrix} = \langle 1, -(3y^2-z^2),\ 2z(y-1)\rangle$$

increasing x, so will make acute angle w/ the positive x-axis

4. We get a normal vector to the surface

$$\vec{n} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -(R_1 + R_2\cos v)\sin u & (R_1 + R_2\cos v)\cos u & 0 \\ -R_2 \sin v \cos u & -R_2 \sin v \sin u & R_2 \cos v \end{vmatrix}$$

$$= \left(R_1R_2\cos v + R_2^2\cos^2 v\right)\cos u\ \hat{\imath} + \left(R_1R_2\cos v + R_2^2\cos^2 v\right)\sin u\ \hat{\jmath}$$
$$+ \left[\left(R_1R_2\sin v + R_2^2\cos v\sin v\right)\sin^2 u + \left(R_1R_2\sin v + R_2^2\cos v\sin v\right)\cos^2 u\right]\hat{k}$$

$$= \langle R_2\cos v\cos u(R_1 + R_2\cos v),\ R_2\cos v\sin u(R_1+R_2\cos v),\ R_2\sin v(R_1+R_2\cos v)\rangle$$

$$\Rightarrow \|\vec{n}\|^2 = R_2^2(R_1+R_2\cos v)^2\left(\cos^2 v\cos^2 u + \cos^2 v\sin^2 u + \sin^2 v\right) = R_2^2(R_1+R_2\cos v)^2$$

$$SA = \int_0^{2\pi}\int_0^{2\pi} R_2(R_1+R_2\cos v)\,du\,dv = 2\pi R_2\int_0^{2\pi}(R_1+R_2\cos v)\,dv = 4\pi^2 R_1 R_2.$$

5. $\int \sin^2 x \, dx = \frac{1}{2} \int \left[1 - \cos(2x)\right] dx = \frac{1}{2} \left[x - \frac{1}{2} \sin(2x)\right] + C$

$= \frac{1}{2} x - \frac{1}{4} \sin(2x) + C$

$= \frac{1}{2} x - \frac{1}{2} \cos x \sin x + C$ using $\sin(2x) = 2 \cos x \sin x$

$\int \cos^2 x \, dx = \frac{1}{2} \int \left[1 + \cos(2x)\right] dx = \frac{1}{2} \left[x + \frac{1}{2} \sin(2x)\right] + C$

$= \frac{1}{2} x + \frac{1}{4} \sin(2x) + C$

$= \frac{1}{2} x + \frac{1}{2} \cos x \sin x + C$