Math 251, Fri 18-Sep-2020 -- Fri 18-Sep-2020

Discrete Mathematics

Fall 2020

Friday, September 18th 2020

Due::   PS04 due at 6 pm

Friday, September 18th 2020

Wk 3, Fr

Topic:: Set operations notes/09-18.pdf

HW[[WW sets2 due Thurs.

HW[[PS05 due Fri.

Read:: Rosen 2.2

I got this backwards last time:

natural nos = {0, 1, 2, ...} $= \mathbb{N}$

Rosen denotes {1, 2, ...} as Z^+ $= \mathbb{Z}^+$

$$\mathbb{C} = \{\text{complex nos.}\}$$

$$= \{a + bi \mid a, b \in \mathbb{R}\}$$

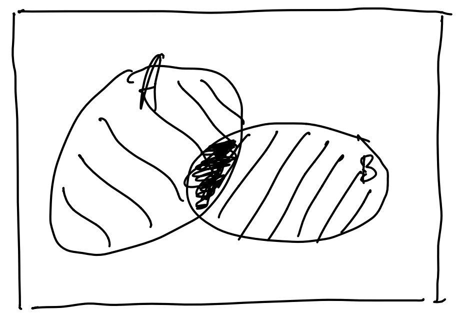
$$\mathbb{R} \times \mathbb{R} = \{(x, y) \mid x, y \in \mathbb{R}\} = \mathbb{R}^2$$

$$\bigcup_{i=1}^{\infty} \left(\frac{1}{n+1}, \frac{1}{n}\right] = \left(\frac{1}{2}, 1\right] \cup \left(\frac{1}{3}, \frac{1}{2}\right] \cup \left(\frac{1}{4}, \frac{1}{3}\right] \cup \cdots$$
$$= (0, 1]$$

Sets built from other sets 2.2

- Union of sets (two, more than two)

$$A \cup B = \{x \mid x \in A \vee x \in B\}$$



$$(A \cup B) \cup C = A \cup (B \cup C)$$
or write $A \cup B \cup C$

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \cdots \cup A_n \qquad \bigcup_{i=1}^{\infty} A_i$$

- Intersection of sets (two, more than two)

$$A \cap B = \{x \mid x \in A \wedge x \in B\}$$

Can apply to more than 2 sets

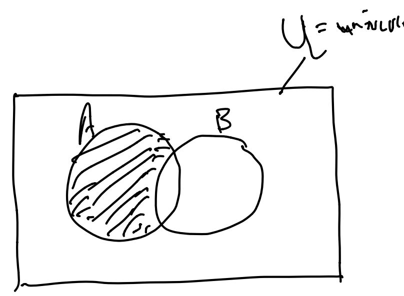
$$(A \cap B) \cap C = A \cap (B \cap C)$$
or write $A \cap B \cap C$

Again $\bigcap_{i=1}^{n} A_i$

$$\bigcap_{i=1}^{\infty} A_i$$

$$\bigcap_{i=1}^{\infty} \left[\frac{1}{i+1}, \frac{1}{i}\right]$$
$$= \left[\frac{1}{2}, 1\right] \cap \left[\frac{1}{3}, \frac{1}{2}\right] \cap \left[\frac{1}{4}, \frac{1}{3}\right] \cap \cdots$$
$$= \emptyset.$$

- Set subtraction and complementation
  - disjoint sets
  - breaking $A \cup B$ into a disjoint union
  - inclusion-exclusion principle
  - complement arises from set subtraction from a universal set



$$A - B \quad (\text{also } A \setminus B) = \{x \in U \mid x \in A \wedge x \notin B\}$$

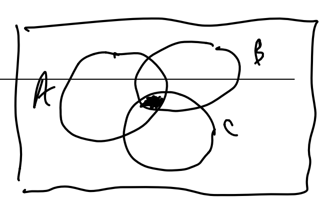
Note: $A = \underbrace{(A - B) \cup (A \cap B)}_{\text{disjoint}}$ — means there are no shared elements

$$A \cup B = (A - B) \cup (A \cap B) \cup (B - A) \quad \text{(3 disjoint pieces)}$$

Complement of A, $\overline{A} = \{x \in U \mid x \notin A\} = U - A$.

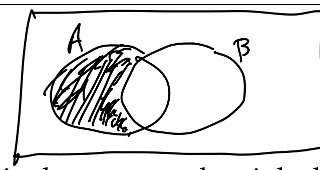
Inclusion-Exclusion

$$|A \cup B| = |A| + |B| - |A \cap B|$$



$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C|$$

$- |A \cap C| + |A \cap B \cap C|$



Identities (akin to logical equivalences in Chapter 1)

Using your intuition, Venn diagrams, etc., present a plausibly equivalent set on the right-hand side, then prove it (first to yourself).

1. $A - B =$ $A \cap \overline{B}$
2. $A \cup \overline{A} =$ $\mathcal{U}$
3. $A \cap A =$ A
4. $(A \cap B) \cap C =$ $A \cap (B \cap C)$ Associative Law
5. $(A \cap B) \cup C =$ $(A \cup C) \cap (B \cup C)$
6. $\overline{A \cup B} =$ $\overline{A} \cap \overline{B}$ DeMorgan
7. $A \cup (A \cap B) =$ A
8. $\overline{A \cup (B \cap C)} =$ $\overline{(A \cup B) \cap (A \cup C)}$

$= \overline{A \cup B} \cup \overline{A \cup C} = (\overline{A} \cap \overline{B}) \cup (\overline{A} \cap \overline{C})$

$= \overline{A} \cap (\overline{B} \cup \overline{C})$

$\frac{x}{x+5}$

Can prove by

- $(A \cap B) \cup C \subseteq (A \cup C) \cap (B \cup C)$ and vice versa $(A \cup C) \cap (B \cup C) \subseteq (A \cap B) \cup C$
- Set-building / propositional equivs.
- Membership table

A	B	C	$(A \cap B) \cup C$	$(A \cup C) \cap (B \cup C)$
0	0	0	0	0
0	0	1	1	1
0	1	0	0	0
1	0	0	0	0
0	1	1	1	1
1	0	1		
1	1	0		
1	1	1		

Methods for proving two sets are equal, i.e., $A = B$

- Show $A \subseteq B$ and $B \subseteq A$ (example: 6)

See Example 10

- Invoke set builder description, use logical equivalences (example: 6)

See Example 11 on p. 131 for one like this

- Show that A and B have the same membership table (example: 8)

Set operations compared with bit operations

$U = \{1, 2, 3, 4, 5\}$

$$\begin{array}{c} 00000 \\ \cup \; 01000 \\ \hline 01000 \end{array}$$