1. (a) $X \sim \text{Binom}(36, 0.79)$

(b) $X \sim \text{Pois}(36)$

(c) Either X is geometric or negative binomial with $s=1$.

(d) $X \sim \text{Hyper}(24, 6, 10)$.

2. $$E((X-\mu_X)(Y-\mu_Y)) = E(XY - \mu_X Y - \mu_Y X + \mu_X\mu_Y)$$
$$= E(XY) - \mu_X E(Y) - \mu_Y E(X) + \mu_X\mu_Y$$
$$= E(XY) - E(X)E(Y) - E(X)E(Y) + E(X)E(Y)$$
$$= E(XY) - E(X)E(Y).$$

3. (a) The probabilities should sum to 1, so the missing values comprise a total $1 - (0.05 + 0.1 + 0.15 + 0.26) = 0.44$.
Splitting this amount so 3 parts go to $X=6$ and 1 part to $X=5$:

$\Pr(X=5) = 0.11$ and $\Pr(X=6) = 0.33$.

(b) $\Pr(X \leq 3) = \Pr(X=1) + \Pr(X=2) + \Pr(X=3) = 0.05 + 0.1 + 0.15 = 0.3$

(c) The event, A = "3 or less," has probability 0.3. That success rate is the same for all $n=60$ trials, so this is a binomial setting. That is,

$Y \sim \text{Binom}(60, 0.3) \Rightarrow \Pr(Y \geq 10) = 1 - \text{pbinom}(9, 60, 0.3)$.

(d) With the original probability table/function,

$$\mu_X = (1)(0.05) + (2)(0.1) + (3)(0.15) + (4)(0.26) + (5)(0.11) + (6)(0.33) = 4.27.$$
$$E(X^2) = (1^2)(0.05) + (2^2)(0.1) + (3^2)(0.15) + (4^2)(0.26) + (5^2)(0.11) + (6^2)(0.33) = 20.59$$
$$\Rightarrow Var(X) = E(X^2) - \mu_X^2 = 20.59 - (4.27)^2 = 2.357.$$

With the substitute table,

$$\mu_X = (1)/9 + (2)/6 + (3)/3 + (4)/6 + (5)/9 + (6)/9 = 3.333, \text{ or } \tfrac{10}{3}.$$
$$Var(X) = (1-\tfrac{10}{3})^2(1/9) + (2-\tfrac{10}{3})^2(1/6) + (3-\tfrac{10}{3})^2(1/3) + (4-\tfrac{10}{3})^2(1/6)$$
$$+ (5-\tfrac{10}{3})^2(1/9) + (6-\tfrac{10}{3})^2(1/9) = 2.111.$$

4. (a) For $0 < x \le 4$,

$$F_X(x) = Pr(X \le x) = \int_0^x \frac{1}{4} t^{-1/2}\, dt = \frac{1}{2} t^{1/2} \Big|_0^x = \frac{1}{2}\sqrt{x}.$$

So,

$$F_X(x) = \begin{cases} 0, & x \le 0 \\ \frac{1}{2}\sqrt{x}, & 0 < x \le 4 \\ 1, & x > 4 \end{cases}$$

(b) $Pr(1 \le x \le 2.25) = F_X(2.25) - F_X(1) = \frac{1}{2}\sqrt{2.25} - \frac{1}{2} = \frac{1}{4}.$

5. (a) $\sigma_W > \sigma_Y$, and that is because any extreme value in X is simply rescaled by 3 for W but, in contrast, extreme values in X_1 need not simultaneously be mirrored in X_2 and X_3, tempering their effect on Y.

(b) $E(Y) = E(X_1) + E(X_2) + E(X_3) = 3(126.3) = 378.9.$

$Var(Y) = \sum Var(X_i) = 3(13) = 39$

$\Rightarrow Y \sim Norm(378.9, \sqrt{39}).$

So, $\quad$ pnorm$(390, 378.9, \sqrt{39})$ - pnorm$(375, 378.9, \sqrt{39})$

6. (a) $M_X'(t) = -4(1-2t)^{-5}(-2) = 8(1-2t)^{-5}$

$\Rightarrow E(X) = M_X'(0) = 8$

$M_X''(t) = -40(1-2t)^{-6}(-2) = 80(1-2t)^{-6}$

$\Rightarrow E(X^2) = M_X''(0) = 80$

$\Rightarrow Var(X) = 80 - 8^2 = 16.$

(b) $M_Y(t) = M_{3X-2}(t) = E(e^{t(3X-2)}) = E(e^{-2t} \cdot e^{(3t)X})$

$= e^{-2t} \cdot E(e^{(3t)X}) = e^{-2t} \cdot M_X(3t) = e^{-2t} \cdot (1-6t)^{-4}.$