2. (a) This is the problem of having 10 slots you fill with letters C, T, V
and D (D for divider) so that precisely two D's are used.
Counting ways to choose 2 slots for D's answers the question:

$$\binom{10}{2} = \frac{10!}{2! \, 8!} = \frac{(10)(9)}{2} = 45.$$

(b) 9! would reflect the number of permutations (strings) one can make
out of 9 distinct letters. To use that as an answer would
overcount strings, but predictably so: there are

$$4! = 24 \text{ equivalent arrangements of the 4 E's}$$
$$2! = 2 \qquad " \qquad " \qquad " \qquad " \quad 2 \text{ N's}$$
$$2! = 2 \qquad " \qquad " \qquad " \qquad " \quad 2 \text{ S's}$$

So, $$\frac{9!}{4! \, 2! \, 2!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}{2^2} = (9)(2)(7)(6)(5) = 3780$$

3. Note: $Pr(A \mid B) + Pr(A^c \mid B) = 1$

$$\implies Pr(A^c \mid B) = 1 - Pr(A \mid B) = 1 - Pr(A) \quad \left(\text{given information}\right)$$

$$= Pr(A^c).$$

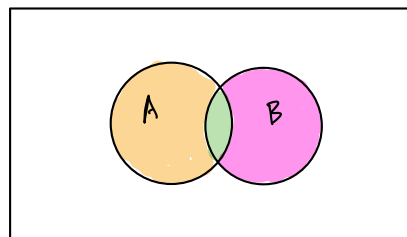Since $Pr(A^c \mid B) = Pr(A^c)$, A^c and B are independent.

4. Let A denote the event that a person has symptom A.
B denote the event that a person has symptom B.

We know $Pr(A \setminus B) = 0.31$ (orange shading)

We know $Pr(B \setminus A) = 0.23$ (pink shading)

We know $Pr(A \cap B) = 0.09$ (green shading)

(a) $Pr(\text{neither symptom}) = 1 - 0.31 - 0.23 - 0.09 = 0.37.$

(b) $Pr(A \cap B \mid B) = Pr(A \mid B) = \dfrac{Pr(A \cap B)}{Pr(B)} = \dfrac{0.09}{0.23 + 0.09} = 0.28125$

5. (a) $E(2X) = 2E(X) = 38.$

   (b) $E(X+Y) = E(X) + E(Y) = 19 - 8 = 11.$

   (c) $E(X-Y) = E(X) - E(Y) = 19 + 8 = 27.$

   (d) $\text{Var}(X+2Y) = \text{Var}(X) + \text{Var}(2Y) \quad (\text{by independence})$

   $\qquad\qquad\qquad = \text{Var}(X) + 4\text{Var}(Y) = 4 + 12 = 16.$

6. (a) We require

   $$1 = \sum_{j=0}^{4} c \cdot \frac{j^2}{10} = \frac{c}{10}(0 + 1 + 4 + 9 + 16) = 3c$$

   So, $c = \frac{1}{3}.$

   (b) $Pr(X \leq 3) = \frac{1}{3} \cdot \frac{1}{10}(0 + 1 + 4 + 9) = \frac{7}{15} \doteq 0.4667$

   (c)
   $$E(X) = \sum_{j=0}^{4} j\, f(j) = 0 + \frac{1}{30} + 2 \cdot \frac{4}{30} + 3 \cdot \frac{9}{30} + 4 \cdot \frac{16}{30}$$

   $$= \frac{10}{3} \doteq 3.3333. \quad (\text{the mean})$$

   $$E(X^2) = \sum_{j=0}^{4} j^2\, f(j) = 0 + \frac{1}{30} + 2^2 \cdot \frac{4}{30} + 3^2 \cdot \frac{9}{30} + 4^2 \cdot \frac{16}{30}$$

   $$= \frac{354}{30} = 11.8.$$

   So, $\text{Var}(X) = E(X^2) - [E(X)]^2 = \frac{354}{30} - \left(\frac{10}{3}\right)^2 = \frac{31}{45} \doteq 0.6889$

7. (a) For X counting defects on a 1 sq-ft piece, $X \sim \text{Pois}(0.5).$

   $$Pr(X \geq 1) = 1 - Pr(X=0) = 1 - e^{-0.5}\frac{(0.5)^0}{0!} = 1 - e^{-1/2} \doteq 0.3935$$

   (b) For Y counting defects on a 4 sq-ft. piece, $Y \sim \text{Pois}(2).$

   $$Pr(Y \geq 2) = 1 - Pr(Y=0) - Pr(Y=1) = 1 - e^{-2}\left(\frac{2^0}{0!}\right) - e^{-2}\left(\frac{2}{1!}\right)$$

   $$= 1 - 3e^{-2} \doteq 0.5940.$$

   (c) Let Z count the number, out of 3, of 1 sq-ft. pieces w/ a defect. Then

   $$Z \sim \text{Binom}(3, 0.3935). \qquad Pr(Z \geq 1) = 1 - Pr(Z=0) = 1 - \binom{3}{0}(1 - 0.3935)^3 \doteq 0.7769.$$

   (d) $1 - \text{pbinom}(0, 3, 0.3935)$ works. So do others.