Solutions

1. (a)

$$\int_C F \circ dr = \int_0^{2\pi} \left\langle \frac{2\sin t}{\sqrt{4\cos^2 t + 4\sin^2 t}}, \frac{-2\cos t}{\sqrt{4\cos^2 t + 4\sin^2 t}} \right\rangle \circ \langle -2\sin t, 2\cos t \rangle \, dt$$

$$= -2\int_0^{2\pi} (\sin^2 t + \cos^2 t)\, dt = -2\int_0^{2\pi} dt = -2\Big[t\Big]_0^{2\pi} = \boxed{-4\pi}$$

(b)

Many correct answers. Two of them are

$\vec{r}(t) = \langle -2\cos t, 2\sin t \rangle, \quad 0 \le t \le 2\pi$

$\vec{r}(t) = \langle 2\sin t, 2\cos t \rangle, \quad -\pi \le t \le \pi$

(c)

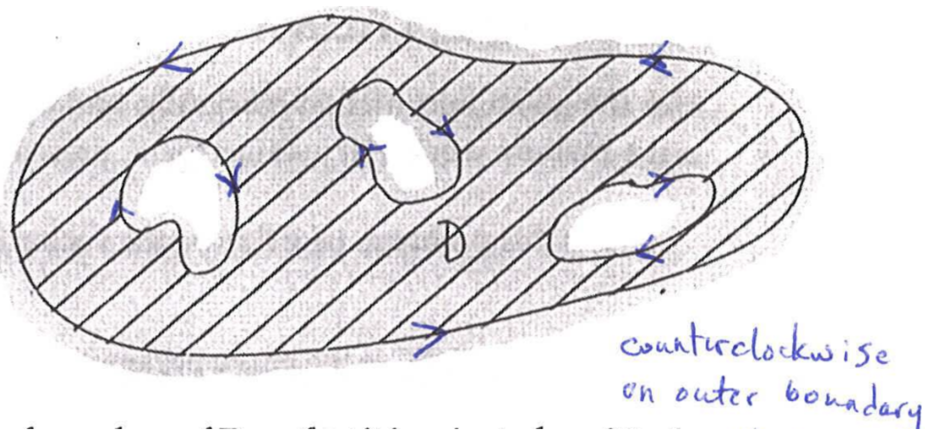
Opposite of answer to part (a): 4π

(d)

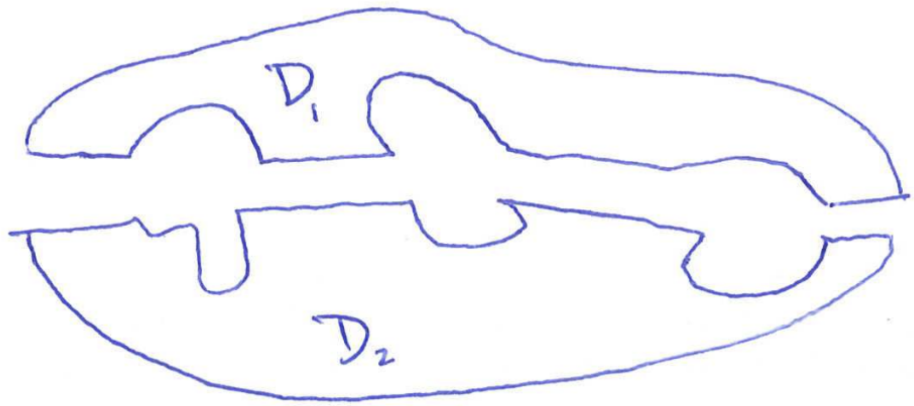
$\vec{F}$ is not conservative since

- a line integral around a closed curve produced a nonzero result.
- the partials $\frac{\partial}{\partial y} \frac{y}{\sqrt{x^2+y^2}}$ and $\frac{\partial}{\partial x} \frac{-x}{\sqrt{x^2+y^2}}$ are unequal

2. (a)



(b) You must slice up D so the new regions, D_1, D_2 are without holes:



(c)

$$\int_{\partial D_1} \vec{F} \circ \vec{N}\, ds \quad \text{or maybe} \quad \int_{\partial D_1} -Q\,dx + P\,dy$$

(d)

$$= \iint_D (Q_x - P_y)\, dA$$