SCOFIELD3894

Edit Quiz

SAVE AND EXIT

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SOC-38681413

MATH 231 Sections 1.1-1.9 quiz

Align quiz to standard

#1

EDIT

If **A** is an m-by-n matrix, then the column space of **A** is a subspace of $\mathbf{R}^n$.

Correct Answer:

True False

Explanation:

Since every column of **A** has m components, the linear combinations of columns of **A** also have m components, and belong to $\mathbf{R}^m$, not necessarily to $\mathbf{R}^n$.

#2

EDIT

If **A** is a matrix, **b** is a vector, and the vector equation **Ax** = **b** has infinitely many solutions, then the nullspace of **A** is nontrivial.

Correct Answer:

True	False

Explanation:

The nullspace of any matrix is either trivial, consisting of the vector **0** only, or it is nontrivial (contains the vector **0** and many others). The deciding factor is whether the matrix has free columns or not. Since we are told **Ax** = **b** has infinitely many solutions, we learn from this that **A** has a free column, and hence its nullspace is nontrivial.

#3

EDIT

If **b** is in the column space of the matrix **A**, then so is (-3**b**).

Correct Answer:

True	False

Explanation:

Since the column space is a vector space, it is closed under scalar multiplication. When we are told **b** is in col(**A**), then every scalar multiple of **b** is in col(**A**), too.

#4

EDIT

If $\mathbf{b}_1$ and $\mathbf{b}_2$ are both in the column space of the matrix **A**, then so is $5\mathbf{b}_1$-$3\mathbf{b}_2$.

Correct Answer:

True | False

Explanation:

The column space of any matrix, which is a vector space, is closed under scalar multiplication and addition, which is equivalent to saying it is closed under linear combinations. So, knowing $\mathbf{b}_1$ and $\mathbf{b}_2$ to be in col($\mathbf{A}$), we know each linear combination, including $5\mathbf{b}_1$-$3\mathbf{b}_2$, is in col($\mathbf{A}$), too.

#5

EDIT

The rank of a matrix gives the dimension of its column space.

Correct Answer:

True | False

#6

EDIT

If $\mathbf{Ax} = \mathbf{0}$ and $\mathbf{Ay} = \mathbf{0}$, then $\mathbf{A}(3\mathbf{x} - 2\mathbf{y}) = \mathbf{0}$.

Correct Answer:

True | False

Explanation:

The nullspace of any matrix, also a vector space, is similarly closed under linear combinations. So, knowing **x** and **y** to be in Null(**A**), we know each linear combination, including 3**x**-2**y**, is in Null(**A**), too.

#7

EDIT

The number of free columns of a matrix **A**, also known as the nullity of **A**, gives the dimension of the nullspace of **A**.

Correct Answer:

True False

#8

EDIT

The set of all vectors $[x\ y\ z]^T$ satisfying the equation $3x + 2y + 5z = 8$ is a subspace of $\mathbf{R}^3$.

Correct Answer:

True False

Explanation:

This equation here is linear, but nonhomogeneous. Though its solutions come from $\mathbf{R}^3$, they do not comprise a subspace of $\mathbf{R}^3$. One problem is that the solutions are not closed under addition. That is, if you had two vectors, say $[x_1\ y_1\ z_1]^T$ and $[x_2\ y_2\ z_2]^T$, which solve this equation, then their sum $[(x_1+x_2)\ (y_1+y_2)\ (z_1+z_2)]^T$ would not satisfy this equation: instead of $3(x_1+x_2) + 2(y_1+y_2) + 5(z_1+z_2)$ being equal to 8, this quantity would equal 16.

#9

EDIT

Suppose **A** is an *m*-by-*n* matrix and, for a given vector **b** in $\mathbf{R}^m$, **Ax** = **b** has exactly one solution. Then the columns of **A** are linearly independent.

Correct Answer:

True | False

Explanation:

A consistent system either has exactly one solution or infinitely many, the latter arising only when **A** has at least one free column. Since there is just one solution for this given **b**, **A** does not have a free column, making its columns linearly independent.

#10

EDIT

Suppose **A** is a square matrix and, for a given **b** in $\mathbf{R}^n$, **Ax** = **b** has exactly one solution. Then **A** is nonsingular.

Correct Answer:

True | False

Explanation:

By the same reasoning as in #7, the columns of **A** are independent. Every one of them is a pivot column. But since **A** has the same number of rows as columns, this means it must be the case that its RREF is the identity matrix. Whenever that happens, the process of using elementary row operations to reduce [**A** | **I**] to an RREF [**I** | $\mathbf{A}^{-1}$] works and yields the inverse matrix.

#11

EDIT

If **A** is a square matrix whose columns are linearly independent, then **A** is nonsingular.

Correct Answer:

True	False

Explanation:

As with #8, the pertinent facts are that **A** does not have a free column, and has exactly the same number of rows as columns. The rest of the explanation in #8 is repeatable here.

#12

EDIT

If **A** is an n-by-n matrix with rank(**A**) = n, then **Ax** = **b** is consistent for every choice of vector **b** in $\mathbf{R}^n$.

Correct Answer:

True	False

Explanation:

Every column is a pivot column, given the rank is n. Being square, **A** has no row of zeros in its RREF, making it square. In fact, in all of questions #8, #9 and #10, we have enough information to know that the columns of **A** form a basis of $\mathbf{R}^n$. Thus, they are linearly independent and span all of $\mathbf{R}^n$.

#13

EDIT

If **A** is a matrix with m rows **Ax** = **b** is consistent for every choice of **b** in $\mathbf{R}^m$, then the RREF of **A** contains no row of zeros.

Correct Answer:

True

False

Explanation:

Were RREF to contain a row of zeros, then there would exist a column vector **b** for which the augmented matrix [**A** | **b**] had RREF with a bottom row containing zeros to the left of the bar but 1 to the right of it. But for that **b**, the system **Ax** = **b** would be inconsistent, and we are told this system is consistent for all choices of **b**. Thus, RREF for **A** (by itself) cannot have a row of zeros.

#14

EDIT

If **A** is a 4-by-n matrix whose rank is 4, then ________

ANSWER CHOICE

A n is exactly 4.

B n is at least 4.

C n is no larger than 4.

Explanation:

The rank of a matrix is the count of its pivot columns. For **A** to have rank 4, then, necessarily means it has at least 4 columns.

#15

EDIT

If **A** is a 4-by-*n* matrix whose rank is 4, then the columns of **A** are linearly independent.

Correct Answer:

True False

Explanation:

A has to have 4 pivot columns, but it may have free ones, too. Indeed, if *n* is larger than 4, it will have at least one free column, and then its columns will be linear dependent.

#16

EDIT

If **A** is a 4-by-*n* matrix whose rank is 4, then the columns of **A** span $\mathbf{R}^4$.

Correct Answer:

True False

Explanation:

Having 4 pivot columns requires the RREF of **A** to have 4 rows containing pivots. Thus, RREF has no row of zeros, which means **Ax** = **b** is consistent for all choices of **b** in $\mathbf{R}^4$. That is, every vector of $\mathbf{R}^4$ can be written as a linear combination of the columns of **A**.

questions

+ MULTIPLE CHOICE

+ TRUE / FALSE

+ SHORT ANSWER