

1. (a) VII (b) II (c) III (d) III (e) III (f) VII (g) VII

3. (a)
$$\int_0^{\pi/2} \vec{r}(t) dt = \frac{-1}{3} \cos(3t) \hat{i} - \frac{1}{3} \sin(3t) \hat{j} - \frac{5}{2} t^2 \hat{k} \Big|_0^{\pi/2}$$

$$= \left(\frac{1}{3} \hat{i} - \frac{5\pi^2}{8} \hat{k} \right) - \left(-\frac{1}{3} \hat{i} \right)$$

$$= \frac{1}{3} \hat{i} + \frac{1}{3} \hat{j} - \frac{5\pi^2}{8} \hat{k} \quad \text{or} \quad \left\langle \frac{1}{3}, \frac{1}{3}, -\frac{5\pi^2}{8} \right\rangle$$

(b) $\vec{r}'(t) = \langle 3 \cos(3t), 3 \sin(3t), -5 \rangle$

$$\Rightarrow \|\vec{r}'(t)\| = \sqrt{9 \cos^2(3t) + 9 \sin^2(3t) + 25} = \sqrt{34}$$

$$\text{length} = \int_1^4 \|\vec{r}'(t)\| dt = \sqrt{34} \int_1^4 dt = 3\sqrt{34}.$$

(c) $\vec{T}(t) = \frac{\vec{r}'(t)}{\|\vec{r}'(t)\|} = \frac{1}{\sqrt{34}} \vec{r}'(t) = \left\langle \frac{3}{\sqrt{34}} \cos(3t), \frac{3}{\sqrt{34}} \sin(3t), \frac{-5}{\sqrt{34}} \right\rangle$

4. (a) $\frac{\partial f}{\partial z} = \frac{-6xye^{-3z}(z-x^2) - 2xye^{-3z}}{(z-x^2)^2} = \frac{-2xye^{-3z}(3z-3x^2+1)}{(z-x^2)^2}$

(b) The domain excludes those points (x, y, z) for which $z = x^2$.

This is a sheet-like set of points, slicing through the xz -plane in the shape of a parabola, and splitting xyz -space into 2 distinct parts that contain only interior points. So, the domain is open.

(c) This, by definition, is $\frac{\partial f}{\partial x}$, so it equals

$$\frac{2ye^{-3z}(z-x^2) - (2x)2xye^{-3z}}{(z-x^2)^2} = \frac{2ye^{-3z}(1-3x^2)}{(z-x^2)^2}$$

5. (a) $\overrightarrow{PQ} = \langle -2, 4, -4 \rangle$, so the line can be written as

$$\vec{r}(t) = \langle 3, -1, 2 \rangle + t \langle -2, 4, -4 \rangle = \langle 3-2t, -1+4t, 2-4t \rangle.$$

(b) A normal vector to this plane is

$$\begin{aligned}\vec{n} &= \vec{OP} \times \vec{OQ} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 2 \\ 1 & 3 & -2 \end{vmatrix} = \begin{vmatrix} -1 & 2 \\ 3 & -2 \end{vmatrix} \hat{i} - \begin{vmatrix} 3 & 2 \\ 1 & -2 \end{vmatrix} \hat{j} + \begin{vmatrix} 3 & -1 \\ 1 & 3 \end{vmatrix} \hat{k} \\ &= -4\hat{i} + 8\hat{j} + 10\hat{k}.\end{aligned}$$

Using the origin as our point, the equation is

$$\vec{n} \cdot \langle x-0, y-0, z-0 \rangle = 0, \quad \text{or} \quad -4x + 8y + 10z = 0$$

(c) The angle θ is the acute angle between normal vectors:

$$\langle -4, 8, 10 \rangle \text{ and } \langle 0, 0, 1 \rangle:$$

$$\cos \theta = \frac{10}{\sqrt{4^2 + 8^2 + 10^2} \cdot \sqrt{1}} = \frac{10}{\sqrt{16 + 64 + 100}} = \frac{10}{\sqrt{180}} = \frac{\sqrt{5}}{3}$$

Or, alternately (w/ the other plane)

$$\cos \theta = \frac{|-4|}{\sqrt{9+4+16}} = \frac{4}{\sqrt{29}}.$$