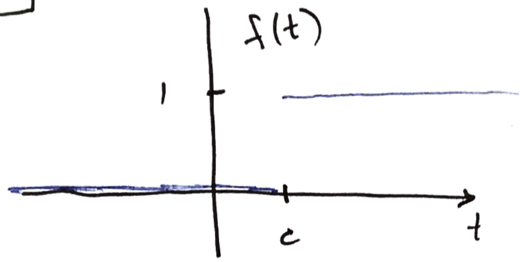
Goal: Modeling switches (and taking Laplace transforms)

Ex.)



$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)\, e^{-st}\, dt$$

$$= \int_0^c 0 \cdot e^{-st}\, dt + \int_c^\infty 1 \cdot e^{-st}\, dt$$

$$= 0 + \int_c^\infty e^{-st}\, dt = -\frac{1}{s} e^{-st} \Big|_c^\infty$$ — diverges if $s < 0$, undefined if $s = 0$

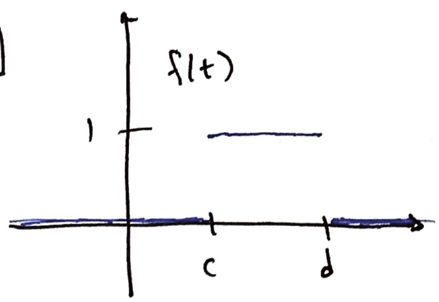
$$\overset{s>0}{=} 0 - \left(-\frac{1}{s} e^{-sc}\right) = \boxed{\frac{1}{s} e^{-cs}}$$

Note: In the instance

$c = 0$, get $\frac{1}{s}$ (same Laplace transform as for $f(t) = 1$ const.)

$c < 0$, same is true (resulting L.T. $1/s$)

Ex.)

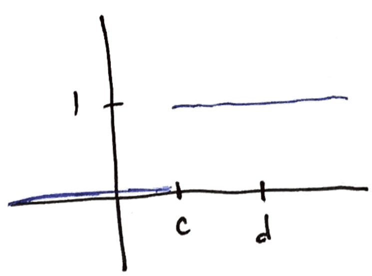


$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)\, e^{-st}\, dt$$

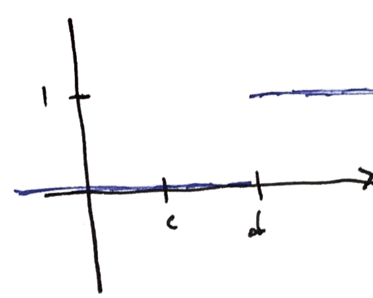
$$= \int_0^c 0 \cdot e^{-st}\, dt + \int_c^d e^{-st}\, dt + \int_d^\infty 0 \cdot e^{-st}\, dt$$

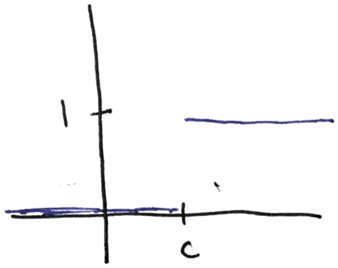
$$= \int_c^d e^{-st}\, dt = -\frac{1}{s} e^{-st} \Big|_c^d = \boxed{\frac{1}{s} e^{-cs} - \frac{1}{s} e^{-ds}}$$

L.T. of

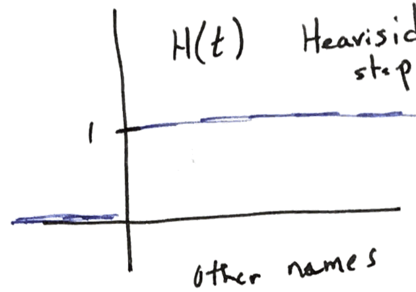


L.T. of





is translation
c units to right of

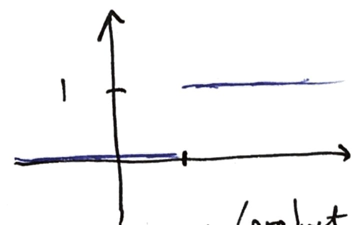


$H(t)$ Heaviside unit step fn.

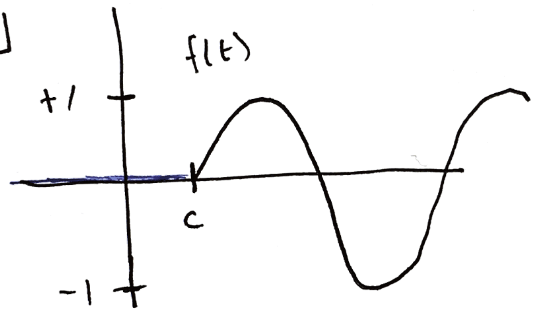
$H(t-c)$
$\mathcal{U}(t-c)$
$u_c(t)$

other names
$\mathcal{U}(t)$
$u_0(t)$

Useful for simplifying notation

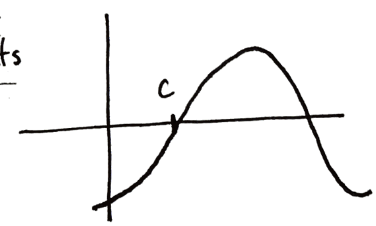


Ex.



$\times$ (product)

shift
c units



$$f(t) = H(t-c)\sin(t-c) \quad \left(\text{or} \quad u_c(t)\sin(t-c)\right)$$

What sort of transforms arise from $H(t-c)f(t-c)$?

– i.e. What effect on the resulting transform does it have to delay f c units and switch it on then?

$$\mathcal{L}\{H(t-c)\,f(t-c)\} = \int_0^\infty H(t-c)\,f(t-c)\,e^{-st}\,dt$$

$$= \int_0^c 0\,dt + \int_c^\infty 1\cdot f(t-c)\,e^{-st}\,dt$$

substitute
$z = t-c$ (so $t = z+c$)
$dz = dt$

$$= \int_0^\infty f(z)\,e^{-s(z+c)}\,dz = \int_0^\infty f(z)\,e^{-sz}\cdot e^{-sc}\,dz$$

$$= e^{-sc} \int_0^\infty f(z)\, e^{-sz}\, dz$$

But, z is a dummy variable of integration, could be named anything

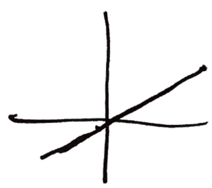
rename z by t

$$= e^{-sc} \underbrace{\int_0^\infty f(t)\, e^{-st}\, dt}_{= \mathcal{L}\{f(t)\}} = e^{-sc} \cdot \mathcal{L}\{f(t)\}$$

Implications:

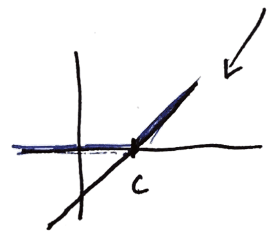
Know from Monday

$$\mathcal{L}\{t\} = \frac{1}{s^2}$$

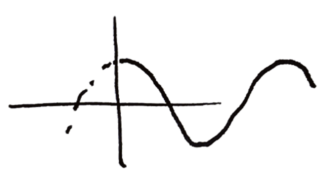


So

$$\mathcal{L}\{H(t-c)\underbrace{(t-c)}\} = e^{-cs} \cdot \frac{1}{s^2}$$

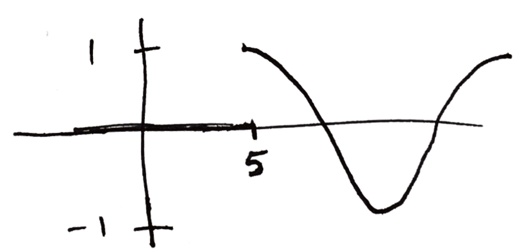


$$\mathcal{L}\{\cos(t)\} = \frac{s}{s^2+1}$$



So

$$\mathcal{L}\{H(t-5)\cos(t-5)\} = e^{-5s}\frac{s}{s^2+1}$$



So far, focus been

take t-domain $f(t)$ $\longrightarrow$ get s-domain fn. $\mathcal{L}\{f(t)\}$

We will need to be able to return

? $\longleftarrow$ Have fn. of s

Ex.) What fn. on the t-side has L.T. $e^{-3s}\dfrac{2}{s^2+4}$?

If my s-fn. were simply $\frac{2}{s^2+4}$, Monday's catalog included

$$\mathcal{L}\{\sin(at)\} = \frac{a}{s^2+a^2}$$

So

$$\mathcal{L}\{\sin(2t)\} = \frac{2}{s^2+4}$$

Our result

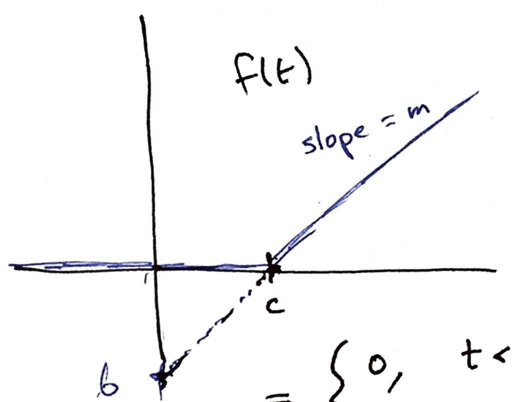
$$\mathcal{L}\{H(t-c)\,f(t-c)\} = e^{-cs}\cdot\mathcal{L}\{f(t)\}$$

now implies that

$$e^{-3s}\frac{2}{s^2+4} \text{ comes from } H(t-3)\,\sin(2(t-3))$$

↑ instead of simple t

Ex.) More ~~w~~writing expressions of t fns. using $H(t-c)$



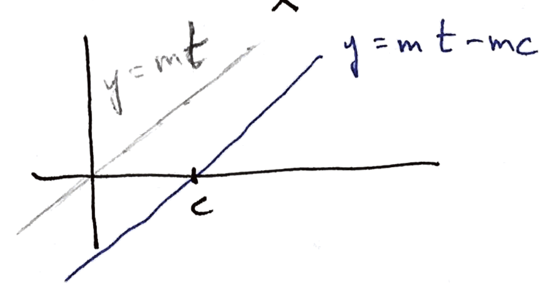
$$= \begin{cases} 0, & t<c \\ mt+\underline{(-mc)}, & t\ge c \end{cases}$$

$$m = \frac{\text{rise}}{\text{run}} = \frac{-b}{c}$$

$$\Rightarrow b = \underline{-mc}$$

Avoid piecewise-style expression using $H(t-c)$

$\times$



Alternate expression: $H(t-c)\cdot(mt-mc)$

$$= H(t-c)\cdot m(t-c)$$