SCOFIELD3894

Edit Quiz **SAVE AND EXIT Enable Sharing** SOC-38681413 MATH 231 Sections 1.1-1.9 quiz Align quiz to standard #1 EDIT If **A** is an *m*-by-*n* matrix, then the column space of **A** is a subspace of \mathbf{R}^n . **Correct Answer:** True **False Explanation:** Since every column of **A** has *m* components, the linear combinations of columns of **A** also have m components, and belong to \mathbf{R}^{m} , not necessarily to \mathbf{R}^n . #2 EDIT

If **A** is a matrix, **b** is a vector, and the vector equation **Ax** = **b**

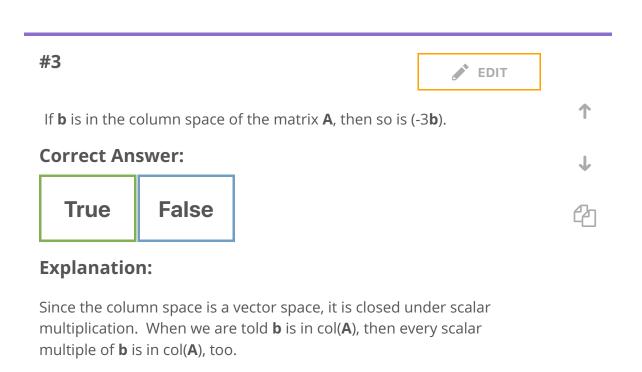
has infinitely many solutions, then the nullspace of A is nontrivial.

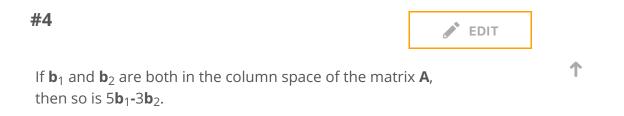
Correct Answer:

True False

Explanation:

The nullspace of any matrix is either trivial, consisting of the vector $\mathbf{0}$ only, or it is nontrivial (contains the vector $\mathbf{0}$ and many others). The deciding factor is whether the matrix has free columns or not. Since we are told $\mathbf{A}\mathbf{x} = \mathbf{b}$ has infinitely many solutions, we learn from this that \mathbf{A} has a free column, and hence its nullspace is nontrivial.





Correct Answer:



True

False



Explanation:

The column space of any matrix, which is a vector space, is closed under scalar multiplication and addition, which is equivalent to saying it is closed under linear combinations. So, knowing \mathbf{b}_1 and \mathbf{b}_2 to be in col(\mathbf{A}), we know each linear combination, including $5\mathbf{b}_1$ - $3\mathbf{b}_2$, is in col(\mathbf{A}), too.

#5



The rank of a matrix gives the dimension of its column space.



Correct Answer:



True

False



#6



If Ax = 0 and Ay = 0, then A(3x - 2y) = 0.



Correct Answer:



True

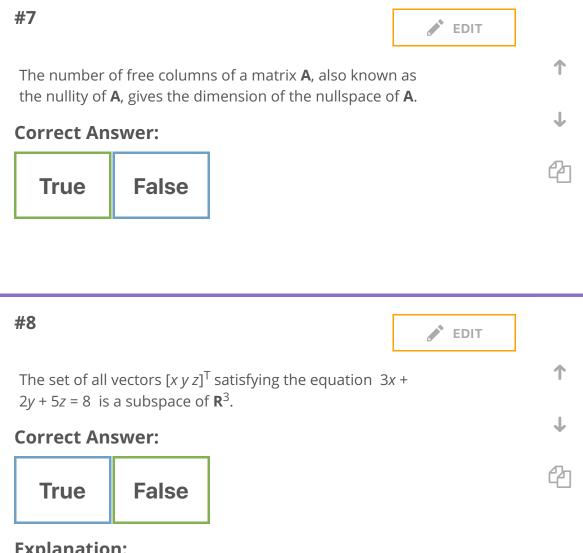
False



Explanation:

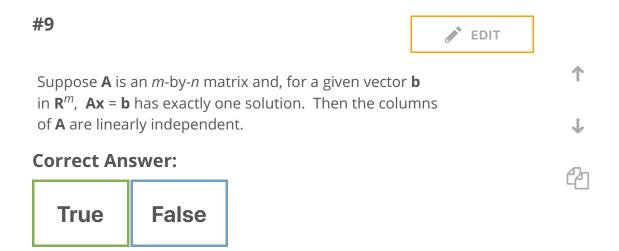
2/20/19, 10:58 PM Socrative

> The nullspace of any matrix, also a vector space, is similarly closed under linear combinations. So, knowing x and y to be in Null(A), we know each linear combination, including 3x-2y, is in Null(A), too.



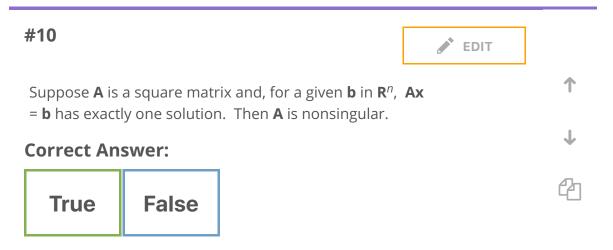
Explanation:

This equation here is linear, but nonhomogeneous. Though its solutions come from \mathbb{R}^3 , they do not comprise a subspace of \mathbb{R}^3 . One problem is that the solutions are not closed under addition. That is, if you had two vectors, say $[x_1 \ y_1 \ z_1]^T$ and $[x_2 \ y_2 \ z_2]^T$, which solve this equation, then their sum $[(x_1+x_2)(y_1+y_2)(z_1+z_2)]^T$ would not satisfy this equation: instead of $3(x_1+x_2) + 2(y_1+y_2) + 5(z_1+z_2)$ being equal to 8, this quantity would equal 16.



Explanation:

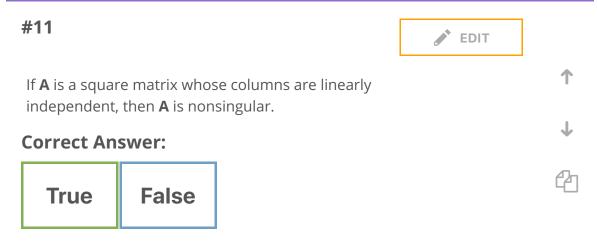
A consistent system either has exactly one solution or infinitely many, the latter arising only when **A** has at least one free column. Since there is just one solution for this given **b**, **A** does not have a free column, making its columns linearly independent.



Explanation:

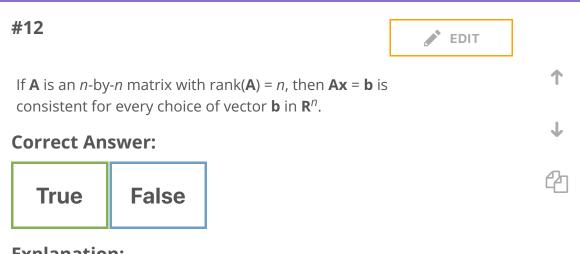
By the same reasoning as in #7, the columns of **A** are independent. Every one of them is a pivot column. But since **A** has the same number of rows as columns, this means it must be the case that its RREF is the identity matrix. Whenever that happens, the process of using elementary row operations to reduce $[A \mid I]$ to an RREF $[I \mid A^{-1}]$ works and yields the inverse matrix.

2/20/19, 10:58 PM Socrative



Explanation:

As with #8, the pertinent facts are that **A** does not have a free column, and has exactly the same number of rows as columns. The rest of the explanation in #8 is repeatable here.



Explanation:

Every column is a pivot column, given the rank is n. Being square, **A** has no row of zeros in its RREF, making it square. In fact, in all of questions #8, #9 and #10, we have enough information to know that the columns of **A** form a basis of \mathbb{R}^n . Thus, they are linearly independent and span all of \mathbf{R}^n .

If **A** is a matrix with m rows $\mathbf{A}\mathbf{x} = \mathbf{b}$ is consistent for every choice of **b** in \mathbf{R}^m , then the RREF of **A** contains no row of zeros.



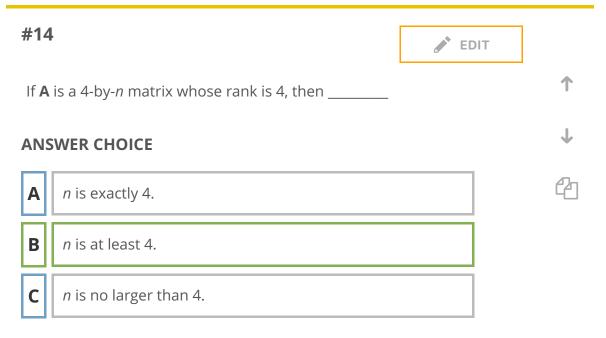
Correct Answer:





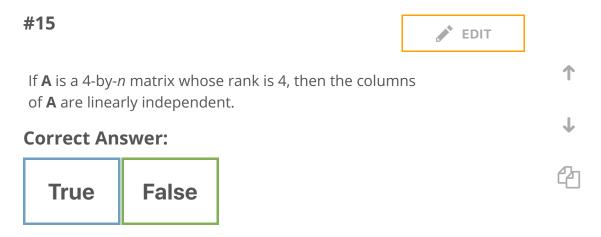
Explanation:

Were RREF to contain a row of zeros, then there would exist a column vector \mathbf{b} for which the augmented matrix $[\mathbf{A} \mid \mathbf{b}]$ had RREF with a bottom row containing zeros to the left of the bar but 1 to the right of it. But for that \mathbf{b} , the system $\mathbf{A}\mathbf{x} = \mathbf{b}$ would be inconsistent, and we are told this system is consistent for all choices of \mathbf{b} . Thus, RREF for \mathbf{A} (by itself) cannot have a row of zeros.



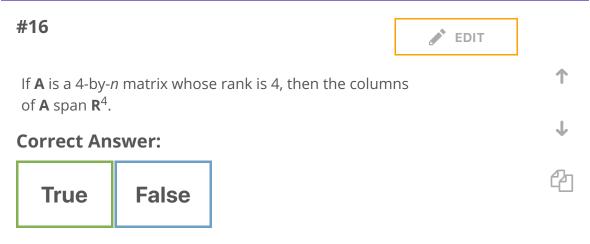
Explanation:

The rank of a matrix is the count of its pivot columns. For $\bf A$ to have rank 4, then, necessarily means it has at least 4 columns.



Explanation:

A has to have 4 pivot columns, but it may have free ones, too. Indeed, if n is larger than 4, it will have at least one free column, and then its columns will be linear dependent.



Explanation:

Having 4 pivot columns requires the RREF of $\bf A$ to have 4 rows containing pivots. Thus, RREF has no row of zeros, which means $\bf Ax = b$ is consistent for all choices of $\bf b$ in $\bf R^4$. That is, every vector of $\bf R^4$ can be written as a linear combination of the columns of $\bf A$.

questions



Socrative Get PRO! Learn More