Form B Solutions

1. (a) Going through the list of elements in A, we have

$$f(-8) = -8, \quad f(-7) = -8, \quad f(-6) = -6, \quad f(-5) = -6, \quad f(1) = 0, \quad f(2) = 2, \quad f(3) = 2.$$

   Thus, $f(A) = \{-8, -6, 0, 2\}$.

   (b) For each integer x, $f(x)$ is the largest even integer that does not exceed x. Since $f(8) = 8$ and $f(9) = 8$, and no other $x \in \mathbb{Z}$ satisfies $f(x) = 8$, the desired preimage is $\{8, 9\}$.

   (c) f is not injective. For instance, $f(2)$ and $f(3)$ are both 2, but $2 \neq 3$.

2. Let us temporarily use propositional variables to rewrite p. Taking

   b: welk is Type B
   r: welk is red
   s: welk has been visible for at least 10 days

   then statement p can be written in these equivalent forms:

$$b \to (r \vee s) \ \equiv \ \neg b \vee r \vee s.$$

   (a) The negation of p, in symbols, is

$$\neg(\neg b \vee r \vee s) \ \equiv \ b \wedge \neg r \wedge \neg s.$$

   Writing this in English, we have "A welk is considered Type B and it is not red and it has not been visible for at least 10 days."

   (b) The contrapositive of p is $\neg(r \vee s) \to \neg b \equiv \neg r \wedge (\neg s \to \neg b)$. In English, this is "If a welk is not red and has not been visible for at least 10 days, then it is not considered Type B."

3. $p \leftrightarrow q \ \equiv \ (p \to q) \wedge (q \to p) \ \equiv \ (\neg p \vee q) \wedge (\neg q \vee p)$.

4. (a) $q \to p \equiv \neg q \vee p$

   (b) $p \oplus q \equiv (p \wedge \neg q) \vee (\neg p \wedge q)$

   (c) $p \vee q$

5. (a) Something like this: "There is precisely one movie that Ellen has not watched."

   (b) "Every student has watched some movie."

   (c) $\exists s\, \exists m_1\, \exists m_2 (m_1 \neq m_2 \wedge R(m_1) \wedge R(m_2) \wedge W(s, m_1) \wedge W(s, m_2))$

   (d) The statement you are out to negate can be written as $\exists s\, \forall m (R(m) \to W(s, m))$. Following our rules of negation,

$$\begin{aligned}\neg\exists s\, \forall m(R(m) \to W(s,m)) \quad &\equiv \quad \forall s\, \neg\, \forall m(R(m) \to W(s,m)) \quad \equiv \quad \forall s\, \exists m\, \neg(R(m) \to W(s,m)) \\ &\equiv \quad \forall s\, \exists m\, \neg(\neg R(m) \vee W(s,m)) \quad \equiv \quad \forall s\, \exists m\, (R(m) \wedge \neg W(s,m))\end{aligned}$$

   This is a trick question, albeit an unintentional one, as the correct option is not in the list. Nothing in the list even binds both variables using the correct quantifiers.

6. (a) $A \subseteq B$

   (b) $B \subseteq A$

7. (a) 5
   (b) $2^6 = 64$
   (d) $|A \times A| = |A|^2 = 25$
   (e) This statement is False. For there to be a bijection f, each element in A would be paired with just one in B, and likewise each element in B would be paired with one in A. That cannot happen when $|A| \neq |B|$, as is the case here.

8. A membership table is one way to carry this out.

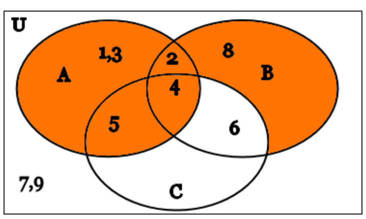
A	B	C	$B-C$	$A \cup (B-C)$	$A \cup B$	$(A \cup B) - C$
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	1	1	1	1
0	1	1	0	0	1	0
1	0	0	0	1	1	1
1	0	1	0	1	1	0
1	1	0	1	1	1	1
1	1	1	0	1	1	0

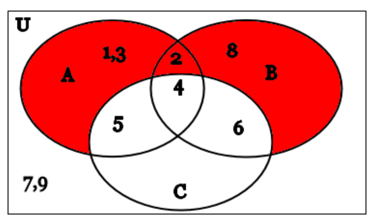
Comparing the $A-(B-C)$ column with the $(A-B) \cup C$ one, we see discrepancies in rows 2 and 4. Thus, these sets are not equal.

Another approach is to use specific sets, or a Venn diagram. We illustrate both, taking

$$U = \{1,2,3,4,5,6,7,8,9\}, \quad A = \{1,2,3,4,5\}, \quad B = \{2,4,6,8\}, \quad C = \{4,5,6\}.$$

Drawing Venn diagrams with these elements inserted, we have $A \cup (B-C) = \{1,2,3,4,5,8\}$ on the left, and $(A \cup B) - C = \{1,2,3,8\}$ on the right:





9. Many answers are correct. Here are several:
   (a) Each of $f(x) = x^2$, $f(x) = |x|$, $f(x) = 0$, or $f(x) = \lfloor x \rfloor$ suffices, as each fails the horizontal line test as a function from $\mathbb{R}$ to $\mathbb{R}$.
   (b) Each of $f(x) = 2x+5$, $f(x) = 1-7x$, or $f(x) = x^3$ suffices, as each passes the horizontal line test and has range $\mathbb{R}$.

10. (a) $a_n = 73 + 28n$ (b) $a_n = 11(4)^n$

11. (i) This sum involves finitely many, $285 - 2 + 1 = 284$, to be exact, terms of an arithmetic series with first term $19 - 7(2) = 5$ and last term $19 - 7(285) = -1976$. The sum, then, is

$$\left(\frac{1}{2}\right)(284)(5 + -1976) = \left(\frac{1}{2}\right)(284)(-1971) = -279882.$$

   (ii) The sum involves infinitely many terms of a geometric series with $a_0 = 57/27$ and $r = 1/3$. Since $|r| < 1$, the series converges to

$$s = \frac{a_0}{1-r} = \frac{57/27}{1-1/3} = \frac{57/27}{2/3} = \frac{57}{27} \cdot \frac{3}{2} = \frac{57}{18} = \frac{19}{6} = 3.1\overline{6}.$$