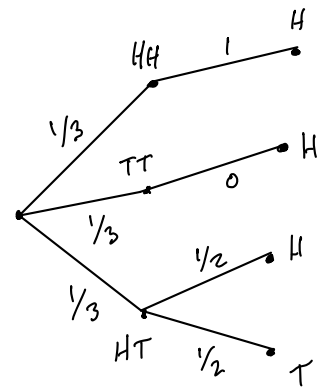2.34 $Pr\left(\text{next two cards are diamonds}\right) = \left(\frac{10}{47}\right)\left(\frac{9}{46}\right) = 0.0416$

2.38   (a)
$$Pr(HH \mid H) = \frac{Pr(HH \text{ and } H)}{Pr(H)}$$

$$= \frac{(1/3)(1)}{(1)(1/3) + (1/2)(1/3)} = \frac{2}{3}$$

(b)
$$Pr(2^{nd} \text{ flip is } H \mid 1^{st} \text{ flip is } H) = \frac{Pr(1^{st} \text{ and } 2^{nd} \text{ flips } H)}{Pr(1^{st} \text{ flip is } H)}$$

$$= \frac{(1/3)(1)(1) + (1/3)(1/2)^2}{(1/3)(1) + (1/3)(1/2)} = 0.833$$

(c) $Pr(HH \mid 1^{st} \text{ and } 2^{nd} \text{ flips are } H) = \dfrac{Pr(HH \text{ and } 1^{st} \text{ and } 2^{nd} \text{ flips are } H)}{Pr(1^{st} \text{ and } 2^{nd} \text{ flips are } H)}$

$$= \frac{1/3}{(1/3)(1)(1) + (1/3)(1/2)(1/2)} = \frac{1}{1 + 0.25} = 0.8$$

2.41 $Pr(\text{at least one six in 4 throws}) = 1 - Pr(\text{no six in 4 throws})$

$$= 1 - dbinom(0, 4, 1/6) = 0.5177.$$

$Pr(\text{a double 6 in 24 rolls}) = 1 - Pr(\text{no double six in 24 rolls})$

$$= 1 - dbinom(0, 24, 1/36) \doteq 0.491$$

2.42   A variable X counting misses here has the $NBinom(100, 0.92)$ distribution. So,

$$Pr(114 \text{ or more attempts}) = Pr(X \geq 14) = 1 - Pr(X \leq 13)$$

$$= 1 - pnbinom(13, 100, 0.92) \doteq 0.0675.$$

2.49 (a) $1 - \text{pbinom}(11, 20, 0.25) = 0.000935$

(b) $1 - \text{pbinom}(11, 20, \frac{1}{3}) = 0.013$

(c) Say the questions are Type I if you know the answer, Type II if you can eliminate one choice, and Type III if you must guess blindly. Then the probability tree for the 1st question is given at right.
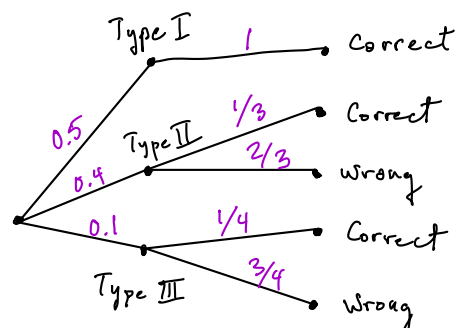
$$Pr(1^{st} \text{ question is correct}) = \left(\frac{1}{2}\right)(1) + \left(\frac{2}{5}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{10}\right)\left(\frac{1}{4}\right)$$

$$\doteq 0.6583.$$

So,

$$Pr(\text{pass test}) = 1 - \text{pbinom}(11, 20, 0.6583) \doteq 0.786.$$

2.52 Take $H_0: \pi = \frac{1}{4}$, $H_a: \pi \neq \frac{1}{4}$ (2-sided)

The data has $X = 8$ in 50 tries. Our P-value

bprobabilities $\leftarrow$ dbinom$(0:50, 50, 0.25)$

sum$($ bprobabilities $[$ bprobabilities $<=$ dbinom$(8, 50, 0.25)]) = 0.19.$

2.70 (a) For $X \sim DUnif(10)$, $Pr(X = 3) = \frac{1}{10}$, $Pr(X = 12) = 0$, $Pr(X \leq 3) = 0.3$.

(b) $E(X) = \left(1\right)\left(\frac{1}{n}\right) + \left(2\right)\left(\frac{1}{n}\right) + \cdots + n\left(\frac{1}{n}\right) = \frac{1}{n} \sum_{i=1}^{n} i$

$$= \left(\frac{1}{n}\right) \frac{n(n+1)}{2} = \frac{n+1}{2}.$$

$$E(X^2) = \frac{1}{n} \sum_{i=1}^{n} i^2 = \frac{1}{n} \cdot \frac{n(n+1)(2n+1)}{6} = \frac{1}{6}\left(2n^2 + 3n + 1\right).$$

$$\Rightarrow Var(X) = \frac{1}{3}n^2 + \frac{1}{2}n + \frac{1}{6} - \frac{1}{4}\left(n^2 + 2n + 1\right)$$

$$= \frac{1}{12}n^2 - \frac{1}{12}.$$

2.81 (a) $E(X + Y) = E(X) + E(Y) = 20$

Var$(X + Y) = $ Var$(X) + $ Var$(Y) = 4$,   because X, Y are independent.

(b) $E(X + X) = E(2X) = 2E(X) = 20$

Var$(X + X) = $ Var$(2X) = 4$ Var$(X) = 8$

(c) When Y is independent of X, one may well have a small value of X paired with a large value of Y, making $X + Y$ moderately-sized. The same cannot be said of $X + X$, which is always twice as large as X. These observations lead to $X + X$ having greater variability than $X + Y$. Independence, however, is not a factor in the expected value of a sum.

2.88 $E(XY) = E(X)$

$= (-1)(\frac{1}{4}) + 0 + (1)(\frac{1}{4})$

$= 0$

$\Rightarrow$ Cov$(X, Y) = E(XY) - E(X)E(Y)$

$= 0.$

x	-1	0	1
Pr$(X=x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
Pr$(Y=x)$	0	$\frac{1}{2}$	$\frac{1}{2}$
Pr$(XY=x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

However, X and Y are not independent, as $f_{X,Y}(x,y) \neq f_X(x) f_Y(y)$.
For example, $f_{X,Y}(1,1) = \frac{1}{4}$, but $f_X(1) f_Y(1) = (\frac{1}{4})(\frac{1}{2}) = \frac{1}{8}$.

2.91 Pr(Z scores at least 44 goals in 89 games)

$= 1 - \text{ppois}(43, (89)(\frac{206}{506})) \doteq 0.1157$

It is not terribly unlikely — not significant even at the 10% level — that he might score 44 goals during any 89-game stretch during the regular season.