Math 251, Wed 16-Sep-2020 -- Wed 16-Sep-2020
Discrete Mathematics
Fall 2020

Wednesday, September 16th 2020

Topic:: Sets
Read:: Rosen 2.1
HW::   WW sets1 due Sat. 6 pm

Sets

float absval (float x)

Sets

- collections of objects known as elements of the set

  Important in computing? sets are involved when thinking about valid arguments/inputs to a function, error-handling, function declaration/typing

- ways of describing
  - enumeration $\{A, B, C, D, \ldots, Z\}$, $\{1, 2, 3, 4, 5, \ldots\}$
  - set builder notation $\{\text{object} \mid \text{criteria objects must meet}\}$

$$(0, \infty) = \{x \in \mathbb{R} \mid x > 0\}$$

$$\{x \in \text{Calvin student} \mid \text{height of } x < 5'\}$$

  - intervals of numbers $[0, 1) = \{x \in \mathbb{R} \mid 0 \leq x < 1\}$
  - words/symbols

$\{\text{days of the week}\}$

$\mathbb{R}$, $\mathbb{C} = \{a + bi \mid a, b \in \mathbb{R}\}$

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, \ldots\}$

$\mathbb{N} = \{1, 2, 3, 4, \ldots\}$

$\mathbb{Z}^+ = \{0, 1, 2, \ldots\}$

- items in a set are called **elements** of that set

  Some notions and how to express them
  - "is/is not an element of"

    Monday $\in$ {days of the week}, Monday $\notin \mathbb{R}$

    Q: Is it accurate to write

    $\{1, 2\} \in \{1, 2, 3, 4, 5\}$ Invalid

    $\{1\} \in \{1, 2, 3\}$ Invalid $\{1\} \in \{\{1\}, \{2\}, \{1, 2\}\}$ Valid

    $1 \in \{1, 2, 3\}$ Valid
  - "is/is not a subset of" Given 2 sets, A and B, "A is a subset of B" means all elements of A are elements of B.

    $x \leq 5$

    $\{1, 2\} \subseteq \{1, 2, 3, 4, 5\}$

    $\emptyset \subseteq$ every other set
  - "equality of sets" $\{1, 2, 3\} = \{3, 1, 2\}$

    $\{1, 1, 1, 1, 2, 3\} = \{1, 3, 1, 2, 3\}$

    Fact: Sets $A = B$ iff $A \subseteq B \wedge B \subseteq A$.

- can be empty, finite or infinite

  cardinality

  $|A|$ = cardinality of A.

  $|\{\text{days of week}\}| = 7$.

  Call a set A satisfying $|A| = 1$ a singleton set.

  The set w/ no elements is the empty set $\{\}$, $\emptyset$.

Sets built from other sets

- the power set $\mathcal{P}(A)$ of a set A
  use of bitstrings to describe subsets of a finite set A

$A = \{0, 1\}$

$\mathcal{P}(A) = \{\{\}, \{0\}, \{1\}, \{0, 1\}\}$

$B = \{\alpha, \gamma, \sigma\}$

$\mathcal{P}(B) = \{\emptyset, \{\alpha\}, \{\gamma\}, \{\sigma\}, \{\alpha, \gamma\}, \{\alpha, \sigma\}, \{\gamma, \sigma\}, B\}$

000 100, 010 001 110 101 011 111

How large (cardinality) is $\mathcal{P}(B)$?

Generally, $|\mathcal{P}(B)| = 2^{|B|}$ when B is finite.

$\mathcal{P}(\emptyset) = \{\{\}\}$

$\mathcal{P}(\mathcal{P}(\emptyset)) = \{\{\}, \{\{\}\}\} = \{\emptyset, \{\emptyset\}\}$

- Cartesian product $A \times B$ of two sets A, B

$A \times B = \{(a, b) \mid a \in A, b \in B\}$

$\{H, T\} \times \{1, 2, 3\} = \{(H, 1), (H, 2), (H, 3), (T, 1), (T, 2), (T, 3)\}$

But $(3, T) \notin A \times B$ though it is in $B \times A$.

$\mathbb{R} \times \mathbb{R} = \mathbb{R}^2$

2.2

- Union of sets

- Intersection of sets

- Set subtraction and complementation