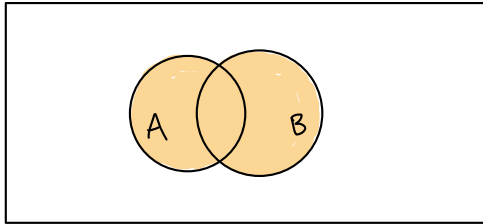
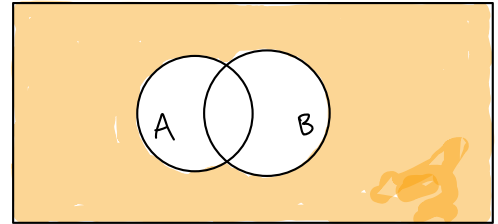


B.2 For  $A \cup B$ , defined as  $\{x \mid x \in A \text{ or } x \in B\}$ , to be the same as  $A$ , there can be nothing in  $B$  except elements which are also in  $A$ . That is,  $B$  must be a subset of  $A$ . A moment's reflection shows that  $B \subset A$  is also a sufficient condition.

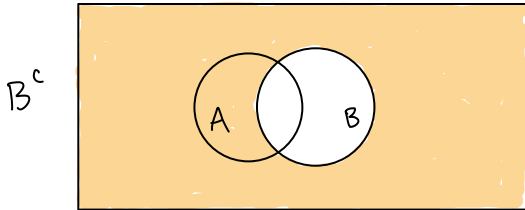
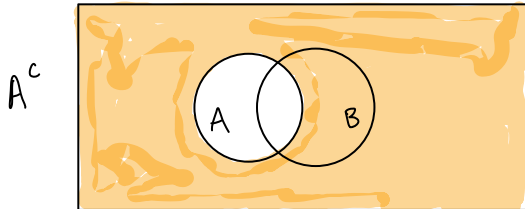
B.4  $A \cup B$  can be depicted as



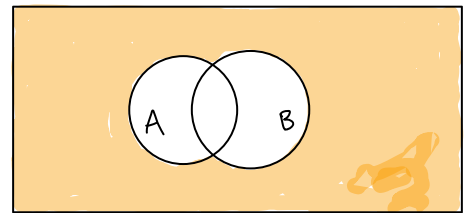
and, thus,  
 $(A \cup B)^c$  is



Whereas  $A^c$ ,  $B^c$  are depicted below



So, taken  
together,  
we obtain  
 $A^c \cap B^c$



- B.9 (a)  $f(x) = 1$  precisely when  $x \in A$  and  $x \in B$ ; that is, when  $x \in A \cap B$ .  
 (b)  $f(x) = 0$  whenever  $x \notin A \cap B$ ; that is, precisely when  $x$  is absent from  $A$  or  $B$ .  
 (c)  $f(x) = \llbracket x \in A \cap B \rrbracket$

B.11 (a)  $\sum_{i=2}^5 i^2 = 4 + 9 + 16 + 25 = 54.$

(b)  $\sum_{n=1}^4 n = 1 + 2 + 3 + 4 = 10.$

(c)  $\sum_{x=1}^5 (2x-1) = 1 + 3 + 5 + 7 + 9 = 25.$

(d)  $\prod_{n=2}^4 n = (2 \times 3) \times 4 = 24.$

B.15 Let  $S = \sum_{i=1}^n i = 1 + 2 + 3 + \dots + n$ . Then

$$S = 1 + 2 + 3 + \dots + n, \quad \text{and}$$

$$S = n + (n-1) + (n-2) + \dots + 1.$$

Adding these equations gives

$$2S = \underbrace{(n+1) + (n+1) + \dots + (n+1)}_{n \text{ summands}} = n(n+1).$$

Dividing by 2 gives the result.

$$\begin{aligned} \text{B.20 (a)} \quad \sum_{a=1}^n (2a-1)x &= 2x \sum_{a=1}^n a - x \sum_{a=1}^n 1 = 2x \cdot \frac{n(n+1)}{2} - nx \\ &= nx(n+1-1) = n^2x. \end{aligned}$$

$$(b) \quad \sum_{y=1}^n xy = x \sum_{y=1}^n y = \frac{1}{2} nx(n+1)$$

$$\begin{aligned} (c) \quad \sum_{x \in S} (x-m) &= \sum_{x \in S} x - m \sum_{x \in S} 1 = \frac{|S|}{|S|} \sum_{x \in S} x - m|S| \\ &= |S|(\bar{x} - m) = |S|(\bar{x} - m). \end{aligned}$$

(d) The assumption that

$$m = \sum_{x \in S} \frac{x}{|S|} = \frac{1}{|S|} \sum_{x \in S} x = \bar{x},$$

leads to

$$\sum_{x \in S} (x-m) = |S|(\bar{x} - m) = 0.$$