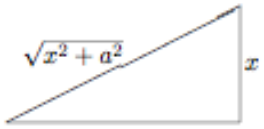
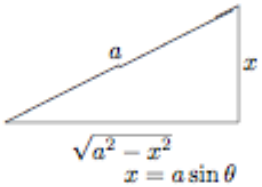
MATH 162: Calculus II
Framework for Fri., Feb. 2
Trigonometric Substitutions

Trigonometric substitution

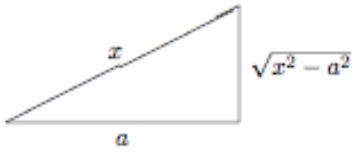
- a technique used in integration
- useful most often when integrands involve $(a^2+x^2)^m$, $(a^2-x^2)^m$ or $(x^2-a^2)^m$, where a and m are constants. m is often, but not exclusively, equal to $1/2$.
  Note: To get one of these forms, often completion of a square is required.
- The usual substitutions



$x = a\tan\theta$
$dx = a\sec^2\theta\,d\theta$
$\sqrt{x^2+a^2} = a\sec\theta$
$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$



$x = a\sin\theta$
$dx = a\cos\theta\,d\theta$
$\sqrt{a^2-x^2} = a\cos\theta$
$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$



$x = a\sec\theta$
$dx = a\sec\theta\tan\theta\,d\theta$
$\sqrt{x^2-a^2} = a|\tan\theta|$
$0 \le \theta < \frac{\pi}{2}$ or $\frac{\pi}{2} < \theta \le \pi$

Some examples:

$$\int \frac{dx}{x^2\sqrt{9-x^2}}$$

$$\int \frac{dx}{(x^2+1)^{3/2}}$$

$$\int \frac{x}{\sqrt{x^2-3}}\,dx$$