MATH 162: Calculus II

Framework for Wed., Jan. 31

Trigonometric Integrals

Types we handle and relevant trig. identities:

integral type	trig. identity
$\int \sin^m x \cos^n x\,dx, \quad m$ or n odd	$\sin^2\theta + \cos^2\theta = 1$
$\int \sin^m x \cos^n x\,dx, \quad m, n$ even	$\sin^2\theta = \frac{1}{2}[1-\cos(2\theta)]$ and $\cos^2\theta = \frac{1}{2}[1+\cos(2\theta)]$
$\int \sqrt{1+\cos(mx)}\,dx$	$2\cos^2\theta = 1+\cos(2\theta)$
$\int \tan^m x \sec^n x\,dx, \quad m, n > 1$	$1+\tan^2\theta = \sec^2\theta$
$\int \cot^m x \csc^n x\,dx, \quad m, n > 1$	$1+\cot^2\theta = \csc^2\theta$
$\int \sin(mx)\sin(nx)\,dx, \quad m \neq n$	$\sin(m\theta)\sin(n\theta) = \frac{1}{2}[\cos((m-n)\theta) - \cos((m+n)\theta)]$
$\int \sin(mx)\cos(nx)\,dx, \quad m \neq n$	$\sin(m\theta)\cos(n\theta) = \frac{1}{2}[\sin((m-n)\theta) + \sin((m+n)\theta)]$
$\int \cos(mx)\cos(nx)\,dx, \quad m \neq n$	$\cos(m\theta)\cos(n\theta) = \frac{1}{2}[\cos((m-n)\theta) + \cos((m+n)\theta)]$

But, frequently the above identities only serve to reduce things to a special case you must be able to handle some other way. Some of these special cases:

$\displaystyle\int \tan x\,dx$, handled via substitution

$\displaystyle\int \sec^3 x\,dx$, handled by parts (see p. 459)

$\displaystyle\int \tan^m x \sec^2 x\,dx$, handled via substitution

$\displaystyle\int \sec x\,dx$, handled via substitution, after multiplying through by $\dfrac{\sec x + \tan x}{\sec x + \tan x}$

$\displaystyle\int \tan x \sec^n x\,dx$, handled via substitution