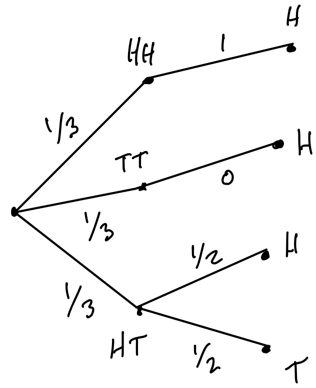
2.34 $\Pr(\text{next two cards are diamonds}) = \left(\frac{10}{47}\right)\left(\frac{9}{46}\right) = 0.0416$

2.38 (a)

$$\Pr(HH \mid H) = \frac{\Pr(HH \text{ and } H)}{\Pr(H)}$$

$$= \frac{(1/3)(1)}{(1)(1/3) + (1/2)(1/3)} = \frac{2}{3}$$



(b)

$$\Pr(2^{nd} \text{ flip is } H \mid 1^{st} \text{ flip is } H) = \frac{\Pr(1^{st} \text{ and } 2^{nd} \text{ flips } H)}{\Pr(1^{st} \text{ flip is } H)}$$

$$= \frac{(1/3)(1)(1) + (1/3)(1/2)^2}{(1/3)(1) + (1/3)(1/2)} = 0.833$$

(c)

$$\Pr(HH \mid 1^{st} \text{ and } 2^{nd} \text{ flips are } H) = \frac{\Pr(HH \text{ and } 1^{st} \text{ and } 2^{nd} \text{ flips are } H)}{\Pr(1^{st} \text{ and } 2^{nd} \text{ flips are } H)}$$

$$= \frac{1/3}{(1/3)(1)(1) + (1/3)(1/2)(1/2)} = \frac{1}{1 + 0.25} = 0.8$$

2.41 $\Pr(\text{at least one six in 4 throws}) = 1 - \Pr(\text{no six in 4 throws})$

$$= 1 - \text{dbinom}(0, 4, 1/6) = 0.5177.$$

$\Pr(\text{a double 6 in 24 rolls}) = 1 - \Pr(\text{no double six in 24 rolls})$

$$= 1 - \text{dbinom}(0, 24, 1/36) \doteq 0.491$$

2.42 A variable X counting misses here has the NBinom$(100, 0.92)$ distribution. So,

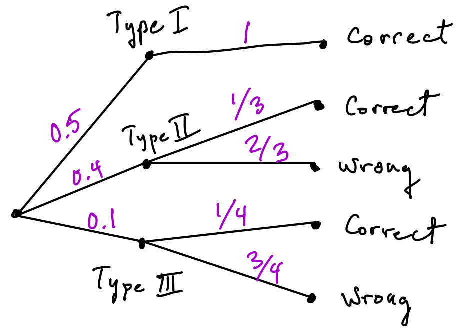
$$\Pr(114 \text{ or more attempts}) = \Pr(X \geq 14) = 1 - \Pr(X \leq 13)$$

$$= 1 - \text{pnbinom}(13, 100, 0.92) \doteq 0.0675.$$

2.49 (a) $1 - \text{pbinom}(11, 20, 0.25) = 0.000935$

(b) $1 - \text{pbinom}(11, 20, 1/3) = 0.013$

(c) Say the questions are Type I if you know the answer, Type II if you can eliminate one choice, and Type III if you must guess blindly. Then the probability tree for the 1^{st} question is given at right.



$$\Pr(1^{st} \text{ question is correct}) = \left(\tfrac{1}{2}\right)(1) + \left(\tfrac{2}{5}\right)\left(\tfrac{1}{3}\right) + \left(\tfrac{1}{10}\right)\left(\tfrac{1}{4}\right) \doteq 0.6583.$$

So,

$$\Pr(\text{pass test}) = 1 - \text{pbinom}(11, 20, 0.6583) \doteq 0.786.$$

2.52 Take $H_0: \pi = 1/4$, $H_a: \pi \neq 1/4$ (2-sided)

The data has $X = 8$ in 50 tries. Our P-value

```
bprobabilities <- dbinom(0:50, 50, 0.25)
sum(bprobabilities[bprobabilities <= dbinom(8, 50, 0.25)]) = 0.19.
```

2.70 (a) For $X \sim \text{DUnif}(10)$, $\Pr(X=3) = 1/10$, $\Pr(X=12) = 0$, $\Pr(X \le 3) = 0.3$.

(b) $$E(X) = (1)\left(\tfrac{1}{n}\right) + (2)\left(\tfrac{1}{n}\right) + \cdots + n\left(\tfrac{1}{n}\right) = \frac{1}{n}\sum_{i=1}^{n} i$$

$$= \left(\frac{1}{n}\right)\frac{n(n+1)}{2} = \frac{n+1}{2}.$$

$$E(X^2) = \frac{1}{n}\sum_{i=1}^{n} i^2 = \frac{1}{n}\cdot\frac{n(n+1)(2n+1)}{6} = \frac{1}{6}(2n^2+3n+1).$$

$$\Rightarrow \text{Var}(X) = \frac{1}{3}n^2 + \frac{1}{2}n + \frac{1}{6} - \frac{1}{4}(n^2+2n+1)$$

$$= \frac{1}{12}n^2 - \frac{1}{12}.$$

2.81 (a) $E(X+Y) = E(X) + E(Y) = 20$

$Var(X+Y) = Var(X) + Var(Y) = 4$, because X, Y are independent.

(b) $E(X+X) = E(2X) = 2E(X) = 20$

$Var(X+X) = Var(2X) = 4\,Var(X) = 8$

(c) When Y is independent of X, one may well have a small value of X paired with a large value of Y, making $X+Y$ moderately-sized. The same cannot be said of $X+X$, which is always twice as large as X. These observations lead to $X+X$ having greater variability than $X+Y$. Independence, however, is not a factor in the expected value of a sum.

2.88 $E(XY) = (-1)\cdot Pr(XY=-1) + (0)\cdot Pr(XY=0) + (1)\cdot Pr(XY=1)$

$= (-1)(0.25) + 0 + (1)(0.25) = 0$

$E(X) = (-1)(0.25) + (0)(0.5) + (1)(0.25) = 0$, $\quad E(Y) = (0)(0.5) + (1)(0.5) = 0.5$

$\Rightarrow \quad Cov(X,Y) = E(XY) - E(X)E(Y) = 0 - (0)(0.5) = 0.$

However, X and Y are not independent, as $f_{XY}(x,y) \neq f_X(x) f_Y(y)$.

For example, $Pr(X=1 \text{ and } Y=1) = \frac{1}{4}$, but $Pr(X=1)Pr(Y=1) = (\frac{1}{4})(\frac{1}{2}) = \frac{1}{8}$.

2.91 $Pr(Z \text{ scores at least 44 goals in 89 games})$

$= 1 - \text{ppois}\left(43, (89)\left(\frac{206}{506}\right)\right) \doteq 0.1157$

It is not terribly unlikely — not significant even at the 10% level — that he might score 44 goals during any 89-game stretch during the regular season.