

Formulas Midterm 2

Coordinate changes:

$$x = \rho \sin \phi \cos \theta$$

$$y = \rho \sin \phi \sin \theta$$

$$z = \rho \cos \phi$$

$$\rho^2 = x^2 + y^2 + z^2$$

$$\tan \theta = \frac{y}{x}$$

$$\phi = \arccos \left(\frac{z}{\sqrt{x^2 + y^2 + z^2}} \right)$$

$$r = \rho \sin \phi$$

$$z = \rho \cos \phi$$

$$\rho = \sqrt{r^2 + z^2}$$

$$\phi = \arccos \left(\frac{z}{\sqrt{r^2 + z^2}} \right)$$

Expansion factors:

$$dA = r dr d\theta$$

$$dV = r dr d\theta dz$$

$$= \rho^2 \sin \phi d\rho d\phi d\theta$$

Center of mass, 2D: $\left(\frac{M_x}{M}, \frac{M_x}{M} \right)$

$$M_y = \iint_D x \rho(x, y) dA$$

$$M_x = \iint_D y \rho(x, y) dA$$

Moment of inertia, 2D:

$$I_y = \iint_D x^2 \rho(x, y) dA$$

$$I_x = \iint_D y^2 \rho(x, y) dA$$

$$I_0 = \iint_D (x^2 + y^2) \rho(x, y) dA = I_y + I_x$$

Center of mass, 3D: $\left(\frac{M_{yz}}{M}, \frac{M_{xz}}{M}, \frac{M_{xy}}{M} \right)$

$$M_{yz} = \iiint_D x \rho(x, y, z) dV$$

$$M_{xz} = \iiint_D y \rho(x, y, z) dV$$

$$M_{xy} = \iiint_D z \rho(x, y, z) dV$$