B.10 Since $0^2 = 0$ and $1^2 = 1$, $g(x) = f(x)$. That is, it is the same indicator function.

B.13 $\sum_{i=1}^{99} [\![i \in S]\!] = |S|$. That is, it gives the cardinality of S.

B.21 (a) From the given pmf, we have

$$\sum_x f(x) = \frac{1}{6} + \frac{1}{3} + \frac{1}{4} + \frac{1}{6} + \frac{1}{12} = 1$$

(b) $$\sum_x x f(x) = (0)\left(\tfrac{1}{6}\right) + \frac{1}{3} + (2)\left(\tfrac{1}{4}\right) + (3)\left(\tfrac{1}{6}\right) + (4)\left(\tfrac{1}{12}\right) = \frac{5}{3}$$

(c) $$\sum_x x^2 f(x) = (0)^2\left(\tfrac{1}{6}\right) + \frac{1}{3} + (2)^2\left(\tfrac{1}{4}\right) + (3)^2\left(\tfrac{1}{6}\right) + (4)^2\left(\tfrac{1}{12}\right)$$

$$= \frac{1}{3} + 1 + \frac{3}{2} + \frac{4}{3} = \frac{25}{6}$$

2.1 (c) $A^c = \{HHH, HHT, HTH, THH\}$

$A \cap B = \{HTT, TTT\}$

$A \cup C = \{HTT, THT, TTH, TTT, THH\}$

2.2 (c) $A \cap B = \{(3,6), (4,5), (4,6), (5,6)\}$

$B \cup C = \{(1,2), (1,3), (1,4), (1,5), (1,6), (2,3), (2,4), (2,5), (2,6), (3,4), (3,5), (3,6), (4,5), (4,6), (5,6), (5,5), (6,5)\}$

$A \cap (B \cup C) = \{(3,6), (4,5), (4,6), (5,6), (5,5), (6,5)\}$

2.5 Using the language of positions and dividers, there are

$12 + 3 - 1 = 14$ positions,

and out of those a need to choose 2 of them for the dividers.

So, the number of orders is $\binom{14}{2} = \frac{14!}{2!\,12!} = 91$.

2.6 There are $\binom{52}{5}$ possible (and equally-likely) hands. Out of those,

$$\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2} = (13)(4)(12)(6) = 3744$$

are full houses. So,
$$\Pr(\text{full house}) = \frac{3744}{{}_{52}C_5} \doteq 0.001440576,$$

or about 1 in 694.

2.7 There are $\binom{52}{5}$ possible (and equally-likely) hands. Out of those,

$$\binom{13}{2}\binom{4}{2}\binom{4}{2}\binom{11}{1}\binom{4}{1} = (78)(6^2)(11)(4) = 123552$$

contain two-pair (but escape being classified as full houses). So

$$\Pr(\text{two pair}) = \frac{123552}{{}_{52}C_5} \doteq 0.04753902 \quad \text{or about } 1 \text{ in } 21$$

2.8 There are $\binom{52}{5}$ possible (and equally-likely) hands. Out of those,

$$\binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{1}\binom{4}{1} = (13)(4)(66)(4^2) = 54912$$

have 3-of-a-kind (but escape being classified as full houses). So
$$\Pr(\text{3-of-a-kind}) = \frac{54912}{{}_{52}C_5} \doteq 0.02112845,$$

or about 1 in 47.

2.9 A software solution might begin with a user-defined function like this one:

```
probNoRepeat = function(n) {
    if (n = 1) { return(1) } else {
        product = 1
        for (i in 1:(n-1)) { product = product * (365-i)/365 }
        return(product) }
}
```

(b) probNoRepeat(22) $\doteq$ 0.524 and probNoRepeat(23) $\doteq$ 0.493. So, 23 people.

2.14 By the Inclusion-Exclusion Principle,

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B).$$

Coupled with the fact that $\Pr(A \cup B) \le 1$, we have

$$\Pr(A) + \Pr(B) - \Pr(A \cap B) \le 1 \implies \Pr(A \cap B) \ge \Pr(A) + \Pr(B) - 1.$$

2.17 (a) $P(\text{bad}) = \frac{2+1}{8+10} = \boxed{\frac{1}{6}}$

(b) $P(\text{bad} \mid \text{assembly line } 1) = \frac{2}{8} = \boxed{\frac{1}{4}}$

(c) $P(\text{assembly line } 1 \mid \text{bad}) = \frac{P(\text{bad} \mid \text{assembly line } 1) \cdot P(\text{assembly line } 1)}{P(\text{bad})}$

$$= \frac{(1/4)(8/18)}{1/6} = \boxed{\frac{2}{3}}$$

2.19 $\Pr(B \mid A) = \frac{\Pr(A \cap B)}{\Pr(A)} = \frac{\Pr(A \mid B)\Pr(B)}{\Pr(A)} = \frac{\Pr(A)\Pr(B)}{\Pr(A)} = \Pr(B).$

2.30 There are 10 letters in the word STATISTICS. If these letters were all distinct, there would be $10!$ permutations of them. To use that count here would overcount by a factor $3!$ because of the three S's, by another factor of $3!$ because of the three T's, and $2!$ because of two I's. Thus, there are

$$\frac{10!}{(3!)^2\,2!} = \frac{(10)(9)(8)(7)\cdot(5)(4)}{2} = 50400$$