MATH 231: Differential Equations with Linear Algebra

Hand-Checked Assignment #3, due date: Wed., Mar. 27, 2019

$\star$16 From ODELA Section 2.5, pp. 125–126, do Exercise 2.5.4.

$\star$17 From ODELA Section 3.2, p. 143, do Exercise 3.2.1, parts (b), (e) and (f).

$\star$18 Consider the 2nd-order system

$$\begin{aligned}
\frac{d^2u_1}{dt^2} &= \frac{k_2}{m_1}(u_2 - u_1) + \frac{c_2}{m_1}\left(\frac{du_2}{dt} - \frac{du_1}{dt}\right) - \frac{k_1}{m_1}u_1 - \frac{c_1}{m_1}\frac{du_1}{dt}, \\
\frac{d^2u_2}{dt^2} &= \frac{f(t)}{m_2} - \frac{k_2}{m_2}(u_2 - u_1) - \frac{c_2}{m_2}\left(\frac{du_2}{dt} - \frac{du_1}{dt}\right).
\end{aligned}$$

which may (it has been proposed by someone on the internet, but I have not verified it) accurately model the mass-spring assembly with dashpots (shock absorbers) pictured at the website

http://upload.wikimedia.org/wikipedia/commons/f/fd/Mass-Spring-Damper_%282_body_system%29.svg

It seems the displacements from equilibrium positions of masses m_1, m_2 are denoted by u_1, u_2 respectively. The spring constants are k_1, k_2, and c_1, c_2 are the damping constants associated with the dashpots. There appears, as well, to be an external force $f(t)$ applied to the mass on the right.

Convert this model (whether it models what it purports to or not) to a first order linear system of DEs in the form $\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{b}(t)$, specifying the entries of $\mathbf{A}$ and $\mathbf{b}(t)$.

$\star$19 From ODELA Section 3.6, pp. 181–183, do Exercise 3.6.12.

$\star$20 On pp. 188-189 of ODELA we learn about a 1st order linear DE system model for quantities of lead stored in 1) the blood, 2) body tissues, and 3) bones. The model is

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \mathbf{A}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} I_L(t) \\ 0 \\ 0 \end{bmatrix}, \qquad \text{where} \qquad \mathbf{A} = \begin{bmatrix} -13/360 & 272/21875 & 7/200000 \\ 1/90 & -1/35 & 0 \\ 7/1800 & 0 & -7/200000 \end{bmatrix}.$$

(a) Approximate eigenpairs for the matrix $\mathbf{A}$ are provided at the top of p. 190. Use them to write the homogeneous solution—i.e., the solution in the case the influx into the bloodstream of lead from the environment $I_L(t) = 0$.

(b) If $I_L(t) = 0$ for a person previously poisoned with lead, what aspect of the model or solution indicates that the lead will be flushed out over time?

(c) Continue assuming that $I_L(t) = 0$, but suppose we have initial conditions $x_1(0) = 50$, $x_2(0) = 0$ and $x_3(0) = 0$; that is, we start with 50 units of lead in the blood and none in tissue nor bone. Solve the (homogeneous) IVP, and use it to write a formula for the amount $x_3(t)$ of lead in the bones. Find the approximate time t (in days) at which the level of lead in the bones is at its peak value. [Give your answer accurate to the tenths place.] Also, find the approximate time, following that peak, when the lead level in the bones has receded to no more than 0.5 units.