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Math W84, Wed 13-Jan-2016 -- Wed 13-Jan-2016

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Wednesday, January 13th 2016
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day05, We

Some issues raised by student responses:
 - harmonics and overtones are both multiples of fundamental
     200, 400, 600, 800, 1000, ...   name 3rd overtone, 2nd harmonic, etc.
     360, 540, 720, 900, 1080, ...   name fundamental
 - 7th harmonic is just another harmonic as above
    may tend to be softer, but is natural in most instruments
    play it in comparison with even temperament
 - curious if 659 is used for E5 because it is being looked up
    do NOT do that unless it is clear even tempering is desired
```

Tunings: a First Look

Just tuning

The basic idea is that, for a given fundamental frequency f_0, the overtone frequencies

$$2f_0,\ 3f_0,\ 4f_0,\ 5f_0,\ \ldots$$

can all be divided by 2^k for some integer k (which amounts to lowering the note k octaves) until it lies in the interval $[f_0, 2f_0)$. Some pairs of overtones, when divided by the appropriate power of 2, produce the same frequency (the first and third overtones, for instance, both result in f_0 once again), but you can produce

- a pentatonic scale using the first 8 overtones. If $f_0 = 440$ (A4), then the pentatonic scale we obtain has frequencies 440 (A4), 495 (B4), 550 (C#5), 660 (E5), and 770 (F#?).
- a heptatonic scale using the first twelve overtones. If $f_0 = 440$ (A4), then the heptatonic scale we obtain has frequencies 440 (A4), 495 (B4), 550 (C#5), 605 (note name?), 660 (E5), 715 (note name?), and 770 (note name?).

Each of the notes/frequencies in these scales is consonant with the fundamental f_0. Useful Octave commands may include

```
> reducedFreqs = unique(sort(reduceToOctave(440*(1:13),440)))
> format rat; reducedFreqs/440
> playFreqsSingly(reducedFreqs)
> format short      % returns to display results as decimal nos
```

The problem with just tuning arises when notes in the scale are played together.

Bootstrapping (essentially(?) Pythagorean tuning)

The idea is to start with the fundamental f_0,

obtain the perfect fifth through multiplication: $f_0 \cdot (3/2)$
go another perfect fifth from the previous: $f_0(3/2) \cdot (3/2) = f_0(3/2)^2$
go another perfect fifth from the previous: $f_0(3/2)^2 \cdot (3/2) = f_0(3/2)^3$
...
the last unrepeated instance *expected* on 12-note scale: $f_0(3/2)^{10} \cdot (3/2) = f_0(3/2)^{11}$

Relevant Octave commands:

```
> pentatonicNotes = reduceToOctave(440*(3/2).^[0:4])
> playFreqsSingly( pentatonicNotes )
> heptatonicNotes1 = reduceToOctave(440*(3/2).^[0:6])
> heptatonicNotes2 = reduceToOctave(440*(3/2).^[−3:3])   % play and compare with previous
```

The problem with this tuning arises from the fact that, on a 12-note scale, multiplication by $(3/2)^{12}$ should return you to the original note (same letter name), 7 octaves higher. In other words, we wish for $(3/2)^{12} = 129.75$ to be the same as $2^7 = 128$.

Assignment

1. In class I did Exercise 3 from the homework of Day 3 using commands such as these:

```
> g = @(x) (mod(x,2*pi) < pi) − (mod(x,2*pi) >= pi)
> xs = −pi/2:.01:7*pi/2;
> plot(xs, g(xs))
> axis([−1.6 11, −1.5 1.5])
> [a, b] = fourierTrigCoeffs(g, 3, 0, 2*pi)
```

calculates the coefficients $\mathbf{a} = [a_0\ a_1\ a_2\ a_3]$, $\mathbf{b} = [b_1\ b_2\ b_3]$ used in a truncated Fourier series (with period $\ell = 2\pi$)

$$\frac{a_0}{2} + \sum_{n=1}^{N} \left[a_n \cos\left(\frac{2\pi m x}{\ell}\right) + b_n \sin\left(\frac{2\pi m x}{\ell}\right) \right]. \tag{1}$$

using formulas

$$a_m = \frac{2}{\ell}\left\langle f, \cos\left(\frac{2m\pi\cdot}{\ell}\right)\right\rangle = \frac{2}{\ell}\int_0^{\ell} f(x)\cos\left(\frac{2\pi m x}{\ell}\right) dx, \quad m = 0, 1, 2, \ldots, \text{ and}$$

$$b_m = \frac{2}{\ell}\left\langle f, \sin\left(\frac{2m\pi\cdot}{\ell}\right)\right\rangle = \frac{2}{\ell}\int_0^{\ell} f(x)\sin\left(\frac{2\pi m x}{\ell}\right) dx, \quad m = 1, 2, \ldots.$$

To evaluate the function (1) I used commands like

```
> truncatedTrigFS(0.2, g, 3, [0 2*pi])   % doesn't require prior call to fourierTrigCoeffs.m
> hold on
> plot(xs, truncatedTrigFS(xs, g, 3, [0 2*pi]), 'r−')
> hold off
```

After getting comfortable with what you see here, and in consultation with group members, do the following:

   (a) Write a function based on the parabola $y = x^2$ that looks like the one on the top of p. 43 and has period 2 (peaks happen at $x = \ldots, -3, -1, 1, 3, 5, \ldots$).

```
> g = @(x)  ...
```

(b) Compute the Fourier coefficients $\mathbf{a} = [a_0\ a_1\ \ \ldots a_5]$, $\mathbf{b} = [b_1\ \ \ldots b_5]$. Are there any that seem predicatably to be zero?

(c) Together plot both g and the truncated Fourier series (1) with $N = 8$. Do they look quite similar in the interval $[-1.5, 1.5]$?

(d) In the plot of the last part, you likely made a call to `truncatedTrigFS.m` in which the final argument was `[-1 1]`. Try it over again, only changing this argument to `[0 1]` and the number of terms N. Discuss the results.

2. Read Chapter 4 of the Benson text, "Music: A Mathematical Offering" and answer the questions in Room SCOFIELD3894 at socrative.com. These questions will remain open through Thurs. am.

Get out in front?

Start reading Chapter 5 of the Benson text. A formal reading assignment will follow.