2.20 Defn. Events A, B are independent precisely when $P(B \mid A) = P(B)$.

Assume that A, B are independent, and show

(a) A and B^c are independent:

$$P(B^c \mid A) = 1 - P(B \mid A) = 1 - P(B) = P(B^c).$$

(b) A^c and B^c are independent

$$P(B^c \mid A^c) = 1 - P(B \mid A^c) = 1 - \frac{P(A^c \mid B)P(B)}{P(A^c)}$$

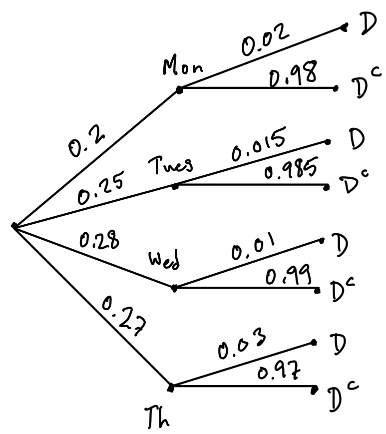
$$= 1 - \frac{P(A^c)P(B)}{P(A^c)} = 1 - P(B) = P(B^c).$$

2.25 Let A = "produced on Monday or Thursday"

D = "part is defective".

(a) $\Pr[A] = 0.2 + 0.27 = 0.47.$

(b) $$\Pr[A \mid D] = \frac{\Pr[A \cap D]}{\Pr[D]}$$



$$= \frac{\Pr[D \text{ and Mon}] + \Pr[D \text{ and Th}]}{\Pr[D \text{ and Mon}] + \Pr[D \text{ and Tu}] + \Pr[D \text{ and Wed}] + \Pr[D \text{ and Th}]}$$

$$= \frac{(0.2)(0.02) + (0.27)(0.03)}{(0.2)(0.02) + (0.25)(0.015) + (0.28)(0.01) + (0.27)(0.03)}$$

$$\doteq 0.6488$$

(c) $$\Pr[A \mid D^c] = \frac{\Pr[A \cap D^c]}{\Pr[D^c]} = \frac{\Pr[A] - \Pr[A \cap D]}{1 - \Pr[D]}$$

$$= \frac{0.47 - [(0.2)(0.02) + (0.27)(0.03)]}{1 - [(0.2)(0.02) + (0.25)(0.015) + (0.28)(0.01) + (0.27)(0.03)]}$$

$$\doteq 0.4666$$

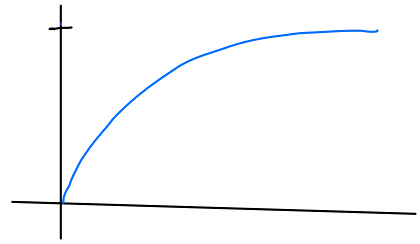
2.31 (a) $\Pr(\text{reject shipment}) = 1 - \Pr(\text{accept shipment}) = 1 - \left(\frac{9}{10}\right)\left(\frac{89}{99}\right)\left(\frac{88}{98}\right)\left(\frac{87}{97}\right) \doteq 0.3484.$

(b) Given the user-defined function

```
asProb <- function(p) {
        nGood <- 100 * (1 - p)
        return( nGood * (nGood - 1) *(nGood-2) * (nGood -3) / (100 * 99 * 98 * 97))
}
```

the desired plot is obtained from

```
gf_fun(1 - asProb(x) ~ x,  xlim(0.1, 0.9))
```



2.36 (a) $\binom{n}{k}$ counts the number of ways you can select k objects from n of them, leaving out the remaining $n-k$ objects. There is a 1-1 correspondence between chosen groups of size k and left-out groups of size $n-k$, counted by $\binom{n}{n-k}$.

2.37 (a) Pr(matching socks) = 1 − Pr(socks all different)

$$= 1 - \frac{\binom{8}{1}\binom{5}{1}\binom{4}{1}}{\binom{17}{3}} = 0.765$$

(b) $$\Pr(\text{at least two black socks}) = \frac{\binom{8}{2}\binom{9}{1} + \binom{8}{3}}{\binom{17}{3}} = 0.453$$

2.43 Let X = the count of pairs until a success, with sample space $\{1, 2, 3, 4\}$.

$$\Pr(X = 1) = 2/5 = 0.4$$
$$\Pr(X = 2) = (3/5)(1/2) = 0.3$$
$$\Pr(X = 3) = (3/5)(1/2)(2/3) = 0.2$$
$$\Pr(X = 4) = (3/5)(1/2)(1/3) = 0.1$$