E3.2

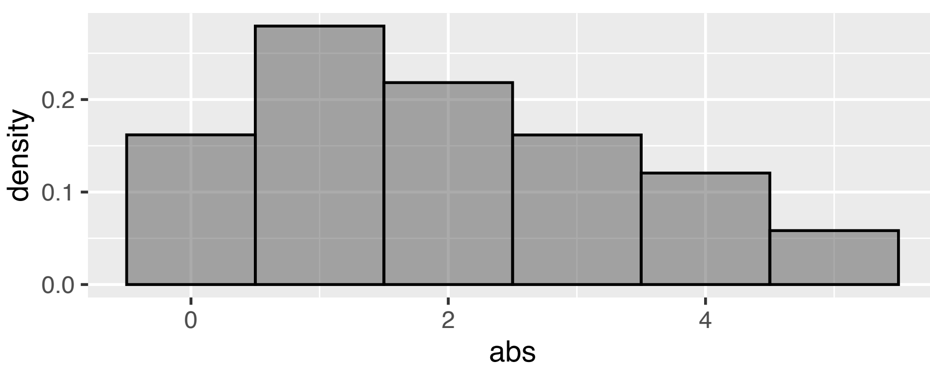
First, we simulate many differences.

```
die <- 1:6
manyTrials <- do(10000) * abs( diff( resample(die, size=2) ) )
tally( ~abs, format="proportion", data=manyTrials )
```

```
abs
     0      1      2      3      4      5
0.1618 0.2794 0.2183 0.1617 0.1205 0.0583
```

The probability of a difference of 2 is approximately 0.21. The approximately probability histogram follows.

```
gf_dhistogram(~ abs, data=manyTrials, bins=6, color="black" )
```



E3.4

Let us define

- event A: "a part passes inspection A"
- event B: "a part passes inspection B"

Then event $A \cup B$ has probability

$$\Pr(A \cup B) \;=\; 1 - \Pr(A^c \cap B^c) \;=\; 1 - 0.05 \;=\; 0.995.$$

The addition rule (Inclusion-Exclusion Principle) says

$$\Pr(A \cup B) \;=\; \Pr(A) + \Pr(B) - \Pr(A \cap B).$$

Rearranging this, we get the desired probability of passing both inspections, or $\Pr(A \cap B)$, as

$$\Pr(A \cap B) \;=\; \Pr(A) + \Pr(B) - \Pr(A \cup B) \;=\; 0.99 + 0.98 - 0.995 \;=\; 0.975.$$

E3.6

For Y, the absolute difference of two fair dice, we may again look at the listing of the sample space in Example 3.4(c).

$$\Pr(Y = 2) \; = \; 4\left(\frac{2}{36}\right) \; = \; \frac{2}{9}.$$

The values of Y must lie in the set $\{0, 1, 2, 3, 4, 5\}$.

The probability function of Y is given by

y	0	1	2	3	4	5
$\Pr(Y = y)$	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{8}{36}$	$\frac{6}{36}$	$\frac{4}{36}$	$\frac{2}{36}$

E3.10

Let's denote events

- S: "is a smoker"
- W: "is a woman"
- M: "is a man"
- C: "has cancer"

The given information is

- $\Pr(C \mid M \cap S) = 23 \cdot \Pr(C \mid M \cap S^c)$
- $\Pr(C \mid W \cap S) = 13 \cdot \Pr(C \mid W \cap S^c)$
- $\Pr(S \mid M) = 0.231$, which means $\Pr(S^c \mid M) = 0.769$
- $\Pr(S \mid W) = 0.183$, which means $\Pr(S^c \mid W) = 0.817$

First,

$$\Pr(W \mid S) \; = \; \frac{\text{number of women smokers}}{\text{number of smokers}} \; = \; \frac{21.1}{21.1 + 24.8} \; = \; 0.46.$$

Next,

$$\begin{aligned}
\Pr(S \mid C \cap W) &= \frac{\Pr(C \cap W \cap S)}{\Pr(C \cap W)} \;=\; \frac{\Pr(C \cap W \cap S)}{\Pr(C \cap W \cap S) + \Pr(C \cap W \cap S^c)} \\
&= \frac{\Pr(C \mid W \cap S) \cdot \Pr(W \cap S)}{\Pr(C \mid W \cap S) \cdot \Pr(W \cap S) + \Pr(C \mid W \cap S^c) \cdot \Pr(W \cap S^c)} \\
&= \frac{13 \cdot \Pr(C \mid W \cap S^c) \cdot \Pr(W \cap S)}{13 \cdot \Pr(C \mid W \cap S^c) \cdot \Pr(W \cap S) + \Pr(C \mid W \cap S^c) \cdot \Pr(W \cap S^c)} \\
&= \frac{13 \cdot \Pr(W \cap S)}{13 \cdot \Pr(W \cap S) + \Pr(W \cap S^c)} \;=\; \frac{13 \cdot \Pr(S \mid W)\Pr(W)}{13 \cdot \Pr(S \mid W)\Pr(W) + \Pr(S^c \mid W)\Pr(W)} \\
&= \frac{13 \cdot \Pr(S \mid W)}{13 \cdot \Pr(S \mid W) + \Pr(S^c \mid W)} \;=\; \frac{13(0.183)}{13(0.183) + 0.817} \;\doteq\; 0.744.
\end{aligned}$$

Thus, after the manner of computing $\Pr(S \mid W \cap C)$, we have

$$\Pr(S \mid M \cap C) \;=\; \frac{23(0.231)}{23(0.231) + 0.769} \;\doteq\; 0.874.$$