Rosen text: parts of

Ch. 1
Ch. 2 } test 1
Ch. 3
Ch. 5
Ch. 8
Ch. 4

Math 251, Fri 30-Oct-2020 -- Fri 30-Oct-2020
Discrete Mathematics
Fall 2020

Friday, October 30th 2020

8.1 Topic:: Recurrences
Read:: Rosen 8.1
HW[[WW Recurrences due Thurs.
Counting problems:
- various things like _nP_r, _nC_r: from Ch.6/M252, not studied in this course
- modeling using Recurrence relations

$_nP_r$ $\quad$ $_nC_r$ $\quad$ don't do

Modeling problems: use recurrences

1. **Posed by Leonardo of Pisa**: A pair of rabbits does not breed until it is 2 months old. At age 2 months, they begin producing a pair of offspring every month.

Let f_n be # of pairs of rabbits after month n.

$f_0 = 1$, $f_1 = 1$ 1, 1, 2, 3, 5, 8, 13, ...

As we get to the end of a month there are

f_{n-1} pairs surviving from previous month

f_{n-2} pairs (from 2 months ago) who can have a pair themselves

$f_n = f_{n-1} + f_{n-2}$ have a recurrence relation

2nd-order recurrence relation (looks two steps back)

2. **Tower of Hanoi**: see http://www.mathsisfun.com/games/towerofhanoi.html
Must move a tapered stack of rings to a different pole, moving only one ring at a time, and never placing a larger ring over a smaller one.

Let H_n = # of moves required to finish the game

$H_n = H_{n-1} + 1 + H_{n-1} = 2 H_{n-1} + 1$

$H_1 = 1$

$H_2 = 2 \cdot H_1 + 1 = 3$

$H_3 = 2H_2 + 1 = 7$

$H_4 = 2H_3 + 1 = 15$

looks like $H_n = 2^n - 1$

proof by induction?

= # of "allowed" bit strings of length n

3. Let b_n represent the number of bit strings (strings of 0s and 1s) of length n not containing consecutive 0s.

0 1 1 0 1 , 1 1 1 1 1 0 1 allowed

1 0 0 not allowed

The strings that b_n counts break into 2 types

- those that end in 1 — as numerous as b_{n-1} (1 gets tacked on)
- " " " " 0 — as numerous as b_{n-2} (10 gets tacked on)

$$b_n = b_{n-1} + b_{n-2} \quad 2^{nd} \text{ order}$$

To start, need 2 initial conds./values

$b_1 = 2$

$b_2 = 3$

4. **Enumerating codewords**: Say a valid codeword is a string from the alphabet "0–9" containing an even number of 0s. Let a_n represent the number of valid codewords of length n.

a_n = # of allowable n-digit codewords

Anything counted by a_n ends in

- 1 – 9, or — as numerous as $9a_{n-1}$
- 0 — as numerous as the disallowed $(n-1)$-digit strings

# of disallowed $(n-1)$-digit strings $= 10^{n-1} - a_{n-1}$

total count of $(n-1)$-digit strings

$$a_n = 9a_{n-1} + 10^{n-1} - a_{n-1} = 10^{n-1} + 8a_{n-1}$$

1^{st}-order recurrence

5. **Catalan numbers**:

$(a_0 a_1) \cdot a_2$ $\quad a_0 \cdot (a_1 a_2)$

C_2 - "# of different groupings for the product $a_0 a_1 a_2$" $= 2$

C_n = # of groupings for multiplying $a_0 a_1 a_2 \cdots a_n$

$(a_0 a_1 a_2)(a_3)$ < $((a_0 a_1) a_2) a_3$, $(a_0 (a_1 a_2)) a_3$

$(a_0 a_1)(a_2 a_3)$

Note: Always a last multiplication op. between two groups

left: $a_0 a_1 a_2 \cdots a_k$ — # of groupings C_k

right: $a_{k+1} \cdots a_n$ — # of groupings C_{n-k-1}

$\longrightarrow C_k \cdot C_{n-k-1}$ groupings

$$C_0 C_{n-1} + C_1 C_{n-2} + C_2 C_{n-3} + \cdots + C_{n-1} C_0 = C_n$$

6. Let w_n represent the number of strings of length n over the alphabet "abcde" with no adjacent e's.