## Some reflections on Taylor series

- Every Taylor series centered at $x = c$ is, itself, an example of a power series $\sum_{n=0}^{\infty} a_n(x-c)^n$ centered at $x = c$.

- The requirements for generating the Taylor series of $f$ centered at $c$ are that

  (i) $f$ be defined in some open interval surrounding $c$—that is, some interval $(a, b)$ containing $c$ lies in the domain of $f$.

  (ii) $f$ be differentiable to arbitrary order at $c$—that is, $f^{(k)}(c)$ exists for every $k = 1, 2, 3, \ldots$.

- Just because some function $f$ and number $c$ fits requirements (i) and (ii) above, does not mean that the Taylor series

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n, \qquad \text{or} \qquad f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \cdots + \frac{f^{(n)}(c)}{n!} + \cdots$$

  *converges* anywhere besides at the center $x = c$ (i.e., there *are* instances when the generated Taylor series has a radius of convergence $R = 0$), nor that, when it converges at some $x \neq c$, it gives the same value as $f(x)$.

- When the choice of center is 0, we use the name **Maclaurin series of** $f$ in place of *Taylor series of $f$ centered at 0*.

- If, in some open interval $(a, b)$ surrounding $x = c$, a given $f$ is equal to a power series $\sum_{n=0}^{\infty} a_n(x-c)^n$ centered at $x = c$, then that power series is the Taylor series of $f$ centered at $x = c$. That is, the coefficients

$$a_n = \frac{f^{(n)}(c)}{n!}, \qquad \text{for each } n = 0, 1, 2, \ldots.$$

- One obtains the $n^{\text{th}}$-degree Taylor polynomial of $f$ centered at $c$ from the Taylor series of $f$ centered at $c$ simply by stopping the infinite sum early, after the term with $(x-c)^n$.