HW07 solutions (Spring 2024)

The exercises here are from the book, "Probability, Statistics and Data: A Fresh Approach Using R", by Speegle and Clair

Exercise 4.10

(a) This probability comes from the difference in cdf values:

```
pnorm(5, 1, 2) - pnorm(3, 1, 2)
```

```
## [1] 0.1359051
```

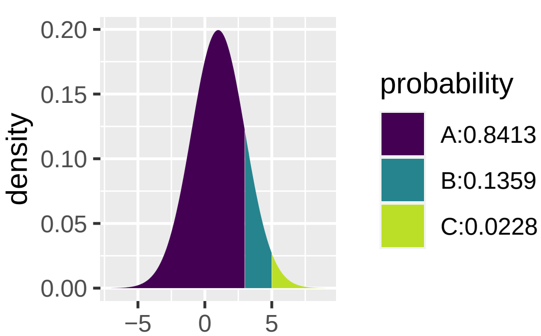
```
# diff( pnorm(c(3,5), 1, 2) )   # this also works
```

The probability is 0.1359.

(b) The function `xpnorm()` offers another, more visually-explicit way of obtaining an answer.

```
xpnorm(c(3,5), 1, 2)
```

```
##
## If X ~ N(1, 2), then
##     P(X <= 3) = P(Z <= 1) = 0.8413  P(X <= 5) = P(Z <= 2) = 0.9772
##     P(X >  3) = P(Z >  1) = 0.15866 P(X >  5) = P(Z >  2) = 0.02275
##
```

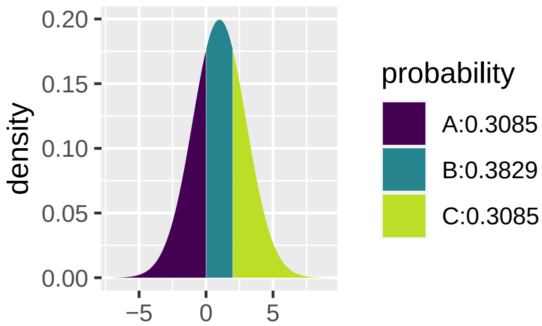



```
## [1] 0.8413447 0.9772499
```

(c) It is the case that, to wish to slice out an area that is maximized while being two units wide, the place to do so is evenly-spaced about the peak—i.e., by taking $a = 0$. That results in the picture below.

```
xpnorm(c(0,2), 1, 2)
```

```
##
## If X ~ N(1, 2), then
##     P(X <= 0) = P(Z <= -0.5) = 0.3085   P(X <= 2) = P(Z <=  0.5) = 0.6915
##     P(X >  0) = P(Z >  -0.5) = 0.6915   P(X >  2) = P(Z >   0.5) = 0.3085
##
```




```
## [1] 0.3085375 0.6914625
```

Exercise 4.11

(a) When exam scores $X \sim \text{Norm}(80, 5)$, the probability $P(X > 85)$ comes from the command

```
1-pnorm(85, 80, 5)
```

```
## [1] 0.1586553
```

The probability is 0.1587.

(b) There is a shift, here, from quantitative scores, to the count of students who succeed at a task. Specifically, the count $X \sim \text{Binom}(10, 0.1587)$, and we want $P(X \geq 4)$:

```
1 - pbinom(3, 10, 0.1587)
```

```
## [1] 0.05979829
```

```
# sum( dbinom(4:10, 10, 0.1587) )     # also works
```

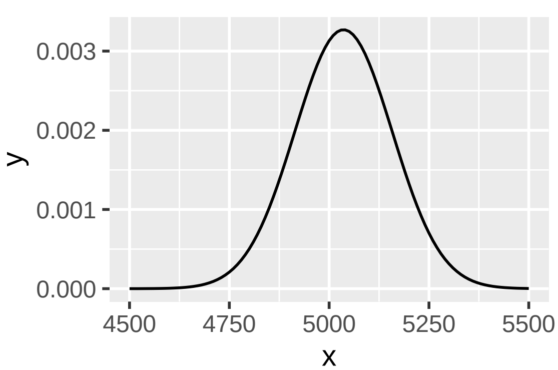
The chance of 4 our more with scores this high is 0.0598.

Exercise 4.12

We are told that the load (in lbs) causing these ropes to break is normally distributed, with $L \sim \text{Norm}(5036, 122)$.

(a) A "sketch" (perhaps a bit more precise than that) can be produced in R in multiple ways:

```
f = makeFun(dnorm(x, 5036, 122) ~ x)
gf_fun(f(x) ~ x, xlim = c(4500, 5500))
```




```
# gf_dist("norm", params=c(5036, 122))  # also works
```

(b) If what is being asked here is the proportion of ropes that breaks exactly under the load $L = 5000$, this L being a continuous r.v., the answer is $P(L = 5000) = 0$. But taking the question to mean $P(L \leq 5000)$, we can evaluate the cdf:

```
pnorm(5000, 5036, 122)
```

```
## [1] 0.3839656
```

The probability is 0.3840.

(c) The question, here, is to find L_0 so that $P(L \leq L_0) = 0.95$. This is answered using `qnorm()`.

```
qnorm(0.95, 5036, 122)
```

```
## [1] 5236.672
```

The 95th percentile for load-to-break is 5236.67 lbs.

Exercise 4.17

Now suppose X Exp(1/4); that is, the *rate* parameter is $\lambda = 0.25$. (Note: Unlike binomial, normal and uniform distributions, exponential distributions require only one parameter.)

(a) When X is exponential, the expected value $E(X) = 1/\lambda = 4$.

(b) A 1-unit-wide slice maximizing area would begin at $a = 0$, and correspond to the interval $[0, 1]$. The mean/expected value is *not* inside this interval.

```
gf_dist("exp", params=1/4, fill=~(x>=0 & x<=1), geom="area") + xlim(-1,15)
```