Math 251, Mon 28-Sep-2020 -- Mon 28-Sep-2020
Discrete Mathematics
Fall 2020

$\log_2 = \lg$
$\log_e = \ln$
$\log_{10} = \log$

1.5: #36

$2^x : \mathbb{R} \to (0, \infty)$

codomain = range

bijective, so has an inverse, called

$\log_2(x)$

Monday, September 28th 2020

Due:: PS06 due at 6 pm

Other calendar items

Monday, September 28th 2020

Wk 5, Mo
Topic:: Logarithms and exponentials
Topic:: Sequences and sums
HW[[PS07 due Thurs.
Read:: Rosen Appendix 2

Take base $b > 1$, get exp. growth fn.

bijective $\mathbb{R} \to (0, \infty)$

inverse: $\log_b x$

Note $b^0 = 1$ so $\log_b 1 = 0$.

Important invertible functions: exponentials: R -> (0,\infty)
view in desmos?
For b > 1, define log_b(x)
Properties of logs
as inverse to b^x

- log_b (xy) = log_b x + log_b y
- log_b (x/y) = log_b x - log_b y
- log_b (x^a) = a log_b x
- log_a x = log_b x / log_b a

$\log_b(xy) = \log_b x + \log_b y \iff a^x \cdot a^y = a^{x+y}$

$\log_b(x/y) = \log_b x - \log_b y \iff a^x / a^y = a^{x-y}$

$a \log_b x = \log_b(x^a) \iff (b^x)^a = b^{ax}$

log(x) as shorthand for log_2(x)

Change-of-base formula

$\log_a x = \frac{\log_b x}{\log_b a}$

Rearranged: Why true?

$(\log_a x)(\log_b a) = \log_b x$

So $\log_7 100 = \frac{\log_e 100}{\log_e 7}$

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Example: Writing n in binary:
  str fn (int n, str binStr) {
     if n is even:
       prepend 0 to binStr
       if n = 0:
         return binStr
       else:
         selfCall(n/2, binStr)
     else:
       prepend 1 to binStr
       selfCall((n-1)/2, binStr)
  }
```

input n = 28 "" "0"

14	"0"	"00"
7	"00"	"100"
3	"106"	"1100"
1	"1106"	"11100"
0	"11100"	"011100"

Note: Number of layers deep into recursion is ceil(log(n)) < 2 log(n)

(Real) sequences a_n
 function from N (or the like) -> R
 examples: make some patterned/un-patterned, arithmetic, geometric, neither
 formulas: explicit (closed-form), recursive
  In Ch.8 get gen'l method to solve linear homogeneous recursion relations
  Note: many solutions to a_n = a_{n-1} + 4; need initial condition
Say you borrow 10K at 8% interest, compounded monthly.
  If at the end of each month you pay $250,
    how much will you owe after one month?
    how much less/more after n months than after n-1?
Summations
 of finite geometric/arithmetic sequences
 Example (cont.): If, after 48 payments, you come into enough money to
   pay it off, how much will you have paid on the loan?

PS4, 1.5 in Rosen, #36

(a) $L(x,y)$: x has lost more than y dollars on the lottery

$$\forall x \neg L(x, 1000) \equiv \neg \exists x \, L(x, 1000)$$

(c) $S(x,y)$: x has sent email to y

$C(x)$: x is a student in this class

$$\exists x \left(C(x) \wedge \exists y \, (y \neq x \wedge S(x,y) \wedge C(y)) \right)$$

$$\equiv \exists x \exists y \left(C(x) \wedge C(y) \wedge (y \neq x) \wedge S(x,y) \right)$$

Our sen domain is students in the class

$$\exists x \exists y \left((y \neq x) \wedge S(x,y) \right)$$

$$S(x,y) \wedge S(x,z) \longrightarrow \exists w, S(x,w)$$

dress up

$$\forall x \forall y \forall z \left(x \neq y \wedge y \neq z \wedge x \neq z \wedge S(x,y) \wedge S(x,z) \longrightarrow \exists w \neq x \text{ or } z \wedge S(x,w) \right)$$

Tricky

Have $\left(\log_b a \right)\left(\log_a x \right) = \log_b \left(a^{\log_a x} \right) = \log_b x$

multiplier

$a^{\log_a x} = x$

f: domain $A \longrightarrow$ codomain B

f injective says

$$f(x) = f(y) \longrightarrow x = y$$

f surjective says

every element $b \in B$ is $f(x)$ for some x,

or said another way,

$$\forall b \in B \ \exists x \in A \ (f(x) = b)$$