Form A

2. (a) III (b) IV (c) I (d) VII (e) V (f) VII (g) I

3. (a)
$$\int_0^{\pi/2} \vec{r}(t)\,dt = \frac{1}{2}\sin(2t)\,\hat{\imath} - \frac{3}{2}t^2\,\hat{\jmath} + \frac{1}{2}\cos(2t)\,\hat{k}\,\Big|_0^{\pi/2}$$
$$= \left(-\frac{3\pi^2}{8}\hat{\jmath} - \frac{1}{2}\hat{k}\right) - \frac{1}{2}\hat{k}$$
$$= -\frac{3\pi^2}{8}\hat{\jmath} - \hat{k} \quad \text{or} \quad \left\langle 0, \frac{-3\pi^2}{8}, -1 \right\rangle$$

(b) $\vec{r}\,'(t) = \langle -2\sin(2t), -3, -2\cos(2t) \rangle$

$$\Rightarrow \|\vec{r}\,'(t)\| = \sqrt{4\sin^2(2t) + 9 + 4\cos^2(2t)} = \sqrt{13}.$$

$$\text{length} = \int_1^4 \|\vec{r}\,'(t)\|\,dt = \sqrt{13}\int_1^4 dt = 3\sqrt{13}.$$

(c) $$\vec{T}(t) = \frac{\vec{r}\,'(t)}{\|\vec{r}\,'(t)\|} = \frac{1}{\sqrt{13}}\vec{r}\,'(t) = \left\langle \frac{-2}{\sqrt{13}}\sin(2t), \frac{-3}{\sqrt{13}}, \frac{-2}{\sqrt{13}}\cos(2t) \right\rangle$$

4. (a)
$$\frac{\partial f}{\partial z} = \frac{-6xye^{-3z}(z - x^2) - 2xye^{-3z}}{(z - x^2)^2} = \frac{-2xye^{-3z}(3z - 3x^2 + 1)}{(z - x^2)^2}$$

(b) The domain excludes those points (x, y, z) for which $z = x^2$. This is a sheet-like set of points, slicing through the xz-plane in the shape of a parabola, and splitting xyz-space into 2 distinct parts that contain only interior points. So, the domain is open.

(c) This, by definition, is $\frac{\partial f}{\partial x}$, so it equals
$$\frac{2ye^{-3z}(z - x^2) + (2x)2xye^{-3z}}{(z - x^2)^2} = \frac{2ye^{-3z}(z + x^2)}{(z - x^2)^2}$$

5. (a) $\overrightarrow{PQ} = \langle -4, 7, 3 \rangle$, so the line can be written as

$$\vec{r}(t) = \langle 1, -3, 2 \rangle + t\langle -4, 7, 3 \rangle = \langle 1 - 4t, -3 + 7t, 2 + 3t \rangle.$$

(b) A normal vector to this plane is

$$\vec{n} = \overrightarrow{OP} \times \overrightarrow{OQ} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ -3 & 4 & 5 \end{vmatrix} = \begin{vmatrix} -3 & 2 \\ 4 & 5 \end{vmatrix} \hat{i} - \begin{vmatrix} 1 & 2 \\ -3 & 5 \end{vmatrix} \hat{j} + \begin{vmatrix} 1 & -3 \\ -3 & 4 \end{vmatrix} \hat{k}$$

$$= -23\hat{i} - 11\hat{j} - 5\hat{k}.$$

Using the origin as our point, the equation is

$$\vec{n} \cdot \langle x-0, y-0, z-0 \rangle = 0, \quad \text{or} \quad -23x - 11y - 5z = 0$$

(c) The angle θ is the acute angle between normal vectors:

$\langle -23, -11, -5 \rangle$ and $\langle 0, 0, 1 \rangle$:

$$\cos\theta = \frac{|-5|}{\sqrt{23^2 + 11^2 + 5^2} \cdot \sqrt{1}} = \frac{5}{\sqrt{529 + 121 + 25}} = \frac{5}{\sqrt{675}} = \frac{1}{3\sqrt{3}}$$

Or, alternately (w/ the other plane)

$$\cos\theta = \frac{2}{\sqrt{25 + 1 + 4}} = \frac{2}{\sqrt{30}}.$$