

1. (a) $\vec{QP} = \langle 2-4, -1+2, -3-1 \rangle = \langle -2, 1, -4 \rangle.$

$$\|\vec{QP}\| = \sqrt{(-2)^2 + 1^2 + (-4)^2} = \sqrt{21}$$

$$\Rightarrow \vec{u} = \frac{\vec{QP}}{\|\vec{QP}\|} = -\frac{2}{\sqrt{21}} \hat{i} + \frac{1}{\sqrt{21}} \hat{j} - \frac{4}{\sqrt{21}} \hat{k}.$$

(b) Take $\vec{v} = \vec{QP}$, found in (a). Take \vec{w} to be the vector

$$\vec{w} = \vec{QR} = \langle 1-4, 3+2, -2-1 \rangle = \langle -3, 5, -3 \rangle.$$

Then

$$\cos \theta = \frac{\vec{w} \cdot \vec{v}}{\|\vec{w}\| \|\vec{v}\|} = \frac{6+5+12}{\sqrt{43} \cdot \sqrt{21}} = \frac{23}{\sqrt{903}}$$

2. (a)

$$\begin{aligned} \vec{w} \times \vec{v} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -2 \\ 4 & -2 & 1 \end{vmatrix} = \begin{vmatrix} 3 & -2 \\ -2 & 1 \end{vmatrix} \hat{i} - \begin{vmatrix} 1 & -2 \\ 4 & 1 \end{vmatrix} \hat{j} + \begin{vmatrix} 1 & 3 \\ 4 & -2 \end{vmatrix} \hat{k} \\ &= \hat{i} - 9\hat{j} - 14\hat{k}. \end{aligned}$$

(b) Volume = $\left| \vec{u} \cdot (\vec{v} \times \vec{w}) \right| = \left| \langle 2, -1, -3 \rangle \cdot \langle -1, -9, 14 \rangle \right| = 49.$