Day 1 Assignment

1. **Read** Sections 1.1–1.6 of the Benson book, "Music: A Mathematical Offering". Visit Room SCOFIELD3894 at socrative.com (you may want to be there already when you begin the reading), by which time a different quiz should be active. Answer the reading questions.

2. Practice using Octave. Try out these tasks:

   - Write functions `phi` and `psi` of two variables, m and x that compute $\phi(m,x) = \cos(2\pi mx)$ and $\psi(m,x) = \sin(2\pi mx)$.
     - Make plots of various instances using your defined functions, perhaps
       $$\phi(2\pi x), \quad \psi(4\pi x), \quad \text{and} \quad \phi(8\pi x).$$
       What is the `frequency` (i.e., the number of cycles per one unit of x) for each?
     - Use the `quad()` function to compute integrals like these (you choose the instances to do):
       $$\text{phi-phi pairings:} \quad \int_0^1 \cos(2m\pi x)\cos(2n\pi x)\,dx$$
       $$\text{psi-psi pairings:} \quad \int_0^1 \sin(2m\pi x)\sin(2n\pi x)\,dx$$
       $$\text{phi-psi pairings:} \quad \int_0^1 \cos(2m\pi x)\sin(2n\pi x)\,dx$$
     - For some of the pairings you computed, make a plot on the interval $[0,1]$ that includes both functions. That is, if you computed $\int_0^1 \cos(2\pi x)\cos(8\pi x)\,dx$, make a plot of the two functions $\cos(2\pi x)$, $\cos(8\pi x)$ on the same coordinate frame with $0 \leqslant x \leqslant 1$. See if the plot confirms, to your eyes, the value you computed in the integral.
       You can make a second plot appear over the top of a first one by typing the command "`hold on`" between your plots.

```
> xs = -1:0.01:1;
> f1 = @(x) x.^2;
> f2 = @(x) x.^3;
> plot(xs, f1(xs), 'k-')
> hold on
> plot(xs, f2(xs), 'b-')
> axis([-1 1 -2 2])    % sets viewing window
> hold off     % without this, further plots continue being added
```

   - Perhaps you recall that the vector projection of one vector $\mathbf{u}$ onto another vector $\mathbf{v}$ is given by
     $$\text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}}\right)\mathbf{v}.$$

Write a function that carries this out. In the skeleton I provide, it is presumed that u, v are vectors.

```
> proj = @(u,v) ...        % the ... is for you to fill in
```

- Consider the vectors $\mathbf{v}_1 = \langle 4, 1, -1\rangle$ and $\mathbf{v}_2 = \langle 2, -5, 3\rangle$. Find the projections $\mathrm{proj}_{\mathbf{v}_1}\mathbf{u}$ and $\mathrm{proj}_{\mathbf{v}_2}\mathbf{u}$, where $\mathbf{u} = \langle 2, 1, 2\rangle$. The linear space $\mathrm{span}(\mathbf{v}_1, \mathbf{v}_2)$ is a plane P. Is the vector $\mathbf{w} = \mathrm{proj}_{\mathbf{v}_1}\mathbf{u} + \mathrm{proj}_{\mathbf{v}_2}\mathbf{u}$ the projection of $\mathbf{u}$ onto the plane P? Check this by finding the dot product of $\mathbf{u} - \mathbf{w}$ and $\mathbf{w}$. What do you expect this dot product to be, if $\mathbf{w}$ is the projection of $\mathbf{u}$ onto P?
- Repeat the previous exercise, using the same vector $\mathbf{u}$, but changing the vectors $\mathbf{v}_1$, $\mathbf{v}_2$ to these:
  $$\mathbf{v}_1 = \langle 0, 11, -7\rangle \quad \text{and} \quad \mathbf{v}_2 = \langle 2, 17, -11\rangle\,.$$
  It is the case (you needn't verify it) that $\mathrm{span}(\mathbf{v}_1, \mathbf{v}_2)$ is the same plane P as before. Is $\mathbf{w} = \mathrm{proj}_{\mathbf{v}_1}\mathbf{u} + \mathrm{proj}_{\mathbf{v}_2}\mathbf{u}$ the projection of $\mathbf{u}$ onto P for this pair of vectors?