CSE 250A HW3

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3.1 Inference in a chain

a) $\mathrm{P}(\mathrm{Y}_1|X_1) = \boxed{\Sigma_{X_0=x_0} P(Y_1|X_1, X_0 = x_0)P(X_0 = x_0)}$

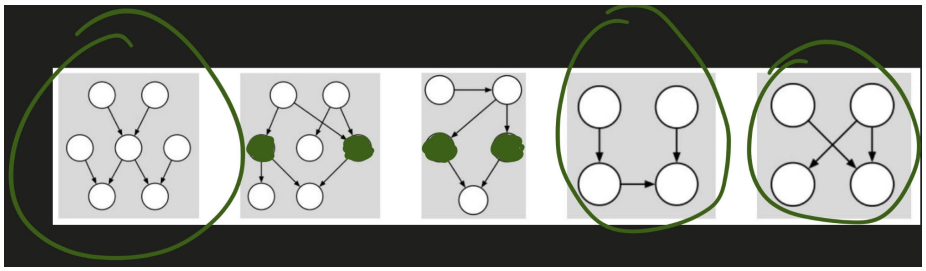
b) $P(Y_1) = \Sigma_{X_0=x_0,X_1=x_1} P(Y_1|X_1 = x_1, X_0 = x_0)P(X_1 = x_1, X_0 = x_0) =$
$\boxed{\Sigma_{X_0=x_0,X_1=x_1} P(Y_1|X_1 = x_1, X_0 = x_0)P(X_1 = x_1)P(X_0 = x_0)}$

c) $P(X_n|Y_1, Y_2, ..., Y_{n-1}) = \boxed{\mathrm{P}(\mathrm{X}_n)}$

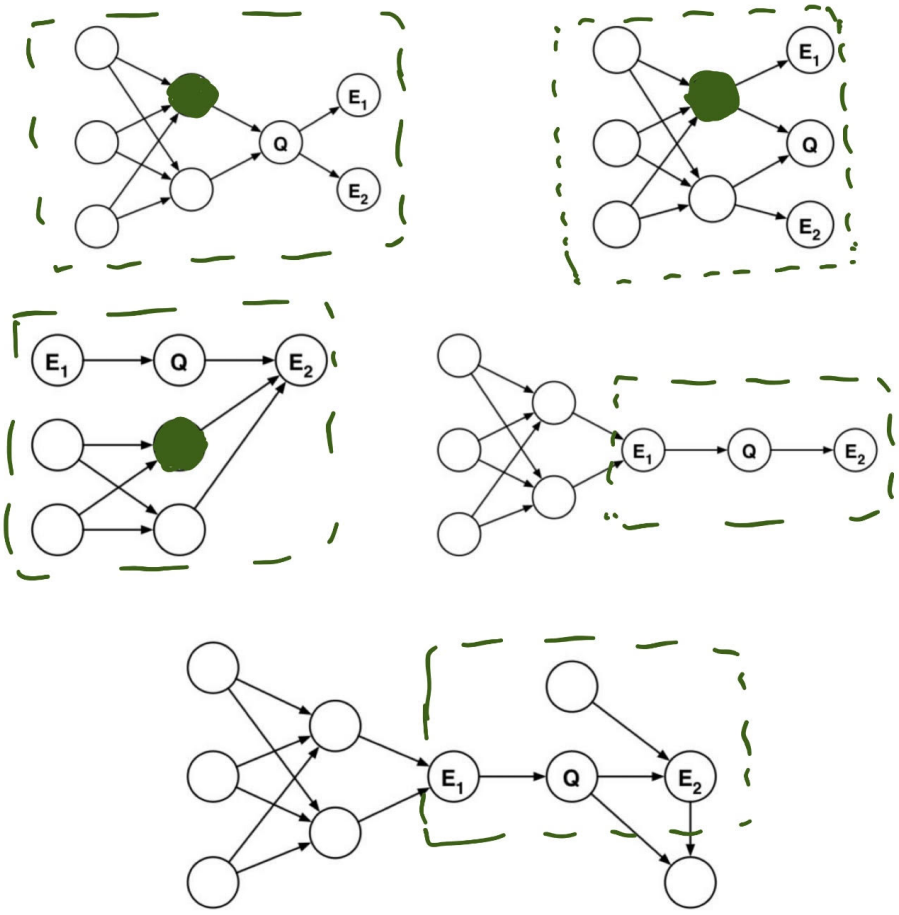
d) $P(Y_n|X_n, Y_1, Y_2, ...Y_{n-1}) = \Sigma_{X_{n-1}=x} P(Y_n|X_{n-1}, X_n, Y_1, Y_2, .., Y_{n-1})P(X_{n-1} =$
$x|X_n, Y_1, Y_2, ..., Y_n) = \boxed{\Sigma_{X_{n-1}=x} P(Y_n|X_{n-1} = x, X_n)P(X_{n-1} = x|X_n, Y_1, Y_2, ..., Y_{n-1})}$

e) $P(Y_n|Y_1, Y_2, ..., Y_{n-1}) = \Sigma_{X_{n-1}=x_{n-1},X_n=x_n} P(X_{n-1}|Y_1, ..., Y_{n-1})P(X_n|Y_1, .., Y_{n-1})$
$P(Y_n|X_{n-1}, X_n, Y_1, ..., Y_{n-1}) = \boxed{\Sigma_{X_{n-1}=x_{n-1},X_n=x_n} P(X_{n-1}|Y_1, ..Y_{n-1})P(X_n)P(Y_n|X_{n-1}, X_n)}$

3.2 Node clustering and polytrees



3.3 Cutsets and polytrees


E1
Q
E2
E1
Q
E2
E1
Q
E2
E1
Q
E2
E1
Q
E2

3.4 Even more inference

a) $P(B|A,C,D) = \frac{P(A,B,C,D)}{P(A)P(C|A)P(D|C,A)} = \frac{P(A)P(B|A)P(C|A,B)P(D|A,B,C)}{P(A)P(C|A)P(D|C,A)} = \frac{P(B|A)P(C|A)P(D|B,C)}{P(C|A)P(D|C,A)}$
$= \frac{P(B|A)P(D|B,C)}{P(D|C,A)} = \frac{P(B|A)P(D|B,C)}{\Sigma_{B=b}P(D|B=b,C,A)P(B=b|C,A)} = \boxed{\frac{P(B|A)P(D|B,C)}{\Sigma_{B=b}P(D|B=b,C,A)P(B=b|C,A)}}$

b) $P(B|A,C,D,E,F) = \frac{P(A,B,C,D,E,F)}{P(A,C,D,E,F)} = \frac{P(A)P(B|A)P(C|B,A)P(D|C,B,A)P(E|D,C,B,A)P(F|E,D,C,B,A)}{P(A)P(C|A)P(D|C,A)P(E|D,C,A)P(F|E,D,C,A)}$
$= \frac{P(B|A)P(C)P(D|B,C)P(E|C)P(F|A)}{P(C)P(D|C)P(E|C)P(F|A)} = \frac{P(B|A)P(D|B,C)}{P(D|C,A)} = \boxed{\frac{P(B|A)P(D|B,C)}{\Sigma_{B=b}P(D|B=b,C,A)P(B=b|C,A)}}$

c) $P(B,E,F|A,C,D) = P(B|E,F,A,C,D)P(E,F|A,C,D) = P(B|A,D)P(E,F|A,C)$
$= P(B|A,D)P(E|F,A,C)P(F|A,C) = \boxed{P(B|A,C,D)P(E|C)P(F|A)}$

3.5 More likelihood weighting

a) $y_t \sim P(Y)$
$z_t \sim P(Z)$
$q_t \sim P(Q|Y = y_t, Z = z_t, E = e)$

$P(Q = q|E = e) = \boxed{\frac{\Sigma_{t=1}^{N} I(q,q_t)P(E=e|Y=y_t,Z=z_t)}{\Sigma_{t=1}^{N} P(E=e|Y=y_t,Z=z_t)}}$

b) $x_t \sim P(X)$
$y_t \sim P(Y)$
$z_t \sim P(Z|X = x_t, Y = y_t)$
$q_{1t} \sim P(Q_1|Z = z_t, Y = y_t, X = x_t, E_1 = e_1, E_2 = e_2)$
$q_{2t} \sim P(Q_2|Z = z_t, Y = y_t, X = x_t, E_1 = e_1, E_2 = e_2)$

$P(Q_1 = q_1, Q_2 = q_2|E_1 = e_1, E_2 = e_2) = \boxed{\frac{\Sigma_{t=1}^{N} I(q,q_{1t})I(q,q_{2t})P(E_1=e|X=x_t,Q=q_{1t})P(E_2=e_2|E_1=e_1,Z=z_t)}{\Sigma_{t=1}^{N} P(E_1=e_1|X=x_t,Q_1=q_{1t})P(E_2=e_2|E_1=e_1,Z=z_t)}}$