CSE 250A HW2

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2.1 Probabilistic inference

a) $P(E=1|A=1) = \frac{P(A=1|E=1)P(E=1)}{P(A=1)} = \frac{P(A=1|E=1)P(E=1)}{P(A=1|E=1)P(E=1)+P(A=1|E=0)P(E=0)}$

P(A=1—E=1) = P(A=1—E=1,B=0)P(B=0)+P(A=1—E=1,B=1)P(B=1)
= (0.29)(0.999)+(0.95(0.001) = 0.2907
P(A=1—E=0) = P(A=1—E=0,B=0)P(B=0)+P(A=1—E=0,B=1)P(B=1)
= (0.001)(0.999)+(0.94)(0.001) = 0.001939
P(E=1) = 0.002 P(E=0) = 0.998

$P(E=1|A=1) = \frac{0.2907(0.002)}{0.297(0.002)+(0.001939)(0.998)} = \boxed{0.231}$

b) $P(E=1|A=1,B=0) = \frac{P(A=1|E=1,B=0)P(E=1|B=0)}{P(A=1|B=0)}$

$P(A=1|E=1,B=0) = 0.29$
$P(E=1|B=0) =$ P(E=1) = 0.002
$P(A=1|B=0) = P(A=1|E=0,B=0)$P(E=0)+
$P(A=1|E=1,B=0)$P(E=1) = (0.001)(1-0.002)+(0.29)(0.002) = 0.001578
$P(E=1|A=1,B=0) = \frac{0.29(0.002)}{0.001578} = \boxed{0.3676}$

c) $P(A=1|M=1) = \frac{P(M=1|A=1)P(A=1)}{P(M=1)}$

$P(M=1|A=1) = 0.7$
P(A=1) = $P(A=1|E=0,B=0)$P(E=0,B=0)+
$P(A=1|E=0,B=1)$P(E=0,B=1)+$P(A=1|E=1,B=0)$P(E=1,B=0)+
$P(A=1|E=1,B=1)$P(E=1,B=1) = (0.001)P(E=0)P(B=0)+(0.94)P(E=0)P(B=1)+
(0.29)P(E=1)P(B=0)+(0.95)P(E=1)P(B=1) = (0.001)(0.998)(0.999)+(0.94)(0.998)(0.001)+
(0.29)(0.002)(0.999)+(0.95)(0.002)(0.001)=0.002516
$P(M=1|A=0) = 0.01$
P(A=0) = 1-P(A=1) = 0.9975
$P(A=1|M=1) = \frac{0.7(0.002516)}{0.7(0.002516)+0.01(0.9975)} = \boxed{0.15}$

d) $P(A=1|M=1,J=0) = \frac{P(A=1,M=1,J=0)}{P(M=1,J=0)} = \frac{P(A=1)P(M=1|A=1)P(J=0|A=1)}{P(M=1,J=0)}$

P(A=1) = 0.002516

P(M=1—A=1) = 0.7
P(J=0—A=1) = 1-P(J=1—A=1) = 1-0.9 = 0.1
P(M=1, J=0) = $\Sigma_a P(A=a, M=1, J=0)$ =$\Sigma_a P(A=a)$
$P(M=1|A=a)P(J=0|M=1,A=a) = \Sigma_a P(A=a)$
$P(M=1|A=a)P(J=0|A=a)$ = P(A=0)$P(M=1|A=0)$
$P(J=0|A=0)$+P(A=1)$P(M=1|A=1)P(J=0|A=1)$ = (0.9975)(0.01)(0.95)+ (0.002516)(0.7)(0.1) = 0.009652

$P(A=1|M=1,J=0) = \frac{0.002516(0.7)(0.1)}{0.009652} = \boxed{0.01825}$

e) $P(A=1|M=0) = \frac{P(M=0|A=1)P(A=1)}{P(M=0|A=1)P(A=1)+P(M=0|A=0)P(A=0)} =$

$\frac{P(A=1)(1-P(M=1|A=1))}{P(A=1)(1-P(M=1|A=1))+P(A=0)(1-P(M=1|A=0))} = \frac{(1-0.7)(0.002516)}{(1-0.7)(0.002516)+(1-0.001)(0.9975)}$

$= \boxed{0.0007639}$

f) $P(A=1|M=0,B=1) = \frac{P(M=0|A=1,B=1)P(A=1|B=1)}{P(M=0|B=1)}$

$P(M=0|A=1,B=1) = P(M=0|B=1) = 0.3$
$P(A=1|B=1) = P(A=1|B=1,E=0)$P(E=0)+
$P(A=1|B=1,E=1)$P(E=1) = (0.94)(0.998)+(0.95)(0.002) = 0.94
$P(M=0|B=1) = \Sigma_a P(M=0, A=a|B=1) =$
$\Sigma_a P(M=0|A=a,B=1)P(A=a|B=1) =$
$\Sigma_a P(M=0|A=a)P(A=a|B=1) = P(M=0|A=0)$
$P(A=0|B=1)+P(M=0|A=1)P(A=1|B=1)$ = (0.99)(1-0.94)+(0.3)(0.94) = 0.3414
$P(A=1|M=0,B=1) = \frac{(0.3)(0.94)}{0.3414} = \boxed{0.8261}$

2.2 Probabilistic reasoning

a) $r_k = \frac{P(D=0|S_1=1,S_2=1,...,S_k=1)}{P(D=1|S_1=1,S_2=1,...,S_k=1)} = \frac{\frac{P(S_1=1,S_2=1,...,S_k=1|D=0)P(D=0)}{P(S_1=1,S_2=1,...,S_k=1)}}{\frac{P(S_1=1,S_2=1,...,S_k=1|D=1)P(D=1)}{P(S_1=1,S_2=1,...,S_k=1)}} = \frac{P(S_1=1,S_2=1,...,S_k=1|D=0)}{P(S_1=1,S_2=1,...,S_k=1|D=1)} =$

$\frac{P(S_1|D=0)P(S_2=1|D=0)P(S_3=1|D=0)....P(S_k=1|D=0)}{P(S_1|D=1)P(S_2=1|D=1)P(S_3=1|D=1)....P(S_k=1|D=1)} = \frac{P(S_2=1|D=0)P(S_3=1|D=0)....P(S_k=1|D=0)}{(\frac{1}{2})^k} =$

$\frac{(\frac{f(1)}{f(2)})(\frac{f(2)}{f(3)})...(\frac{f(k-1)}{f(k)})}{(\frac{1}{2})^k} = \frac{(\frac{f(1)}{f(k)})}{(\frac{1}{2})^k} = \frac{(\frac{1}{2^k+(-1)^k})}{(\frac{1}{2})^k} = \frac{2^k}{2^k+(-1)^k}$

$\boxed{r_k = \frac{2^k}{2^k+(-1)^k}}$

$\boxed{\text{On odd numbered days, the diagnosis will be D=0 and on even numbered days, the diagnosis will be D=1.}}$

b) As more symptoms are observed, the diagnosis becomes less certain because r_k gets closer to 1, which could make the diagnosis either D=0 or D=1 with almost equal probability.

2.3 True or false

1) False
2) True
3) True
4) True
5) True
6) True
7) False
8) True
9) True
10) False

2.4 More on Belief Networks

a) False
b) False
c) True
d) True
e) False
f) True
g) True
h) True
i) True
j) True

2.5 Conditional independence

1) X=rain, Y=sprinkler, E=month
2) X=rain, Y=fall, E=puddle
3) X=rain, Y=fall, E={month, puddle}
4) X=rain, Y=fall, E={sprinkler, puddle}
5) X=rain, Y=fall, E={month, sprinkler, puddle}
6) X=sprinkler, Y=fall, E=puddle
7) X=sprinkler, Y=fall, E={rain, puddle}
8) X=sprinkler, Y=fall, E={month, puddle}
9) X=sprinkler, Y=fall, E={month, rain, puddle}
10) X=month, Y=fall, E=puddle
11) X=month, Y=fall, E={rain, sprinkler}
12) X=month, Y=fall, E={sprinkler, puddle}
13) X=month, Y=fall, E={rain, puddle}
14) X=month, Y=fall, E={rain, sprinkler, puddle}
15) X=month, Y=puddle, E={rain, sprinkler}
16) X=month, Y=puddle, E={rain, sprinkler, fall}

2.6 Noisy-OR

a) $P(Z=1|X=0,Y=0)$=1-(1-p$_x$)$^0(1-p_y)^0=1-1=0$
$P(Z=1|X=0,Y=1)=1-(1-p_x)^0(1-p_y)^1=1-(1-p_y)=p_y$
$\boxed{P(Z=1|X=0,Y=0)<P(Z=1|X=0,Y=1)}$

b) $P(Z=1|X=1,Y=0)=1-(1-p_x)^1(1-p_y)^0=1-(1-p_x)=p_x$
$P(Z=1|X=0,Y=1)=1-(1-p_x)^0(1-p_y)^1=1-(1-p_y)=p_y$
$\boxed{P(Z=1|X=1,Y=0)<P(Z=1|X=0,Y=1)}$

c) $P(Z=1|X=1,Y=0)=p_x$
$P(Z=1|X=1,Y=1)=1-(1-p_x)^1(1-p_y)^1=1-(1-p_y-p_x+p_xp_y)=p_x+p_y-p_xp_y$
$P(Z=1|X=1,Y=0)-P(Z=1|X=1,Y=1)=p_x-(p_x+p_y-p_xp_y)=p_xp_y-p_y=p_y(p_y-1)<0$
$\boxed{P(Z=1|X=1,Y=0)<P(Z=1|X=1,Y=1)}$

d) $P(X=1)=p_x$
$P(X=1|Z=1)=\frac{P(Z=1|X=1)P(X=1)}{P(Z=1)}$

$P(Z=1|X=1)=P(Z=1|X=1,Y=0)P(Y=0)+P(Z=1|X=1,Y=1)P(Y=1)=p_x(1-p_y)+p_y(p_x+p_y-p_xp_y)=p_x-p_xp_y+p_xp_y+p_y^2-p_xp_y^2=p_x+p_y^2-p_xp_y^2$
$P(X=1|Z=1)=\frac{p_x(p_x+p_y^2-p_xp_y^2)}{p_y^2+p_x^2-p_x^2p_y^2}=\frac{p_x^2+p_xp_y^2-p_x^2p_y^2}{p_x^2+p_y^2-p_x^2p_y^2}$

$P(X=1)-P(X=1|Z=1)=p_x-\frac{p_x+p_xp_y^2-p_x^2p_y^2}{p_x^2+p_y^2-p_x^2p_y^2}<0$

$\boxed{P(X=1)<P(X=1|Z=1)}$

e) $P(X=1)=p_x$
$P(X=1|Y=1)=\frac{P(X=1,Y=1)}{P(Y=1)}=\frac{P(X=1)P(Y=1)}{P(Y=1)}=P(X=1)=p_x$
$\boxed{P(X=1)=P(X=1|Y=1)}$

f) $P(X=1|Z=1)=\frac{p_x^2+p_xp_y^2-p_x^2p_y^2}{p_x^2+p_y^2-p_x^2p_y^2}$

$P(X=1|Y=1,Z=1)=\frac{P(X=1,Y=1,Z=1)}{P(Y=1,Z=1)}$

$P(X=1,Y=1,Z=1)=P(Y=1)P(Z=1|Y=1)$
$P(Z=1|Y=1)=P(Z=1|Y=1,X=0)P(X=0)+P(Z=1|Y=1,X=1)P(X=1)=p_y(1-p_x)+p_x(p_x+p_y-p_xp_y)=p_y-p_xp_y+p_x^2+p_xp_y-p_x^2p_y=p_x^2+p_y-p_x^2p_y$
$P(Y=1,Z=1)=p_y(p_x^2+p_y-p_x^2p_y)=p_x^2p_y+p_y^2-p_x^2p_y=p_y^2$
$P(X=1|Y=1,Z=1)=\frac{p_x^2+p_xp_y-p_x^2p_y}{p_y}$

$P(X=1|Z=1) - P(X=1|Y=1,Z=1) > 0$
$\boxed{P(X=1|Z=1) > P(X=1|Y=1,Z=1)}$

g) $P(X=1)P(Y=1)P(Z=1) = p_x p_y(p_x^2 + p_y^2 - p_x^2 p_y^2) = p_x^3 p_y + p_x p_y^3 - p_x^3 p_y^3$
$P(X=1)P(Y=1)P(Z=1) - P(X=1,Y=1,Z=1) < 0$
$\boxed{P(X=1)P(Y=1)P(Z=1) < P(X=1,Y=1,Z=1)}$