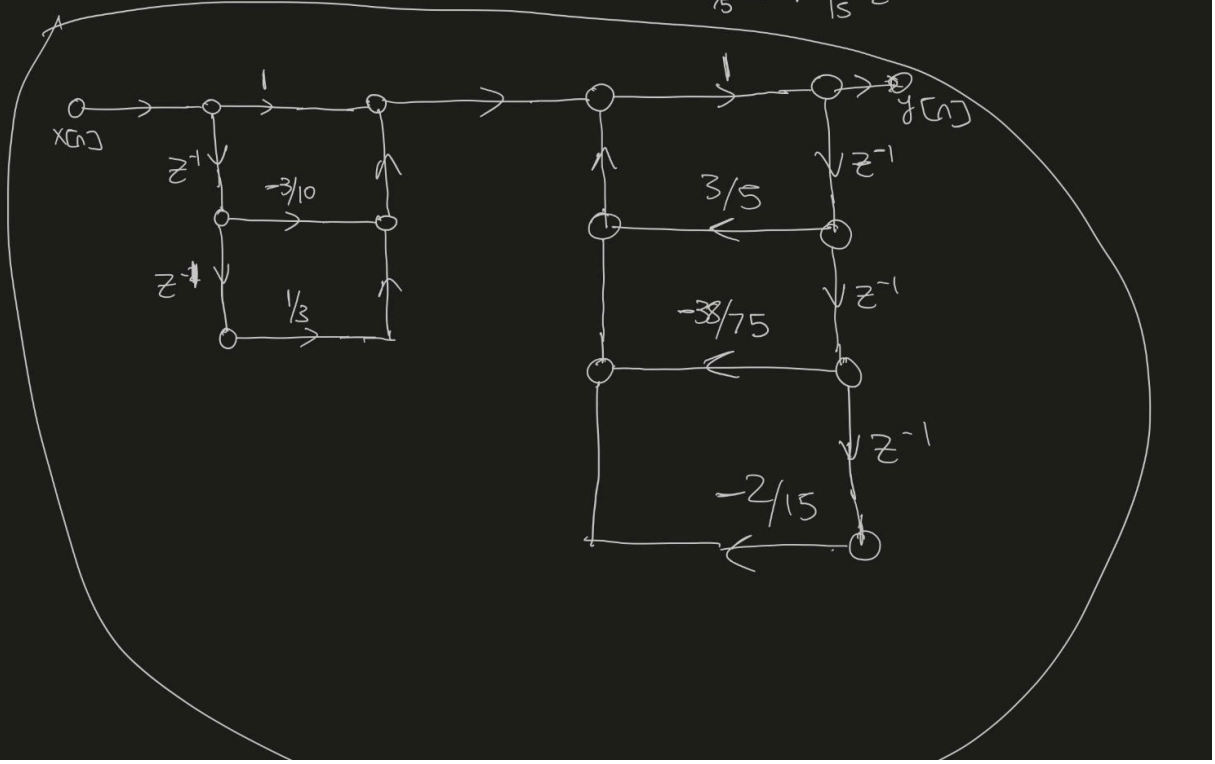
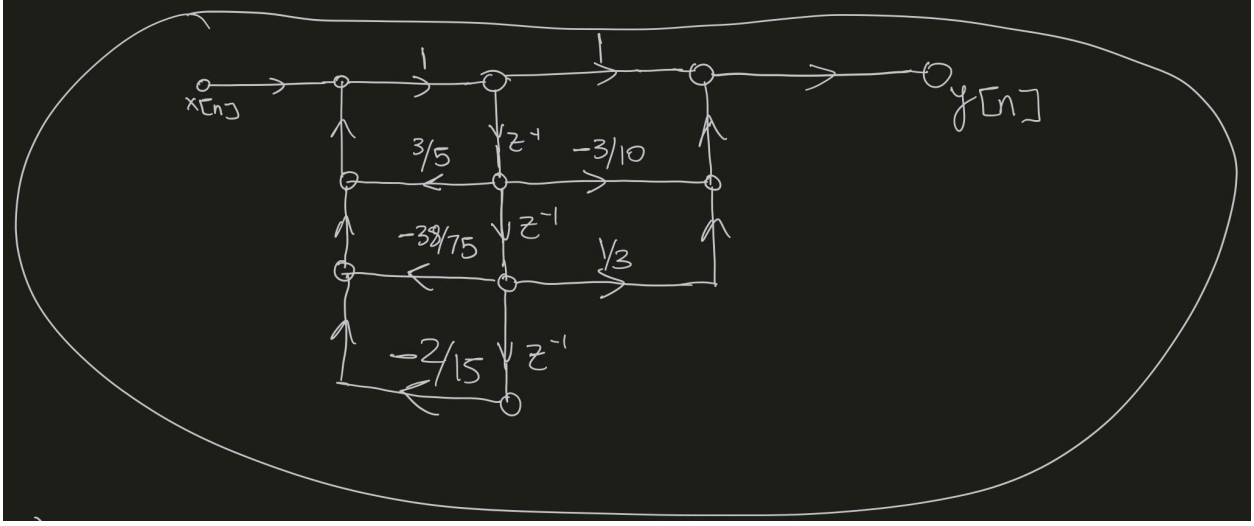
6.24) a) $H(z) = \dfrac{1-\frac{3}{10}z^{-1}+\frac{1}{3}z^{-2}}{(1-\frac{4}{5}z^{-1}+\frac{2}{3}z^{-2})(1+\frac{1}{5}z^{-1})} = \dfrac{1-\frac{3}{10}z^{-1}+\frac{1}{3}z^{-2}}{1-\frac{4}{5}z^{-1}+\frac{2}{3}z^{-2}+\frac{1}{5}z^{-1}-\frac{4}{25}z^{-2}+\frac{2}{15}z^{-3}}$

$= \dfrac{1-\frac{3}{10}z^{-1}+\frac{1}{3}z^{-2}}{1-\frac{3}{5}z^{-1}+\frac{38}{75}z^{-2}+\frac{2}{15}z^{-3}}$

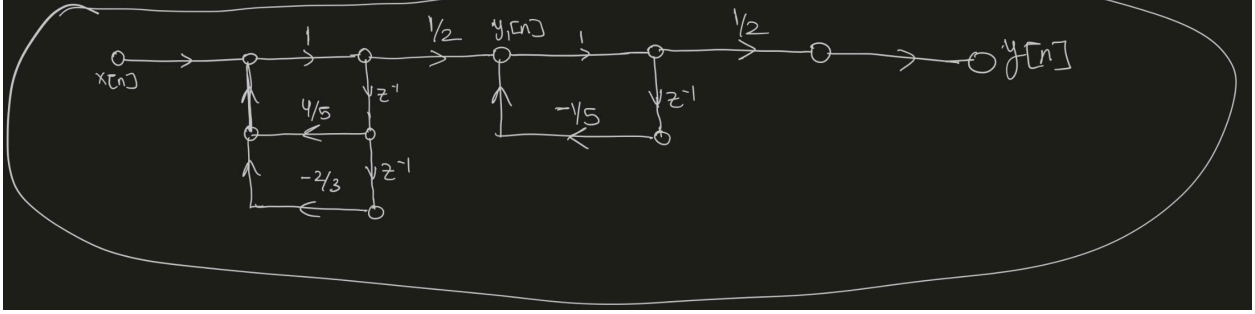
i)



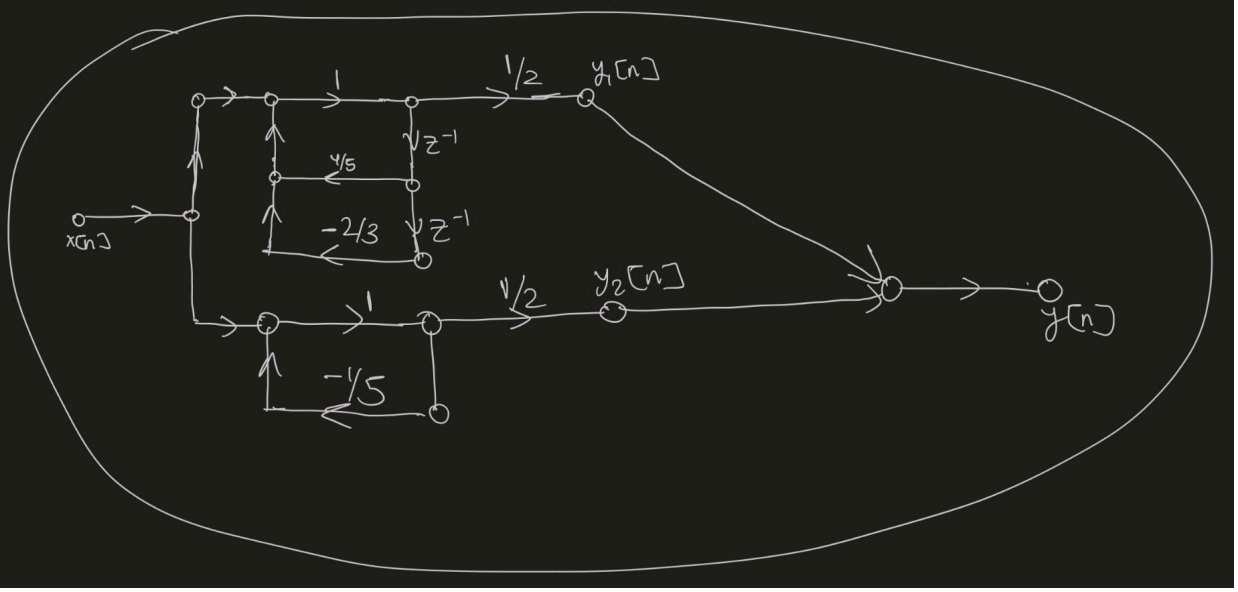
ii) $H(z) = \dfrac{1 - \frac{3}{10}z^{-1} + \frac{1}{3}z^{-2}}{1 - \frac{3}{5}z^{-1} + \frac{38}{75}z^{-2} + \frac{2}{15}z^{-3}}$



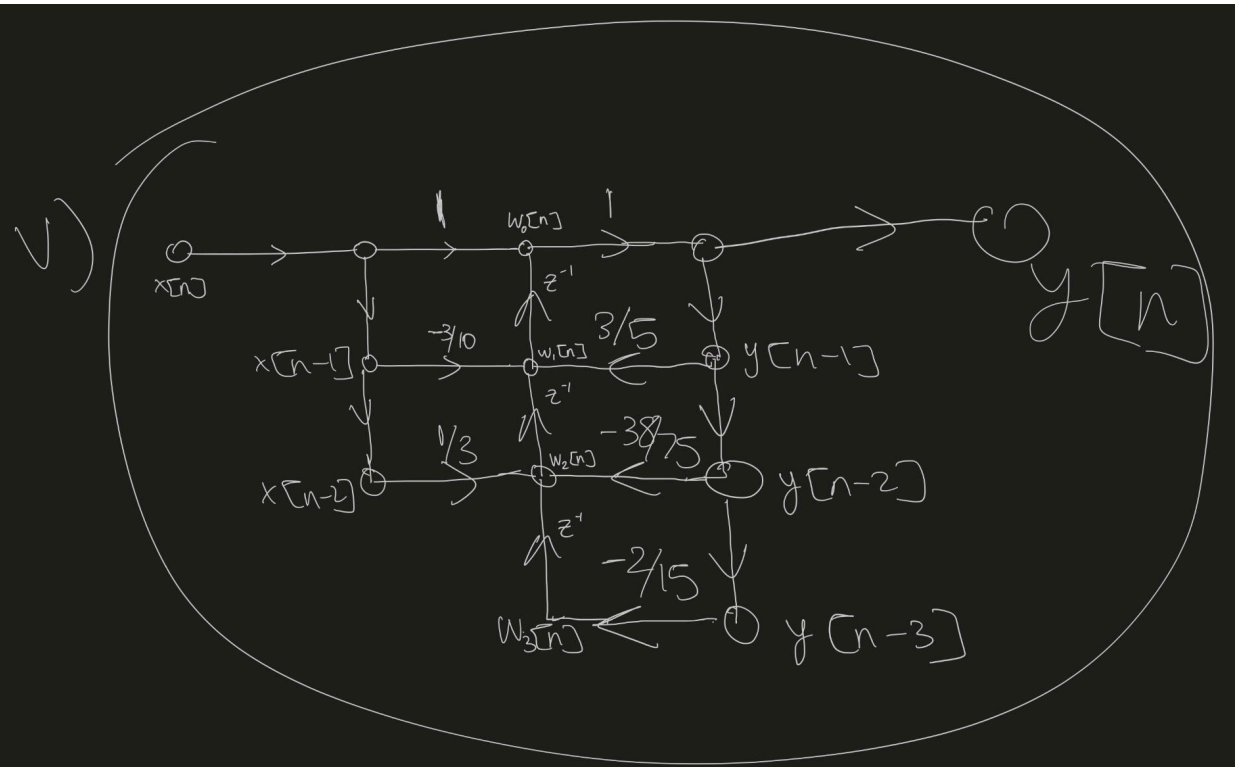
iii) $H(z) = \dfrac{\frac{1}{2}}{1 - \frac{4}{5}z^{-1} + \frac{2}{3}z^{-2}} + \dfrac{\frac{1}{2}}{1 + \frac{1}{5}z^{-1}} = H_1(z) + H_2(z)$



(iv) $H(z) = \frac{1/2}{1 - \frac{4}{5}z^{-1} + \frac{2}{3}z^{-2}} + \frac{1/2}{1 + 1/5\, z^{-1}}$



v)



b) $y[n] = w_0[n] \xrightarrow{Z} Y(z) = W_0(z)$

$w_0[n] = x[n] + w_1[n-1] \xrightarrow{Z} W_0(z) = X(z) + z^{-1}W_1(z)$

$w_1[n] = -\frac{3}{10}x[n] + \frac{3}{5}y[n] + w_2[n-1] \xrightarrow{Z} W_1(z) = -\frac{3}{10}X(z) + \frac{3}{5}Y(z) + z^{-1}W_2(z)$

$w_2[n] = \frac{1}{3}x[n] - \frac{38}{75}y[n] + w_3[n-1] \xrightarrow{Z} W_2(z) = \frac{1}{3}X(z) - \frac{38}{75}Y(z) + z^{-1}W_3(z)$

$w_3[n] = -\frac{2}{15}y[n] \xrightarrow{Z} W_3(z) = -\frac{2}{15}Y(z)$

$$Y(z) = X(z) + z^{-1}\left[-\frac{3}{10}X(z) + \frac{3}{5}Y(z) + z^{-1}\left[\frac{1}{3}X(z) - \frac{38}{75}Y(z) + z^{-1}\left[-\frac{2}{15}Y(z)\right]\right]\right]$$

$$= X(z) + z^{-1}\left[-\frac{3}{10}X(z) + \frac{3}{5}Y(z) + z^{-1}\left[\frac{1}{3}X(z) - \frac{38}{75}Y(z) - \frac{2}{15}z^{-1}Y(z)\right]\right]$$

$$= X(z) + z^{-1}\left[-\frac{3}{10}X(z) + \frac{3}{5}Y(z) + \frac{1}{3}z^{-1}X(z) - \frac{38}{75}z^{-1}Y(z) - \frac{2}{15}z^{-2}Y(z)\right]$$

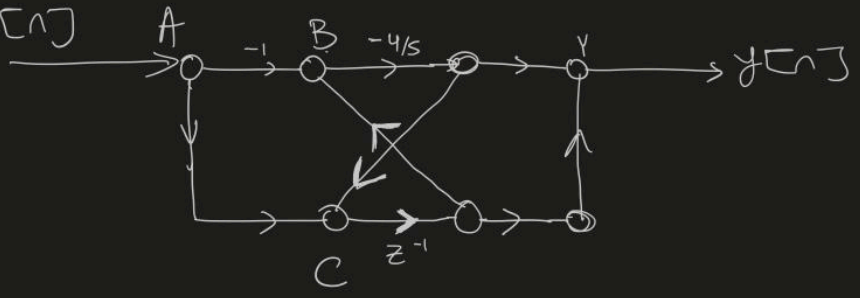
$$= X(z) - \frac{3}{10}z^{-1}X(z) + \frac{3}{5}z^{-1}Y(z) + \frac{1}{3}z^{-2}X(z) - \frac{38}{75}z^{-2}Y(z) - \frac{2}{15}z^{-3}Y(z)$$

$$Y(z) - \frac{3}{5}z^{-1}Y(z) + \frac{38}{75}z^{-2}Y(z) + \frac{2}{15}z^{-3}Y(z) = X(z) - \frac{3}{10}z^{-1}X(z) + \frac{1}{3}z^{-2}X(z)$$

$$Y(z)\left[1 - \frac{3}{5}z^{-1} + \frac{38}{75}z^{-2} + \frac{2}{15}z^{-3}\right] = X(z)\left[1 - \frac{3}{10}z^{-1} + \frac{1}{3}z^{-2}\right]$$

$$\frac{Y(z)}{X(z)} = H(z) = \frac{1 - \frac{3}{10}z^{-1} + \frac{1}{3}z^{-2}}{1 - \frac{3}{5}z^{-1} + \frac{38}{75}z^{-2} + \frac{2}{15}z^{-3}}$$

6.28) a)



$$B = -A + z^{-1}C$$

$$C = A - \frac{4}{5}B$$

$$Y = -\frac{4}{5}B + z^{-1}C$$

$$B = -A + z^{-1}\left(A - \frac{4}{5}B\right)$$

$$= -A + z^{-1}A - \frac{4}{5}z^{-1}B$$

$$B + \frac{4}{5}z^{-1}B = -A + z^{-1}A$$

$$B\left(1 + \frac{4}{5}z^{-1}\right) = A\left(-1 + z^{-1}\right)$$

$$B = A\left(\frac{-1+z^{-1}}{1+\frac{4}{5}z^{-1}}\right)$$

$$C = A - \frac{4}{5}\left(-A + z^{-1}C\right)$$

$$= A + \frac{4}{5}A - \frac{4}{5}z^{-1}C$$

$$C + \frac{4}{5}z^{-1}C = \frac{9}{5}A$$

$$C\left(1 + \frac{4}{5}z^{-1}\right) = \frac{9}{5}A$$

$$C = A\left(\frac{9/5}{1+\frac{4}{5}z^{-1}}\right)$$

$$Y = -\frac{4}{5}A\left(\frac{-1+z^{-1}}{1+\frac{4}{5}z^{-1}}\right) + z^{-1}A\left(\frac{9/5}{1+\frac{4}{5}z^{-1}}\right)$$

$$H_1(z) = \frac{Y}{A} = -\frac{4}{5}\left(\frac{-1+z^{-1}}{1+\frac{4}{5}z^{-1}}\right) + z^{-1}\left(\frac{9/5}{1+\frac{4}{5}z^{-1}}\right)$$

$$= \frac{\frac{4}{5} - \frac{4}{5}z^{-1} + \frac{9}{5}z^{-1}}{1+\frac{4}{5}z^{-1}} = \frac{\frac{4}{5} + \frac{4}{5}z^{-1}}{1+\frac{4}{5}z^{-1}}$$

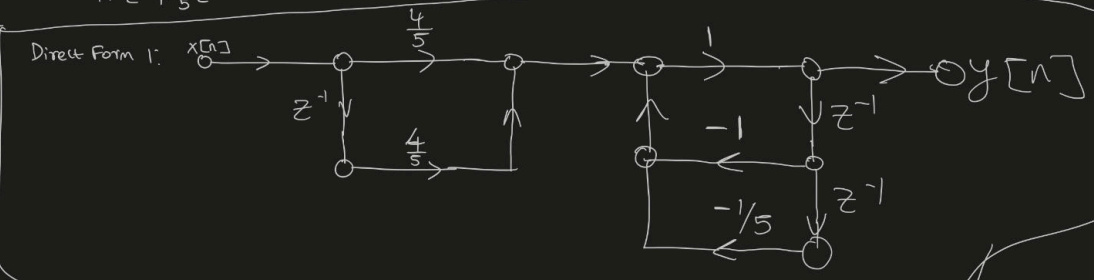
$$H(z) = \frac{H_1(z)}{1-(-\frac{1}{4}z^{-1})H_1(z)} = \left(\frac{\frac{4}{5}+\frac{4}{5}z^{-1}}{1+\frac{4}{5}z^{-1}}\right) \Big/ 1-(-\tfrac{1}{4}z^{-1})\left(\frac{\frac{4}{5}+\frac{4}{5}z^{-1}}{1+\frac{4}{5}z^{-1}}\right)$$

$$= \frac{\left(\frac{\frac{4}{5}+\frac{4}{5}z^{-1}}{1+\frac{4}{5}z^{-1}}\right)}{1-\left(\frac{-\frac{1}{5}z^{-1}-\frac{1}{5}z^{-2}}{1+\frac{4}{5}z^{-1}}\right)} = \frac{\left(\frac{\frac{4}{5}+\frac{4}{5}z^{-1}}{1+\frac{4}{5}z^{-1}}\right)}{\left(\frac{1+\frac{4}{5}z^{-1}+\frac{1}{5}z^{-1}+\frac{1}{5}z^{-2}}{1+\frac{4}{5}z^{-1}}\right)} = \frac{\left(\frac{\frac{4}{5}+\frac{4}{5}z^{-1}}{1+\frac{4}{5}z^{-1}}\right)}{\left(\frac{1+z^{-1}+\frac{1}{5}z^{-2}}{1+\frac{4}{5}z^{-1}}\right)} = \frac{\frac{4}{5}+\frac{4}{5}z^{-1}}{1+z^{-1}+\frac{1}{5}z^{-2}}$$

$$H(z) = \frac{\frac{4}{5}+\frac{4}{5}z^{-1}}{1+z^{-1}+\frac{1}{5}z^{-2}}$$

b) $H(z) = \dfrac{\frac{4}{5} + \frac{4}{5}z^{-1}}{1 + z^{-1} + \frac{1}{5}z^{-2}}$

Direct Form 1:



Direct Form 2: