1) $\tilde{x}[n] \xrightarrow{N} \tilde{X}[k]$

$$\sum_{n=0}^{N-1} |\tilde{x}[n]|^2 = \sum_{n=0} \tilde{x}[n]\,\tilde{x}^*[n]$$

$$\tilde{x}[n] = \frac{1}{N} \sum_{k=0}^{N-1} \tilde{X}[k]\, e^{-j\frac{2\pi}{N}kn}$$

$$\tilde{x}^*[n] = \frac{1}{N} \sum_{k=0}^{N-1} \tilde{X}^*[k]\, e^{j\frac{2\pi}{N}kn}$$

$$\sum_{n=0}^{N-1} |\tilde{x}[n]|^2 = \sum_{n=0}^{N-1} \tilde{x}[n] \left(\frac{1}{N} \sum_{k=0}^{N-1} \tilde{X}^*[k]\, e^{j\frac{2\pi}{N}kn} \right)$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} \tilde{X}^*[k] \left(\sum_{n=0}^{N-1} \tilde{x}[n]\, e^{-j\frac{2\pi}{N}kn} \right)$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} \tilde{X}^*[k]\, \tilde{X}[k] = \frac{1}{N} \sum_{k=0}^{N-1} |\tilde{X}[k]|^2$$

2) $\tilde{X}(e^{j\omega}) = \sum_{r=-\infty}^{\infty} 4\delta[\omega - \frac{2\pi}{3} - 2\pi r] + \sum_{r=-\infty}^{\infty} 3\delta[\omega - \frac{3\pi}{2} - 2\pi r]$

$$\tilde{X}(e^{j\omega}) = \sum_{k=-\infty}^{\infty} \frac{2\pi}{N} \tilde{X}[k] \delta\left[\omega - \frac{2\pi k}{N}\right]$$

$\omega_1 = \frac{2\pi}{3}, \quad \omega_2 = \frac{3\pi}{2} \longrightarrow \boxed{N=6}$

$$\boxed{\tilde{X}[k] = \begin{cases} 4, & k=2 \\ 3, & k=3 \\ 0, & \text{otherwise} \end{cases}}$$

$$\tilde{x}[n] = \frac{1}{N} \sum_{k=0}^{N-1} \tilde{X}[k] e^{j\frac{2\pi}{N}kn}$$

$$\boxed{\tilde{x}[n] = \frac{1}{6}\left(4e^{j\frac{2\pi}{3}n} + 3e^{j\pi n}\right)}$$

3) a) $\tilde{X}[k] = \sum_{n=0}^{N-1} \tilde{x}[n] e^{-j\frac{2\pi}{N}kn}$

$$\tilde{X}_3[k] = \sum_{n=0}^{3N-1} \tilde{x}[n] e^{-j\frac{2\pi}{3N}kn}$$

$$= \sum_{n=0}^{N-1} \tilde{x}[n] e^{-j\frac{2\pi}{3N}kn} + \sum_{n=N}^{2N-1} \tilde{x}[n] e^{-j\frac{2\pi}{3N}kn} + \sum_{n=2N}^{3N-1} \tilde{x}[n] e^{-j\frac{2\pi}{3N}kn}$$

$$= \sum_{n=0}^{N-1} \tilde{x}[n] e^{-j\frac{2\pi}{3N}kn} + e^{-j\frac{2\pi}{3N}kN} \sum_{n=0}^{N-1} \tilde{x}[n+N] e^{-j\frac{2\pi}{3N}kn} + e^{-j\frac{2\pi}{3}2kN} \sum_{n=0}^{N-1} \tilde{x}[n+2N] e^{-j\frac{2\pi}{3N}kn}$$

$$= \left(1+e^{-j\frac{2\pi}{3}k} + e^{-j\frac{4\pi}{3}k}\right) \sum_{n=0}^{N-1} \tilde{x}[n] e^{-j\frac{2\pi}{3N}\left(\frac{k}{3}n\right)}$$

$$= \left(1+e^{-j\frac{2\pi}{3}k} + e^{-j\frac{4\pi}{3}k}\right) \tilde{X}[k]$$

$$= \boxed{\begin{cases} 3\tilde{X}\left[\frac{k}{3}\right] & k = 3l \\ 0, & \text{otherwise} \end{cases}}$$

b) $\tilde{X}[k] = \sum_{n=0}^{N-1} \tilde{x}[n] e^{-j\frac{2\pi}{N}kn}$

$= \sum_{n=0}^{1} \tilde{x}[n] e^{-j\pi k n}$

$= \tilde{x}[0] + \tilde{x}[1] e^{-j\pi k}$

$= 1 + 2(-1)^k$

$$\tilde{X}[k] = \begin{cases} 3, & k=0 \\ -1, & k=1 \end{cases}$$

$\tilde{X}_3[k] = \sum_{n=0}^{5} \tilde{x}[n] e^{-j\frac{\pi}{3}kn}$

$= \left(1 + e^{-j\frac{2\pi}{3}k} + e^{-j\frac{4\pi}{3}k}\right) \tilde{X}\left[\frac{k}{3}\right]$

$$\tilde{X}_3[k] = \begin{cases} 9, & k=0 \\ -3, & k=3 \\ 0, & \text{otherwise} \end{cases}$$

4)a) $x_c(t) = \sum_{k=-9}^{9} a_k e^{j\left(\frac{2\pi}{10^{-3}}kt\right)}$

$x[n] = x_c\left(n\frac{10^{-3}}{6}\right) = \sum_{k=-9}^{9} a_k e^{j\frac{2\pi}{10^{-3}}k\left(\frac{10^{-3}}{6}\right)}$

$= \sum_{k=-9}^{9} a_k e^{j\frac{2\pi}{6}k}$

$\boxed{N = 6}$

$\boxed{x[n] \text{ is Periodic with Period } 6.}$

b) $T = \frac{10^{-3}}{6}\,s \rightarrow f_s = \frac{6}{10^{-3}}\,Hz = 6000\,Hz$

$f_N = \frac{9}{10^{-3}}\,Hz = 9000\,Hz$

c) $\tilde{X}[k] = \sum_{n=0}^{5} \tilde{x}[n]\, e^{-j\frac{2\pi}{6}kn}$

$$= \sum_{n=0}^{5} \left(\sum_{\ell=-9}^{9} a_\ell e^{j\frac{2\pi}{6}\ell n} \right) e^{-j\frac{2\pi}{6}kn}$$

$$= \sum_{n=0}^{5} \sum_{\ell=-9}^{9} a_\ell e^{j\frac{2\pi}{6}(\ell-k)n}$$

$$= 6 \sum_{\ell=-9}^{9} a_\ell \sum_{r=-\infty}^{\infty} \delta[\ell-k+6r]$$

$$= \boxed{6 \sum_{r=-\infty}^{\infty} a_{k-6r}}$$

5) $x_3[n] = x_1[n] \circledast_8 x_2[n] = \sum_{m=0}^{7} x_1[m]\, x_2[((n-m))_8]$

$x_1[n] = \{1, 2, 1, 1, 2, 1, 1, 2\}$

$x_2[n] = \{1, 3, 2\}$

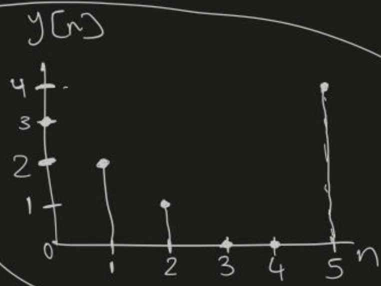
$x_1(n)$	1	2	1	1	2	1	1	2			
$x_2(n)$	0	1	3	2							
	0	0	0	0	0	0	0	0			
		1	2	1	1	2	1	1	2		
			3	6	3	3	6	3	3	6	
				2	4	2	2	4	2	2	4
	0	1	5	9	8	7	9	8	7	8	4
	7	8	4								
	7	9	9	9	8	7	9	8			

$\boxed{x_3[n] = \{7, 9, 9, 9, 8, 7, 9, 8\}}$

$\boxed{x_3[3:7] \text{ corresponds to linear convolution}}$

6) 8.23) a) $Y[k] = W_6^{5k} X[k]$

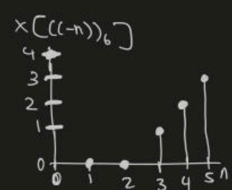
$$y[n] = x[((n-5))_6]$$



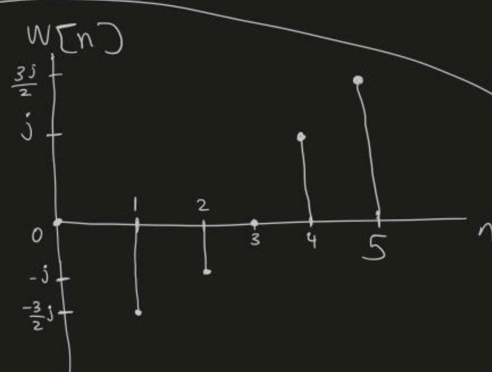
b) $W[k] = \mathrm{Im}\{X[k]\}$

$$j\,\mathrm{Im}\{X[k]\} \xrightarrow{DFT^{-1}} x_{op}[n] = \frac{1}{2}\{x[n] - x^*[((-n))_N]\}$$

$$w[n] = -j\left[\frac{1}{2}\{x[n] - x^*[((-n))_6]\}\right]$$



$$w[n] = \left\{-\frac{j}{2}(0),\ -\frac{j}{2}(3),\ -\frac{j}{2}(2),\ -\frac{j}{2}(0),\ -\frac{j}{2}(-2),\ -\frac{j}{2}(-3)\right\}$$



c) $Q[k] = X[2k+1] \qquad k = 0, 1, 2$

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$$

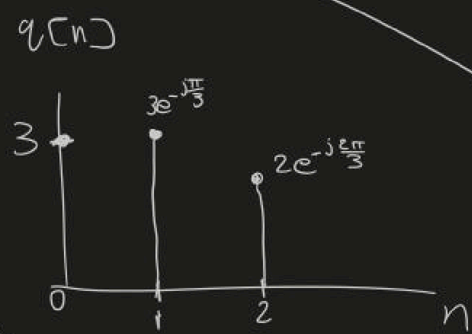
$$= \sum_{n=0}^{5} x[n] e^{-j\frac{2\pi}{6}kn}$$

$$= 4 + 3e^{-j\frac{\pi}{3}k} + 2e^{-j\frac{2\pi}{3}k} + e^{-j\pi k}$$

$$Q[k] = X[2k+1] = 4 + 3e^{-j\frac{\pi}{3}(2k+1)} + 2e^{-j\frac{2\pi}{3}(2k+1)} + e^{-j\pi(2k+1)}$$

$$= 4 + 3e^{-j\frac{\pi}{3}} e^{-j\frac{2\pi}{3}k} + 2e^{-j\frac{2\pi}{3}} e^{-j\frac{4\pi}{3}k} + e^{-j\pi} e^{-j2\pi k}$$

$$= 3 + 3e^{-j\frac{\pi}{3}} e^{-j\frac{2\pi}{3}k} + 2e^{-j\frac{2\pi}{3}} e^{-j\frac{4\pi}{3}k}$$



8.30) a) $N_1 = 21 + 18 = 39$ $\quad N_1 = 39$

$N_2 = 31 + 31 = 62$ $\quad N_2 = 62$

b) $y_1[n] \xrightarrow{32} Y_1[k]$

$$y_1[n] = \sum_{r=-\infty}^{\infty} y[n+32r]$$

$$= \boxed{y[n+32]}, \quad n = 0, \dots, 31$$

c) $N > 62$

8.36) a) $x[n] = \left(\frac{1}{2}\right)^n u[n] \xrightarrow{5} X[k] = X(e^{j\omega})\Big|_{\omega = \frac{2\pi}{N}k}$

$g[n] \xrightarrow{5} G[k]$

$$g[n] = \sum_{r=-\infty}^{\infty} x[n+5r] \quad , \quad 0 \le n \le 4$$

$$= \sum_{r=0}^{\infty} \left(\frac{1}{2}\right)^{n+5r}$$

$$= \left(\frac{1}{2}\right)^n \sum_{r=0}^{\infty} \left(\frac{1}{2}\right)^{5r} \qquad S = \frac{a_1}{1-r} \rightarrow a_1 = 1, \; r = \left(\frac{1}{2}\right)^5$$

$$= \boxed{\frac{32}{31}\left(\frac{1}{2}\right)^n} \qquad S = \frac{1}{1-\frac{1}{32}} = \frac{32}{31}$$

b) $w[n] \ne 0 \, , \quad 0 \le n \le 9$

$w[n] = 0 \, , \quad$ otherwise

$$W(e^{j\frac{2\pi k}{5}}) = X(e^{j\frac{2\pi k}{5}}) \qquad k = 0, 1, 2, 3, 4$$

$$w[n] = \sum_{r=-\infty}^{\infty} x[n+10r]$$

$$= \sum_{r=-\infty}^{\infty} \left(\frac{1}{2}\right)^{n+10r}$$

$$= \left(\frac{1}{2}\right)^n \sum_{r=-\infty}^{\infty} \left(\frac{1}{2}\right)^{10r} \qquad S = \frac{a_1}{1-r} = \frac{1}{1-\left(\frac{1}{2}\right)^{10}} = \frac{1}{1-\frac{1}{1024}} = \frac{1024}{1023}$$

$$= \boxed{\frac{1024}{1023}\left(\frac{1}{2}\right)^n}$$