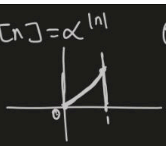
3.3) a) $x_a[n] = \alpha^{|n|}$ $\quad 0 < |\alpha| < 1$



$$X_a(z) = \sum_{n=-\infty}^{-1} \alpha^{-n} z^{-n} + \sum_{n=0}^{\infty} \alpha^{n} z^{-n}$$

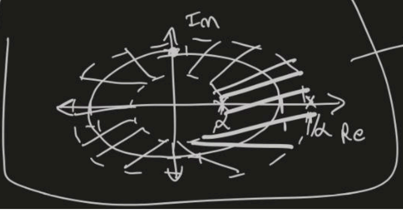
$$= \sum_{n=1}^{\infty} \alpha^{n} z^{n} + \sum_{n=0}^{\infty} \alpha^{n} z^{-n}$$

$$= \frac{\alpha z}{1-\alpha z} + \frac{1}{1-\alpha z^{-1}} = \frac{\alpha z(1-\alpha z^{-1}) + 1(1-\alpha z)}{(1-\alpha z)(1-\alpha z^{-1})}$$

$$= \frac{\alpha \cancel{z} - \alpha^2 + 1 - \alpha \cancel{z}}{(1-\alpha z)(1-\alpha z^{-1})}$$

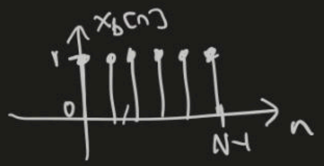
$$= \boxed{\frac{1-\alpha^2}{(1-\alpha z)(1-\alpha z^{-1})}}$$

$$\boxed{\text{ROC: } R_x = \left\{ z : |\alpha| < |z| < \frac{1}{|\alpha|} \right\}}$$



→ |z| is in shaded region

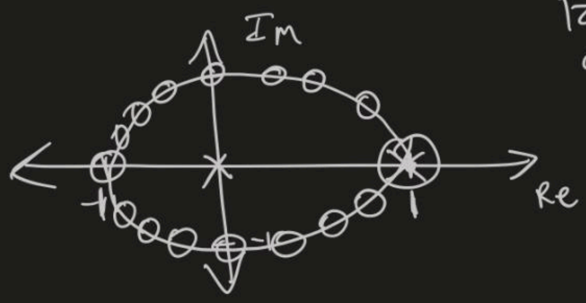
b) $x_b[n] = \begin{cases} 1, & 0 \leq n \leq N-1 \\ 0, & \text{Otherwise} \end{cases}$



$$X_b(z) = \sum_{n=0}^{N-1} z^{-n} = \frac{1(1-z^{-N})}{1-z^{-1}} = \boxed{\frac{1-z^{-N}}{1-z^{-1}}}$$

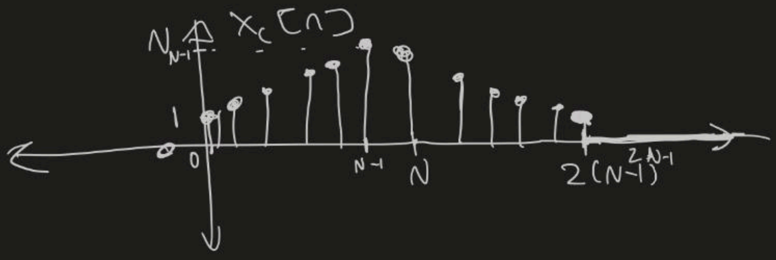
$$\boxed{X_b(z) = \frac{1-z^{-N}}{1-z^{-1}}}$$

$\boxed{\text{ROC: } R_x = \{z : |z| \neq 0\}}$ → 0 is a pole just like 1 but 1 cancels out because it's a zero



|z| can take any value except 0.

c) $x_c[n] = \begin{cases} n+1, & 0 \le n \le N-1 \\ 2N-1-n, & N \le n \le 2(N-1) \\ 0, & \text{otherwise} \end{cases}$

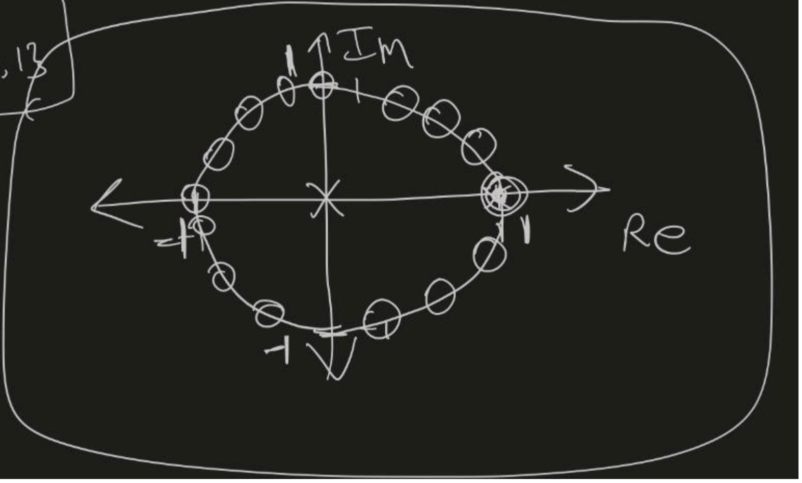


A triangular function is a convolution of 2 rectangular functions.

$$x_c[n] = x_b[n] * x_b[n-1]$$

$$X_c(z) = X_b(z)\, z^{-1} X_b(z) = \left(\frac{1-z^{-N}}{1-z^{-1}}\right)^2 z^{-1} = \boxed{\frac{z^{-1}(1-z^{-N})^2}{1-z^{-1}}}$$

$$\boxed{\text{ROC: } R_x = \{z : |z| \neq 0, 1\}}$$



3.21) a) $H(z) = \dfrac{4+0.25z^{-1}-0.5z^{-2}}{(1-0.25z^{-1})(1+0.5z^{-1})}$

$$\boxed{\text{ROC: } R_H = \{z : \{|z| > 0.5\}\}}$$

b) If a system is stable, then the unit circle has to be contained in the ROC. The ROC R_H contains the unit circle, therefore the system is stable.

c) $H(z) = \dfrac{4+0.25z^{-1}-0.5z^{-2}}{(1-0.25z^{-1})(1+0.5z^{-1})} = \dfrac{4+0.25z^{-1}-0.5z^{-2}}{1+0.5z^{-1}-0.25z^{-1}-0.125z^{-2}}$

$$= \frac{4+0.25z^{-1}-0.5z^{-2}}{1+0.25z^{-1}-0.125z^{-2}} = \frac{Y(z)}{X(z)}$$

$$Y(z)\left[1+0.25z^{-1}-0.125z^{-2}\right] = X(z)\left[4+0.25z^{-1}-0.5z^{-2}\right]$$

$$Y(z) + 0.25z^{-1}Y(z) - 0.125z^{-2}Y(z) = 4X(z) + 0.25z^{-1}X(z) - 0.5z^{-2}X(z)$$

$$\downarrow z^{-1}$$

$$\boxed{y[n] + 0.25y[n-1] - 0.125y[n-2] = 4x[n] + 0.25x[n-1] - 0.5x[n-2]}$$

d) $H(z) = \dfrac{4+0.25z^{-1}-0.5z^{-2}}{1+0.25z^{-1}-0.125z^{-2}}$

$$\begin{array}{r|l} & \quad 4 \\ \hline 1+0.25z^{-1}-0.125z^{-2} & \cancel{4}+0.25z^{-1}-\cancel{0.5}z^{-2} \\ & -(\cancel{4}+z^{-1}-\cancel{0.5}z^{-2}) \\ \hline & \quad -0.75z^{-1} \end{array}$$

$$H(z) = 4 - \frac{0.75z^{-1}}{(1-0.25z^{-1})(1+0.5z^{-1})}$$

$$\frac{0.75z^{-1}}{(1-0.25z^{-1})(1+0.5z^{-1})} = \frac{A}{1-0.25z^{-1}} + \frac{B}{1+0.5z^{-1}}$$

$$A(1+0.5z^{-1}) + B(1-0.25z^{-1}) = 0.75z^{-1}$$

If $z^{-1} = -2$:

$B(1-0.25(-2)) = 0.75(-2)$

$B(1.5) = -1.5 \longrightarrow B = -1$

If $z^{-1} = 4$:

$A(1+0.5(4)) = 0.75(4)$

$A(3) = 3$

$A = 1$

$$H(z) = 4 - \frac{1}{1-0.25z^{-1}} + \frac{1}{1+0.5z^{-1}}$$

$\downarrow Z^{-1}$

$$\boxed{h[n] = 4\delta[n] - \left(\frac{1}{4}\right)^n u[n] + \left(\frac{-1}{2}\right)^n u[n]}$$

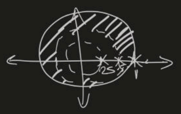
e) $x[n] = u[-n-1] \xrightarrow{z} X(z) = \frac{-1}{1-z^{-1}}$

$$H(z) = \frac{4+0.25z^{-1} - 0.5z^{-2}}{(1-0.25z^{-1})(1+0.5z^{-\text{[illegible]}})}$$

$$Y(z) = X(z)\,H(z) = \frac{-(4+0.25z^{-1}-0.5z^{-2})}{(1-z^{-1})(1-0.25z^{-1})(1+0.5z^{-\text{[illegible]}})}$$

$$\boxed{Y(z) = \frac{-4-0.25z^{-1}+0.5z^{-2}}{(1-z^{-1})(1-0.25z^{-1})(1+0.5z^{-1})}}$$

$$\boxed{\text{ROC: } R_Y = \{z : 0.5<|z|<1\}}$$



f) $$Y(z) = \frac{-4-0.25z^{-1}+0.5z^{-2}}{(1-z^{-1})(1-0.25z^{-1})(1+0.5z^{-1})} = \frac{A}{1-z^{-1}} + \frac{B}{1-0.25z^{-1}} + \frac{C}{1+0.5z^{-1}}$$

$A(1-0.25z^{-1})(1+0.5z^{-1}) + B(1-z^{-1})(1+0.5z^{-1}) + C(1-z^{-1})(1-0.25z^{-1}) = -4-0.25z^{-1}+0.5z^{-2}$

if $z^{-1} = 4$:

$B(1-4)(1+2) = -4-0.25(4)+0.5(16)$

$B(-3)(3) = -4-1+8$

$B(-9) = 3 \longrightarrow B = -1/3$

if $z^{-1} = -2$:

$C(1+2)(1+0.5) = -4-0.25(-2)+0.5(4)$

$C(3)(1.5) = -4+0.5+2$

$C(4.5) = -1.5$

$C = -1/3$

if $z^{-1} = 1$:

$A(1-0.25)(1+0.5) = -4-0.25+0.5$

$A(0.75)(1.5) = -3.75$

$A(1.125) = -3.75$

$A = \frac{-10}{3}$

$$Y(z) = \frac{-10/3}{1-z^{-1}} - \frac{1/3}{1-0.25z^{-1}} - \frac{1/3}{1+0.5z^{-1}} \xrightarrow{z^{-1}} \boxed{y[n] = \frac{-10}{3}u[-n-1] - \frac{1}{3}\left(\frac{1}{4}\right)^n u[n] - \frac{1}{3}\left(\frac{-1}{2}\right)^n u[n]}$$

3.31) a) $X(z) = \dfrac{1 - \frac{1}{3}z^{-1}}{1 + \frac{1}{3}z^{-1}}$ (x[n] is a right sided sequence)

$\hookrightarrow$ Sequence is 0 for $n < N_1 < \infty$

$1 - \frac{2}{3}z^{-1} + \frac{2}{9}z^{-2} + \ldots$

$1 + \frac{1}{3}z^{-1} \,\big)\, 1 - \frac{1}{3}z^{-1}$

$-(1 + \frac{1}{3}z^{-1})$

$-\frac{2}{3}\cancel{z^{-1}}$

$-(-\frac{2}{3}z^{-1} - \frac{2}{9}z^{-2})$

$\frac{2}{9}z^{-2}$

$$X(z) = \sum_{n=[illegible]}^{\infty} 2\left(\frac{-1}{3}\right)^n z^{-n}$$

$$\hookrightarrow \boxed{x[n] = 2\left(\frac{-1}{3}\right)^n u[n] - \delta[n]}$$

b) $X(z) = \dfrac{3}{z - \frac{1}{4} - \frac{1}{8}z^{-1}}$ x[n] stable

$$= \frac{3}{z(1 - \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2})} = \frac{3}{z(1+\frac{1}{4}z^{-1})(1-\frac{1}{2}z^{-1})} = \frac{3z^{-1}}{(1+\frac{1}{4}z^{-1})(1-\frac{1}{2}z^{-1})} = \frac{A}{1+\frac{1}{4}z^{-1}} + \frac{B}{1-\frac{1}{2}z^{-1}}$$

$A(1 - \frac{1}{2}z^{-1}) + B(1 + \frac{1}{4}z^{-1}) = 3z^{-1}$

if $z^{-1} = 2$:

$B(1 + \frac{1}{4}(2)) = 3(2)$

$B(1.5) = 6$

$B = 4$

if $z^{-1} = -4$:

$A(1 - \frac{1}{2}(-4)) = 3(-4)$

$A(3) = -12$

$A = -4$

$$X(z) = \frac{4}{1 - \frac{1}{2}z^{-1}} - \frac{4}{1 + \frac{1}{4}z^{-1}}$$

$$\boxed{x[n] = 4\left(\frac{1}{2}\right)^n u[n] - 4\left(\frac{-1}{4}\right)^n u[n]}$$

c) $X(z) = \ln(1-4z) \qquad |z| < \frac{1}{4}$

$$X(z) = -\sum_{k=1}^{\infty} \frac{(-4)^k (-z)^k}{k} = -\sum_{k=1}^{\infty} \frac{(4z)^k}{k} = -\sum_{m=-\infty}^{-1} \frac{1}{m} (4)^{-m} z^{-m}$$

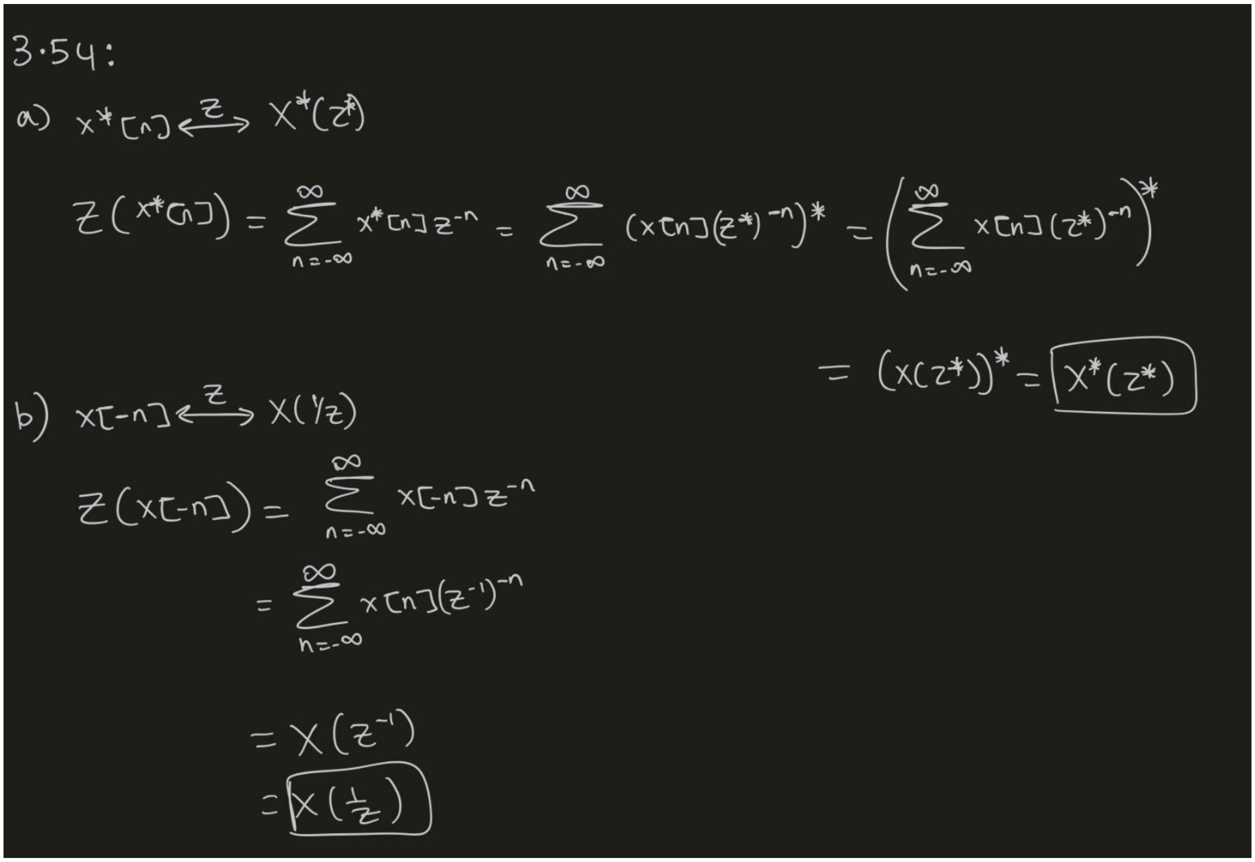
$$\boxed{x[n] = \frac{1}{n} (4)^{-n} u[-n-1]}$$

d) $X(z) = \frac{1}{1-\frac{1}{3}z^{-3}} \qquad |z| > (3)^{-1/3}$

$$\begin{array}{r|l} & 1 + \frac{1}{3}z^{-3} + \frac{1}{9}z^{-6} \\ \hline 1-\frac{1}{3}z^{-3} & \cancel{1} \\ & -(\cancel{1} - \frac{1}{3}z^{-3}) \\ \hline & \cancel{\frac{1}{3}z^{-3}} \\ & -(\cancel{\frac{1}{3}z^{-3}} - \frac{1}{9}z^{-6}) \\ \hline & \frac{1}{9}z^{-6} \end{array}$$

$$X(z) = \sum_{\substack{k=0 \\ n=3k}}^{\infty} \left(\frac{1}{3}\right)^{n/3} z^{-n}$$

$$\boxed{x[n] = \begin{cases} \left(\frac{1}{3}\right)^{n/3} & n = 0, 3, 6, \ldots \\ 0, & \text{otherwise} \end{cases}}$$



3.54:

a) $x^*[n] \xleftrightarrow{z} X^*(z^*)$

$$Z(x^*[n]) = \sum_{n=-\infty}^{\infty} x^*[n] z^{-n} = \sum_{n=-\infty}^{\infty} (x[n](z^*)^{-n})^* = \left(\sum_{n=-\infty}^{\infty} x[n](z^*)^{-n}\right)^*$$

$$= (x(z^*))^* = \boxed{X^*(z^*)}$$

b) $x[-n] \xleftrightarrow{z} X(1/z)$

$$Z(x[-n]) = \sum_{n=-\infty}^{\infty} x[-n] z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} x[n](z^{-1})^{-n}$$

$$= X(z^{-1})$$

$$= \boxed{X\left(\frac{1}{z}\right)}$$

c) $x_R[n] \xleftrightarrow{z} \frac{1}{2}[X(z)+X^*(z)]$

$$x[n] = x_R[n] + j x_I[n]$$

$$x^*[n] = x_R[n] - j x_I[n]$$

$$x[n] + x^*[n] = 2 x_R[n]$$

$$x_R[n] = \frac{1}{2}(x[n] + x^*[n])$$

$$\mathcal{Z}(x_R[n]) = \sum_{n=-\infty}^{\infty} \frac{1}{2}(x[n] + x^*[n]) z^{-n}$$

$$= \frac{1}{2} \sum_{n=-\infty}^{\infty} (x[n] + x^*[n]) z^{-n}$$

$$= \frac{1}{2}\left(\sum_{n=-\infty}^{\infty} x[n] z^{-n} + \sum_{n=-\infty}^{\infty} x^*[n] z^{-n}\right)$$

$$= \frac{1}{2}\left(\sum_{n=-\infty}^{\infty} x[n] z^{-n} + \sum_{n=-\infty}^{\infty} x^*[n] (z^*)^{-n}\right)$$

$$= \frac{1}{2}\left(\sum_{n=-\infty}^{\infty} x[n] z^{-n} + \left(\sum_{n=-\infty}^{\infty} x[n] (z^*)^{-n}\right)^*\right)$$

$$= \frac{1}{2}\left(X(z) + (X(z^*))^*\right)$$

$$= \boxed{\frac{1}{2}\left(X(z) + X^*(z^*)\right)}$$

d) $x_I[n] \overset{Z}{\longleftrightarrow} \frac{1}{2j}\left[X(z) - X^*(z^*)\right]$

$$X[n] = x_R[n] + j x_I[n]$$

$$X^*[n] = x_R[n] - j x_I[n]$$

$$X[n] - X^*[n] = 2j\, x_I[n]$$

$$x_I[n] = \frac{1}{2j}(x[n] - x^*[n])$$

$$Z(x_I[n]) = \sum_{n=-\infty}^{\infty} \frac{1}{2j}(x[n] - x^*[n])\, z^{-n}$$

$$= \frac{1}{2j}\left(\sum_{n=-\infty}^{\infty} x[n] z^{-n} - \sum_{n=-\infty}^{\infty} x^*[n] z^{-n}\right)$$

$$= \frac{1}{2j}\Big(X(z) - \Big(\sum_{n=-\infty}^{\infty} x[n] (z^*)^{-n}\Big)^*$$

$$= \boxed{\frac{1}{2j}\left(X(z) - X^*(z^*)\right)}$$