Are these expressions equivalent?

Annotation

Amy shows relational thinking by using appropriate mathematical statements that demonstrate her understanding of rational expressions. In seeing the fraction as a statement of one number divided by another, she recognises the need for the quadratics in the rational expression to be in factorised form in order to carry out the process of simplifying. Amy also demonstrates the ability to use a systematic approach to factorising quadratics successfully, and she can justify her working.

Problem: Are these expressions equivalent?

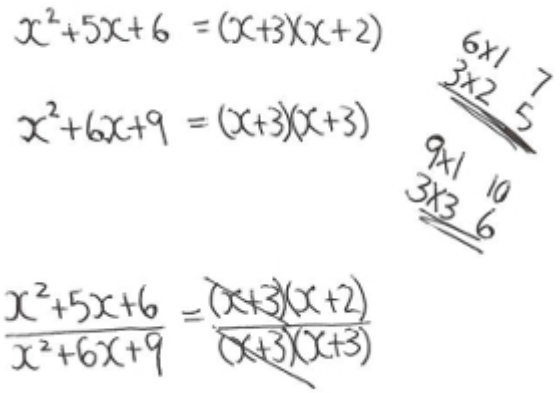
The teacher writes two expressions on the board.

$$\frac{x^2+5x+6}{x^2+6x+9} \quad \text{and} \quad \frac{x+2}{x+3}$$

The teacher poses the following challenge:

"*I'm going to tell you that these two 'rational expressions', algebraic fractions, are equivalent. My challenge to you is to prove me right.*

Student Response



Teacher: Tell me about your working here.

Amy: Well the first fraction has quadratics and the other one has only x, so I thought that maybe if I factorised the quadratic equations...it would be a good place to start.

Teacher: I'm pleased to see that you began by factorising.

Amy: Well I can't just cancel out x's in the expanded form because there's a whole lot of terms added together and they don't all have x's in them. I can only cancel out common factors because I'd be saying x over x is the same as one and anything times one is itself.Factorising was the hard bit, but I followed through the steps with seeing x squared means two sets of brackets and put x at the start. Then the sign at the end was + so the signs were the same…in the brackets I mean. That happened both times. Then I looked at the factors of the end number and found the ones that add up to the middle number.

Teacher: I see that you have factorised both quadratics correctly. Well done. Now, what did you do next?

Amy: I wrote the big fraction out again and put the quadratic bits in brackets…factorised. Then I could straight away see the answer. I could cross out some (x+3)'s and I was there.

Teacher: What makes that crossing out a valid step? Amy: Here, I'll show you.

$$\frac{(x+3)(x+2)}{(x+3)(x+3)} = \frac{(x+3)}{(x+3)} \times \frac{(x+2)}{(x+3)}$$
$$= 1 \times \frac{(x+2)}{(x+3)}$$
$$= \frac{x+2}{x+3}$$

Amy: I've expanded out the fraction and put x plus 3 over x plus 3 out on its own. I know that any number over itself is the same as one. so it means the whole thing is one times the answer. And then you don't have to put the brackets on at the end because there's only one bit at the top and one bit at the bottom.

Teacher: Bit?

Amy: Factor