Level 3 – AS91585 – 4 Credits – External

Probability Concepts

Written by J Wills – MathsNZ – jwills@mathsnz.com

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability concepts in solving problems	Apply probability concepts, using relational thinking, in solving problems.	Apply probability concepts, using extended abstract thinking, in solving problems.

Contents

Introduction

This topic covers a number of concepts relating to probability including tables, trees, Venn diagrams and risk. There is a number of new terminology that you will come across in this standard. It is important that you learn what each of these words mean.

This booklet covers all of the content required for the Level 3 Standard. This standard is externally assessed, and is worth 4 credits.

It is worth noting, that in addition to doing this booklet it is extremely worthwhile to work through the past exams which are lined in section 9.

There is an appendix that outlines all of the details of the standard, as well as the related curriculum objectives.

Part 1: Randomness, Theoretical, Experimental and True Probability

With Mathematics and Statistics there are two types of models we can have when looking at situations: **deterministic** and **probabilistic**.

Deterministic models always produce the same result for a particular input. They do not take into account **randomness** and are often expressed as a formula.
An example of this is a formula for compound interest (compounding monthly) is $P\left(1+\frac{r}{12}\right)^n$, where P is the principal, r is the rate of return, and n is the total number of months the money is invested for. If we know what P, r and n are, we can know the amount of money that is in the account. We normally make some assumptions around the situation, for example we are assuming that no other money will be added or removed from the account during the time period.

Probabilistic models take uncertainties into account. This is often done by assigning probabilities to each of the different outcomes. These models take into account **randomness**.
An example of this is flipping a coin. When we flip a coin we can either get heads or tails, and we can work out the probability of each of these events happening.

Randomness is the idea that although each outcome of a process has a fixed probability, the actual outcome of any trial of the process cannot be predicted.

In this paper we focus solely on **probabilistic** models. What we try and do with our **probabilistic** model is estimate the **true probability** of an event occurring.

True probability is the (almost always) unknown actual probability that an event will happen. The **true probability** of a particular coin landing heads up may be affected by the shape of the coin, errors in its manufacture and many other factors, so may not be exactly as we work out.

There are two types of probability we use to estimate the **true probability**… these are: **theoretical probability** and **experimental probability**.

Theoretical probability is what we expect to happen in "theory". If we stay looking at our coin example, we expect that a coin will land on heads 50% of the time, or 0.5. This is a **theoretical** estimate of the **true** probability.

Experimental probability is the probability based on a number of trials or simulations of the event: how many times the event happens divided by the number of trials or simulations we did. The more trials or simulations you do the closer the **experimental probability** will get to the **true probability**, and often it will get close to the **theoretical probability** as well, assuming the theoretical model was a good model. Looking at our coin example, if we took a coin and flipped it 20 times, and 11 of those times it came up with a head, we would have an **experimental probability** of 11/20 or 0.55.

Depending on the situation sometimes it is easier to run a **simulation** rather than doing the trials in real life. This is often done using random number generators on either calculators or computers.

Part 1.1: Matching Definitions

Cut out the cars below and match the definitions to the words:

Deterministic Model	A process or study that results in the collection of data, the outcome of which is unknown.
Experiment	A model that takes uncertainty in outcomes into account. This is often done by associating a probability with each possible outcome.
Experimental Probability	The probability that an event will occur based on a probability model.
Probabilistic Model	The idea that although each outcome of a process has a fixed probability, the actual outcome of any trial of the process cannot be predicted.
Randomness	The actual probability that an event will occur.
Simulation	An estimate of the probability that an event will occur calculated from trials of a probability activity by dividing the number of times the event occurred by the total number of trials.
Theoretical Probability	A technique for imitating the behaviour of a situation that involves elements of chance or a probability activity.
True Probability	A model that will always produce the same result for a given set of input values. Does not include elements of randomness.

Part 1.1: Answers

Deterministic Model	A model that will always produce the same result for a given set of input values. Does not include elements of randomness.
Experiment	A process or study that results in the collection of data, the outcome of which is unknown.
Experimental Probability	An estimate of the probability that an event will occur calculated from trials of a probability activity by dividing the number of times the event occurred by the total number of trials.
Probabilistic Model	A model that takes uncertainty in outcomes into account. This is often done by associating a probability with each possible outcome.
Randomness	The idea that although each outcome of a process has a fixed probability, the actual outcome of any trial of the process cannot be predicted.
Simulation	A technique for imitating the behaviour of a situation that involves elements of chance or a probability activity.
Theoretical Probability	The probability that an event will occur based on a probability model.
True Probability	The actual probability that an event will occur.

Part 2: Probability Distribution Tables and Graphs

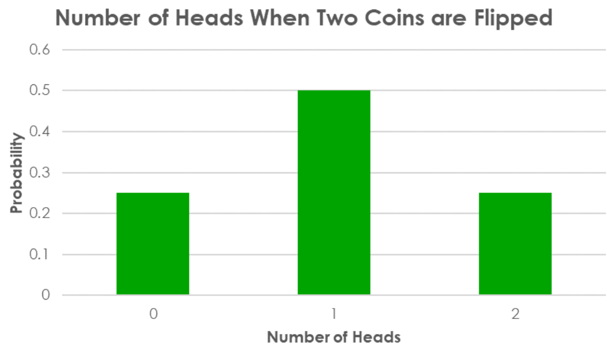
There are a few different ways we can represent probability outcomes. The first two we will look at are a table and a graph. It is worth remembering that probabilities always add up to one.

Let's look at the possible number of heads when two coins are flipped:

We can represent this as a table:

x	0	1	2
P(X=x)	0.25	0.5	0.25

Or as a graph:



Both of these have the same information, just represented in a different way.

Let's look at answering a couple of questions about this data:

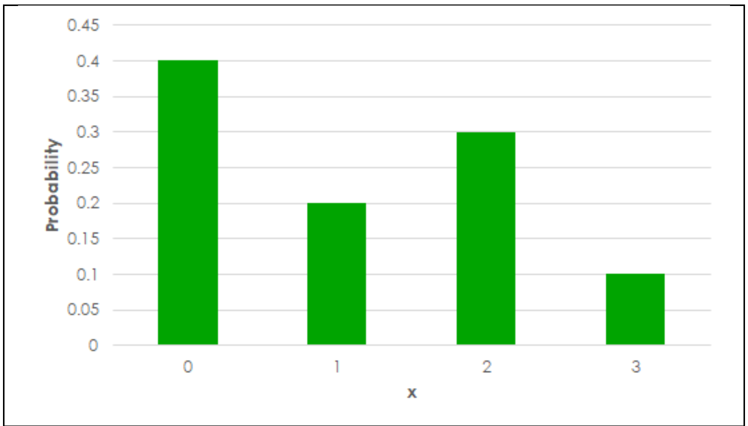
a) What is the probability of getting no heads?
   P(no heads) = 0.25

b) What is the probability of getting exactly 1 head?
   P(1 head) = 0.5

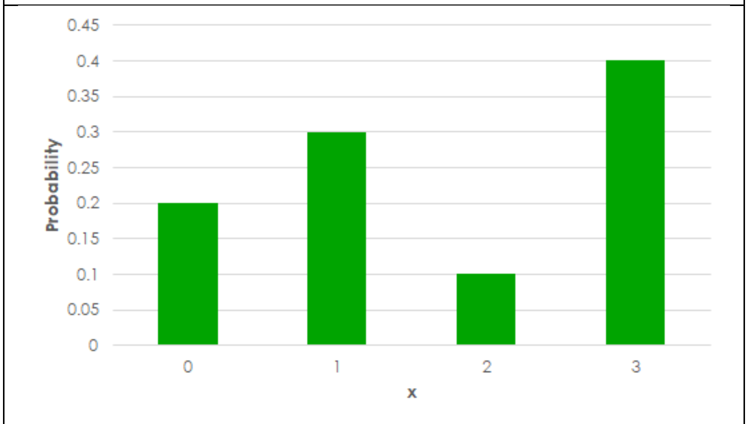
c) What is the probability of getting one or more head?
   P(1 or more heads) = 0.5 + 0.25 = 0.75

Part 2.1: Matching Tables and Graphs

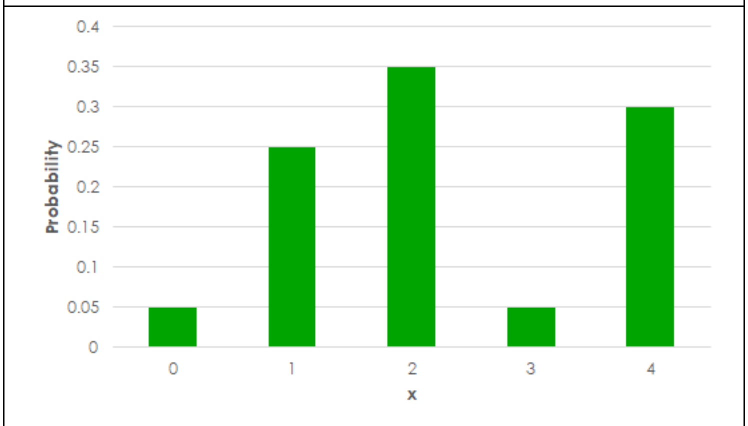
Match the tables and graphs below:



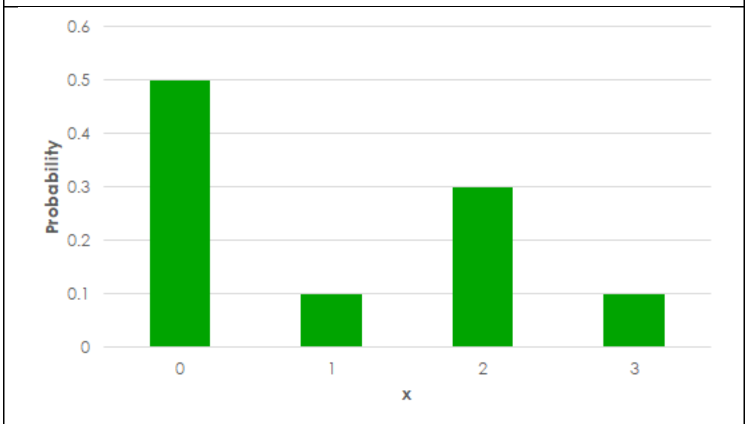
x	0	1	2	3
P(X=x)	0.2	0.3	0.1	0.4



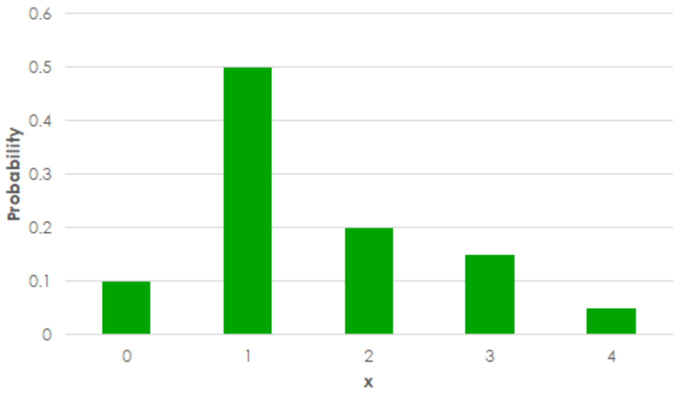
x	0	1	2	3	4
P(X=x)	0.15	0.05	0.3	0.2	0.3



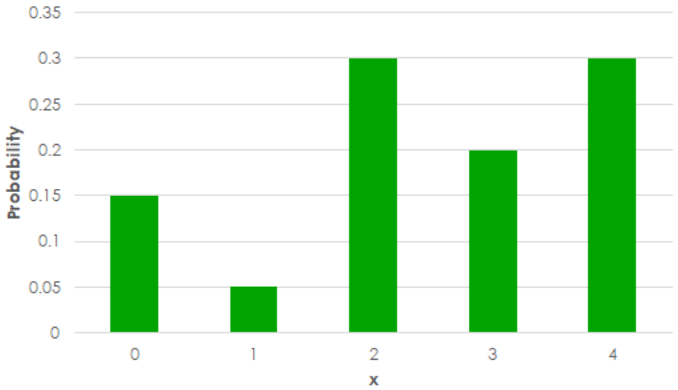
x	0	1	2	3
P(X=x)	0.4	0.2	0.3	0.1



x	0	1	2	3	4
P(X=x)	0.1	0.5	0.2	0.15	0.05

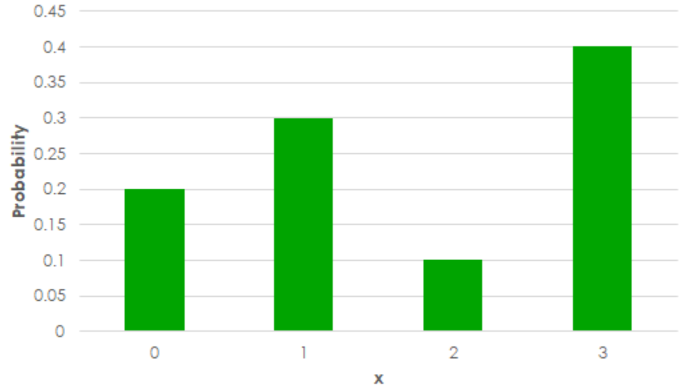


x	0	1	2	3	4
P(X=x)	0.05	0.25	0.35	0.05	0.3

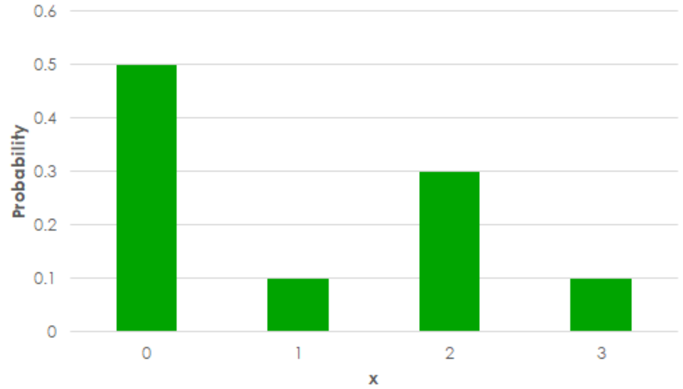


x	0	1	2	3
P(X=x)	0.5	0.1	0.3	0.1

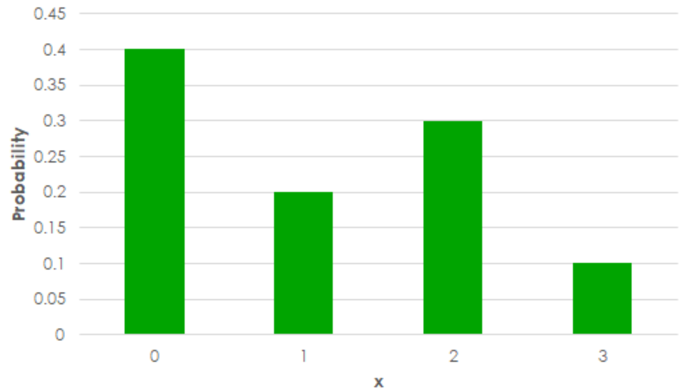
Part 2.1 Answers



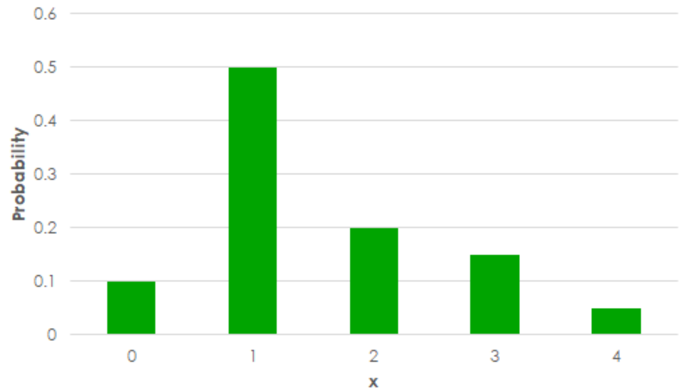
x	0	1	2	3
P(X=x)	0.2	0.3	0.1	0.4



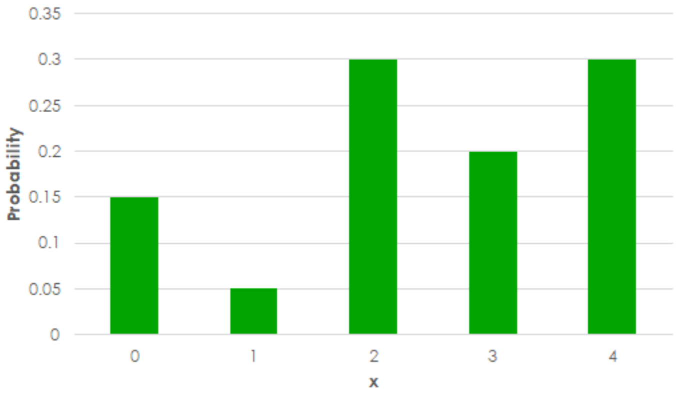
x	0	1	2	3
P(X=x)	0.5	0.1	0.3	0.1



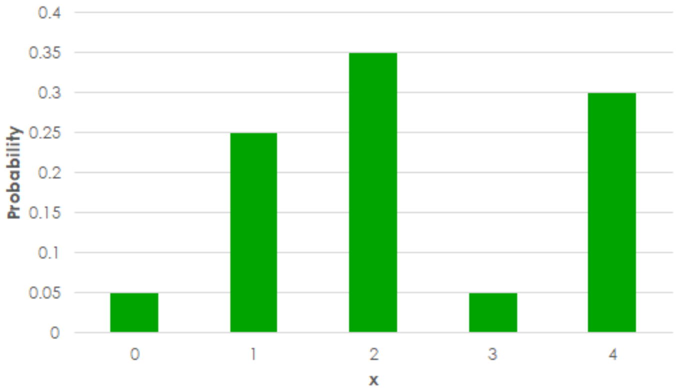
x	0	1	2	3
P(X=x)	0.4	0.2	0.3	0.1



x	0	1	2	3	4
P(X=x)	0.1	0.5	0.2	0.15	0.05



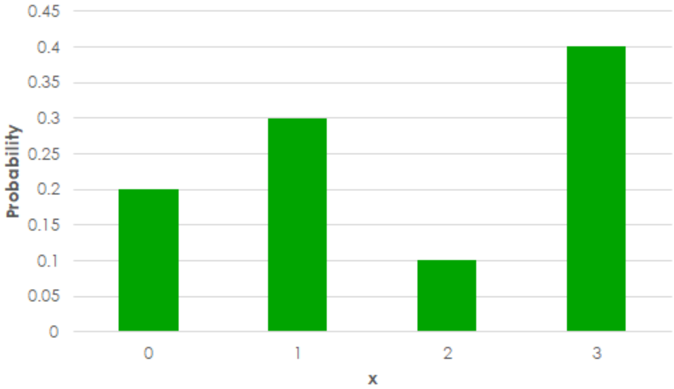
x	0	1	2	3	4
P(X=x)	0.15	0.05	0.3	0.2	0.3



x	0	1	2	3	4
P(X=x)	0.05	0.25	0.35	0.05	0.3

Part 2.2: Reading Probabilities from Tables and Graphs

1. Looking at the graph below, what is the probability



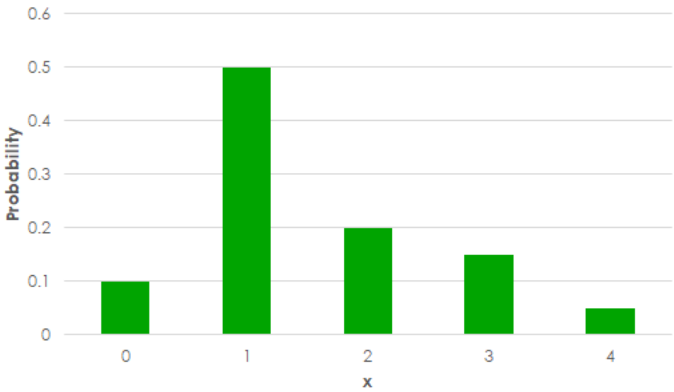
   a. P(X=0)
   b. P(there is exactly one)
   c. There are two or more?

2. Looking at the table below, what is the probability

x	0	1	2	3
P(X=x)	0.5	0.1	0.3	0.1

   a. P(X=1)
   b. P(there is exactly 2)
   c. There is one or less?

3. Looking at the graph below, what is the probability



   a. P(X=3)
   b. P(there is exactly 4)
   c. There is one or more?

4. Looking at the table below, what is the probability

x	0	1	2	3	4
P(X=x)	0.15	0.05	0.3	0.2	0.3

   a. P(X=4)
   b. P(there is exactly 3)
   c. There is 3 or less?

Part 2.2 Answers

1a. 0.2
1b. 0.3
1c. 0.5

2a. 0.1
2b. 0.3
2c. 0.6

3a. 0.15
3b. 0.05
3c. 0.9

4a. 0.3
4b. 0.2
4c. 0.7

Part 2.3: When to Add vs When to Multiply

In probability we are often presented with two events that occur (or could occur). It could be one event followed by another event, or one event happening or another event happening.

When we have:
One event then another event = **multiply**
One event or another event = **add**

For example, I have 10 marbles in a bag,

- 4 labelled A
- 2 labelled B
- 1 labelled C and
- 3 labelled D

We could show this on a probability table:

x	A	B	C	D
P(X=x)	0.4	0.2	0.1	0.3

Whenever I draw a marble out of the bag I always put it back in.

What is the probability I:

a) Pull out marble labelled A then I pull out a marble labelled B?
If we look at the sample space we could have:

		Ball 1			
		A	**B**	**C**	**D**
Ball 2	**A**	AA	**AB**	AC	AD
	B	BA	BB	BC	BD
	C	CA	CB	CC	CD
	D	DA	DB	DC	DD

Looking at that, we can see we just want AB… so P(A then B) = 0.4 x 0.2 = 0.08

b) Pull out one marble and it is labelled A or B?
Looking at the sample space we just have A, B, C and D
We want two of these, so we add them together.
P (A or B) = 0.4 + 0.2 = 0.6

Part 2.4: When to Add vs When to Multiply Exercise

1. I have 10 marbles in a bag. They all have a letter on them, either A, B, C or D.

x	A	B	C	D
P(X=x)	0.4	0.2	0.1	0.3

After drawing a ball out and looking at it, I always return it to the bag.
   a. What is the probability of pulling out a ball labelled C then a ball labelled D?
   b. What is the likelihood of pulling out a ball labelled C or a ball labelled D?
   c. Calculate the probability of pulling ball C out twice in a row.
   d. What is the probability you pull out the same letter twice in a row?

2. I have a number of 4 sided dice with numbers 1 to 4 on them. What is the probability:
   a. I get a total of 2 when I throw two dice (both land on 1)?
   b. I get 3 or 4 when I throw once die?
   c. I get a total of 7 when I throw two dice? (either 3 and 4, or 4 and 3)
   d. I get a total of 6 when I throw two dice?

3. The number of spas sold during a week at a successful spa shop is shown in the table below.

x	0	1	2	3	4
P(X=x)	0.1	0.4	0.3	0.1	k

   a. What is the missing value (k)?
   b. What is the probability of selling 2 or 3 spas in a week?
   c. What is the probability of selling 2 spas in two consecutive weeks?
   d. What is the probability of selling no spas 3 weeks in a row?

4. A student has gone into a multi choice test without doing any preparation. There are two questions he has no idea on, therefore needs to guess the answer to the questions.
One has 5 possible answers, with two of them being correct (only needs to choose one of the correct answers), and the other one has 4 possible answers, with only one of the answers being correct.
What is the probability:
   a. In the question with 5 possible answers, they get it correct?
   b. They get both questions correct?
   c. The get one of the two questions correct?

Part 2.4 Answers

1a. 0.03
1b. 0.4
1c. 0.01
1d. 0.3

2a. 0.0625
2b. 0.5
2c. 0.125
2d. 0.1875

3a. 0.1
3b. 0.4
3c. 0.09
3d. 0.001

4a. 0.4
4b. 0.1
4c. 0.45

Part 3: Two Way Tables

Often in statistics we look at data where we know information about two (or more variables). We often represent this using a two way table like this:

	Number of Students		
Eye Colour	**Male**	**Female**	**Total**
Brown	6	7	13
Blue	4	3	7
Total	10	10	20

We can show the same data as percentages or decimals

	Number of Students		
Eye Colour	**Male**	**Female**	**Total**
Brown	30%	35%	65%
Blue	20%	15%	35%
Total	50%	50%	100%

	Number of Students		
Eye Colour	**Male**	**Female**	**Total**
Brown	0.3	0.35	0.65
Blue	0.2	0.15	0.35
Total	0.5	0.5	1

All three of these tables have the same information, just written in a slightly different way.

This means we can then ask some questions like:

a. What percentage of the students are male?
   We can see the total for the male column is 50%. We could also work this out as 10/20.

b. What is the probability if choose a student at random they have blue eyes?
   We can see the total for the blue eyes column is 0.35. We could also work this out as 7/20.

c. How many female students are there with brown eyes?
   We can see the box that is in the brown row and the female column has 7 in it.

You will notice that for all of the rows, and all of the columns, the total row is just what the different parts add up to. This means if we are given a table with missing values we can often work out the missing values. For example if I was given this table:

	Number of Students		
Eye Colour	**Male**	**Female**	**Total**
Brown	0.3	0.35	*d*
Blue	0.2	*f*	*e*
Total	*b*	*c*	*a*

I could work out all the missing values:

a. This is 1, as probabilities always add up to 1.
b. This is 0.5, as 0.3 + 0.2 = 0.5
c. This is 0.5 as well, as b and c need to add to make 1.
d. This is 0.65 as 0.3 + 0.35 = 0.65
e. This is 0.35, as d and e need to add up to 1.
f. This is 0.15, as 0.2 and e need to make 0.35.

Occasionally you are given tables without a total column. The first thing you should do if this is the case is add the total column on.

Part 3.1: Tables with Proportions or Percentages

1. This table shows data from the 2013 Census for New Zealand, and the percentage of males and females in each ethnicity group.

	Number of People		
Ethnicity	**Male**	**Female**	**Total**
European	33.0%	36.4%	69.4%
Maori	5.4%	6.1%	11.5%
Pacific Peoples	2.7%	2.9%	5.5%
Asian	5.2%	5.7%	10.9%
Middle Eastern/Latin American/African	0.5%	0.5%	1.0%
Other ethnicity	0.9%	0.7%	1.6%
Total	47.7%	52.3%	100.0%

data extracted on 05 Aug 2018 from NZ.Stat

What is the percentage of New Zealanders are:

a. European?
b. Male?
c. European or Maori?
d. Asian and a Female?
e. A male and either Asian or Middle Eastern/Latin American/African

2. This table shows data from the 2013 Census for New Zealand, and the proportion of males and females and how long they have been living in their current residence.

	Number of People		
Years at Residence	**Male**	**Female**	**Total**
0 years	0.109	0.113	0.223
1-4 years	0.145	0.154	0.300
5-9 years	0.101	0.106	0.208
10-14 years	0.053	0.056	0.109
15-29 years	0.055	0.060	0.115
30 years or more	0.022	0.025	0.047
Total	0.486	0.514	1.000

data extracted on 05 Aug 2018 from NZ.Stat

What is the proportion of New Zealanders:

a. Are female?
b. Have lived at their current residence for 10-14 years?
c. Are male and lived at their residence for less than 1 year?
d. Are female and lived in their residence for 30 or more years?
e. Have lived at their current residence for less than 10 years?

3. The table below shows the number of offences recorded during 2014 in New Zealand. Note: some values are deliberately missing from the table.

	Number of Offences		
Offence	Unresolved	Resolved	Total Recorded
Homicide and related offences	0.0000	0.0002	0.0002
Acts intended to cause injury	0.0315	0.0825	
Sexual assault and related offences	0.0061	0.0055	0.0116
Dangerous or negligent acts endangering persons	0.0009	0.0012	0.0021
Abduction, harassment and other related offences against a person	0.0187	0.0242	0.0429
Robbery, extortion and related offences	0.0037	0.0025	
Unlawful entry with intent/burglary, break and enter	0.1336	0.0184	0.1520
Theft and related offences	0.2667	0.0738	
Fraud, deception and related offences	0.0130	0.0128	0.0258
Illicit drug offences	0.0043	0.0429	0.0472
Prohibited and regulated weapons and explosives offences	0.0018	0.0148	0.0165
Property damage and environmental pollution	0.0826	0.0326	0.1152
Public order offences		0.0607	0.0763
Offences against justice procedures, government security and government operations	0.0053	0.0406	0.0459
Miscellaneous offences	0.0015	0.0021	0.0036
Total	0.5851	0.4149	1.0000

data extracted on 05 Aug 2018 from NZ.Stat

1. Fill in the blanks in the table
2. What proportion of offences from 2014 were:
   i. Unresolved?
   ii. Acts intended to cause injury?
   iii. Either theft and related offences or robbery, extorsion and related offences?
   iv. Unresolved public order offences?

4. This table shows data from the 2012 New Zealand General Social Survey (NZGSS), and the percentage of people who voted in the most recent general election, and the most recent local body election. Note: some values are deliberately missing from the table.

	Voted in General Election		
Voted in Local Body Election	Yes	No	Total
Yes	61.1%		
No		18.9%	
Total	78.7%		

data extracted on 05 Aug 2018 from NZ.Stat

a. Fill in the blanks in the table
b. What percentage of New Zealanders:
   i. Voted in the most recent general election?
   ii. Voted in both the most recent local body and general elections?
   iii. Voted in at least one of the elections?
   iv. Voted in neither of the elections?

Part 3.1 Answers

1a. 69.4%
1b. 47.7%
1c. 80.9&
1d. 5.7%
1e. 5.7%

2a. 0.514
2b. 0.109
2c. 0.109
2d. 0.025
2e. 0.731

3a.

	Number of Offences		
Offence	Unresolved	Resolved	Total Recorded
Homicide and related offences	0.0000	0.0002	0.0002
Acts intended to cause injury	0.0315	0.0825	0.1140
Sexual assault and related offences	0.0061	0.0055	0.0116
Dangerous or negligent acts endangering persons	0.0009	0.0012	0.0021
Abduction, harassment and other related offences against a person	0.0187	0.0242	0.0429
Robbery, extortion and related offences	0.0037	0.0025	0.0062
Unlawful entry with intent/burglary, break and enter	0.1336	0.0184	0.1520
Theft and related offences	0.2667	0.0738	0.3405
Fraud, deception and related offences	0.0130	0.0128	0.0258
Illicit drug offences	0.0043	0.0429	0.0472
Prohibited and regulated weapons and explosives offences	0.0018	0.0148	0.0165
Property damage and environmental pollution	0.0826	0.0326	0.1152
Public order offences	0.0156	0.0607	0.0763
Offences against justice procedures, government security and government operations	0.0053	0.0406	0.0459
Miscellaneous offences	0.0015	0.0021	0.0036
Total	0.5851	0.4149	1.0000

3bi. 0.5851
3bii. 0.1140
3biii. 0.3467
3biv. 0.0156

4a.

	Voted in General Election		
Voted in Local Body Election	Yes	No	Total
Yes	61.1%	2.4%	63.5%
No	17.6%	18.9%	36.5%
Total	78.7%	21.3%	100.0%

4bi. 78.7%
4bii. 61.1%
4biii. 81.1%
4biv. 18.9%

Part 3.2: Tables with Numbers

1. This table shows data from the 2013 Census for New Zealand, and the number of males and females in each age group.

	Number of People		
Age in Years	Male	Female	Total
0-14	444,624	425,865	870,489
15-29	438,294	441,135	879,429
30-44	403,485	445,758	849,243
45-59	424,254	457,005	881,259
60-74	295,242	312,285	607,527
75+	113,571	151,686	265,257
Total	2,119,470	2,233,734	4,353,204

data extracted on 05 Aug 2018 from NZ.Stat

How many New Zealanders:
a. Are aged 0-14?
b. Are male?
c. Are female and aged 15-29?
What proportion of New Zealanders:
d. Are under 30?
e. Are 60 or over and female?

2. This table shows data from the 2013 Census for New Zealand, and the number of children born alive to each women based on their age group.

	Age of Woman in Years			
Number of children	15-29	30-64	65+	Total
No children	299,919	165,036	26,124	491,079
One child	43,134	136,182	19,827	199,143
Two children	24,408	313,971	81,231	419,610
Three children	8,328	179,205	78,162	265,695
Four children	2,502	69,615	46,158	118,275
Five children	786	24,138	20,061	44,985
Six or more children	369	18,057	18,543	36,969
Object to answering	5,952	29,031	10,587	45,570
Total	385,398	935,235	300,693	1,621,326

data extracted on 05 Aug 2018 from NZ.Stat

How many New Zealand women:
a. Have no children?
b. Are aged over 65?
c. Have five or more children?
What proportion of New Zealand women:
d. Are under 30 and have one child?
e. Are 65 or over and have no children?

3. This table shows data from the 2012 New Zealand General Social Survey (NZGSS), and the number of people who have high and low life satisfaction based on their family type. Note:

	Number of People			
Overall life satisfaction	Couple without child(ren)	Couple with child(ren)	One parent with child(ren)	Not in a family nucleus
Low	133,000	210000	107,000	165,000
High	922,000	1,205,000	245,000	541,000
Total	1,055,000	1,415,000	352,000	706,000

data extracted on 05 Aug 2018 from NZ.Stat

a. Add a total column to the right of the table
b. How many New Zealanders:
   i. Have high life satisfaction?
   ii. Are not in a family nucleus?
   iii. Have high life satisfaction and are a couple without children?
   iv. Have high life satisfaction and are not in a family nucleus?

4. The table below shows data from 2014, and the number of fatal and serious non-fatal injuries by sex, and severity of injury.

	Number of Injuries		
Severity of injury	Male	Female	Total
Fatal	1,125		1,797
Serious non-fatal			
Total		5,295	11,295

data extracted on 05 Aug 2018 from NZ.Stat

a. Fill in the blanks in the table
b. How many injuries:
   i. Were on a male?
   ii. Were fatal?
   iii. Were on a male and fatal?
   iv. Were on a female and serious (non-fatal)?

Part 3.2 Answers

1a. 870,489
1b. 2,119,470
1c. 441,135
1d. 1,749,918/4,353,204 = 0.402 (3sf)
1e. 463,971/4,353,204 = 0.107 (3sf)

2a. 491,079
2b. 300,693
2c. 81,954
2d. 43,134/1,621,326 = 0.0266 (3sf)
2e. 26,124/1,621,326 = 0.0161 (3sf)

3a.

Overall life satisfaction	Couple without child(ren)	Couple with child(ren)	One parent with child(ren)	Not in a family nucleus	Total
Low	133,000	210000	107,000	165,000	615,000
High	922,000	1,205,000	245,000	541,000	2,913,000
Total	1,055,000	1,415,000	352,000	706,000	3,528,000

3bi. 2,913,000
3bii. 706,000
3biii. 922,000
3biv. 541,000

4a.

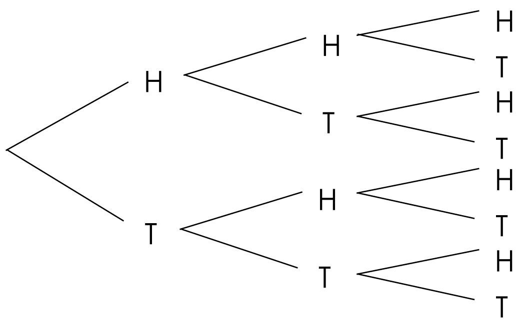
	Number of Injuries		
Severity of injury	Male	Female	Total
Fatal	1,125	672	1,797
Serious non-fatal	4,875	4,623	9,498
Total	6,000	5,295	11,295

4bi. 6,000
4bii. 1,797
4biii. 1,125
4bvi. 4,623

Part 4: Probability Trees

Probability trees are great for when you have more than one event which follow on from each other. There are two types of tree diagrams: one where all the probabilities are equal, and one where they can change. In Level 3 we will usually get situations where the probabilities change.

Example 1 (Probabilities all the Same):
I flip three coins. For each coin we can either get heads (H) or tails (T). The tree would look like this:



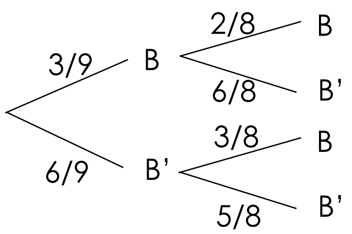
The first split represents the first coin, the second split the second coin, and so on.
Because all the probabilities are the same, and there are 8 different branches, we know each one has a 1/8 chance of occurring. Let's answer some questions about this tree:

a. What is the probability of getting 3 heads?
   There is only one branch with 3 heads (HHH) so would be 1/8

b. What is the probability of getting 2 heads?
   We can get two heads as either HHT, HTH or THH, so there are three ways, so 3/8

c. What is the probability of getting 1 or fewer heads?
   We could have TTT, TTH, THT or HTT, so 4/8 or 1/2

Example 2 (Probabilities Change):
The letters in BUMBLEBEE are written on 9 balls and put into a bag. Two are drawn out, and they are either a "B" or not a "B". In probability we will often write the opposite of an event as the event with a ' (pronounced dash). So in this case we have B and B' (B-dash).The tree would look like this:



The 3/9 on the first branch comes from there being 3 "B" balls in the bag. The 6/9 comes from the fact each branch always needs to add to one.

If we take out one "B" ball, there are only 2 left out of the 8 in total, so we get 2/8 for the top branch on the second, and likewise we get 3/8 for the top branch coming off B'.

Let's answer some questions about this situation:

a. What is the probability of getting two "B"s?
   As it is one event, then another, we times as we go along, so 3/9 x 2/8 = 1/12

b. What is the probability of getting exactly one "B"?
   This can be BB' or B'B, so would be 3/9 x 6/8 + 6/9 x 3/8 = 1/2

c. What is the probability of getting at least one "B"?
   This is either 1 "B" or two "B"s, so the answer from a, plus the answer from b = 1/12 + 1/2 = 7/12

Tip: always draw the tree diagram (or the part you need) you are much less likely to make mistakes!

Part 4.1: Probability Trees

1. I have a standard pack of 52 cards.
    a. If I draw two cards out what is the probability of them both being black?
    b. If I draw three cards out what is the probability of them all being black?
    c. If I draw two cards out, how likely is it they are both a face card (King, Queen or Jack)?

2. I have one four sided (numbered 1-4) and one six sided (numbered 1-6) dice. What is the probability when I roll them:
    a. They are both 1?
    b. They are both even?
    c. They are both the same number?

3. I flip 4 coins. Fill in the blanks contingency table below for the number of heads

Number of heads (x)	0	1	2	3	4
P(X=x)	0.0625				

4. Jane and Vikki are playing each other in tennis. The probability that Jane wins the first set is 0.5. If Jane wins a set the probability she wins the next set is 0.6, otherwise it is 0.35. In a women's tennis game you play the best of three sets, so you stop playing once one player has won two sets. What is the probability:
    a. Jane wins the first two sets?
    b. The game goes to three sets?
    c. Jane wins the game?

5. Two students are chosen at random from a class of 12 boys and 13 girls to be office messengers / runners for the school for the day. What is the probability:
    a. They are both boys?
    b. They are both girls?
    c. At least one of the students is a girl?

6. A company manufactures a jellybean game where each colour of jellybean has two possible flavours, one that tastes nice, and the other that is not. In the packet there are 10 green jelly beans, 5 of them are green apple flavoured, and the other 5 are snot flavoured.
    a. If I eat 3 green jellybeans, what is the probability they are all green apple flavoured?
    b. If I eat 2 green jellybeans, what is the probability they are all green apple flavoured?
    c. If I eat 5 green jellybeans, what is the probability I get no snot flavoured jellybeans?

7. In a particular game show, children are given the chance to choose between a valuable prize and one that is quite tempting for the kids, for example a stuffed toy vs a coffee maker, or a family cruise vs a ride on car. They do this while their parents are watching from another room, and the children can't see or hear what the parents are saying. The children get three pairs of prizes to choose from, with the value of the expensive prize increasing each time. The probability the child chooses the expensive prize for each round is 0.35, 0.3 and 0.4 for each round. What is the probability the child chooses:
    a. No expensive prizes?
    b. Exactly one expensive prize?
    c. Two or more expensive prizes?

8. Whenever a particular developer releases an update to their website, the probability of their being a bug is 0.3. What is the probability:
    a. In three updates there are bugs in all of them?
    b. In two updates there is a bug in exactly one of them?
    c. In ten updates there are no bugs?

Part 4.1 Answers

1a. 25/102 = 0.245 (3sf)
1b. 2/17 = 0.118 (3sf)
1c. 11/221 = 0.0498 (3sf)

2a. 1/24 = 0.0417 (3sf)
2b. 1/4 = 0.25
2c. 1/6 = 0.167 (3sf)

3.

Number of heads (x)	0	1	2	3	4
P(X=x)	0.0625	0.25	0.375	0.25	0.0625

4a. 0.3
4b. 0.375
4c. 0.475

5a. 11/50 = 0.22
5b. 13/50 = 0.26
5c. 39/50 = 0.78

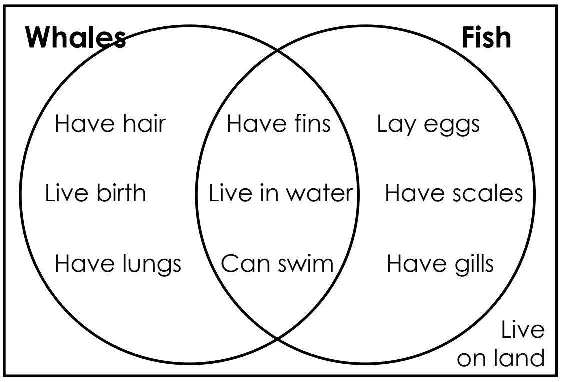
6a. 1/12 = 0.0833 (3sf)
6b. 2/9 = 0.222 (3sf)
6c. 1/252 = 0.00397 (3sf)

7a. 0.273
7b. 0.446
7c. 0.281

8a. 0.027
8b. 0.42
8c. 0.0282 (3sf)

Part 5: Venn Diagrams

Venn diagrams (named after John Venn) are a way of representing the different sets of information. They can be used in situations other than probability for example:



We can see:

- The things that apply to whales only are having hair, live birth and having lungs.
- The things that apply only to fish are laying eggs and having scales and gills.
- The things that apply to both are having fins, living in water and being able to swim
- Living on land doesn't apply to either fish or whales.

We can also use Venn diagrams to represent probabilities.

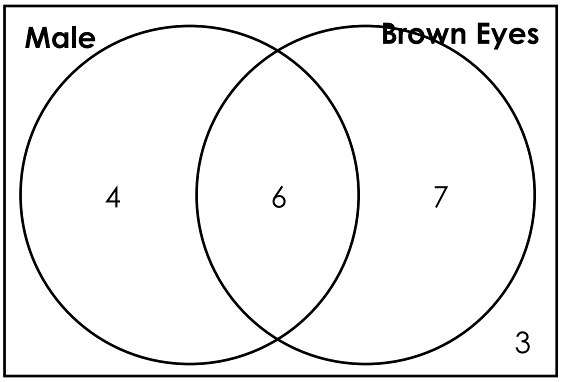
Let's look at this table:

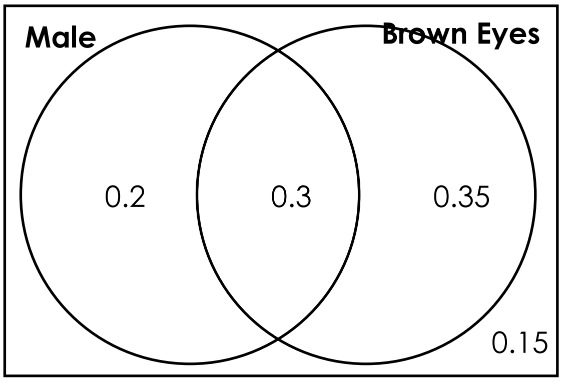
	Number of Students		
Eye Colour	**Male**	**Female**	**Total**
Brown	6	7	13
Blue	4	3	7
Total	10	10	20

We could change the headings to:

	Number of Students		
Eye Colour	**Male**	**Not Male**	**Total**
Brown	6	7	13
Not Brown	4	3	7
Total	10	10	20

And then we can show this in the Venn diagram, either as numbers or as probabilities. It is important to remember the numbers must always add up to the total, and the probabilities must add to one.





We can see the data matches what is in the table:

- 6 students are male and have brown eyes
- 4 students are male but don't have brown eyes
- 7 students have brown eyes but aren't male
- 3 students aren't male and don't have brown eyes
- In the male circle there is 10 students in total (6+4)
- In the brown eye circle there is 13 in total (6+7)

This means we can then ask some questions like:

a. What percentage of the students are male?
   We can see the total for the male circle is 10 of the 20 in total, so this gives us 10/20 = 50% (or using the probability Venn diagram we could do 0.2 + 0.3).

b. What is the probability if choose a student at random they have brown eyes?
   We can see the total for the brown eyes circle is 13. There are 20 students in total, so this gives us 13/20 or 0.65. (or using the probability Venn diagram we could do 0.3 + 0.35)

Part 5.1: Venn Diagrams Exercise 1

1. An insurance company does a survey of their customers (all of whom have either car or house insurance). They find the proportion of customers that have house insurance is 0.7, and the proportion that have both house and car insurance is 0.6. What is the proportion that:
   a. Just have car insurance?
   b. Have car insurance?
   c. Don't have car insurance?

2. When customers come into a dairy, 45% of them buy milk and 55% of them buy bread. 20% of them buy neither bread nor milk. What percentage of customers:
   a. Buy both bread and milk?
   b. Do not buy milk?
   c. Buy bread or milk but not both?

3. In a survey done of students at a school, 0.3 were found to have iPhones and 0.45 were found to have Chromebooks. 0.2 had a Chromebook but did not have an iPhone. What proportion of students:
   a. Did not have an iPhone?
   b. Had neither an iPhone nor a Chromebook?
   c. Had both a Chromebook and an iPhone?

4. In 2014, the number of fatal injuries in New Zealand was 1,797. Of these 1,125 were males. Males also were involved in 4,875 serious (but non-fatal) injuries. In total there were 11,295 serious or fatal injuries in 2014.
   a. How many males were injured in total?
   b. How many serious (non-fatal) injuries were there in 2014?
   c. How many of the serious (non-fatal) injuries were not male?

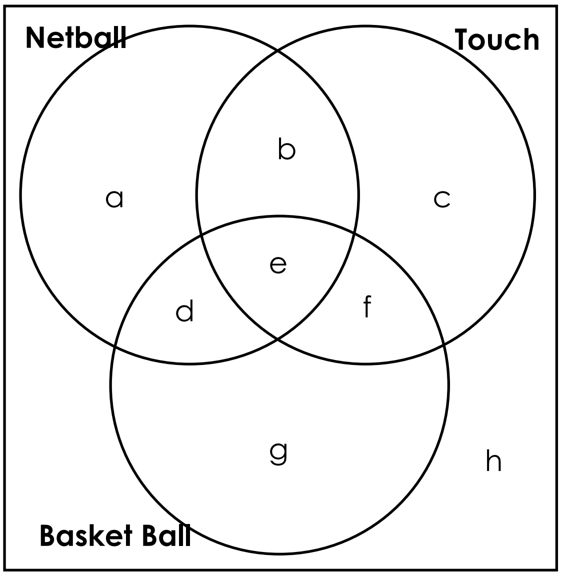
Part 5.1 Answers

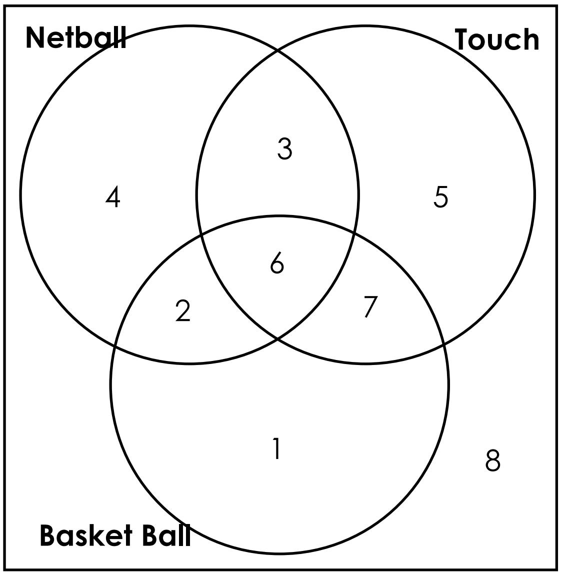
1a. 0.3	2a. 20%	3a. 0.7	4a. 6,000
1b. 0.9	2b. 55%	3b. 0.5	4b. 9,498
1c. 0.1	2c. 60%	3c. 0.25	4c. 4,623

Part 5.2: Three Way Venn Diagrams

Venn diagrams do not always have 2 circles in them. They can have 3 (or more).

Let's look at an example with 3 circles:





We are given the following information about what sports are played at a school:

- 6 students play netball, touch and basketball
- 8 students play netball and basketball
- 13 students play touch and basketball
- 18 students play more than 1 sport
- 15 students play netball
- 27 students netball and or touch
- 8 students don't play anything
- There are 36 students in total

Let's start by completing the Venn diagram:

- e = 6 (as 6 students play all 3 sports
- d = 2 (as 8 students play netball and basketball)
- f = 7 (as 13 students play touch and basketball
- b = 3 (as 18 students play more than 1 sport)
- a = 4 (as 15 students play netball)
- c = 5 (as 27 students play netball and or touch)
- h = 8 (as 8 students don't play anything)
- g = 1 (as there are 36 students in total)

In this example we could work through the information in order to fill in all the blanks. Often you will need to pick and choose which pieces of information you can use in order to be able to fill in the whole diagram.

Let's answer some questions about this Venn diagram:

a. How many students play exactly 2 sports?
   There are 2 + 3 + 7 = 12 students

b. How many students play only 1 sport?
   There are 4 + 5 +1 = 10 students

c. What is the probability a randomly chosen student plays no sports?
   There are 8 students out of 36 in total, so 8/36 or 0.222 (3sf)

Part 5.3: Venn Diagrams Exercise 2

1. At a restaurant one night 25 groups of people come through.
    - All of them order something
    - None of them just order entrees
    - 2 groups have entrees, mains and desserts
    - 8 have both entrees and mains
    - 12 have entrees
    - 3 have both mains and desserts
    - 12 have desserts

    a. How many groups have mains?
    b. How many groups have entrees and desserts?
    c. What is the probability a randomly chosen group has exactly 2 courses?

2. A car dealership sells cars.
    - 55% of them have tow bars
    - 45% of them are manual
    - 40% of them have sunroofs
    - 30% of them are manuals with tow bars
    - 20% don't have either a tow bar or a sun roof and 20% are manuals with sunroofs
    - 15% of them have a tow bar and a sun roof
    - 10% of them are manuals with both a tow bar and a sun roof

    What percentage of cars:
    a. Are not manual but have a tow bar?
    b. Do not have a sunroof but do have a tow bar?
    c. Are not manual and do not have a sun roof or tow bar?

3. Fifty students in a year level are asked if they have been to Africa, North America and Asia.
    - The same number of students have been to just Africa as have been to just North America
    - Double the number of students have been to just Asia as have been to just Africa
    - The same number of students have been to both North America and Asia as have not been to any of the three continents.
    - 23 students have been to more than 1 of the continents.
    - 8 students have been to all 3 continents
    - 2 more students have been to Africa and Asia than have been to Africa and North America
    - 19 students have been to Africa
    - 11 students have travelled to North America and Africa

    What is the probability student chosen at random
    a. Has travelled to all 3 continents?
    b. Has only Africa?
    c. Has travelled to Africa and Asia?

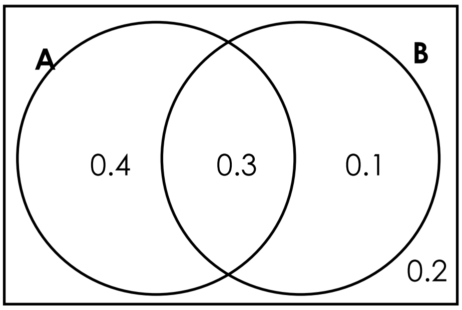
Part 5.3 Answers

1a. 16	2a. 25%	3a. 0.16
1b. 6	2b. 40%	3b. 0.38
1c. 0.44	2c. 15%	3c. 0.26

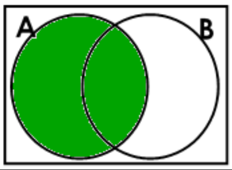
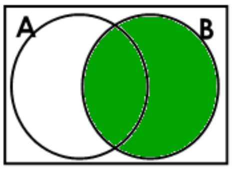
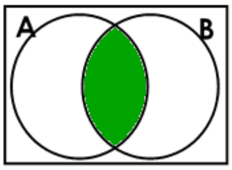
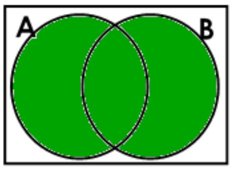
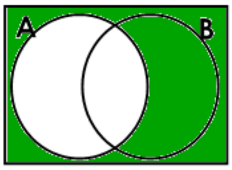
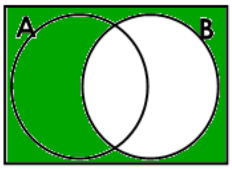
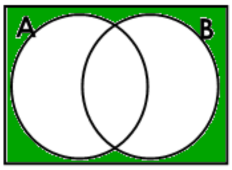
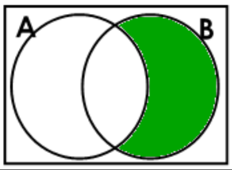
Part 5.4: Venn Diagrams Terminology

When looking at Venn diagrams (or probability in general) we have a few different ways of representing information.

Let's use this Venn diagram to help us look at things:

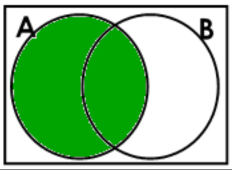
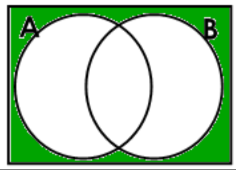
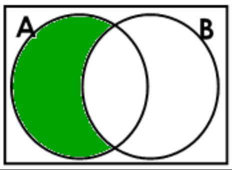
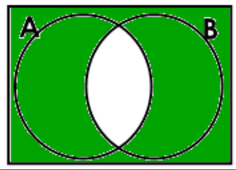
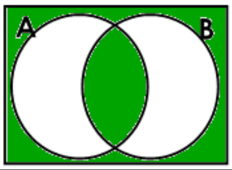
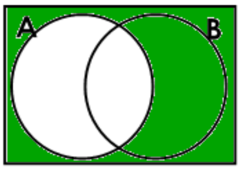
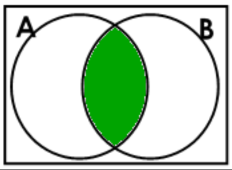
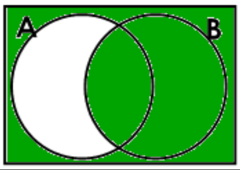
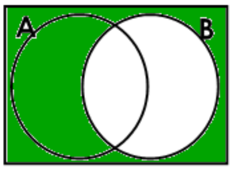
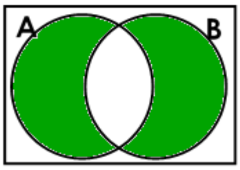
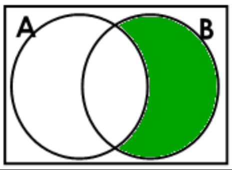
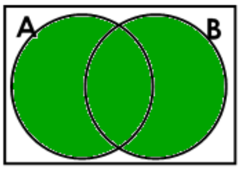
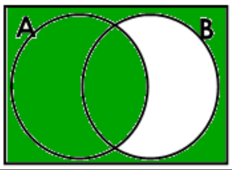
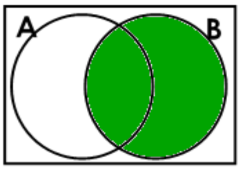


Below are core terminologies, and the last two are just a couple of ways of combining them. They can be combined in a huge number of different ways.

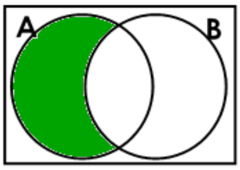
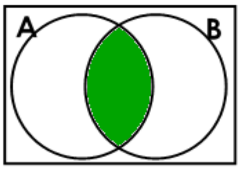
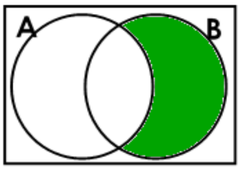
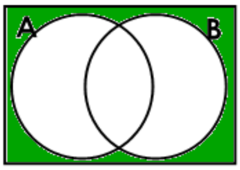
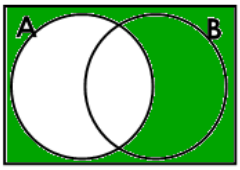
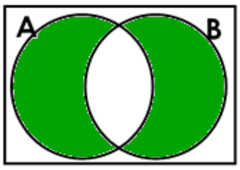
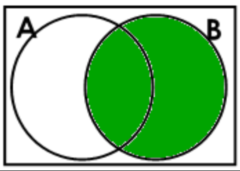
Probability Way of Writing It	How you say it	What it means	Shaded Venn Diagram	Probability
P(A)	Probability of A	Probability of A occurring		0.4 + 0.3 = 0.7
P(B)	Probability of B	Probability of B occurring		0.1 + 0.3 = 0.4
P(A ∩ B)	Probability of A intersection B	Probability of both A and B occurring		0.3
P(A U B)	Probability of A union B	Probability of A or B or both occurring		0.4 + 0.3 + 0.1 = 0.8
P(A')	Probability of A dash	Probability of A NOT occurring		0.1 + 0.2 = 0.3 (or 1 – 0.7)
P(B')	Probability of B dash	Probability of B NOT occurring		0.4 + 0.2 = 0.6 (or 1 – 0.4)
P(A U B)'	Probability of A union B dash	Probability of A or B or both NOT occurring		0.2 (or 1 – 0.8)
P(A' ∩ B)	Probability of A dash intersection B	Probability of both A NOT occurring and B occurring		0.1

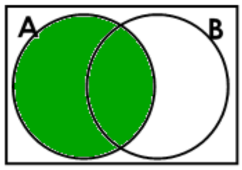
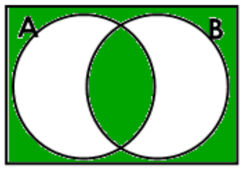
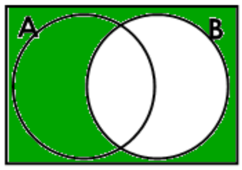
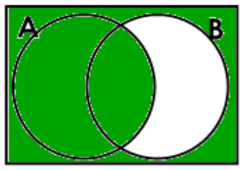
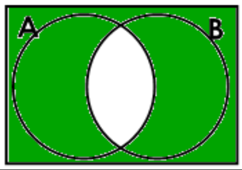
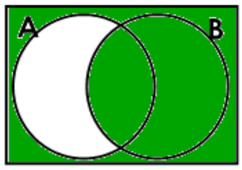
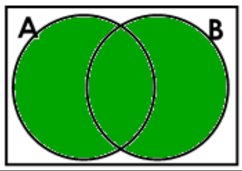
Part 5.5: Venn Diagrams Terminology Exercise

Match the diagrams with the probability way of writing it.

P(A ∩ B')		P(A)	
P(A ∩ B)		P((A ∩ B) U (A' ∩ B'))	
P(A' ∩ B)		P(B')	
P(A U B)'		P(A U B')	
P(A')		P(A ∩ B)'	
P((A ∩ B') U (A' ∩ B))		P(A' U B)	
P(B)		P(A U B)	

Part 5.5 Answers

P(A ∩ B’)	
P(A ∩ B)	
P(A’ ∩ B)	
P(A U B)’	
P(A’)	
P((A ∩ B’) U (A’ ∩ B))	
P(B)	

P(A)	
P((A ∩ B) U (A’ ∩ B’))	
P(B’)	
P(A U B’)	
P(A ∩ B)’	
P(A’ U B)	
P(A U B)	

Part 6: Complementary, Independent and Mutually Exclusive Events

There are three other pieces of terminology that we need to know that relate to types of events in probability. These are **complementary**, **independent** and **mutually exclusive** events

Complementary events are events that are the opposite to each other. For example heads and tails on a coin, or A and A'.

Independent events are events where the outcome of one does do not affect the probability of the other. There is a statistical formula to check if two events are independent, this is

$P(A) \times P(B) = P(A \cap B)$

Example 1: are a "day being sunny" and a "day being a weekday" **independent**?

To check if these events are **independent** or not we can check if the formula $P(A) \times P(B) = P(A \cap B)$ holds true.
P(A) = P(Day is a Weekday) = 5/7
P(B) = P(Day is Sunny) = 1/5
P(A ∩ B) = P(Weekday ∩ Sunny) = **1/7**
P(A) x P(B) = 5/7 x 1/5 = **1/7**
Therefore they are **independent**.

	Day is Sunny		
Day is a Weekday	**Yes**	**No**	**Total**
Yes	1/7	4/7	5/7
No	2/35	8/35	2/7
Total	1/5	4/5	1

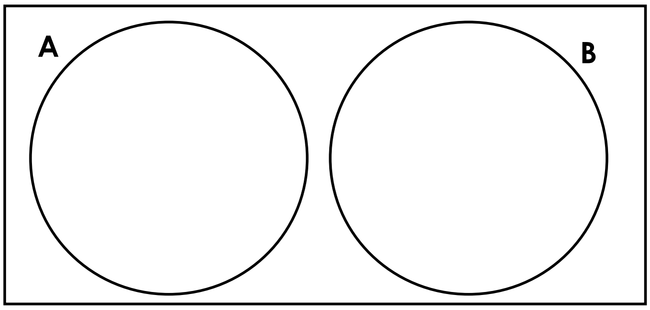
Example 2: are a "student having brown eyes" and a "student being female" **independent**?

We check if P(A) x P(B) = P(A ∩ B) is true.
P(A) = P(Has Brown Eyes) = 13/20
P(B) = P(Is Female) = 10/20
P(A ∩ B) = P(Brown Eyes ∩ Female) = **7/20**
P(A) x P(B) = 13/20 x 10/20 = **13/40**
$7/20 \neq 13/40$
Therefore they are **not independent**.

	Number of Students		
Eye Colour	**Male**	**Female**	**Total**
Brown	6	7	13
Blue	4	3	7
Total	10	10	20

Mutually Exclusive events are events that cannot both happen. **Complementary** events must be mutually exclusive. For **mutually exclusive** events P(A ∩ B) = 0

This means if you were to draw a Venn diagram for it the circles would not overlap:



Example 3:
A student could be in a Maths class at the moment or an English class, but couldn't be both in an English and a Maths class at the same time (but could be in another class like Science) If we were to look at this table:

	Currently in English		
Currently In Maths	**Yes**	**No**	**Total**
Yes	0	25	25
No	27	23	50
Total	27	48	75

We can see these events are **mutually exclusive** as P(English and Maths) = 0

Part 6.1: Complementary, Complementary, Independent and Mutually Exclusive Events Exercise

1. $P(A) = 0.5$, $P(B) = 0.6$ and $P(A \cap B) = 0.4$
    a. Are A and B independent?
    b. Are A and B mutually exclusive?
    c. Find $P(A \cup B)$

2. Events C and D are mutually exclusive. $P(C) = 0.35$ and $P(D) = 0.45$
    a. Find $P(C \cup D)$
    b. Find $P(C \cap D)$

3. Events E and F are independent. $P(E) = 0.2$ and $P(F) = 0.3$
    a. Find $P(E \cap F)$
    b. Find $P(E \cup F)$

4. The table below shows the proportion of students that live more than 1km away from school, and the number of students who catch a bus.

	Catches the bus to school	Does not catch the bus to school
Lives more than 1km away from school	0.4	0.25
Lives more than 1km away from school	0.2	0.15

   Are the events "a student lives more than 1km away from school" and "a student catches a bus to school":
    a. Mutually exclusive?
    b. Independent?

5. Two movies come out in the same week and the local movie theatre is screening both of them. Of the 1420 people that go to the movies that week: Captain Africa: The First Nemesis is seen by 852 people, Awesome Man: Dawn of Fairness is seen by 710 people. 284 people do not see either of those movies. Are the events "Going to Captain Africa: The First Nemesis" and "Going to Awesome Man: Dawn of Fairness"
    a. Mutually exclusive?
    b. Independent?

6. Two servers are running at a school. The likelihood of the first (older one) failing in the next week is 0.01. The likelihood of the second (newer one) failing in the next week is 0.005. The likelihood of both servers failing in the next week is 0.001. Explain if these two events are independent or not.

Part 6.1 Answers

1a. No ($0.5 \times 0.6 \neq 0.4$)
1b. No ($0.4 \neq 0$)
1c. 0.7

2a. 0.8
2b. 0

3a. 0.06
3b. 0.44

4a. No
4b. No

5a. No
5b. Yes

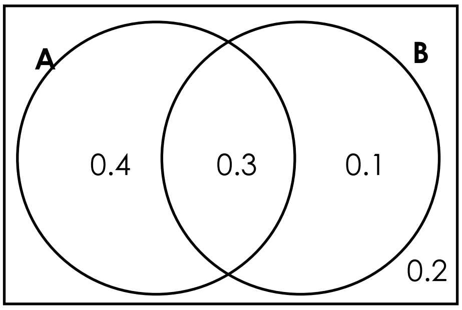
6. No

Part 7: Conditional Probability

Conditional probability is the probability **given** we know something else. We write this as P(A | B) which is said "probability of A given B." Another way of saying it would be "on the condition that B happens, what is the probability of A."

The formula is:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$



Example 1:
Let's look at the Venn diagram we had earlier:

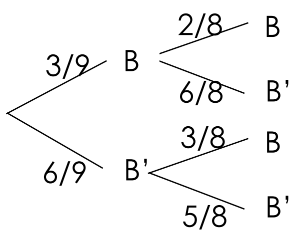
$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.3}{0.4} = 0.75$$

We can also see this off a table:

	A	A'	Total
B	**0.3**	0.1	**0.4**
B'	0.4	0.2	0.6
Total	0.7	0.3	1

We just get the 0.3 and the 0.4 out of the B line, as we know that B has happened, so what is the probability that A occurs, which is exactly the same 0.3 ÷ 0.4 = 0.75.

Example 2:
If we look at our BUMBLEBEE example from our probability trees, where we put the 9 letters onto 9 balls and draw two of them out of a bag which are either B or not a B (B').



a. What is the probability the second ball is a B given the first ball is a B?
From the diagram we can see on the second split there is a 2/8 = 1/4 chance of getting a B.
We could also work out $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(2 \text{ B balls})}{P(\text{second ball is B})} = \frac{\frac{3}{9} \times \frac{2}{8}}{\frac{3}{9} \times \frac{2}{8} + \frac{6}{9} \times \frac{3}{8}} = \frac{1}{4}$ which is the same as before.

b. What is the probability the first ball is a B given he second ball is not a B?

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(\text{1st ball is B and second ball is not B})}{P(\text{second ball is not a B})} = \frac{\frac{3}{9} \times \frac{6}{8}}{\frac{3}{9} \times \frac{6}{8} + \frac{6}{9} \times \frac{5}{8}} = \frac{1}{6}$$

Part 7.1: Conditional Probability Exercise

1. This table shows data from the 2013 Census for New Zealand, and the number of males and females in each ethnicity group.

	Number of People		
Ethnicity	Male	Female	Total
European	1,135,746	1,250,496	2,386,242
Maori	184,923	211,362	396,285
Pacific Peoples	91,344	99,084	190,428
Asian	177,192	197,316	374,508
Middle Eastern/Latin American/African	17,763	17,211	34,974
Other ethnicity	32,376	22,656	55,032
Total	1,639,344	1,798,125	3,437,469

data extracted on 05 Aug 2018 from NZ.Stat

What is the probability a randomly chosen person

a. Is male given they are of European descent?
b. Is European given they are a male?
c. Is either of Maori or Pacific descent, given they are female?

2. This table shows data from the 2013 Census for New Zealand, and the number of males and females and how long they have been living in their current residence.

	Number of People		
Years at Residence	Male	Female	Total
0 years	0.109	0.113	0.223
1-4 years	0.145	0.154	0.300
5-9 years	0.101	0.106	0.208
10-14 years	0.053	0.056	0.109
15-29 years	0.055	0.060	0.115
30 years or more	0.022	0.025	0.047
Total	0.486	0.514	1.000

data extracted on 05 Aug 2018 from NZ.Stat

What is the probability a randomly chosen person

a. Is male given they have only lived in their current residence for less than 1 year?
b. Who has lived in their house for more than 30 years is female?
c. Has lived in their address for 5-14 years given they are female?

3. The table below shows the number of offences recorded during 2014 in New Zealand.

	Number of Offences		
Offence	Unresolved	Resolved	Total Recorded
Homicide and related offences	8	58	66
Acts intended to cause injury	11,036	28,908	39,944
Sexual assault and related offences	2,124	1,932	4,056
Dangerous or negligent acts endangering persons	303	434	737
Abduction, harassment and other related offences against a person	6,541	8,483	15,024
Robbery, extortion and related offences	1,279	861	2,140
Unlawful entry with intent/burglary, break and enter	46,801	6,464	53,265
Theft and related offences	93,453	25,870	119,323
Fraud, deception and related offences	4,556	4,481	9,037
Illicit drug offences	1,512	15,031	16,543
Prohibited and regulated weapons and explosives offences	620	5,171	5,791
Property damage and environmental pollution	28,927	11,431	40,358
Public order offences	5,469	21,282	26,751
Offences against justice procedures, government security and government operations	1,866	14,219	16,085
Miscellaneous offences	527	742	1,269
Total	205,022	145,367	350,389

data extracted on 05 Aug 2018 from NZ.Stat

a. What is the likelihood of a homicide and related offences being unresolved?
b. What is the likelihood of an unresolved offence being a theft or related offence?
c. Given the offence is "unlawful entry with intent/burglary, break and enter", what is the probability it is resolved?

4. An insurance company does a survey of their customers (all of whom have either car or house insurance). They find the proportion of customers that have house insurance is 0.7, and the proportion that have both house and car insurance is 0.6.
   a. Given a person has house insurance what is the probability they have car insurance?
   b. What is the probability they do not have house insurance, given they have car insurance?

5. At a restaurant one night 25 groups of people come through.
   - All of them order something
   - None of them just order entrees
   - 2 groups have entrees, mains and desserts
   - 8 have both entrees and mains
   - 12 have entrees
   - 3 have both mains and desserts
   - 12 have desserts

   What is the probability
   a. A randomly chosen group had mains, given they had desert?
   b. Given a group had an entrée, they also had desert?

6. Two students are chosen at random from a class of 12 boys and 13 girls to be office messengers / runners for the school for the day. What is the probability:
    a. If the first students is a boy, the second is a girl?
    b. The first student is a boy, given the second student is also a boy?

7. I have a standard pack of 52 cards
    a. What is the probability, that given I have drawn two black cards out, the next one will also be black?
    b. The next card is an Ace, given I have already drawn out 4 cards that were not aces?

8. Jane and Vikki are playing each other in tennis. The probability that Jane wins the first set is 0.5. If Jane wins a set the probability she wins the next set is 0.6, otherwise it is 0.35. In a women's tennis game you play the best of three sets, so you stop playing once one player has won two sets.
    a. Vikki wins the second set, given Jane wins the first?
    b. Given Jane wins the game overall, the game goes to three sets?

Part 7.1 Answers

1a. 1,135,746 / 2,386,242 = 0.476 (3sf)
1b. 1,135,746 / 1,639,344 = 0.693 (3sf)
1c. (211,362 + 99,084) / 1,798,125 = 0.173 (3sf)

2a. 0.109 / 0.223 = 0.489 (3sf)
2b. 0.025 / 0.047 = 0.532 (3sf)
2c. (0.106 + 0.056) / 0.514 = 0.315 (3sf)

3a. 8 / 66 = 0.121 (3sf)
3b. 93,453 / 205,022 = 0.456 (3sf)
3c. 6,464 / 53,265 = 0.121 (3sf)

4a. 0.6 / 0.7 = 0.857 (3sf)
4b. 0.3 / 0.9 = 0.333 (3sf)

5a. 3 / 12 = 0.25
5b. 6 / 12 = 0.5

6a. 13 / 24 = 0.542 (3sf)
6b. 12 / 24 = 0.458

7a. 24 / 50 = 0.48
7b. 4 / 48 = 0.0833 (3sf)

8a. 0.4
8b. 0.175 / 0.475 = 0.368 (3sf)

Part 8: Risk and Relative Risk

Risk is the probability of something happening. **Relative Risk** is the two risks compared.

Risk is often associated with things that have a *risk*, or are negative, however it can be used for any situation, positive or negative. You can have the "risk" of winning a major prize, or getting an excellent result in a test.

Example:
Aspirin is thought to be beneficial in preventing heart attacks. In order to test this a group of 1000 doctors were split into two even groups, and they either took aspirin daily or not at all, and recorded if they had a heart attack or not. The table below shows the results

	Aspirin	No Aspirin	Total
Heart Attack	5	12	17
No Heart Attack	495	488	983
Total	500	500	1000

The **risk** of having a heart attack = 17 / 1000 = 0.017
The **risk** of a heart attack if taking aspirin is 5 / 500 = 0.01
The **risk** of a heart attack if not taking aspirin is 12 / 500 = 0.024

We can see just from comparing the probabilities you are more likely to have a heart attack if you are not taking aspirin. To work out the **relative risk** we divide one probability by the other.

Therefore the **relative risk** of having a heart attack if you are not taking aspirin (compared to if you are) is 0.024 / 0.01 = 2.4

Therefore you are 2.4 times as likely to have a heart attack if you are not taking aspirin compared to if you are.

Note: this data in this question is fictional, and there are benefits and risks of taking aspirin daily. If you are concerned about the risk of either yourself or someone else having a heart attack it is best to consult your health professional.

Part 8.1: Risk and Relative Risk Exercise

1. Complete and use the table below to answer the following questions:

	A	A'	Total
B	1	3	
B'			
Total	8		12

Calculate the
   a. Risk of A
   b. Risk of B
   c. Risk of A given B
   d. Risk of A given B'
   e. Relative risk of A when B is compared to B'

2. This table shows data from the 2012 New Zealand General Social Survey (NZGSS), and the percentage of people who voted in the most recent general election, and the most recent local body election.

	Voted in General Election		
Voted in Local Body Election	Yes	No	Total
Yes	1,980,000	79000	2,059,000
No	572,000	612,000	1,184,000
Total	2,552,000	691,000	3,243,000

data extracted on 05 Aug 2018 from NZ.Stat

   a. How likely is someone to vote in the general elections?
   b. How likely is someone to vote in the local body elections?
   c. How many times more likely are you to vote in the local body elections if you vote in the general election to if you do not vote in the general election?

3. The table below shows data from 2014, and the number of fatal and serious non-fatal injuries by sex, and severity of injury.

	Number of Injuries		
Severity of injury	Male	Female	Total
Fatal	1,125	672	1,797
Serious non-fatal	4,875	4,623	9,498
Total	6,000	5,295	11,295

data extracted on 05 Aug 2018 from NZ.Stat

   a. What is the risk an injury is fatal?
   b. What is the risk an injury is on a female?
   c. What is the relative risk of an injury being fatal if it is a male vs a female?

4. This table shows data from the 2012 New Zealand General Social Survey (NZGSS), and the number of people who have high and low life satisfaction based on their family type.

	Number of People				
Overall life satisfaction	Couple without child(ren)	Couple with child(ren)	One parent with child(ren)	Not in a family nucleus	Total
Low	133,000	210000	107,000	165,000	615,000
High	922,000	1,205,000	245,000	541,000	2,913,000
Total	1,055,000	1,415,000	352,000	706,000	3,528,000

data extracted on 05 Aug 2018 from NZ.Stat

a. What is the relative risk of not being in a family nucleus if you have low life satisfaction compared to high life satisfaction?
b. Which of these statements is more likely: "A person has high life satisfaction given they are in a couple without children" or "A person has high life satisfaction given they are in a couple with child(ren)"?

5. The table below shows the risk of developing lung disease based on if you are a smoker or a non-smoker

	Smoker	Non-Smoker
Lung Disease	0.12	0.03
No Lung Disease	0.19	0.66

What is the relative risk of developing lunch disease when smokers are compared to non-smokers?

Exercise 8.1 Answers

1a. 0.667 (3sf)
1b. 0.333 (3sf)
1c. 0.25
1d. 0.875
1e. 0.286 (3sf) (A is 0.286 times as likely to happen if B occurs compared to B not occurring)

2a. 0.789 (3sf)
2b. 0.635 (3sf)
2c. 6.79 (3sf) (someone votes in the general election is 6.79 times as likely to vote in the local body elections compared to someone who didn't vote in the general election)

3a. 0.159 (3sf)
3b. 0.469 (3sf)
3c. 1.47 (3sf) (It is 1.47 times more likely an injury is fatal if it is a male involved compared to a female)

4a. 1.44 (3sf) (New Zealanders are 1.44 times as likely to not be in a family nucleus if they have low life satisfaction compared to if you have high life satisfaction)
4b. "A person has high life satisfaction given they are in a couple without children" is more likely (New Zealanders are 1.03 times as likely to have high life satisfaction if they are in a couple without children compared to if you are in a couple with child(ren))

5. 8.52 (3sf) (A person are 8.52 times as likely to develop lung disease if they smoke compared to if they do not smoke)

Part 9: Past Exams

One of the most important things you can do to prepare for the exam is do lots of practice exam questions. Fortunately NZQA publishes all their exams online, along with answers.

You can access the exams from the links below:

2017: Exam Answers
2016: Exam Answers
2015: Exam Answers
2014: Exam Answers
2013: Exam Answers

You can access all the past exams as well as resources such as markers reports and students papers here: https://www.nzqa.govt.nz/ncea/assessment/view-detailed.do?standardNumber=91585

Appendix: Standard & Curriculum Details

Below is the details from the standard and the curriculum documents relating to the content of this standard.

Standard Details

Methods are selected from those related to
- true probability versus model estimates versus experimental estimates
- randomness
- independence
- mutually exclusive events
- conditional probabilities
- probability distribution tables and graphs
- two-way tables
- probability trees
- Venn diagrams.

Curriculum Elaborations

A. Calculating probabilities of independent, combined, and conditional events:
Students learn that some situations involving chance produce discrete numerical variables, that situations involving real data from statistical investigations can be investigated from a probabilistic perspective. These have probability distributions. They can be investigated by making assumptions about the situation and applying probability rules and/or by doing repeated trials of the situation and collecting frequencies.
- Selects and uses appropriate methods to investigate probability situations including experiments, simulations, and theoretical probability, distinguishing between deterministic and probabilistic models.
- Interprets results of probability investigations, demonstrating understanding of the relationship between true probability (unknown and unique to the situation), model estimates (theoretical probability), and experimental estimates.
- Selects and uses appropriate tools to solve problems in probability, including two-way tables, Venn diagrams, and tree diagrams, including combined events.
- Solves probability problems involving conditional probabilities, randomness, independence, and mutually exclusive events.

More details on the Senior Secondary Guide on TKI