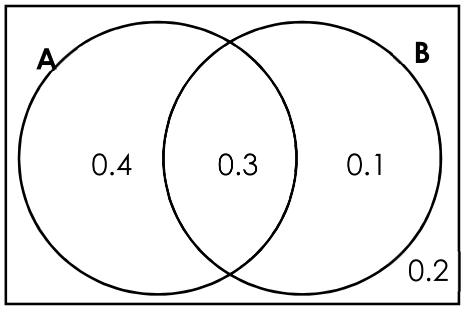
Part 7: Conditional Probability

Conditional probability is the probability **given** we know something else. We write this as P(A | B) which is said "probability of A given B." Another way of saying it would be "on the condition that B happens, what is the probability of A."

The formula is:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$



Example 1:
Let's look at the Venn diagram we had earlier:

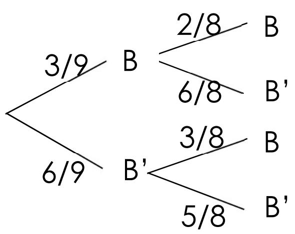
$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.3}{0.4} = 0.75$$

We can also see this off a table:

	A	A'	Total
B	**0.3**	0.1	**0.4**
B'	0.4	0.2	0.6
Total	0.7	0.3	1

We just get the 0.3 and the 0.4 out of the B line, as we know that B has happened, so what is the probability that A occurs, which is exactly the same 0.3 ÷ 0.4 = 0.75.

Example 2:
If we look at our BUMBLEBEE example from our probability trees, where we put the 9 letters onto 9 balls and draw two of them out of a bag which are either B or not a B (B').



a. What is the probability the second ball is a B given the first ball is a B?
From the diagram we can see on the second split there is a 2/8 = 1/4 chance of getting a B.

We could also work out $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(2\text{ B balls})}{P(\text{second ball is B})} = \frac{\frac{3}{9}\times\frac{2}{8}}{\frac{3}{9}\times\frac{2}{8}+\frac{6}{9}\times\frac{3}{8}} = \frac{1}{4}$ which is the same as before.

b. What is the probability the first ball is a B given he second ball is not a B?

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(\text{1st ball is B and second ball is not B})}{P(\text{second ball is not a B})} = \frac{\frac{3}{9}\times\frac{6}{8}}{\frac{3}{9}\times\frac{6}{8}+\frac{6}{9}\times\frac{5}{8}} = \frac{1}{6}$$