Part 8.2: Probability it is Less Than or Up To

As well as working out the probability that it is exactly an amount, we need to know the probability it is up to or less than a certain amount. Let's look at an example:

Example
My car has broken down 10 times in the last 8 years. What is the probability it breaks down less than 3 times in the next year?

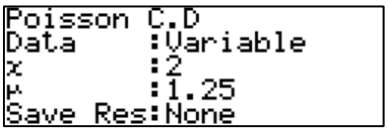
Answer
The first thing I always do is draw a number line and highlight the numbers I want, this helps avoid making silly mistakes… remember the number line stats at zero.

0 1 2 3 4 5 …

This is super important… do it every time! Otherwise you will make mistakes.

We then work out that the mean per year is $\frac{10}{8} = 1.25$ and the number we want up to (x) is 2.

(Graphics Calculator)
We go to STAT (2) → DIST (F5) → Across (F6) → Poisn (F1). We want Pcd as we are after up to. In this case x = 2 and μ = 1.25 which looks like this:



When we press calculate we get 0.868 (3sf) which is the probability of getting less than three break downs in the next year.

(Formula)
We can see that x = 0, 1 or 2 and λ = 1.25. This means we need to substitute into the formula 3 times, once for each vaule of x… we get

$$\mathrm{P}(X = 2) = \frac{1.25^2 e^{-1.25}}{2!} = 0.22383$$

$$\mathrm{P}(X = 1) = \frac{1.25^1 e^{-1.25}}{1!} = 0.35813$$

$$\mathrm{P}(X = 0) = \frac{1.25^0 e^{-1.25}}{0!} = 0.28650$$

We then add these up giving 0.868 (3sf).
As you can see these two answers match up.

Exercise 8.2

1. There are approximately 300 flights per day from Auckland airport. What is the probability there is:
   a. Less than 275 flights in a given day?
   b. Up to 310 flights in a given day?
   c. Exactly 300 flights in a given day?
   d. Less than 500 flights in two days?
2. The number of police callouts to a particular neighbourhood is on average 3 per week (7 days). What is the probability that there are:
   a. Up to 2 callouts in the next 2 days?
   b. Less than 4 callouts in the next 2 days?
   c. Exactly 1 callout in the next 2 days?
   d. No callouts in the next week?
3. The number of trees planted per hectare in a pine plantation is on average 300. What is the probability that in a forestry block there are:
   a. Less than 3100 trees in a 10 hectares?
   b. Up to 4400 trees in a 15 hectares?
   c. Exactly 3000 trees in a 10 hectares?
   d. Less than 100 trees in a half hectare block?
4. The number of bacteria in a probiotic tablet is on average 20,000. What is the probability there are:
   a. Less than 19,900 bacteria in the tablet?
   b. Up to 19,900 bacteria in the tablet?
   c. Exactly 20,000 bacteria in the tablet?
   d. Less than 40,000 bacteria in two tablets?