Part 6.2: Finding the Standard Deviation

The next thing we need to do is find the standard deviation. This is a two-step process where we need to find E(X²) (pronounced expected value of x-squared) and E(X) (the expected value of x, which is also the mean), this enables us to find the variance which is the standard deviation squared, so we just square root to get the standard deviation.

The formula you are given is $\sigma = \sqrt{E(X^2) - [E(X)]^2}$.

Example

The likelihood of winning different prizes in a raffle is shown below.

x	$100	$20	$0
P(X=x)	1/100	3/100	q

Find q and calculate the mean and standard deviation of the prize amount.

Answer

In part 6.1 we worked out $q = \frac{96}{100}$ and the mean, or E(X) = 1.6. This means to use our formula we need to calculate E(X²). To do this we need to work out what x² is. It is best to add a new row to the table.

x	$100	$20	$0
P(X=x)	1/100	3/100	q
x²	10000	400	0

To work out E(X²) it is the same process as with the mean except instead of multiplying the probability and x we multiply the probability and x². This gives $E(X^2) = 10000 \times \frac{1}{100} + 400 \times \frac{3}{100} + 0 \times \frac{96}{100} = 112$.

We then substitute into the formula $\sigma = \sqrt{E(X^2) - [E(X)]^2} = \sqrt{112 - [1.6]^2} = \sqrt{109.44} = 10.46$ (4sf).

Therefore the mean = $1.60 and the standard deviation = 10.46.

Note: If you are asked to calculate the variance you just do not square root at the end, so for this example the variance is 109.44.

Exercise 6.2

Note, to save doing double calculations these are the same situations as exercise 6.1.

1. The prizes in a lottery are shown in the table below.

x	$20	$10	$0
P(X=x)	0.02	0.1	0.88

What is the standard deviation of the prize from the raffle?

2. Biscuits come in 3 packet sizes, small (12), medium (18) and large (30).

x	12	18	30
P(X=x)	0.3	0.5	0.2

Calculate the standard deviation of the number of biscuits in a packet based on the table above.

3. Toyota Corollas come in 3 engine sizes, 35% are 1.5 L engines, 50% are 1.8 L engines and the rest (15%) are 2.0 L engines. Calculate the standard deviation of the size of Toyota Corolla engines.

4. Stockings come in three lengths, small (147 – 161 cm), average (162 – 171 cm) and Tall (172 – 183 cm).

x	147 – 161	162 – 171	172 – 183
P(X=x)	0.6	m	0.2

Calculate the standard deviation of the length of the stockings.