Part 4.1: The Left Slope

The first thing we need to know, is that just like the uniform distribution, the probability we are trying to find is always the area under the graph. The formula you will need to know for the area of a triangle is

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

In the pdf formula above there are 4 parts. The first part '0' means the probability when x is less than the minimum is just zero. The part we are going to be focusing on in this section is the part when x is between a and c, this is the left slope and the height is given by the formula: $\frac{2(x-a)}{(b-a)(c-a)}$. To find the area up to a point on this slope you work out the area of the triangle up to this point. Let's look at an example.

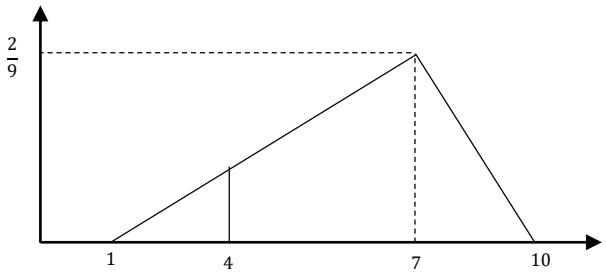
Example

A manager knows the total sales of a product are going to be between \$1 million and \$10 million, and the most likely amount of sales will be \$7 million.

a. What is the probability the sales are less than \$4 million?
b. What is the probability the sales are more than \$4 million?

Answer

The first thing we need to do is draw a diagram and fill in the values.
a = 1 (minimum)
b = 10 (maximum)
c = 7 (most likely)
x = 4 (the point we are looking at)



a. The area of a triangle is $\frac{1}{2} \times \text{base} \times \text{height}$
The base is the difference between 1 and 4 which is 3
Height = $\frac{2(x-a)}{(b-a)(c-a)} = \frac{2(4-1)}{(10-1)(7-1)} = \frac{6}{54} = \frac{1}{9}$
So probability = $\frac{1}{2} \times 3 \times \frac{1}{9} = \frac{1}{6}$
b. The probability that it is **more** than 4 is just 1 minus the probability that it is **less** than 4 so this is $1 - \frac{1}{6} = \frac{5}{6}$

Exercise 4.1

1. The minimum amount of time spent changing a tire is 1 minute and the maximum is 10 minutes. The most likely time is 3 minutes. What is the probability it takes:
    a. Less than 2 minutes to change?
    b. More than 2 minutes to change?
    c. Less than 1.5 minutes to change?
    d. More than 3 minutes to change?
2. The minimum amount a company will spend on advertising is \$100,000 and the maximum is \$800,000. They think they are most likely to spend \$600,000. What is the probability they spend:
    a. Less than \$150,000?
    b. More than \$200,000?
    c. Less than \$300,000?
    d. More than \$500,000?
3. A company knows that the time spent on a project will be between 300 and 800 hours, with the expected time being 500 hours. What is the probability the project takes:
    a. Less than 350 hours?
    b. More than 500 hours?
    c. Less than 450 hours?
    d. More than 200 hours?
4. The time the cookies spent in the oven is between 20 and 22 minutes with the most likely time being 21.5 minutes. What is the probability they spend:
    a. Less than 21 minutes in the oven?
    b. More than 20.5 minutes in the oven?
    c. More than 21.5 minutes in the oven?
    d. Less than 19 minutes in the oven?