

Part 6: Complementary, Independent and Mutually Exclusive Events

There are three other pieces of terminology that we need to know that relate to types of events in probability. These are **complementary**, **independent** and **mutually exclusive** events

Complementary events are events that are the opposite to each other. For example heads and tails on a coin, or A and A'.

Independent events are events where the outcome of one does not affect the probability of the other. There is a statistical formula to check if two events are independent, this is

$$P(A) \times P(B) = P(A \cap B)$$

Example 1: are a "day being sunny" and a "day being a weekday" **independent**?

To check if these events are **independent** or not we can check if the formula $P(A) \times P(B) = P(A \cap B)$ holds true.

$$P(A) = P(\text{Day is a Weekday}) = 5/7$$

$$P(B) = P(\text{Day is Sunny}) = 1/5$$

$$P(A \cap B) = P(\text{Weekday} \cap \text{Sunny}) = 1/7$$

$$P(A) \times P(B) = 5/7 \times 1/5 = 1/7$$

Therefore they are **independent**.

	Day is Sunny		
Day is a Weekday	Yes	No	Total
Yes	1/7	4/7	5/7
No	2/35	8/35	2/7
Total	1/5	4/5	1

Example 2: are a "student having brown eyes" and a "student being female" **independent**?

We check if $P(A) \times P(B) = P(A \cap B)$ is true.

$$P(A) = P(\text{Has Brown Eyes}) = 13/20$$

$$P(B) = P(\text{Is Female}) = 10/20$$

$$P(A \cap B) = P(\text{Brown Eyes} \cap \text{Female}) = 7/20$$

$$P(A) \times P(B) = 13/20 \times 10/20 = 13/40$$

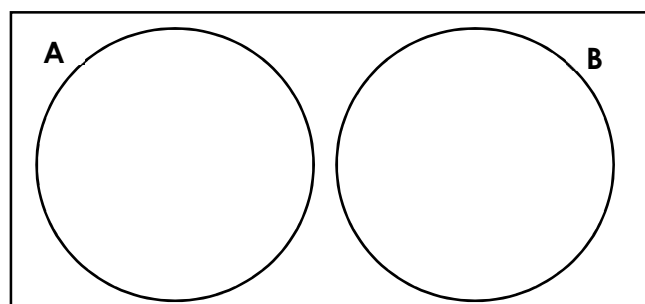
$$7/20 \neq 13/40$$

Therefore they are **not independent**.

	Number of Students		
Eye Colour	Male	Female	Total
Brown	6	7	13
Blue	4	3	7
Total	10	10	20

Mutually Exclusive events are events that cannot both happen. **Complementary** events must be mutually exclusive. For **mutually exclusive** events $P(A \cap B) = 0$

This means if you were to draw a Venn diagram for it the circles would not overlap:



Example 3:

A student could be in a Maths class at the moment or an English class, but couldn't be both in an English and a Maths class at the same time (but could be in another class like Science)

If we were to look at this table:

	Currently in English		
Currently In Maths	Yes	No	Total
Yes	0	25	25
No	27	23	50
Total	27	48	75

We can see these events are **mutually exclusive** as $P(\text{English and Maths}) = 0$