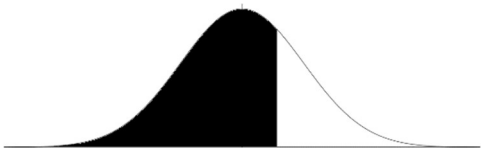
Part 2.1: Up To

The first type of question that we are going to look at is when we want to know the probability that something is less than (or up to) a certain number. We are going to step through an example together using both the graphics calculator and the standard normal tables. The first thing we do is draw a diagram if what we want like the one shown below.



On the graphics calculator you need to go to STAT (2) and then go to DIST (F5) and for this section we want the NORM (F1). This will give us three options: Npd Ncd and InvN. For all the questions where we are trying to find a probability for normal distributions I find it easiest just to use the Ncd (F2). So now… our question.

Example

The weights of Great Spotted kiwi birds are normally distributed with a mean of 2.8 kg and a standard deviation of 0.58 kg. Calculate the probability that a randomly selected kiwi weighs less than 2 kg.

Answer (Graphics Calculator)

As we are after everything **less** than 2kg, our Lower (or minimum amount) is the biggest negative number we can think of… so -99999999999 (the negative button and then a whole bunch of nines). The Upper (or maximum amount) is 2, the σ (or standard deviation) is 0.58 and the μ (or mean) is 2.8.

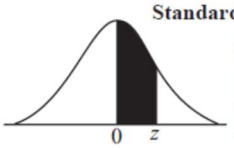
```
Normal C.D
Lower    :-9.999E+11
Upper    :2
σ        :0.58
μ        :2.8
Save Res:None
Execute
```

Pressing the execute button gives us the following screen:

```
Normal C.D
P     =0.08389954
z:Low=-1.724E+12
z:Up =-1.3793103
```

You can ignore the z:Low and z:Up.
So we can see the probability is 0.0839 (3sf) which is our answer.

Standard Normal Distribution



$\left(Z = \frac{X-\mu}{\sigma}\right)$

Each entry gives the probability that the standardised normal random variable Z lies between 0 and z.

z	0	1	2	3	4	5	6	7	8	9	Differences 1 2 3	4 5 6	7 8 9
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359	4 8 12	16 20 24	28 32 36
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754	4 8 12	16 20 24	28 32 36
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141	4 8 12	15 19 22	27 31 35
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517	4 8 11	15 19 22	26 30 34
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879	4 7 11	14 18 22	25 29 32
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224	3 7 10	14 17 21	24 27 31
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549	3 6 10	13 16 19	23 26 29
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852	3 6 9	12 15 18	21 24 27
0.8	.2881	.2910	.2939	.2967	.2996	.3023	.3051	.3078	.3106	.3133	3 6 8	11 14 17	19 22 25
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389	3 5 8	10 13 15	18 20 23
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621	2 5 7	9 12 14	16 18 21
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830	2 4 6	8 10 12	14 16 19
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015	2 4 5	7 9 11	13 15 16
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177	2 3 5	6 8 10	11 13 14
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319	1 3 4	6 7 8	10 11 13
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441	1 2 4	5 6 7	8 10 11
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545	1 2 3	4 5 6	7 8 9
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633	1 2 3	3 4 5	6 7 8
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706	1 1 2	3 4 4	5 6 6
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767	1 1 2	2 3 4	4 5 5
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817	0 1 1	2 2 3	3 4 4
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857	0 1 1	2 2 2	3 3 4
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890	0 1 1	1 2 2	2 3 3
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916	0 0 1	1 1 2	2 2 2
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936	0 0 1	1 1 1	1 2 2
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952	0 0 0	1 1 1	1 1 1
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964	0 0 0	0 1 1	1 1 1
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974	0 0 0	0 0 1	1 1 1
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981	0 0 0	0 0 0	0 0 1
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986	0 0 0	0 0 0	0 0 1
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990	0 0 0	0 0 0	0 0 0
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993	0 0 0	0 0 0	0 0 0
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995	0 0 0	0 0 0	0 0 0
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997	0 0 0	0 0 0	0 0 0
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998	.4998	0 0 0	0 0 0	0 0 0
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	0 0 0	0 0 0	0 0 0
3.6	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	0 0 0	0 0 0	0 0 0
3.7	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	0 0 0	0 0 0	0 0 0
3.8	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.5000	.5000	.5000	0 0 0	0 0 0	0 0 0
3.9	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	0 0 0	0 0 0	0 0 0

Answer (Tables)

The first thing we need to do is work out the Z value. We do this using the formula $Z = \frac{X-\mu}{\sigma}$.
With our data X=2, μ=2.8, σ=0.58… when we put this into our formula we get:

$$Z = \frac{2 - 2.8}{0.58} = -1.379$$

What we do is we pretend this value is positive and look it up in the table above… we look up the 1.3 in the left column, and then go along to the 7 column giving us .4147… we then look up the 9 in the right group of columns, which is 14. We need to add this onto the last two digits, giving us 0.4147 + 0.0014 which is 0.4161… now what does this number mean?
This number is the area under the graph between half way and our point… given that our point is less than the mean, we need to subtract this off 0.5 to find our answer.
Therefore: 0.5 - 0.4161 = 0.0839 which is the same as the answer from the graphics calculator.

Hint: always draw the distribution graph… it helps.

Exercise 2.1

1. The North Island Brown kiwi birds' weights are normally distributed with a mean weight of 2.5 kg and a standard deviation of 0.4 kg. What is the probability that a North Island Brown kiwi will weigh less than 2.7 kg?
2. The length of a pounamu necklaces that have been hand crafted are normally distributed with a mean of 5.3 cm and a standard deviation of 1.1 cm. What is the probability that the pounamu is less than 4.1 cm?
3. The height of women's high heels are normally distributed with a mean of 7.5 cm and a standard deviation of 2.1 cm. What is the probability that the heel is less that 2.9 cm?
4. The amount of milk that a cow produces each day is normally distributed with a mean of 30.3 L and a standard deviation of 3.6 L. What is the probability that the cow produces less than 30 L of milk?
5. The price of bananas in the supermarket is normally distributed with a mean of $2.10 and a standard deviation of $0.40. What is the probability that bananas cost less than $3.10?
6. The weight of a full grown elephant is normally distributed with a mean of 5,200 kg and a standard deviation of 800 kg. What is the probability that the elephant weighs less than 5,000 kg?
7. The length of the leaves on a tree is normally distributed with a mean of 10.4 cm and a standard deviation of 2.1 cm. What is the probability that a leaf is less than 13 cm?
8. The amount of fizzy drink in a bottle is normally distributed with a mean of 1.53 L with a standard deviation of 0.02 L. What is the probability that the bottle has less than 1.50 L of drink in it?