Part 4.3: Combining the Two

Sometimes the triangles we are using look slightly different (e.g.: a right angle triangle) or the probability we are after spans both the left and the right slopes of the triangle. Here are some examples for you to look at:

Example 1

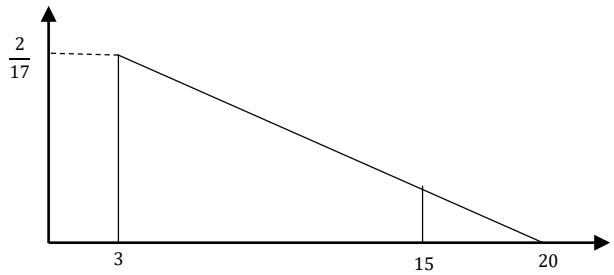
The amount of time it takes to clean a bathroom has a triangular distribution with a minimum time of 3 minutes and a maximum time of 20 minutes. The most likely amount of time it takes is 3 minutes.

a. What is the probability it takes more than 15 minutes to clean?
b. What is the probability it takes between 10 and 15 minutes to clean?

Answer

The first thing we need to do is draw a diagram and fill in the values.
a = 3 (minimum)
b = 20 (maximum)
c = 3 (most likely)
x = 15, and then 10 (the points we are looking at)

We have a right angled triangle and the left part of the triangle is non-existent… so that means that we just need to look at the right part of the triangle.



a. The area of a triangle is $\frac{1}{2} \times \text{base} \times \text{height}$.
The base is the difference between 15 and 20 which is 5.
Height = $\frac{2(b-x)}{(b-a)(b-c)} = \frac{2(20-15)}{(20-3)(20-3)} = \frac{10}{289}$
So probability = $\frac{1}{2} \times 5 \times \frac{10}{289} = \frac{25}{289}$

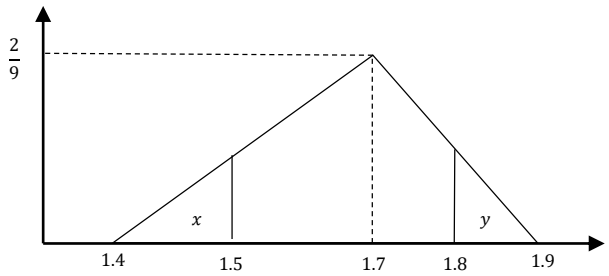
b. To find this we find the difference in the two triangles… the one from 10 to 20 and the one we just worked out from 15 to 20.
Height = $\frac{2(b-x)}{(b-a)(b-c)} = \frac{2(20-10)}{(20-3)(20-3)} = \frac{20}{289}$
So probability = $\frac{1}{2} \times 10 \times \frac{20}{289} = \frac{100}{289}$
Therefore the probability that we are interested in is $\frac{100}{289} - \frac{25}{289} = \frac{75}{289}$.

Example 2

The height of a bookcase is somewhere between 1.4 m and 1.9 m with the most common height being 1.7 m. What is the probability the bookcase is between 1.5 m and 1.8 m tall?

Answer

Again, the first thing we need to do is draw a diagram and fill in the values.
a = 1.4 (minimum)
b = 1.9 (maximum)
c = 1.7 (most likely)
x = 1.5 and 1.8 (the points we are looking at)



This time we need to look at the two triangles. What is probably easiest is to work out the probability of the parts we do not want (marked x and y), and then subtract them from one.

Area x:
Height = $\frac{2(x-a)}{(b-a)(c-a)} = \frac{2(1.5-1.4)}{(1.9-1.4)(1.7-1.4)} = \frac{4}{3}$
Probability = $\frac{1}{2} \times 0.1 \times \frac{4}{3} = \frac{1}{15}$

Area y:
Height = $\frac{2(b-x)}{(b-a)(b-c)} = \frac{2(1.9-1.8)}{(1.9-1.4)(1.9-1.7)} = 2$
Probability = $\frac{1}{2} \times 0.1 \times 2 = \frac{1}{10}$

Therefore the probability the height is between 1.5 m and 1.8 m tall is $1 - \frac{1}{15} - \frac{1}{10} = \frac{5}{6}$

Exercise 4.3

1. The gap between the seats on Trans-Tasman flights is always between 30 cm and 60 cm with the most common gap being 40 cm. What is the probability the gap is:
    a. More than 50 cm?
    b. Between 45 cm and 50 cm?
    c. Between 35 cm and 45 cm?
2. The weight of a pillow is known to be somewhere between 100 g and 250 g, with the most likely weight being 250 g. What is the probability the weight is:
    a. Less than 200 g?
    b. Between 150 and 200 g?
3. I know the amount of water in a cup is between 100 mL and 250 mL, but I think there is 220 mL in the glass. What is the probability the amount of water in the cup is:
    a. Less than 150 mL?
    b. Between 150 mL and 200 mL?
    c. Between 200 mL and 240 mL?
4. The weight of a suitcase must be less than 32 kg. A suitcase is never less than 5 kg. The most common weight of suitcase is 15 kg. What is the probability the suitcase weights:
    a. Less than 10 kg?
    b. Between 4 kg and 10 kg?
    c. Less than the limit for extra baggage charge of 20 kg?
5. The thickness of a magazine is between 3 mm and 6 mm with the most common thickness being 3 mm. What is the probability that the magazine is:
    a. Less than 4 mm thick?
    b. Between 4 mm and 5 mm thick?
    c. More than 5 mm thick?
6. My watch is always within 1 minute of the school bell (in either direction), and it is normally exactly correct. What is the probability my watch is:
    a. Less than 15 seconds out?
    b. Less than 30 seconds out?
7. The amount spent on weddings in New Zealand is normally between $5,000 and $60,000 with the most common amount spent being $28,000. What is the probability that a couple spends:
    a. Less than $10,000?
    b. Between $20,000 and $30,000?
8. The time taken for a person to get ready in the morning is known to be between 10 minutes and 1 hour. Most mornings it takes about 20 minutes. What is the probability it takes:
    a. Less than 15 minutes?
    b. Between 15 minutes and 30 minutes?
    c. Between 30 minutes and 45 minutes?