1 INVERSE LAPLACE TRANSFORMATION

If we consider the costs are proportional to the price, then our boundary conditions,

$$V_q(x,s,q) = \exp(x+s) - \exp(x+s)\mu(q) = \exp(s)\left(\exp(x) - \exp(x)\mu(q)\right),$$

and

$$V_q(x,s,q) = \exp(x+s) + \exp(x+s)\lambda(q) = \exp(s)\left(\exp(x) + \exp(x)\lambda(q)\right),$$

can be separated as s and x,q too.
It is reasonable to assume that

$$V(x,s,q) = \int_{-\infty}^{+\infty} g(x,q;\eta)\,h(s;\eta)\,t(\eta)\,d\eta. \tag{1.1}$$

We can solve the equation

$$\int_{-\infty}^{+\infty} h(s;\eta)\,t(\eta)\,d\eta = \exp(s),$$

to have $t(\eta)$. Then solve

$$\int_{-\infty}^{+\infty} g(x,q;\eta)\,t(\eta)\,d\eta = \exp(x) + \exp(x)\lambda(q).$$

to have $g(x,q;\eta)$. **This is wrong because I can't separate the integral of (1.1) as**

$$\int_{-\infty}^{+\infty} g(x,q;\eta)h(s;\eta)\,t(\eta)\,d\eta = \int_{-\infty}^{+\infty} h(s;\eta)\,t(\eta)\,d\eta \times \int_{-\infty}^{+\infty} g(x,q;\eta)\,t(\eta)\,d\eta$$