7 When futures are redundant?

Let $F_{t,T}$ denote the price at time t of future whose maturity is T. Q, P are risk-neutral and historical measure respectively. If the prices are consistent with underlying asset price $exp(X_t)$,

$$\exp(-\beta t)F_{t,T} = E_Q(\exp(-\beta T)\exp(X_T)|\mathcal{F}_t)$$

The discounted profit of selling one unit commodity via this future is

$$\begin{aligned}
&\exp(-\beta t)F_{t,T} - \exp(-\beta T)\int_Q^{Q+1}\mu(q)dq \\
=&E_Q(\exp(-\beta T)\exp(X_T)|\mathcal{F}_t) - \exp(-\beta T)\int_Q^{Q+1}\mu(q)dq \\
=&E_Q(\exp(-\beta T)\exp(X_T) - \exp(-\beta T)\int_Q^{Q+1}\mu(q)dq|\mathcal{F}_t). \\
=&E_Q(e^{-\beta T}(e^{X_T} - \mu(Q_T^1)|\mathcal{F}_t).
\end{aligned}$$

where $\mu(Q_T^1) = \int_Q^{Q+1}\mu(q)dq$.

Similarly total discounted cost of one unit commodity bought via this future is

$$\begin{aligned}
&\exp(-\beta t)F_{t,T} + \exp(-\beta T)\int_Q^{Q+1}\lambda(q)dq \\
=&E_Q(\exp(-\beta T)\exp(X_T)|\mathcal{F}_t) + \exp(-\beta T)\int_Q^{Q+1}\lambda(q)dq \\
=&E_Q(\exp(-\beta T)\exp(X_T) + \exp(-\beta T)\int_Q^{Q+1}\lambda(q)dq|\mathcal{F}_t) \\
=&E_Q(e^{-\beta T}(e^{X_T} + \lambda(Q_T^2)|\mathcal{F}_t).
\end{aligned}$$

where $\lambda(Q_T^2) = \int_Q^{Q+1}\lambda(q)dq)$.

Recall the objective function without futures.

$$V(x,q) = \max_{(L,U)\in\mathcal{U}}\mathbb{E}_P\left(\int_0^\infty e^{-\beta t}(e^{X_t} - \mu(Q_t^1))dU_t - \int_0^\infty e^{-\beta t}(e^{X_t} + \lambda(Q_t^2))dL_t\right) \tag{28}$$

Notice, this expectation is under historical measure. If $P = Q$, the future is redundant. If $P \neq Q$, it is not.