1. The difference between Shreve and ours.

   (a) The value function there is homotheticity which is $v(ax, ay) = a^p v(x, y)$. This property makes this problem into a one dimension problem instead of two.

   (b) The value function there is concave from which the value function is proven to be C^1 in the buying stock and selling stock region.

   (c) The value function is $C^(1, 2)$ via the boundary, however, with two equality holds the same time, it is still impossible to solve for the ratio without knowing the expression of the value function(because there is a nonlinear term that can not be eliminated). Unlike this, ours can be solved directly without knowing the value function.

   (d) We actually assume that V_{qxx} is continuous at the boundary which is not proven in the paper. They only show that V_x, V_{yy} are both continuous at the boundary and says nothing about V_{qxx}.

2. The value functions via solving PDE and simulation are quite similar. I do simulation on 1600 points of one fixed boundary and the results are similar.

3. Since the proof of moving boundary used the W_{qq} and W_{qxx}, I want to make sure both of them exist.

4. I want to come back to the constant transaction cost case.

   (a) In this case, is the analytical way of solving boundary right?

   (b) If it is not right, does W_{qq} and W_{qxx} exist?

   (c) How to discretize the PDE in this problem? It is always singular matrix.

5. Is it possible that this problem can be somehow turned into a one dimensional problem? Maybe not, because the price above average and below average are not the same.

6. I want to do the moving boundary method for this problem right now and check all the things after getting the solution of it.