

# Higher Top-Up Prelim Revision by Topic

The [Higher Webpage](#) contains [hints](#) and [worked solutions](#) for these questions, as well as [practice papers](#) to help you prepare for the March Top-Up Prelim.



## FORMULAE LIST

### Circle

$x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre  $(-g, -f)$ , radius  $\sqrt{g^2 + f^2 - c}$ .

$(x - a)^2 + (y - b)^2 = r^2$  represents a circle centre  $(a, b)$  and radius  $r$ .

### Scalar product

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta, \text{ where } \theta \text{ is the angle between } \mathbf{a} \text{ and } \mathbf{b}$$

or

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 \text{ where } \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

### Trigonometric formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

### Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

### Table of standard integrals

$f(x)$	$\int f(x) \, dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

# Top-Up Prelim Topic List

13. The Circle (Continued)

14. Wave Function

15. Logs and Exponentials

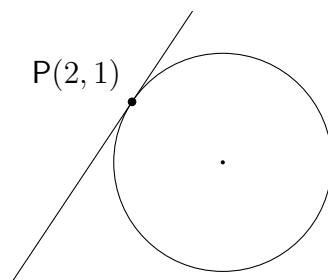
16. Further Calculus

## \* Answers

**Note:** Questions which require a **calculator** are marked with .

### 13 The Circle (Continued)

75. Find the coordinates of the points of intersection of the circle with equation  $x^2 + y^2 - 8x + 4y - 5 = 0$  and the line with equation  $x + y = 3$ .
76. Show that the line with equation  $y = x - 3$  is a tangent to the circle with equation  $x^2 + y^2 - 6x + 4y + 11 = 0$  and find the coordinates of the point of contact.
77. Point P(2, 1) lies on circle C with equation  $x^2 + y^2 - 16x + 6y + 21 = 0$ .



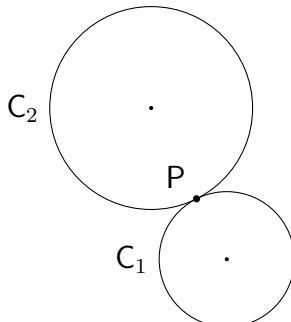
Find the equation of the tangent to the circle at P.

78. Circle C has equation  $(x - 5)^2 + (y + 3)^2 = 17$ .

(a) State the radius and the centre of circle C.

(b) Show that the point Q(9, -8) lies outside of circle C.

79. Circles  $C_1$  and  $C_2$  are shown in the diagram below.



Their equations are:

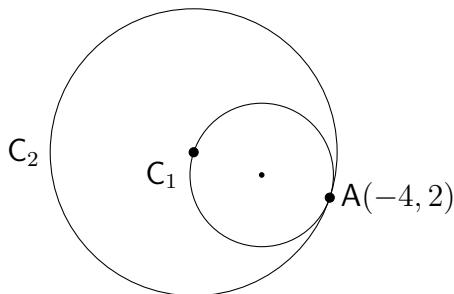
- $C_1 : (x - 3)^2 + (y + 7)^2 = 20$
- $C_2 : x^2 + y^2 + 4x - 6y - 32 = 0$

(a) Determine the distance between the centres of  $C_1$  and  $C_2$ .

(b) Show that circles  $C_1$  and  $C_2$  touch externally.

(c) The point of contact, P, is shown on the diagram.  
Determine the coordinates of P.

80. Circle  $C_1$  touches circle  $C_2$  internally at point A(-4, 2) and passes through the centre of circle  $C_2$ , as shown in the diagram below.



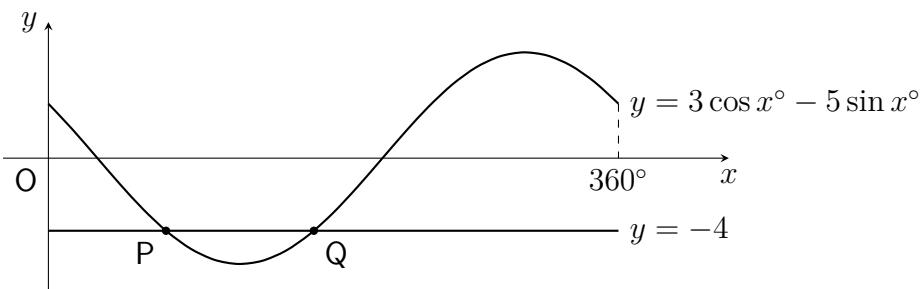
Circle  $C_1$  has equation  $x^2 + y^2 + 14x - 6y + 48 = 0$ .

Determine the equation of circle  $C_2$ .

## 14 Wave Function

For all questions,  $k > 0$  and  $\boxed{\text{ }}.$

81. Express  $8 \sin x^\circ + 7 \cos x^\circ$  in the form  $k \sin(x + a)^\circ$ ,  $0 < a < 360$ .
82. Express  $\sqrt{5} \sin x + \cos x$  in the form  $k \cos(x - a)$ ,  $0 < a < 2\pi$ .
83. (a) Express  $2 \sin x^\circ - 4 \cos x^\circ$  in the form  $k \sin(x - a)^\circ$ ,  $0 < x < 360$ .  
(b) Hence state:  
i. The maximum value of  $2 \sin x^\circ - 4 \cos x^\circ$ .  
ii. The value of  $x$  for which it occurs, where  $0 < x < 360$ .
84. (a) Express  $2 \cos x^\circ - \sin x^\circ$  in the form  $k \cos(x + a)^\circ$ ,  $0 < a < 360$ .  
(b) Hence solve  $2 \cos x^\circ - \sin x^\circ = -1$  where  $0 < x < 360$ .
85. Part of the graphs of  $y = 3 \cos x^\circ - 5 \sin x^\circ$  and  $y = -4$  are shown in the diagram below:



Points P and Q are points of intersection.

- (a) Express  $3 \cos x^\circ - 5 \sin x^\circ$  in the form  $k \cos(x + a)^\circ$ ,  $0 < a < 360$ .  
(b) Hence determine the coordinates of P and Q.

## 15 Logs and Exponentials

86. Express  $2 \log_p 10 + \log_p 2 - \log_p 50$  in the form  $\log_p q$ .

87. Evaluate the following:

(a)  $\log_3 \frac{1}{4} + 2 \log_3 6$

(b)  $\frac{1}{2} \log_5 64 - \log_5 40$

88. Solve each equation:

(a)  $\log_6 x + \log_6 4 = 2$

(b)  $\log_2 x + \log_2 (x + 7) = 3$

89. Given that  $\log_p 4 + 3 = \log_p 32$ , find the value of  $p$ .

90. A population of mice is discovered living in some woodland. The number of mice can be modelled by:

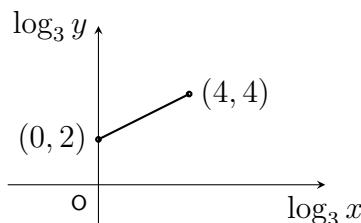
$$M = 80e^{kt}$$

where  $M$  is the estimated number of mice in the population,  $t$  is the number of weeks since the population was discovered and  $k$  is a constant.

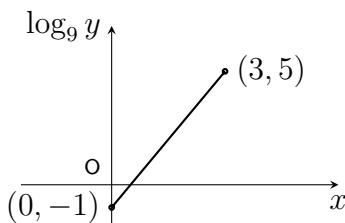
- (a) State the estimated number of mice in the population when it was first discovered.
- (b) It is estimated there will be 120 mice in the population after 3 weeks. Find the value of  $k$ .
- (c) Calculate the time taken for the population to double in size.

91. Variables  $x$  and  $y$  are linked by the equations given below. Find the values of  $a$  and  $b$  for each:

(a)  $y = ax^b$



(b)  $y = ab^x$



## 16 Further Calculus

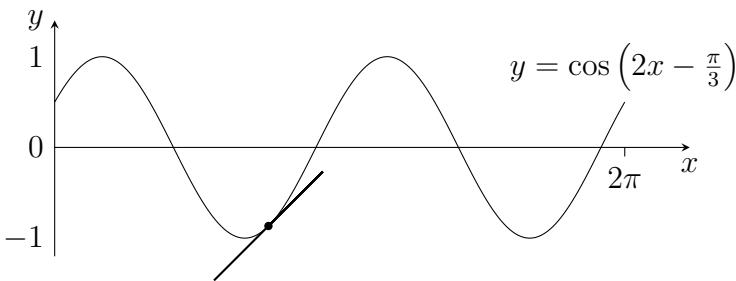
92. Given that  $y = (6x - 5)^3$ , find  $\frac{dy}{dx}$ .

93. A function,  $f$ , is defined on a suitable domain by  $f(x) = \frac{1}{(3x - 5)^2}$ . Determine the rate of change of  $f$  when  $x = 2$ .

94. Given  $g(x) = 4 \cos 3x + 2 \sin 2x$ , find the value of  $g' \left(\frac{\pi}{2}\right)$ .

95. Differentiate  $\sqrt[3]{6x^2 - 1}$  with respect to  $x$ .

96. Find the gradient of the tangent to  $y = \cos \left(2x - \frac{\pi}{3}\right)$  when  $x = \frac{3\pi}{4}$ .



97. Find  $\int (2x - 3)^5 dx$ .

98. Evaluate  $\int_0^1 2 \cos 3x dx$ .  

99. Find  $\int \frac{3}{(x+4)^{\frac{1}{2}}} dx$ .

100. Find the value of  $\int_0^{\frac{\pi}{3}} \sin \left(x + \frac{\pi}{6}\right) dx$ .

# Answers

## Answers 13 The Circle

75.  $(1, 2)$  and  $(8, -5)$

76.  $(2, -1)$  and statement ("Show that...")

77.  $y = \frac{3}{2}x - 2$  or  $2y = 3x - 4$

78. (a) Centre  $(5, -3)$  and radius  $\sqrt{17}$

(b)  $\sqrt{41} > \sqrt{17}$  and conclusion, or other valid approach

79. (a)  $\sqrt{125}$  or  $5\sqrt{5}$

(b)  $3\sqrt{5} + 2\sqrt{5} = 5\sqrt{5}$  and conclusion, or other valid approach

(c)  $(1, -3)$

80.  $(x + 10)^2 + (y - 4)^2 = 40$

## Answers 14 Wave Function

81.  $\sqrt{113} \sin(x + 41.2)^\circ$

82.  $\sqrt{6} \cos(x - 1.15)$

83. (a)  $2\sqrt{5} \sin(x - 63.4)^\circ$

(b) i. max value is  $2\sqrt{5}$

ii. occurs when  $x = 153.4$

84. (a)  $\sqrt{5} \cos(x + 26.6)^\circ$

(b)  $x = 90, 216.8$

85. (a)  $\sqrt{34} \cos(x + 59.0)^\circ$

(b) P( $73.3^\circ, -4$ ) and Q( $167.7^\circ, -4$ )

## Answers 15 Logs and Exponentials

86.  $\log_p 4$

87. (a) 2

(b) -1

88. (a)  $x = 9$

(b)  $x = 1$

89.  $p = 2$

90. (a) 80

(b)  $k = 0.135$

(c) 5.13 weeks

91. (a)  $a = 9$

$b = \frac{1}{2}$

(b)  $a = \frac{1}{9}$

$b = 81$

## Answers 16 Further Calculus

92.  $\frac{dy}{dx} = 18(6x - 5)^2$

93. -6

94. 8

95.  $4x(6x^2 - 1)^{-\frac{2}{3}}$

96. 1

97.  $\frac{(2x-3)^6}{12} + C$

98. 0.0941

99.  $6(x + 4)^{\frac{1}{2}} + C$

100.  $\frac{\sqrt{3}}{2}$