H National Qualifications

Practice Exam B

1 hour 30 mins

Total marks: 65

Mathematics

Paper 2

Calculator

FORMULAE LIST

Circle

$x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$, radius $\sqrt{g^2 + f^2 - c}$.

$(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta, \text{ where } \theta \text{ is the angle between } \mathbf{a} \text{ and } \mathbf{b}$$

or

$$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 \text{ where } \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae

$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2 \sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

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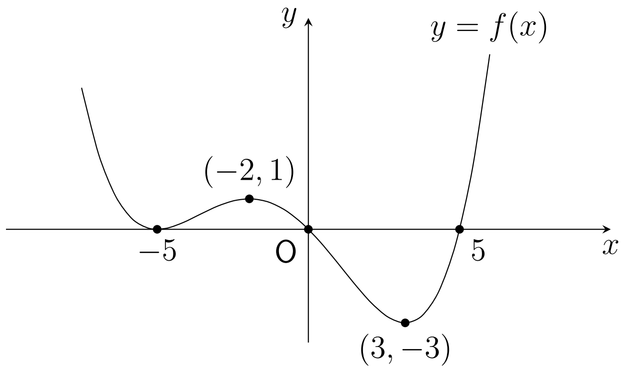
1. A curve has equation $y = 2x^3 - 3x + 7$. (4)
   Find the equation of the tangent to the curve at the point where $x = 1$.

2. Triangle PQR has vertices P$(-3, 2)$, Q$(0, 5)$ and R$(4, -1)$. (3)
   Determine the equation of the altitude through P.

3. Solve $2\cos\left(2x - \frac{\pi}{6}\right) = 1$ where $0 < x < 2\pi$. (4)

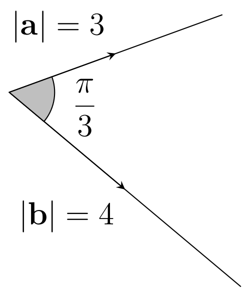
4. The diagram below shows part of the graph of $y = f(x)$.



(a) Sketch the graph of $y = 1 - f(x)$. (3)

(b) Sketch the graph of $y = f'(x)$. (3)

5. The angle between $\mathbf{a}$ and $\mathbf{b}$ is $\dfrac{\pi}{3}$ as shown in the diagram below.



(a) Determine the value of $\mathbf{a} \cdot (\mathbf{b} - \mathbf{a})$. (3)

(b) State the value of k such that $k\mathbf{a}$ is a unit vector. (1)

6. A sequenced generated by $u_{n+1} = pu_n + 5$ has consecutive terms 3 and 4.

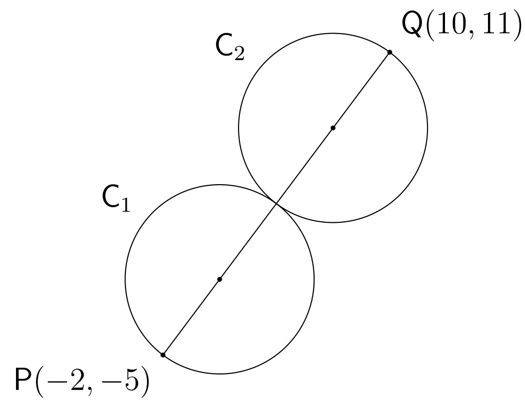
(a) Determine the value of p. (2)

(b) Explain whether the sequence converges to a limit as $x \to \infty$. (1)

7. (a) Express $-3x^2 - 6x - 4$ in the form $p(x + q)^2 + r$. (3)

(b) Hence show that $f(x) = -x^3 - 3x^2 - 4x + 7$ is strictly decreasing for all $x \in \mathbb{R}$. (3)

8. Congruent circles C_1 and C_2 touch and points $P(-2,-5)$ and $Q(10,11)$ lie on the circumferences of C_1 and C_2 respectively. Line PR passes through the centres of C_1 and C_2, as shown in the diagram.

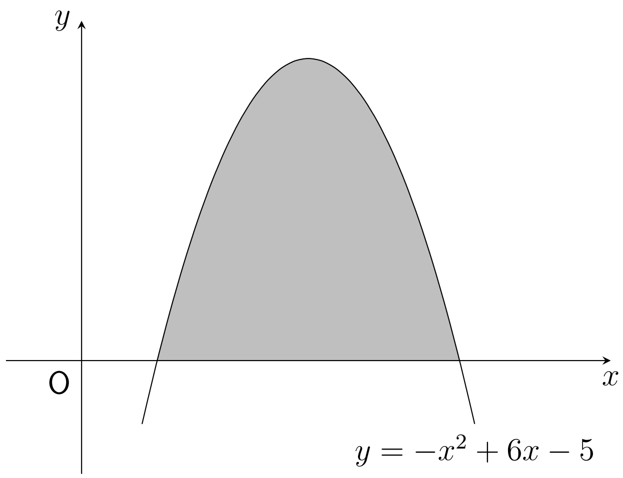


(a) Determine the coordinates of the point where the circles touch. (1)

(b) Find the equation of circle C_1. (4)

9. Show that the line with equation $y = 2x - 2$ meets the curve with equation $y = x^2 - 4x + 7$ at a tangent, and find the coordinates of the point of tangency. (5)

10. The shaded region in the diagram below is enclosed by the x-axis and the curve with equation $y = -x^2 + 6x - 5$. (6)



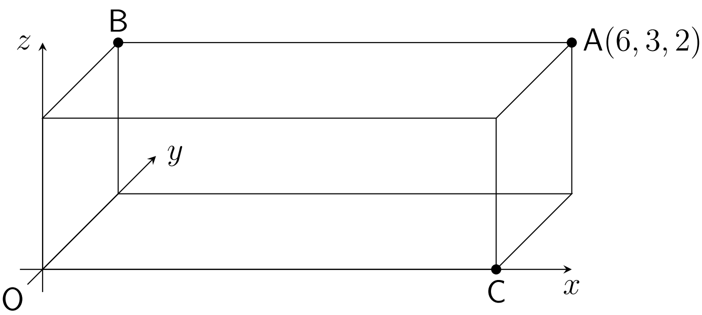
Calculate the area of the shaded region.

11. A function f is defined on a suitable domain such that: (4)

- $f'(x) = 4x + 5$
- The curve with equation $y = f(x)$ passes through the point $(-2, 7)$.

Determine an expression for $f(x)$.

12. A cuboid is placed along the coordinate axes as shown below.

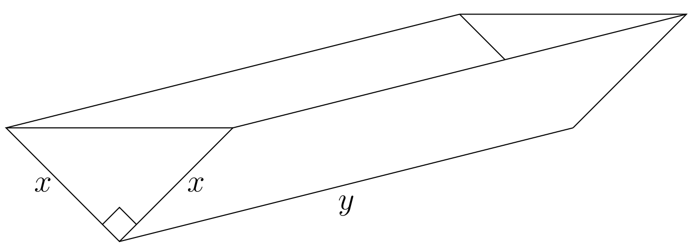


(a) Express $\overrightarrow{AB}$ and $\overrightarrow{AC}$ in component form. (2)

(b) Calculate the size of angle CAB. (5)

13. A trough is to be made from sheets of plastic in the shape of a triangular prism of length y with an open top, as shown in the diagram below. Each end of the prism is a right-angled, isosceles triangle with its shorter sides having length x.

The total area of plastic required is 12 square metres.



(a) Show that the volume of the trough, V, is given by: (3)

$$V(x) = 3x - \frac{1}{4}x^3$$

(b) Calculate the value for x which the volume of the trough is maximised. (5)

Question	Topic	Marks Available	Marks Awarded
1	Differentiation I	4	
2	Straight Line	3	
3	Trigonometry	4	
4	Graph Transformations/Differentiation II	6	
5	Vectors	4	
6	Recurrence Relations	3	
7	Quadratics/Differentiation II	6	
8	The Circle	5	
9	Quadratics	5	
10	Integration	6	
11	Integration	4	
12	Vectors	7	
13	Differentiation II	8	
Total		**65**	

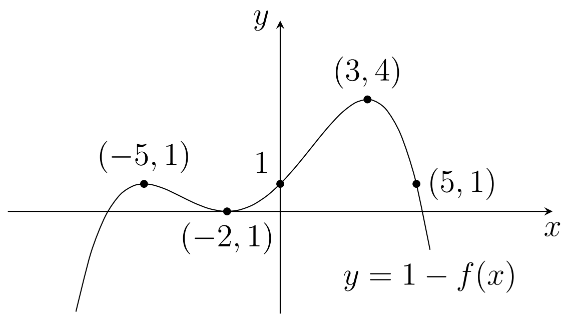
ANSWERS - Practice Exam B Paper 2

1. $y = 3x + 3$

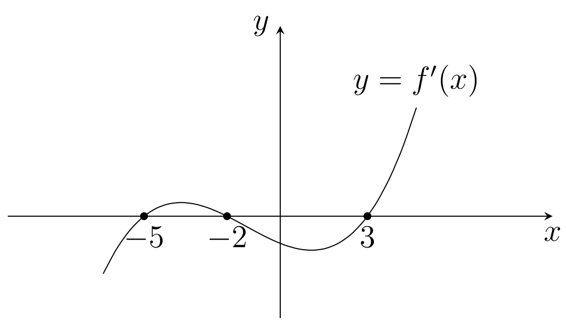
2. $y = \frac{2}{3}x + 4$ or $3y = 2x + 12$

3. $x = \frac{\pi}{4}, \frac{11\pi}{12}, \frac{5\pi}{4}, \frac{23\pi}{12}$

4. (a) Sketch of $y = 1 - f(x)$ with five points shown:



(b) Sketch of $y = f'(x)$ with roots shown:



5. (a) -3
   (b) $k = \frac{1}{3}$
6. (a) $p = -\frac{1}{3}$
   (b) Since $-1 < -\frac{1}{3} < 1$, the sequence converges towards a limit.
7. (a) $-3(x+1)^2 - 1$
   (b) Maximum value of $f'(x)$ is -1, and since $-1 < 0$ then $f'(x) < 0$ for all x. Hence, $f(x)$ is strictly decreasing for all $x \in \mathbb{R}$.
8. (a) $(4, 3)$
   (b) $(x-1)^2 + (y+1)^2 = 25$
9. Repeated factor therefore the line meets the parabola at a tangent, at $(3, 4)$.
10. $\frac{32}{3}$ square units
11. $f(x) = 2x^2 + 5x + 9$
12. (a) Step-by-step proof, likely beginning with $V = \frac{1}{2}x^2y$ and using $12 = x^2 + xy$
    (b) $x = 2$