

# Higher Prelim Revision by Topic

The Higher Prelim Webpage contains **hints and worked solutions** for these questions, as well as **full practice papers** to help you prepare for the Prelim.



## FORMULAE LIST (*only includes formulae already met in course*)

### Circle

$x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre  $(-g, -f)$ , radius  $\sqrt{g^2 + f^2 - c}$ .

$(x - a)^2 + (y - b)^2 = r^2$  represents a circle centre  $(a, b)$  and radius  $r$ .

### Scalar product

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$ , where  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$

or

$$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 \text{ where } \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

### Trigonometric formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

# Prelim Topic List

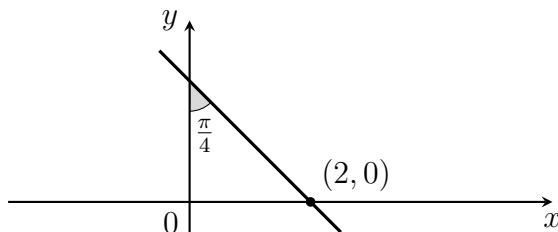
1. Straight Line
2. Recurrence Relations
3. Differentiation I
4. Quadratic Theory
5. Sets and Functions
6. Trigonometry
7. Graph Transformations
8. Vectors
9. Differentiation II
10. Polynomials
11. Integration
12. Addition Formulae
13. The Circle

## ★ Answers

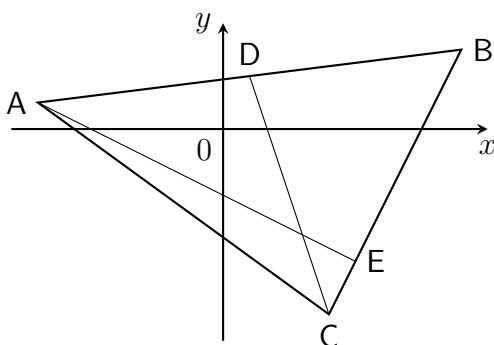
**Note:** Questions which require a **calculator** are marked with .

## 1 Straight Line

- A and B are the points  $(4, -2)$  and  $(10, 6)$ .  
Find the equation of the perpendicular bisector of AB.
- Determine the equation of the line perpendicular to  $6x - 3y + 7 = 0$ , passing through  $(3, -4)$ .
- A line makes an angle of  $\frac{\pi}{4}$  radians with the  $y$ -axis, as shown in the diagram below.



- Determine the equation of the line.
- Show that the points  $P(-3, 7)$ ,  $Q(-1, 1)$  and  $R(3, -11)$  are collinear.
  - Triangle ABC has vertices  $A(-7, 1)$ ,  $B(9, 3)$  and  $C(4, -7)$ .



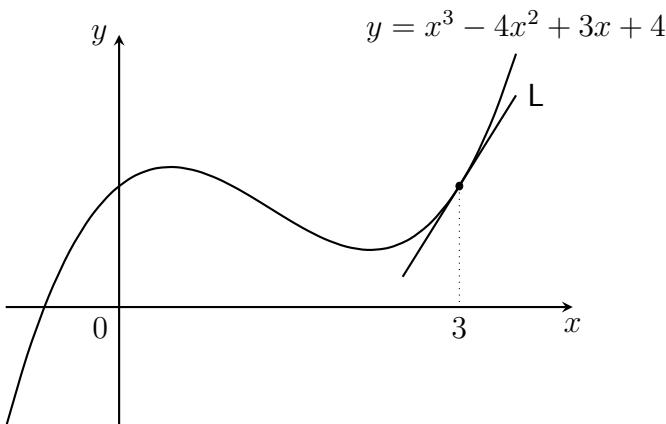
- Find the equation of the median CD.
- Find the equation of the altitude AE.
- Find the coordinates of the point of intersection of CD and AE.

## 2 Recurrence Relations

6. A sequence is defined by the recurrence relation  $u_{n+1} = \frac{1}{3}u_n + 6$  with  $u_0 = 27$ .
- Calculate the value of  $u_2$ .
  - i. Explain why this sequence approaches a limit as  $x \rightarrow \infty$ .  
ii. Calculate this limit.
7. For a sequence generated by  $u_{n+1} = -2u_n + 5$ , one of its terms is  $u_3 = 13$ .
- Find the value of  $u_2$ .
  - Explain why this sequence does not approach a limit as  $x \rightarrow \infty$ .
8. A sequence is generated by the recurrence relation  $u_{n+1} = ku_n - 4$ , where  $k$  is a constant, and  $u_4 = 6$ .
- Express  $u_5$  in terms of  $k$ .
  - Hence or otherwise, determine the value of  $k$  given  $u_5 = -1$ .
9. A population of red squirrels in a local area is declining by 12% each year. To support the population, wildlife conservationists create a scheme to relocate red squirrels from other areas with strong populations. They plan to release 30 squirrels into the area yearly.
- If  $u_n$  is the estimated number of red squirrels  $n$  years after the start of the scheme, it is believed that the population size can be modelled with the recurrence relation:
- $$u_{n+1} = au_n + b$$
- State the values of  $a$  and  $b$ .
- Explain why the population size will stabilise in the long term, and calculate the expected long term population.
10. A sequence generated by the recurrence relation  $u_{n+1} = pu_n + q$  has consecutive terms 24, -10 and 7. Find  $p$  and  $q$ .

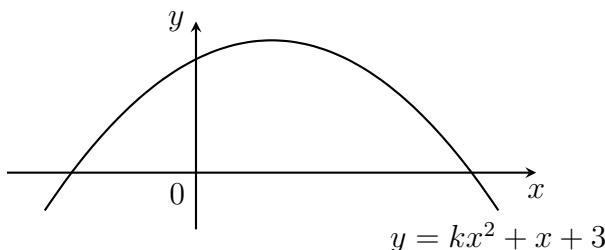
### 3 Differentiation I

11. Function  $f$  is defined by  $f(x) = 3x^2 - 7x + 5$ . Find the value of  $f'(-2)$ .
12. Differentiate  $y = \frac{2x^5 - 3}{3x^2}$ , where  $x \neq 0$ .
13. Find the gradient of the tangent to the curve with the equation  $y = 4x^3 - 2x^2 - 7x + 5$  at the coordinate  $(-1, 6)$ .
14. Part of the graph of  $y = x^3 - 4x^2 + 3x + 4$  is shown below.  
Line L is a tangent to the curve at the point where  $x = 3$ .



Determine the equation of line L.

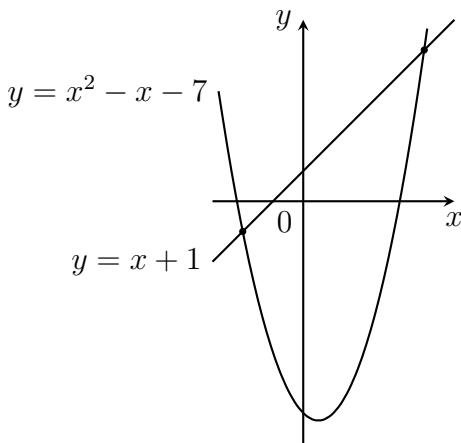
15. Calculate the rate of change of  $h(t) = 12\sqrt{t} - 3$  when  $t = 9$ .
16. Part of the graph of  $y = kx^2 + x + 3$  is shown below.



The tangent to the curve where  $x = 2$  has a gradient  $-1$ . Find  $k$ .

## 4 Quadratic Theory

17. Write  $3x^2 - 24x + 57$  in the form  $p(x + q)^2 + r$ .
18. Write  $-2x^2 + 8x - 1$  in the form  $a(x + b)^2 + c$ .
19. Solve the inequation  $2x^2 + 8x - 10 > 0$ .
20. Solve  $m^2 - m - 20 \leq 0$ .
21. Given that the equation  $3x^2 - px + 3 = 0$  has equal roots, where  $p$  is a constant, determine the possible values of  $p$ .
22. Find the range of values of  $q$  such that the equation  $5x^2 - 8x + 2 - q = 0$  has no real roots.
23. Find the range of values of  $k$  such that  $x^2 + (k - 2)x + 4 = 0$  has real, distinct roots.
24. Find the coordinates of the points of intersection of the curve with equation  $y = x^2 - x - 7$  and the line with equation  $y = x + 1$ .



25. Show that the line with equation  $y = 4x - 3$  is a tangent to the parabola with equation  $y = 2x^2 - 4x + 5$ , and find the point of intersection.

## 5 Sets and Functions

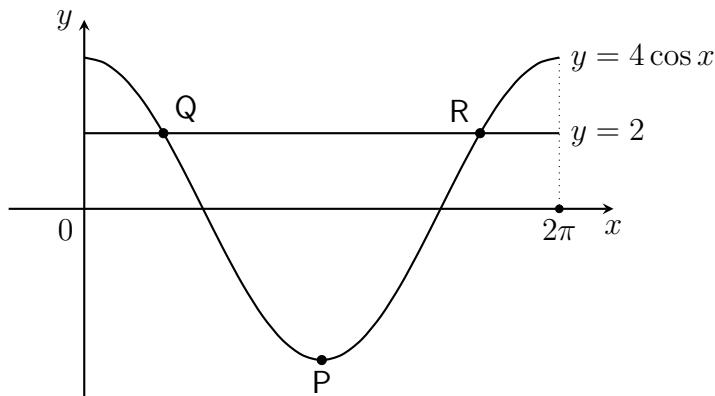
26. A function,  $h$ , is defined on the set of real numbers by  $h(x) = \frac{x-4}{3}$ .  
Find the inverse function,  $h^{-1}(x)$ .
27. Function  $f$  is defined by  $f(x) = 2\sqrt[3]{x} - 3$  where  $x \in \mathbb{R}$ .  
Find the inverse function  $f^{-1}(x)$ .
28. Determine the domain of the function  $g(x) = \frac{5}{2x-1}$ .
29. State the range of values for which  $g(x) = \sqrt{x-3}$  is undefined.
30. Functions  $f$  and  $g$  are defined on  $\mathbb{R}$  by:
- $f(x) = \frac{x+1}{2}$
  - $g(x) = 4x - 3$
- Determine an expression for  $f(g(x))$ .
31. Functions  $f$  and  $g$  are defined on  $\mathbb{R}$  by:
- $f(x) = 2(x-1)$
  - $g(x) = \frac{x+2}{2}$
- (a) Find  $g(f(x))$ .
- (b) Hence state the relationship between functions  $f$  and  $g$ .
32. Functions  $f$  and  $g$  are given by  $f(x) = x^2 - 1$  and  $g(x) = x - 3$ .
- (a) Find an expression for  $k(x)$  where  $k(x) = f(g(x))$ .
- (b) State the range of  $k(x)$ , where  $x \in \mathbb{R}$ .
- (c) Solve  $k(x) = 0$ .

## 6 Trigonometry

33. Solve  $2 \sin x - 1 = 0$  where  $0 \leq x \leq 2\pi$ .

34. Solve  $\sqrt{3} \tan x + 1 = 0$  where  $0 \leq x \leq 2\pi$ .

35. Part of the graph of  $y = 4 \cos x$  is shown below.



The point P is a minimum turning point on  $y = 4 \cos x$ . The line with equation  $y = 2$  intercepts  $y = 4 \cos x$  at points Q and R.

(a) State the coordinates of P.

(b) Determine the coordinates of Q and R.

36. Solve  $2 \cos 2x^\circ - 1 = 0$  where  $0 \leq x \leq 360$ .

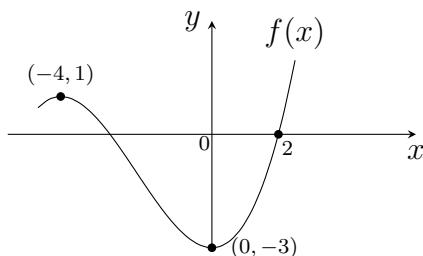
37. Solve  $2 \sin 2x^\circ + \sqrt{3} = 0$  where  $0 \leq x \leq 360$ .

38. Solve  $\tan(x - \frac{\pi}{6}) + 1 = 0$  where  $0 \leq x \leq 2\pi$ .

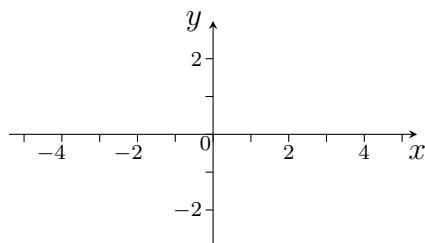
39. Solve  $2 \cos(2x + \frac{\pi}{3}) + \sqrt{3} = 0$  where  $0 \leq x \leq 2\pi$ .

## 7 Graph Transformations

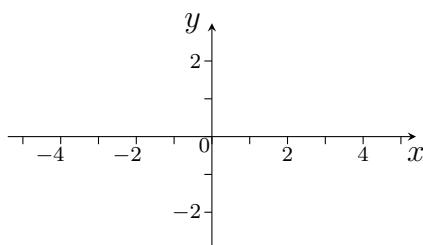
40. Given  $y = f(x) \dots$



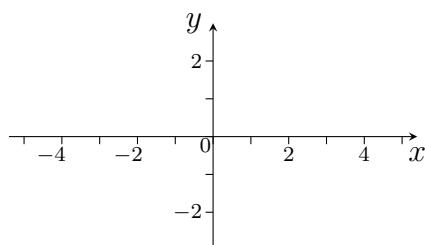
(a) Sketch  $y = f(x - 2) + 1$



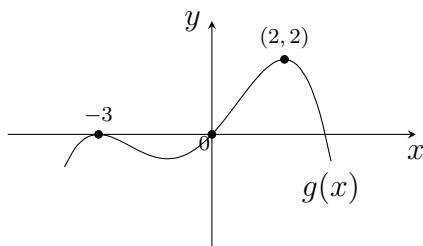
(b) Sketch  $y = -f(x) - 1$



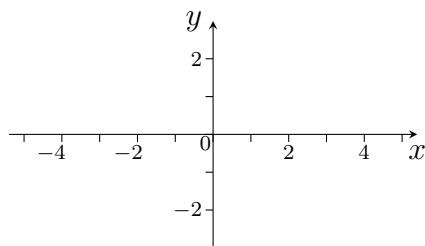
(c) Sketch  $y = f(-x) + 2$



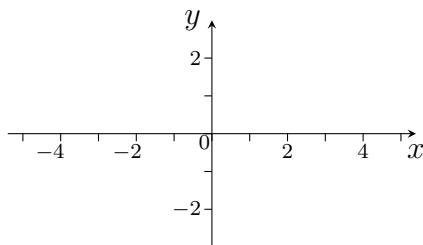
41. Given  $y = g(x) \dots$



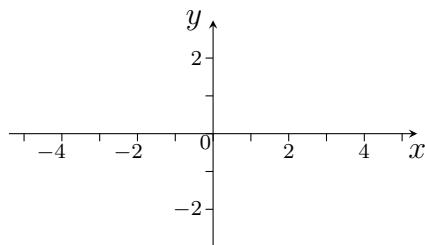
(a) Sketch  $y = g(x + 1) - 2$



(b) Sketch  $y = 2 - g(x)$



(c) Sketch  $y = \frac{1}{2}g(x) + 3$



## 8 Vectors

42. (a) Shows that points  $P(-3, 5, -7)$ ,  $Q(1, 2, -6)$  and  $R(9, -5, -4)$  are collinear.  
(b) State the ratio in which the points Q divides the lines PR.
43. D and F have coordinates  $(8, -7, 4)$  and  $(3, 3, 4)$  respectively.  
Find the coordinates of point E which divides DF in the ratio  $3 : 2$ .
44. The first of three festive balloons spelling M, E and R are attached to pegs in a room whose positions can be described by the coordinates  $(4, -1, 7)$ ,  $(2, 5, 8)$  and  $(-1, 3, 11)$  respectively.

(a) Express  $\overrightarrow{EM}$  and  $\overrightarrow{ER}$  in component form.

(b) Calculate  $\overrightarrow{EM} \cdot \overrightarrow{ER}$ .

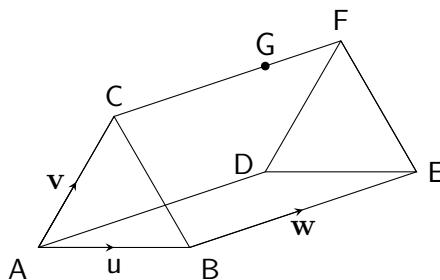
(c) Calculate the size of angle MER. 

45. Given  $\mathbf{u} = 2\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$ :

(a) Calculate  $|\mathbf{u}|$ .

(b) Hence find the components of the unit vector  $\mathbf{a}$  parallel to  $\mathbf{u}$ .

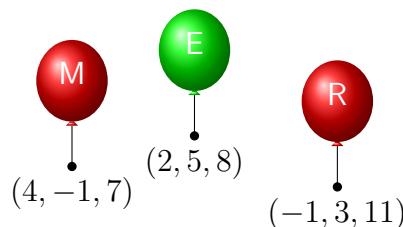
46. The prism below has an equilateral triangle as its cross-section.  
 $|\mathbf{u}| = 3$  and G divides CF in the ratio 2:1.



(a) Express vector  $\overrightarrow{BF}$  in terms of  $\mathbf{u}$ ,  $\mathbf{v}$  and  $\mathbf{w}$ .

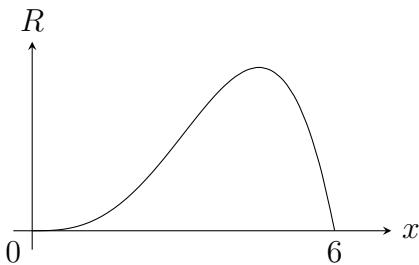
(b) Express vector  $\overrightarrow{GB}$  in terms of  $\mathbf{u}$ ,  $\mathbf{v}$  and  $\mathbf{w}$ .

(c) Evaluate: (i)  $\mathbf{u} \cdot (\mathbf{u} - \mathbf{v})$       (ii)  $\mathbf{v} \cdot (\mathbf{u} + \mathbf{w})$



## 9 Differentiation II

47. Find the range of values of  $x$  for which  $f(x) = \frac{3}{2}x^2 + 6x - 7$  is increasing.
48. Determine the range of values of  $x$  for which the curve with equation  $y = x^3 - 3x^2 - 24x - 1$  is decreasing.
49. A function  $f$  is defined on  $\mathbb{R}$  by  $f(x) = \frac{1}{3}x^3 + x^2 - 15x + 7$ . Determine the  $x$ -coordinates of the stationary points of  $f(x)$ .
50. Find the coordinates of the stationary points on the curve with equation  $y = 2x^3 - 3x^2 - 36x + 5$  and determine their nature.
51. A function  $g$ , defined by  $g(x) = x^3 - x^2 + 2x - 1$  is strictly increasing for  $-1 \leq x \leq 2$ . Determine the greatest and least values of  $g$ .
52. Find the value of  $x$  which minimises  $f(x) = x^2 + \frac{54}{x}$  for  $x > 0$ .
53. A company has determined that a mathematical equation can model the amount of revenue in thousands of pounds,  $R$ , which may be earned by setting the price for a new product as  $x$  pounds.

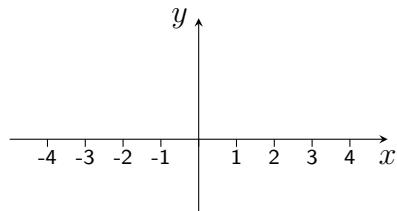
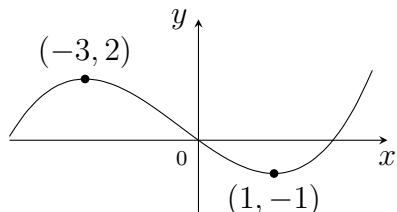


The diagram illustrates the model used by the company, which is:

$$R(x) = 6x^3 - x^4 \text{ for } 0 < x \leq 6$$

Find the value of  $x$  which gives the maximum revenue for the product.

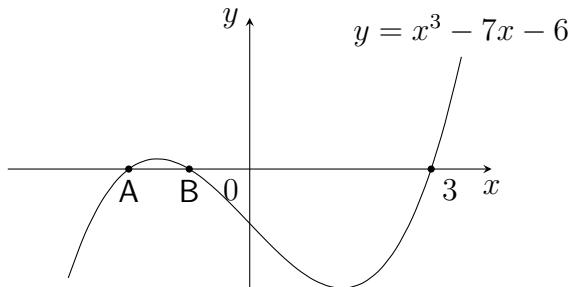
54. Part of the graph of  $y = f(x)$  for the cubic function  $f(x)$  is below.



Sketch  $y = f'(x)$  using the set of axes to the right as a guide.

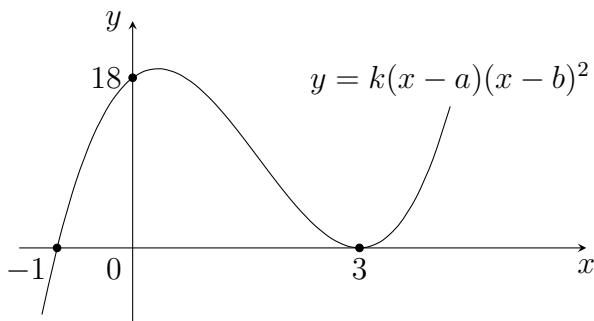
## 10 Polynomials

55. Determine the remainder when  $2x^3 - 5x^2 + 7x - 3$  is divided by  $(x - 2)$ .
56. (a) Show that  $(x + 1)$  is a factor of  $x^3 - x^2 - 10x - 8$ .  
(b) Hence, or otherwise, factorise  $x^3 - x^2 - 10x - 8$  fully.
57. A function  $f$  is defined on a suitable domain by  $f(x) = 2x^3 + x^2 - 7x - 6$ .  
(a) Show that  $(x - 2)$  is a factor of  $f(x)$ .  
(b) Solve  $f(x) = 0$ .
58. Part of the graph of  $y = x^3 - 7x - 6$  is shown below.



Determine the coordinates of points A and B.

59. Given  $(x + 2)$  is a factor of  $2x^3 + kx^2 - 14x + 8$ , find  $k$ .
60. Part of the graph of  $y = k(x - a)(x - b)^2$  is shown below.



Determine the values of  $k$ ,  $a$  and  $b$ .

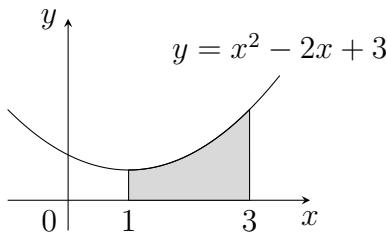
## 11 Integration

61. Calculate:  $\int (x^3 - 6x^2 + 10x - 7) dx$ .

62. Find:  $\int (12x^3 + 6\sqrt{x}) dx$ , where  $x \geq 0$ .

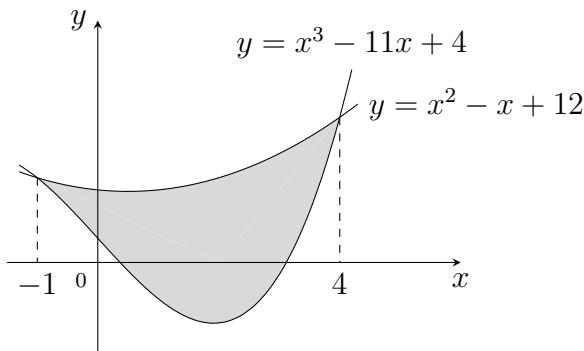
63. Calculate:  $\int_1^2 \left(3x^2 + \frac{1}{x^2}\right) dx$ ,  $x \neq 0$ .

64. The diagram shows part of the graph of  $y = x^2 - 2x + 3$ .



Calculate the shaded area.

65. In the diagram below, the region enclosed by two curves is shaded.

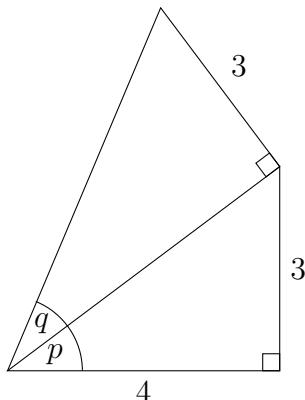


Calculate the area of the shaded region. █

66. Given that function  $f$  is defined such that  $f'(x) = 3x^2 - 8x + 7$  and that the graph of  $y = f(x)$  passes through  $(2, 10)$ , find an expression for  $f(x)$ .

## 12 Addition Formulae

67. The diagram below shows two right-angled triangles placed together.



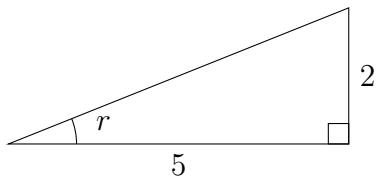
(a) Determine the value of:

- i.  $\sin p$
- ii.  $\sin q$

(b) Hence determine the value of:

- i.  $\sin(p+q)$
- ii.  $\cos(p+q)$

68. The right-angled triangle in the diagram below is such that  $0 < r < \frac{\pi}{4}$ .



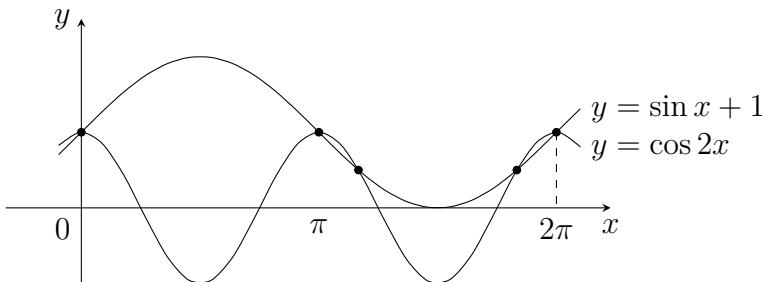
Determine the value of:

- (a)  $\cos r$
- (b)  $\cos 2r$

69. Solve the equation  $\sin 2x^\circ - \cos x^\circ = 0$  for  $0 \leq x \leq 360$ .

70. Solve the equation  $\cos 2x - \cos x = 0$  for  $0 \leq x \leq 2\pi$ .

71. Part of the graphs of  $y = \cos 2x$  and  $y = \sin x + 1$  are shown below.



Determine the  $x$ -coordinates of the five points of intersection shown.

## 13 The Circle

72. Determine the radius and centre for each of the following circles:

(a)  $(x - 6)^2 + (y + 1)^2 = 20$

(b)  $x^2 + y^2 - 8x - 6y + 21 = 0$

73.  $C_1$  is the circle with equation  $x^2 + y^2 + 10x - 4y - 7 = 0$ .

(a) Find the centre of circle  $C_1$ .

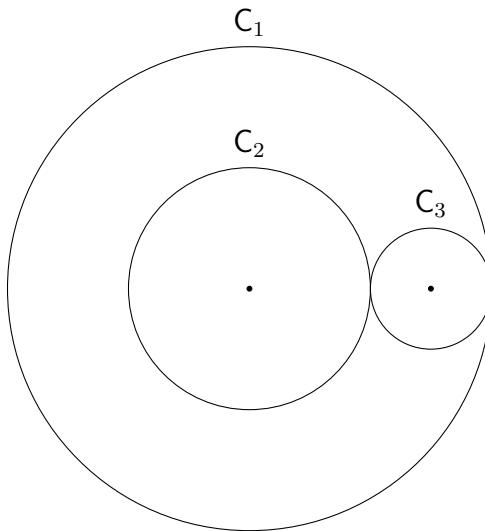
(b) Find the radius of circle  $C_1$ .

$C_2$  is a circle with a diameter half that of  $C_1$ , and centre  $(-7, 1)$ .

(c) Determine the equation of  $C_2$

74. The diagram below shows two concentric circles which each touch a smaller circle.

- The largest circle  $C_1$  has equation  $x^2 + y^2 + 12x - 8y - 12 = 0$
- The smallest circle  $C_3$  has equation  $x^2 + (y - 4)^2 = 4$



(a) Determine the equation of the remaining circle,  $C_2$ .

(b) Find the coordinates of the point where circles  $C_2$  and  $C_3$  touch.

# Answers

## Answers: 1 Straight Line

1.  $4y = 3x - 6$  or  $y = \frac{3}{4}x - \frac{3}{2}$
2.  $2y = -x - 5$  or  $y = -\frac{1}{2}x - \frac{5}{2}$
3.  $y = -x - 2$
4.  $m_{PQ} = m_{QR} = -3$ , valid statement
5. (a)  $y = -3x - 5$   
(b)  $2y = -x - 5$  or  $y = -\frac{1}{2}x - \frac{5}{2}$   
(c)  $(3, -4)$

## Answers 2 Recurrence Relations

6. (a) 11  
(b) i.  $-1 < \frac{1}{3} < 1$ , valid statement  
ii. 9
7. (a) -4  
(b)  $-2 > 1$ , valid statement
8. (a)  $6k - 4$   
(b)  $\frac{1}{2}$
9. (a)  $a = 0.88, b = 30$   
(b)  $-1 < 0.88 < 1$ , valid statement, limit= 250
10.  $p = -\frac{1}{2}, q = 2$

### Answers 3 Differentiation I

11.  $-19$

12.  $\frac{dy}{dx} = 2x^2 + 2x^{-3}$

13.  $9$

14.  $y = 6x - 14$

15.  $2$

16.  $k = -\frac{1}{2}$

### Answers 4 Quadratic Theory

17.  $3(x - 4)^2 + 9$

18.  $-2(x - 2)^2 + 7$

19.  $x < -5, x > 1$

20.  $-4 \leq m \leq 5$

21.  $p = \pm 6$

22.  $q < -\frac{6}{5}$

23.  $k < -2, k > 6$

24.  $(-2, -1)$  and  $(4, 5)$

25.  $(x - 2)$  is a repeated root,  $(2, 5)$

## Answers 5 Sets and Functions

26.  $h^{-1}(x) = 3x + 4$

27.  $f^{-1}(x) = \left(\frac{x+3}{2}\right)^3$

28.  $x \neq \frac{1}{2}$

29.  $x < 3$

30.  $2x - 1$

31. (a)  $x$

(b)  $f$  and  $g$  are inverse to each other

32. (a)  $x^2 - 6x + 8$

(b)  $k(x) \geq -1$

(c)  $x = 2, x = 4$

## Answers 6 Trigonometry

33.  $x = \frac{\pi}{6}, \frac{5\pi}{6}$

34.  $x = \frac{5\pi}{6}, \frac{11\pi}{6}$

35. (a)  $(\pi, -4)$

(b)  $Q\left(\frac{\pi}{3}, 2\right), R\left(\frac{5\pi}{3}, 2\right)$

36.  $x = 30, 150, 210, 330$

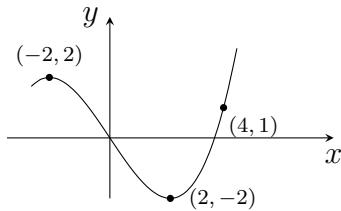
37.  $x = 120, 150, 300, 330$

38.  $x = \frac{11}{12}, \frac{23\pi}{12}$

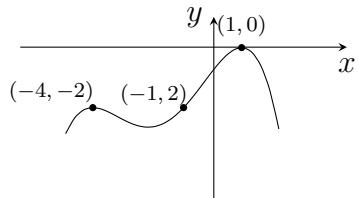
39.  $x = \frac{2\pi}{3}, \pi, \frac{8\pi}{3}, 3\pi$

## Answers 7 Graph Transformations

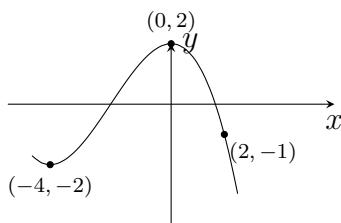
40. (a)  $y = f(x - 2) + 1$



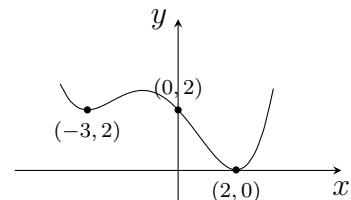
41. (a)  $y = g(x + 1) - 2$



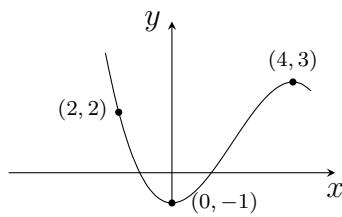
(b)  $y = -f(x) - 1$



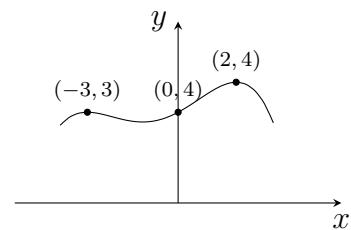
(b)  $y = 2 - g(x)$



(c)  $y = f(-x) + 2$



(c)  $y = \frac{1}{2}g(x) + 3$



## Answers 8 Vectors

42. (a)  $2\overrightarrow{PQ} = \overrightarrow{QR}$  or equivalent, valid statement

(b) 1 : 2

43. (5, -1, 4)

44. (a)  $\overrightarrow{EM} = \begin{pmatrix} 2 \\ -6 \\ -1 \end{pmatrix}$  and  $\overrightarrow{ER} = \begin{pmatrix} -3 \\ -2 \\ 3 \end{pmatrix}$

(b) 3

(c)  $84.3^\circ$

45. (a)  $\sqrt{21}$

(b)  $\begin{pmatrix} 1/3 \\ -2/3 \\ 2/3 \end{pmatrix}$

46. (a)  $\overrightarrow{BF} = -\mathbf{u} + \mathbf{v} + \mathbf{w}$

(b)  $\overrightarrow{GB} = -\frac{2}{3}\mathbf{w} - \mathbf{v} + \mathbf{u}$

(c) i.  $\frac{9}{2}$

ii.  $\frac{9}{2}$

## Answers 9 Differentiation II

47.  $x > -2$

48.  $-2 < x < 4$

49.  $x = -5, x = 3$

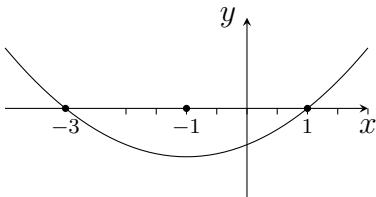
50. maximum turning point at  $(-2, 49)$ , minimum turning point at  $(3, -76)$

51. minimum value =  $-5$ , maximum value =  $7$

52.  $x = 3$

53.  $x = 4.5$

54.  $y = f'(x)$  must be a parabola with turning point when  $x = -1$



## Answers 10 Polynomials

55. 7

56. (a) Remainder = 0 therefore  $(x + 1)$  is a factor

(b)  $(x + 1)(x + 2)(x - 4)$

57. (a) Remainder = 0 therefore  $(x - 2)$  is a factor

(b)  $x = -\frac{3}{2}, x = -1, x = 2$

58. A( $-2, 0$ ) and B( $-1, 0$ )

59.  $k = -7$

60.  $a = -1, b = 3, k = 2$

## Answers 11 Integration

61.  $\frac{1}{4}x^4 - 2x^3 + 5x^2 - 7x + C$

62.  $3x^4 + 4x^{\frac{3}{2}} + C$

63.  $\frac{15}{2}$

64.  $\frac{20}{3}$  square units

65.  $\frac{875}{12}$

66.  $f(x) = x^3 - 4x^2 + 7x + 4$

## Answers 12 Addition Formulae

67. (a) i.  $\frac{3}{5}$   
ii.  $\frac{3}{\sqrt{34}}$

(b) i.  $\frac{3}{5\sqrt{34}}$   
ii.  $\frac{11}{5\sqrt{34}}$

68. (a)  $\frac{5}{\sqrt{29}}$

(b)  $\frac{21}{29}$

69.  $x = 30, 90, 150, 270$

70.  $x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi$   
item  $x = 0, \frac{7\pi}{6}, \pi, \frac{11\pi}{6}, 2\pi$

## Answers 13 The Circle

71. (a) radius=  $\sqrt{20}$ , centre= (6, -1)

(b) radius= 2, centre= (4, 3)

72. (a) centre= (-5, 2)

(b) radius= 6

(c)  $(x + 7)^2 + (x - 1)^2 = 9$

73. (a)  $(x + 6)^2 + (y - 4)^2 = 16$

(b) (-2, 4)