FORMULAE LIST

Circle

$x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$, radius $\sqrt{g^2 + f^2 - c}$.

$(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta, \text{ where } \theta \text{ is the angle between } \mathbf{a} \text{ and } \mathbf{b}$$

or

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 \text{ where } \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

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1. (a) Show that $(x - 3)$ is a factor of $x^3 - 6x^2 + 5x + 12$. (2)

   (b) Hence, or otherwise, solve $x^3 - 6x^2 + 5x + 12 = 0$. (3)

2. P and Q are the points $(7, 2)$ and $(1, 6)$. (4)
   Find the equation of the perpendicular bisector of PQ.

3. Given that $f(x) = x^{\frac{3}{2}} - \frac{5}{x^6}$, where $x > 0$, find $f'(x)$. (3)

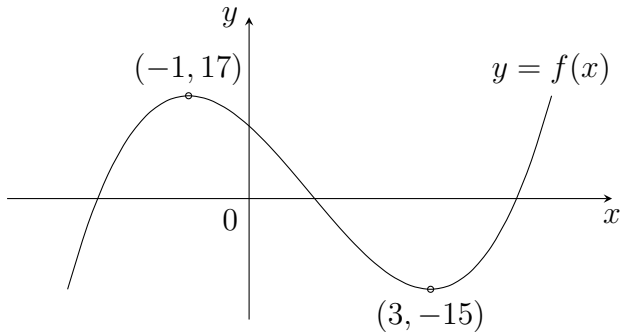
4. A sequence is defined by the recurrence relation $u_{n+1} = -\frac{1}{3}u_n + 8$ with $u_3 = 15$.

   (a) Calculate the value of u_4. (1)

   (b) i. Explain why this sequence approaches a limit as $x \to \infty$. (1)

   ii. Calculate this limit. (2)

5. The diagram shows part of the graph of a cubic function with equation $y = f(x)$. The curve has stationary points at $(-1, 17)$ and $(3, -15)$.

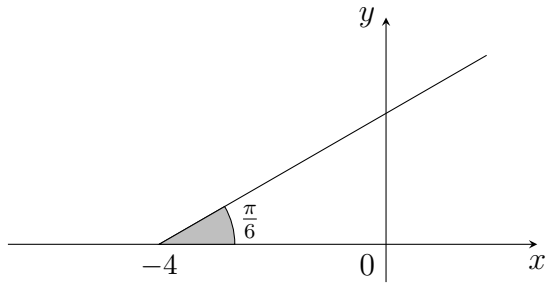


   (a) Sketch the graph of $y = 1 + f(-x)$. (3)

   (b) State the coordinates of the minimum turning point on the graph of $y = -f(x)$. (1)

6. Express $3x^2 - 12x + 25$ in the form $p(x + q)^2 + r$. (3)

7. A line passing through $(-4, 0)$ makes an angle of $\frac{\pi}{6}$ with the x-axis, as shown. (3)



   Determine the equation of the line.

8. Point Q divides the line PR in the ratio $1 : 3$, where $P(-2, 5, 1)$ and $Q(-6, -3, 5)$. (3)
Determine the coordinates of Q.

9. A function, g, is defined by $g(x) = \dfrac{2 - x}{5}$ where $x \in \mathbb{R}$. (3)
Find the inverse function, $g^{-1}(x)$.

10. The equation $x^2 + (2k - 1)x + k^2 = 0$ has real, distinct roots. (3)
Determine the range of values for k.

11. Functions f and g are defined by:

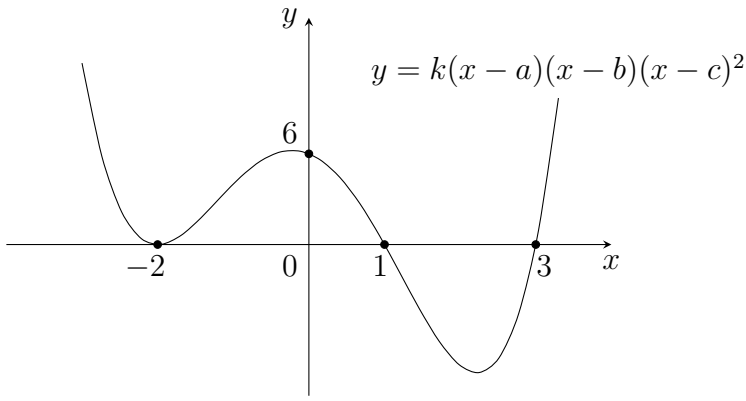
- $f(x) = 2x$
- $g(x) = 2\cos x$

(a) Determine an expression for $g\left(f(x)\right)$. (1)

(b) Hence, or otherwise, evaluate $g\left(f\left(\frac{\pi}{3}\right)\right)$. (2)

(c) Solve $g\left(f(x)\right) = \sqrt{3}$ where $0 < x < 2\pi$. (3)

12. The diagram shows $y = f(x)$ where $f(x)$ is a quartic function. (3)



Express $f(x)$ in the form $f(x) = k(x - a)(x - b)(x - c)^2$.

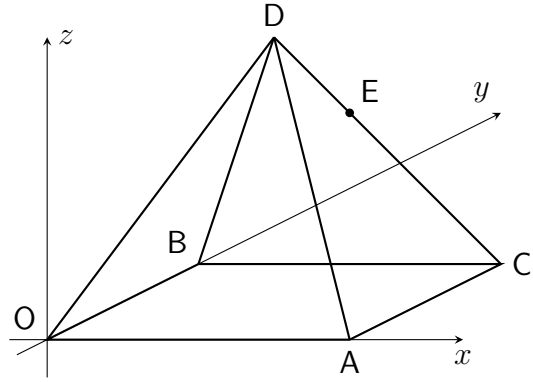
13. (a) Find the x-coordinates of the stationary points on $y = x^3 - \frac{9}{2}x^2 + 6x$. (3)

(b) Determine their nature. (2)

(c) Hence find the greatest and least values of y on the interval $1 \leqslant x \leqslant 2$. (2)

14. A square based pyramid is shown on the diagram below.

- $\overrightarrow{OD} = 3\mathbf{i} + 3\mathbf{j} + 9\mathbf{k}$
- $\overrightarrow{OA} = 6\mathbf{i}$
- $\overrightarrow{OB} = 6\mathbf{j}$
- E divides CD in the ratio 2:1



(a) Express vector $\overrightarrow{CD}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. (2)

(b) Hence calculate $|\overrightarrow{ED}|$. (2)

Question	Topic	Marks Available	Marks Awarded
1	Polynomials	5	
2	Straight Line	4	
3	Differentiation	3	
4	Recurrence Relations	4	
5	Graph Transformations	4	
6	Quadratic Theory	3	
7	Straight Lines	3	
8	Vectors	3	
9	Sets and Functions	3	
10	Quadratic Theory	3	
11	Sets and Functions/Trigonometry	6	
12	Polynomials	3	
13	Differentiation II	7	
14	Vectors	4	
Total		55	

ANSWERS - Practice Exam A Paper 1

1. (a) Remainder of 0 so $(x-3)$ is a factor.
   (b) $x = -1, 3, 4$
2. $2y = 3x - 4$ or $y = \frac{3}{2}x - 2$
3. $f'(x) = \frac{3}{2}x^{\frac{1}{2}} + 30x^{-7}$
4. (a) $u_4 = 3$
      i. $-1 < -\frac{1}{3} < 1$ so a limit exists.
      ii. Limit $= 12$
5. (a) Positive cubic with minimum turning point at $(-3, -14)$ and maximum turning point at $(1, 18)$.
   (b) $(-1, -17)$
6. $3(x-2)^2 + 13$
7. $y = \frac{1}{\sqrt{3}}x + \frac{4}{\sqrt{3}}$
8. Q$(-3, 3, 2)$
9. $g'(x) = 2 - 5x$
10. $\frac{1}{4} > k$
11. (a) $g(f(x)) = 2\cos 2x$
    (b) -1
    (c) $x = \frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12}$
12. $f(x) = \frac{1}{2}(x-1)(x-3)(x+2)^2$
13. (a) $x = 1, x = 2$
    (b) Maximum turning point when $x = 1$
    Minimum turning point when $x = 2$
    (c) Max value $= \frac{5}{2}$, min value $= 2$
14. (a) $-3\mathbf{i} - 3\mathbf{j} + 9\mathbf{k}$
    (b) $\sqrt{11}$