H National Qualifications

Practice Exam C

1 hour 15 mins

Total marks: 55

Mathematics

Paper 1

Non-Calculator

FORMULAE LIST

Circle

$x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$, radius $\sqrt{g^2 + f^2 - c}$.

$(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or

$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Trigonometric formulae

$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2 \sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

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1. A triangle has vertices P$(1, 5)$, Q$(-2, 3)$ and R$(-3, 1)$.

   (a) Find the equation of the altitude through Q. (3)

   (b) Determine the angle that the altitude through Q makes with the positive direction of the x-axis. (1)

2. A function g is defined by $g(x) = 6 - \frac{2}{3}x$ where $x \in \mathbb{R}$. (3)

   Find an expression for the inverse function, $g^{-1}(x)$.

3. Find $\int \left(\sqrt[3]{x} - \dfrac{6}{x^5} \right) \, dx$. (5)

4. A sequence is defined by the recurrence relation $u_{n+1} = \frac{1}{5} u_n - 8$ and $u_0 = 40$.

   (a) Find u_2. (2)

   (b) i. Explain why this sequences converges to a limit as $x \to \infty$. (1)

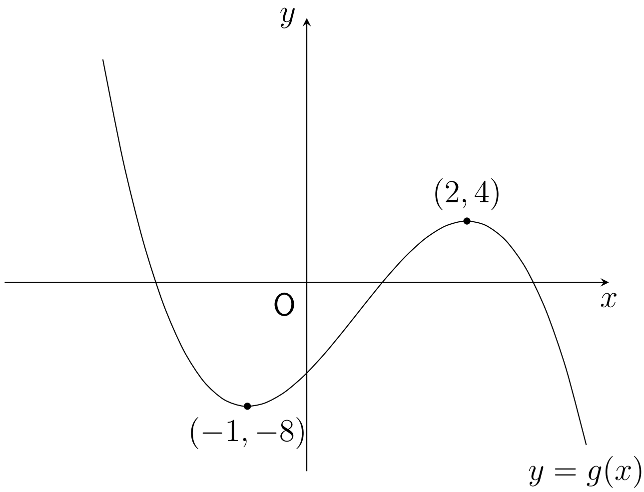
   ii. Calculate this limit. (2)

5. A circle has its centre at $(-4, 6)$ and passes through the point P$(2, 0)$. (3)

   Determine the equation of the circle.

6. Calculate the rate of change of $r(x) = \dfrac{x^5 - 1}{x^2}$ when $x = -2$. (5)

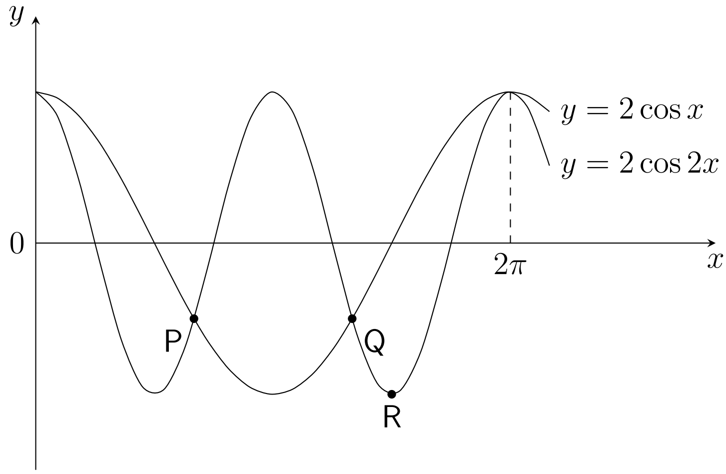
7. Part of the graph of $y = g(x)$, where g is a cubic function, is shown below. (3)



   Sketch the graph of $y = g'(x)$.

8. Given that $(x + 2)$ is a factor of $2x^3 + px + 10$, determine the value of p. (3)

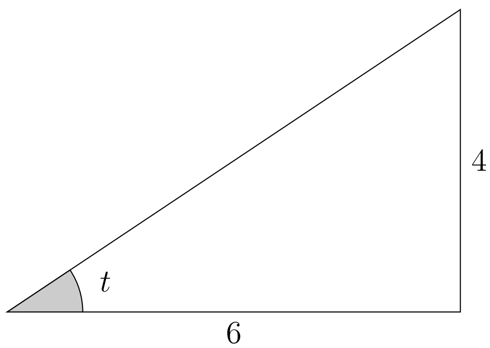
9. Parts of the graphs of $y = 2\cos 2x$ and $y = 2\cos x$ are shown below.



(a) State the coordinates of R, a minimum turning point of $y = 2\cos 2x$. (1)

(b) Determine the coordinates of P and Q, where the two curves intersect. (5)

10. The diagram below shows angle t within a right-angled triangle, where $0 < t < \frac{\pi}{4}$.



(a) Find the value of $\sin t$. (1)

(b) Show that $\sin 2t = \dfrac{12}{13}$. (3)

(c) Hence, or otherwise, find the value of $\tan 2t$. (2)

11. The equation $5x - 2x^2 = k$ has real roots. Determine the range of values for k. (3)

12. Show that the the curve $y = x^3 - 4x^2 + 7x - 2$ does not have any stationary points. (5)

13. Coordinates A, B, C, D and E are such that: (4)

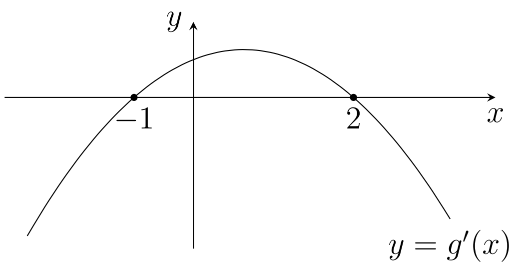
- $\overrightarrow{\mathsf{AB}} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$ and $\overrightarrow{\mathsf{BC}} = \begin{pmatrix} -4 \\ 0 \\ 1 \end{pmatrix}$
- $\overrightarrow{\mathsf{CD}} = 2\overrightarrow{\mathsf{AB}}$
- D divides CE in the ratio $1 : 2$.

Find the components of vector $\overrightarrow{\mathsf{AE}}$.

Question	Topic	Marks Available	Marks Awarded
1	Straight Line	4	
2	Sets and Functions	3	
3	Integration	5	
4	Recurrence Relations	5	
5	The Circle	3	
6	Differentiation I	5	
7	Differentiation II	3	
8	Polynomials	3	
9	Trigonometry / Addition Formulae	6	
10	Addition Formulae	6	
11	Quadratic Theory	3	
12	Differentiation II	5	
13	Vectors	4	
Total		**55**	

ANSWERS - Practice Exam C Paper 1

1. (a) $y = -x + 1$
   (b) $135°$
2. $g^{-1}(x) = \frac{18-3x}{2}$ or $g^{-1}(x) = 9 - \frac{3}{2}x$
3. $\frac{3}{4}x^{\frac{4}{3}} + \frac{3}{2}x^{-4} + C$
4. (a) $u_2 = -8$
   (b) i. Limit exists since $-1 < \frac{1}{5} < 1$.
   ii. Limit$= -10$
5. $(x+4)^2 + (y-6)^2 = 72$
6. $r'(-2) = \frac{47}{4}$
7. Sketch must by a parabola (ie symmetrical)



8. $p = -3$
9. (a) R$(\frac{3\pi}{2}, -2)$
   (b) P$(\frac{2\pi}{3}, -1)$ and Q$(\frac{4\pi}{3} - 1)$
10. (a) $\sin t = \frac{2}{\sqrt{5}}$ or $\sin t = \frac{2\sqrt{5}}{5}$ ($\sin t = \frac{4}{\sqrt{20}}$ should be simplified)
    (b) Step-by-step process leading to the answer, with all working **shown clearly**.
    (c) $\tan 2t = \frac{12}{5}$
11. $\frac{25}{8} \geqslant k$
12. Once the derivative has been obtained, proof must involved the discriminant or obtaining the minimum value of the derivative. Not being able to factorise the derivative by inspection is not sufficient.
13. $\overrightarrow{\text{AE}} = \begin{pmatrix} 17 \\ -7 \\ 15 \end{pmatrix}$