

FORMULAE LIST**Circle**

$x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$, radius $\sqrt{g^2 + f^2 - c}$.

$(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar product

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta, \text{ where } \theta \text{ is the angle between } \mathbf{a} \text{ and } \mathbf{b}$$

or

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 \text{ where } \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals

$f(x)$	$\int f(x) \, dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

1. Given $y = (4x^2 - 3)^5$, find $\frac{dy}{dx}$. (2)

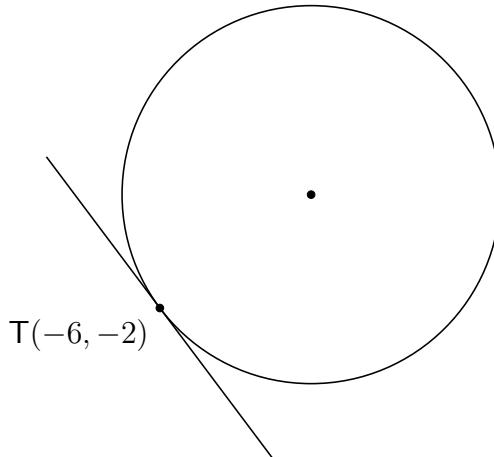
2. Simplify $\log_p 75 - 2 \log_p 5$. (2)

3. Show that the line with equation $y = 2x - 10$ is a tangent to the circle with equation $x^2 + y^2 - 18x + 14y + 85 = 0$ and find the coordinates of the point of contact. (5)

4. Express $8 \cos x + 5 \sin x$ in the form $k \sin(x - a)$ where $k > 0$ and $0 < a < 2\pi$. (4)

5. Find $\int \sqrt[3]{4x+5} dx$ where $x \geq -\frac{5}{4}$. (4)

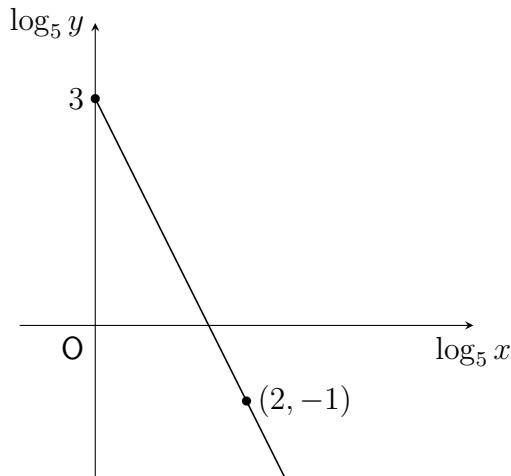
6. The point $T(-6, -2)$ lies on the circle with equation $x^2 + y^2 + 4x - 2y - 20 = 0$. (4)



Find the equation of the tangent to the circle at T.

7. Two variables, x and y , are connected by the equation $y = kx^n$. (5)

The graph of $\log_5 y$ against $\log_5 x$ is a straight line as shown below.



Find the values of k and n .

8. Find $\int_0^1 (4 \sin 2x + 3 \cos 5x) dx$. (4)

9. Workers in a factory leave a piece of hot metal to cool outside on a cold day. The temperature T °C after t minutes after it is put outside is given by the equation:

$$T = 90e^{-0.326t}$$

(a) State the temperature of the metal when it is first taken outside. (1)

(b) Find the time taken for the temperature of the metal to drop by 70%. (3)

END OF QUESTIONS

ANSWERS - Practice Exam D

1. $\frac{dy}{dx} = 40x(4x^2 - 3)^4$

2. $\log_p 3$

3. Repeated factor of $(x - 3)$ therefore the line is a tangent to the circle, with point of contact $(3, -4)$.

4. $\sqrt{89} \sin(x - 5.27)$

5. $\frac{3(4x + 5)^{\frac{4}{3}}}{16} + C$

6. $3y = -4x - 30$ or $y = -\frac{4}{3}x - 10$

7. $k = 125$ and $n = -2$

8. 2.257

9. (a) 90°C

(b) 3.69 minutes