FORMULAE LIST

Circle

$x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$, radius $\sqrt{g^2 + f^2 - c}$.

$(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta, \text{ where } \theta \text{ is the angle between } \mathbf{a} \text{ and } \mathbf{b}$$

or

$$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 \text{ where } \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae

$$\begin{aligned} \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin 2A &= 2 \sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

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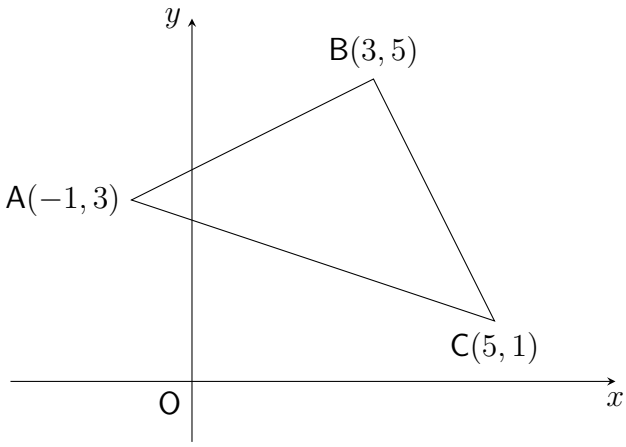
1. Find $\int \left(6\sqrt{x} - 10x^3\right)\, dx$. (4)

2. Find the coordinates of the stationary points on the curve $y = x^3 - 3x + 2$. (4)

3. A function f is defined on a suitable domain by $f(x) = \sqrt[3]{4 + x}$. (3)
   Find an expression for $f^{-1}(x)$.

4. Triangle ABC has vertices A$(-1, 3)$, B$(3, 5)$ and C$(5, 1)$.



(a) Find the equation of the perpendicular bisector of AB. (3)

(b) Find the equation of the median through B. (3)

(c) Determine the coordinates of the point where these lines meet. (2)

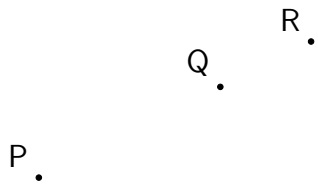
(d) Show that triangle ABC is right-angled at B. (2)

5. Functions f and g are defined on $\mathbb{R}$ by $f(x) = 3x^2 - 3$ and $g(x) = x - 4$.

(a) Find an expression for $f(g(x))$. (2)

(b) Solve $f(g(x)) = 0$. (2)

6. The diagram below shows points P, Q and R where P$(-5, -2)$, Q$(-1, 0)$ and R$(1, 1)$.



(a) Show that points P, Q and R are collinear. (3)

(b) State the ratio in which Q divides PR. (1)

(c) Point S divides PQ in the ratio 1:3. Find the coordinates of S. (3)

7. Solve $3\cos 2x + 3\sin x = 0$ for $0 \leqslant x \leqslant 2\pi$. (5)

8. During the summer, a small pond with no protection from the sun loses 10% of its water daily due to evaporation. The owner is able to put 8 litres of water back into the pond at the end of each day, using unused household water. The volume of water w in the pond can be described by the recurrence relation:

$$w_{n+1} = aw_n + b$$

(a) State the values of a and b. (2)

The water in the pond is measured one morning to be 120 litres.

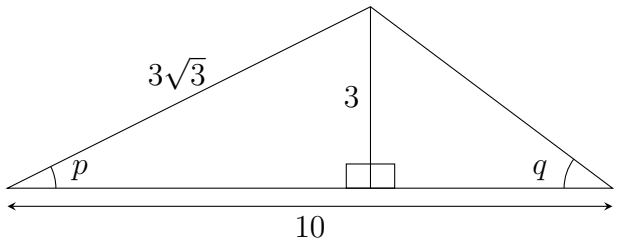
(b) Calculate the expected volume of water in the pond the next morning. (2)

The plants in the pond should survive the summer provided the volume of water in the long-run remains above 70 litres.

(c) Explain whether the plants are likely to survive the summer. (2)

9. The equation $px^2 - 8x - p = 0$ has no real roots. (4)
Determine the range of values for p.

10. The diagram below shows angles p and q within two right-angled triangles.



(a) Find the values of $\sin p$ and $\sin q$. (2)

(b) Hence find the value of $\sin(q - p)$. (3)

11. The function $f(x) = \sqrt[3]{x-2}$ is defined on a suitable domain such that it is strictly increasing with range $1 \leqslant f(x) \leqslant 5$.

(a) Determine the domain of f. (2)

(b) State the range of function g given $g(x) = f(x) + 3$ (1)

Question	Topic	Marks Available	Marks Awarded
1	Integration	4	
2	Differentiation II	4	
3	Sets and Functions	3	
4	Straight Line	10	
5	Sets and Functions	4	
6	Straight Lines/Vectors	7	
7	Addition Formulae	5	
8	Recurrence Relations	6	
9	Quadratic Theory	4	
10	Addition Formulae	5	
11	Sets and Functions	3	
Total		55	

ANSWERS - Practice Exam B Paper 1

1. $4x^{\frac{3}{2}} - \frac{5}{2}x^4 + C$
2. $(-1, 4)$ and $(1, 0)$
3. $f^{-1}(x) = x^3 - 4$
4. (a) $y = -2x + 6$
   (b) $y = 3x - 4$
   (c) $(2, 2)$
   (d) $m_{AB} \times m_{BC} = -1$, hence AB,BC perpendicular $\implies$ ABC right-angled.
5. (a) $f(g(x)) = 3x^2 - 24x + 45$ (b) $x = 3, x = 5$
6. (a) $m_{PQ} = m_{QR} = \frac{1}{2}$ and full collinearity conclusion.
   (b) $2 : 1$
   (c) $(-4, -\frac{3}{2})$
7. $x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$
8. (a) $a = 0.9, b = 8$
   (b) 116 litres
   (c) Limit is 80 litres $\implies$ plants are likely to survive
9. $p < -4, p > 4$ with a sketch drawn
10. (a) $\sin p = \frac{1}{\sqrt{3}}$ and $\sin q = \frac{3}{5}$ (b) $\sin(q - p) = -\frac{2}{5\sqrt{3}}$
11. (a) $3 \leqslant x \leqslant 127$
    (b) $4 \leqslant g(x) \leqslant 8$