Higher Top-Up Prelim Revision by Topic

The Higher Webpage contains hints and worked solutions for these questions, as well as **practice papers** to help you prepare for the March Top-Up Prelim.

FORMULAE LIST

Circle

$x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$, radius $\sqrt{g^2 + f^2 - c}$.

$(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta, \text{ where } \theta \text{ is the angle between } \mathbf{a} \text{ and } \mathbf{b}$$

or

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 \text{ where } \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2 \cos^2 A - 1$$
$$= 1 - 2 \sin^2 A$$

Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals

$f(x)$	$\int f(x) \, dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

Top-Up Prelim Topic List
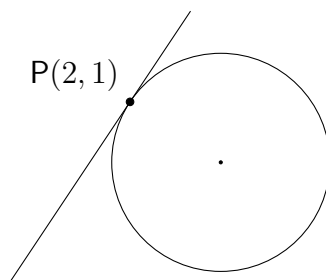
Note: Questions which require a **calculator** are marked with ⌨.

13 The Circle (Continued)

75. Find the coordinates of the points of intersection of the circle with equation $x^2 + y^2 - 8x + 4y - 5 = 0$ and the line with equation $x + y = 3$.

76. Show that the line with equation $y = x - 3$ is a tangent to the circle with equation $x^2 + y^2 - 6x + 4y + 11 = 0$ and find the coordinates of the point of contact.

77. Point $P(2, 1)$ lies on circle C with equation $x^2 + y^2 - 16x + 6y + 21 = 0$.
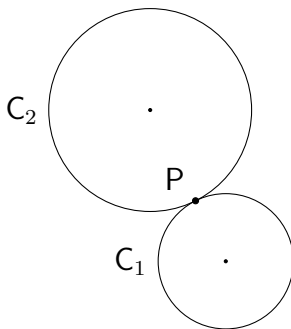

Find the equation of the tangent to the circle at P.

78. Circle C has equation $(x - 5)^2 + (y + 3)^2 = 17$.

    (a) State the radius and the centre of circle C.

    (b) Show that the point $Q(9, -8)$ lies outside of circle C.

79. Circles C_1 and C_2 are shown in the diagram below.

Their equations are:
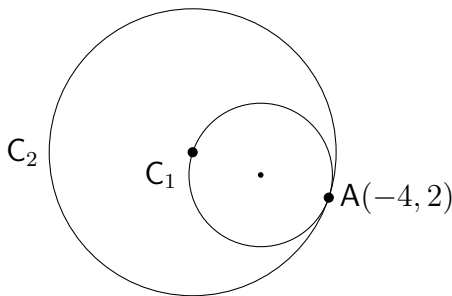
- C_1 : $(x - 3)^2 + (y + 7)^2 = 20$

- C_2 : $x^2 + y^2 + 4x - 6y - 32 = 0$

    (a) Determine the distance between the centres of C_1 and C_2.

    (b) Show that circles C_1 and C_2 touch externally.

    (c) The point of contact, P, is shown on the diagram.
        Determine the coordinates of P.

80. Circle C_1 touches circle C_2 internally at point $A(-4, 2)$ and passes through the centre of circle C_2, as shown in the diagram below.
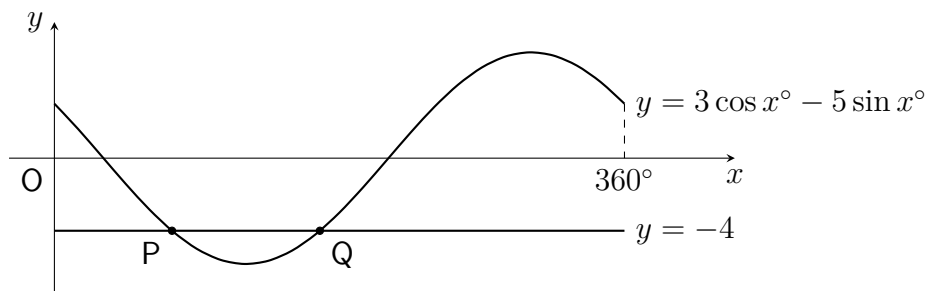

Circle C_1 has equation $x^2 + y^2 + 14x - 6y + 48 = 0$.
Determine the equation of circle C_2.

14 Wave Function

For all questions, $k > 0$ and ▦.

81. **Express** $8 \sin x° + 7 \cos x°$ in the form $k \sin (x + a)°$, $0 < a < 360$.

82. **Express** $\sqrt{5} \sin x + \cos x$ in the form $k \cos (x - a)$, $0 < a < 2\pi$.

83. (a) **Express** $2 \sin x° - 4 \cos x°$ in the form $k \sin (x - a)°$, $0 < x < 360$.

    (b) **Hence state:**
      - i. The maximum value of $2 \sin x° - 4 \cos x°$.
      - ii. The value of x for which it occurs, where $0 < x < 360$.

84. (a) **Express** $2 \cos x° - \sin x°$ in the form $k \cos (x + a)°$, $0 < a < 360$.

    (b) **Hence solve** $2 \cos x° - \sin x° = -1$ where $0 < x < 360$.

85. Part of the graphs of $y = 3 \cos x° - 5 \sin x°$ and $y = -4$ are shown in the diagram below:

Points P and Q are points of intersection.

    (a) **Express** $3 \cos x° - 5 \sin x°$ in the form $k \cos (x + a)°$, $0 < a < 360$.

    (b) **Hence determine the coordinates of P and Q.**

15 Logs and Exponentials

86. Express $2\log_p 10 + \log_p 2 - \log_p 50$ in the form $\log_p q$.

87. Evaluate the following:

    (a) $\log_3 \frac{1}{4} + 2\log_3 6$

    (b) $\frac{1}{2}\log_5 64 - \log_5 40$

88. Solve each equation:

    (a) $\log_6 x + \log_6 4 = 2$

    (b) $\log_2 x + \log_2 (x + 7) = 3$

89. Given that $\log_p 4 + 3 = \log_p 32$, find the value of p.

90. A population of mice is discovered living in some woodland. The number of mice can be modelled by:

$$M = 80e^{kt}$$

where M is the estimated number of mice in the population, t is the number of weeks since the population was discovered and k is a constant.

    (a) State the estimated number of mice in the population when it was first discovered.
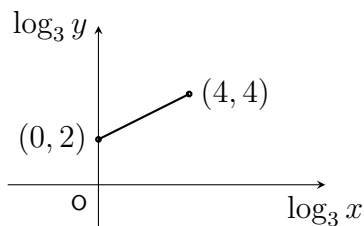
    (b) It is estimated there will be 120 mice in the population after 3 weeks. Find the value of k. ▦

    (c) Calculate the time taken for the population to double in size. ▦

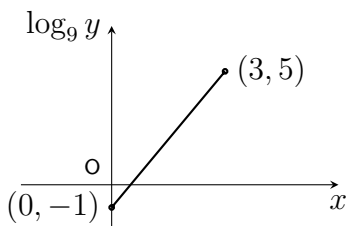91. Variables x and y are linked by the equations given below. Find the values of a and b for each:

    (a) $\qquad\qquad y = ax^b$

$\log_3 y$

$(4, 4)$

$(0, 2)$

O $\qquad\qquad \log_3 x$

    (b) $\qquad\qquad y = ab^x$

$\log_9 y$

$(3, 5)$

O

$(0, -1)$ $\qquad\qquad x$
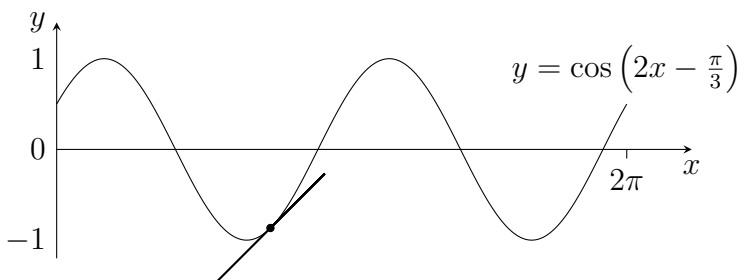
16 Further Calculus

92. Given that $y = (6x - 5)^3$, find $\dfrac{dy}{dx}$.

93. A function, f, is defined on a suitable domain by $f(x) = \dfrac{1}{(3x - 5)^2}$.
    Determine the rate of change of f when $x = 2$.

94. Given $g(x) = 4\cos 3x + 2\sin 2x$, find the value of $g'\left(\frac{\pi}{2}\right)$.

95. Differentiate $\sqrt[3]{6x^2 - 1}$ with respect to x.

96. Find the gradient of the tangent to $y = \cos\left(2x - \frac{\pi}{3}\right)$ when $x = \frac{3\pi}{4}$.

$y = \cos\left(2x - \frac{\pi}{3}\right)$

97. Find $\displaystyle\int (2x - 3)^5 \, dx$.

98. Evaluate $\displaystyle\int_0^1 2\cos 3x \, dx$. ▦

99. Find $\displaystyle\int \dfrac{3}{(x + 4)^{\frac{1}{2}}} \, dx$.

100. Find the value of $\displaystyle\int_0^{\frac{\pi}{3}} \sin\left(x + \frac{\pi}{6}\right) \, dx$.

Answers

Answers 13 The Circle

75. $(1, 2)$ and $(8, -5)$

76. $(2, -1)$ and statement (*"Show that..."*)

77. $y = \frac{3}{2}x - 2$ or $2y = 3x - 4$

78. (a) Centre $(5, -3)$ and radius $\sqrt{17}$

    (b) $\sqrt{41} > \sqrt{17}$ and conclusion, or other valid approach

79. (a) $\sqrt{125}$ or $5\sqrt{5}$

    (b) $3\sqrt{5} + 2\sqrt{5} = 5\sqrt{5}$ and conclusion, or other valid approach

    (c) $(1, -3)$

80. $(x + 10)^2 + (y - 4)^2 = 40$

Answers 14 Wave Function

81. $\sqrt{113} \sin(x + 41.2)°$

82. $\sqrt{6} \cos(x - 1.15)$

83. (a) $2\sqrt{5} \sin(x - 63.4)°$

    (b)   i. max value is $2\sqrt{5}$

        ii. occurs when $x = 153.4$

84. (a) $\sqrt{5} \cos(x + 26.6)°$

    (b) $x = 90, 216.9$

85. (a) $\sqrt{34} \cos(x + 59.0)°$

    (b) P$(74.3°, -4)$ and Q$(167.7°, -4)$

Answers 15 Logs and Exponentials

86. $\log_p 4$

87. (a) 2

    (b) -1

88. (a) $x = 9$

    (b) $x = 1$

89. $p = 2$

90. (a) 80

    (b) $k = 0.135$

    (c) 5.13 weeks

91. (a) $a = 9$
    $b = \frac{1}{2}$

    (b) $a = \frac{1}{9}$
    $b = 81$

Answers 16 Further Calculus

92. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 18(6x - 5)^2$

93. -6

94. 8

95. $4x(6x^2 - 1)^{-\frac{2}{3}}$

96. 1

97. $\frac{(2x-3)^6}{12} + C$

98. 0.0941

99. $6(x + 4)^{\frac{1}{2}} + C$

100. $\frac{\sqrt{3}}{2}$