Higher Prelim Revision by Topic

The Higher Prelim Webpage contains hints and worked solutions for these questions, as well as **full practice papers** to help you prepare for the Prelim.

FORMULAE LIST *(only includes formulae already met in course)*

Circle

$x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$, radius $\sqrt{g^2 + f^2 - c}$.

$(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta, \text{ where } \theta \text{ is the angle between } \mathbf{a} \text{ and } \mathbf{b}$$

or

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 \text{ where } \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

Prelim Topic List

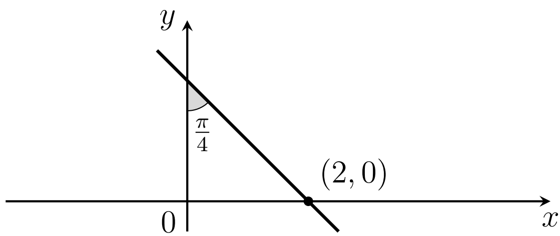
Note: Questions which require a **calculator** are marked with .

1 Straight Line

1. A and B are the points $(4, -2)$ and $(10, 6)$.
   Find the equation of the perpendicular bisector of AB.

2. Determine the equation of the line perpendicular to $6x - 3y + 7 = 0$, passing through $(3, -4)$.

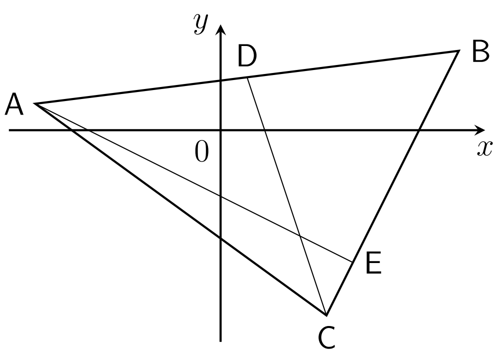
3. A line makes an angle of $\frac{\pi}{4}$ radians with the y-axis, as shown in the diagram below.



   Determine the equation of the line.

4. Show that the points P$(-3, 7)$, Q$(-1, 1)$ and R$(3, -11)$ are collinear.

5. Triangle ABC has vertices A$(-7, 1)$, B$(9, 3)$ and C$(4, -7)$.



   (a) Find the equation of the median CD.

   (b) Find the equation of the altitude AE.

   (c) Find the coordinates of the point of intersection of CD and AE.

2 Recurrence Relations

6. A sequence is defined by the recurrence relation $u_{n+1} = \frac{1}{3}u_n + 6$ with $u_0 = 27$.

   (a) Calculate the value of u_2.

   (b) i. Explain why this sequence approaches a limit as $x \to \infty$.

   ii. Calculate this limit.

7. For a sequence generated by $u_{n+1} = -2u_n + 5$, one of its terms is $u_3 = 13$.

   (a) Find the value of u_2.

   (b) Explain why this sequence does not approach a limit as $x \to \infty$.

8. A sequence is generated by the recurrence relation $u_{n+1} = ku_n - 4$, where k is a constant, and $u_4 = 6$.

   (a) Express u_5 in terms of k.

   (b) Hence or otherwise, determine the value of k given $u_5 = -1$.

9. A population of red squirrels in a local area is declining by 12% each year. To support the population, wildlife conservationists create a scheme to relocate red squirrels from other areas with strong populations. They plan to release 30 squirrels into the area yearly.

   (a) If u_n is the estimated number of red squirrels n years after the start of the scheme, it is believed that the population size can be modelled with the recurrence relation:

   $$u_{n+1} = au_n + b$$

   State the values of a and b.

   (b) Explain why the population size will stabilise in the long term, and calculate the expected long term population.

10. A sequence generated by the recurrence relation $u_{n+1} = pu_n + q$ has consecutive terms $24, -10$ and 7. Find p and q.

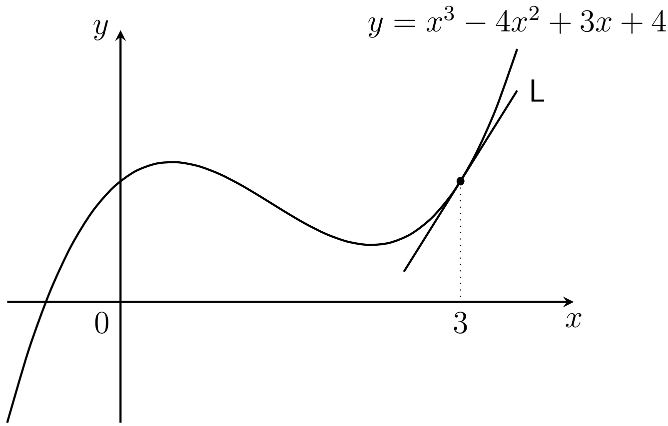
3 Differentiation I

11. Function f is defined by $f(x) = 3x^2 - 7x + 5$. Find the value of $f'(-2)$.

12. Differentiate $y = \dfrac{2x^5 - 3}{3x^2}$, where $x \neq 0$.

13. Find the gradient of the tangent to the curve with the equation $y = 4x^3 - 2x^2 - 7x + 5$ at the coordinate $(-1, 6)$.

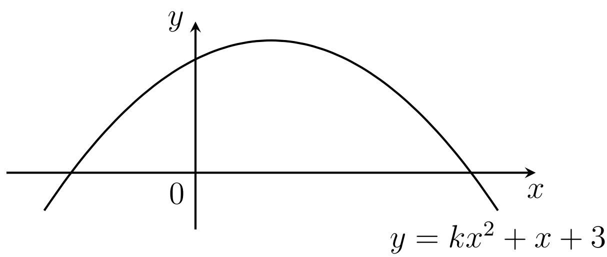
14. Part of the graph of $y = x^3 - 4x^2 + 3x + 4$ is shown below. Line L is a tangent to the curve at the point where $x = 3$.



Determine the equation of line L.

15. Calculate the rate of change of $h(t) = 12\sqrt{t} - 3$ when $t = 9$.

16. Part of the graph of $y = kx^2 + x + 3$ is shown below.



The tangent to the curve where $x = 2$ has a gradient -1. Find k.

4 Quadratic Theory

17. Write $3x^2 - 24x + 57$ in the form $p(x+q)^2 + r$.

18. Write $-2x^2 + 8x - 1$ in the form $a(x+b)^2 + c$.

19. Solve the inequation $2x^2 + 8x - 10 > 0$.

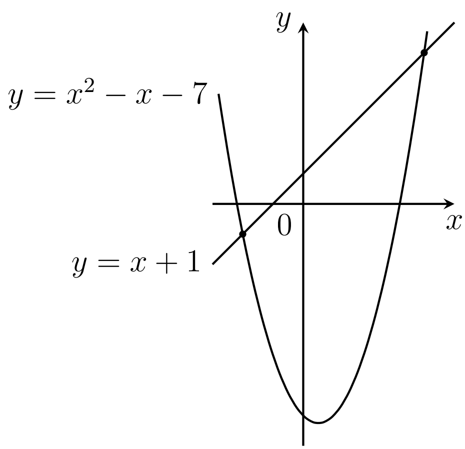
20. Solve $m^2 - m - 20 \leqslant 0$.

21. Given that the equation $3x^2 - px + 3 = 0$ has equal roots, where p is a constant, determine the possible values of p.

22. Find the range of values of q such that the equation $5x^2 - 8x + 2 - q = 0$ has no real roots.

23. Find the range of values of k such that $x^2 + (k-2)x + 4 = 0$ has real, distinct roots.

24. Find the coordinates of the points of intersection of the curve with equation $y = x^2 - x - 7$ and the line with equation $y = x + 1$.



25. Show that the line with equation $y = 4x - 3$ is a tangent to the parabola with equation $y = 2x^2 - 4x + 5$, and find the point of intersection.

5 Sets and Functions

26. A function, h, is defined on the set of real numbers by $h(x) = \dfrac{x-4}{3}$.
    Find the inverse function, $h^{-1}(x)$.

27. Function f is defined by $f(x) = 2\sqrt[3]{x} - 3$ where $x \in \mathbb{R}$.
    Find the inverse function $f^{-1}(x)$.

28. Determine the domain of the function $g(x) = \dfrac{5}{2x-1}$.

29. State the range of values for which $g(x) = \sqrt{x-3}$ is undefined.

30. Functions f and g are defined on $\mathbb{R}$ by:

    - $f(x) = \dfrac{x+1}{2}$
    - $g(x) = 4x - 3$

    Determine an expression for $f(g(x))$.

31. Functions f and g are defined on $\mathbb{R}$ by:

    - $f(x) = 2(x-1)$
    - $g(x) = \dfrac{x+2}{2}$

    (a) Find $g(f(x))$.

    (b) Hence state the relationship between functions f and g.

32. Functions f and g are given by $f(x) = x^2 - 1$ and $g(x) = x - 3$.

    (a) Find an expression for $k(x)$ where $k(x) = f(g(x))$.

    (b) State the range of $k(x)$, where $x \in \mathbb{R}$.

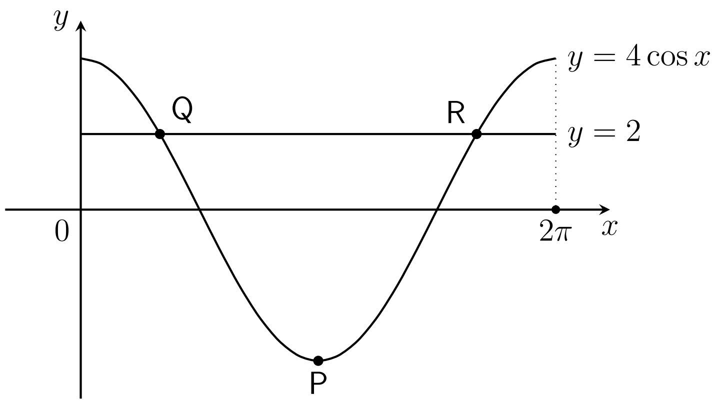
    (c) Solve $k(x) = 0$.

6 Trigonometry

33. Solve $2\sin x - 1 = 0$ where $0 \leqslant x \leqslant 2\pi$.

34. Solve $\sqrt{3}\tan x + 1 = 0$ where $0 \leqslant x \leqslant 2\pi$.

35. Part of the graph of $y = 4\cos x$ is shown below.



The point P is a minimum turning point on $y = 4\cos x$. The line with equation $y = 2$ intercepts $y = 4\cos x$ at points Q and R.

(a) State the coordinates of P.

(b) Determine the coordinates of Q and R.

36. Solve $2\cos 2x^\circ - 1 = 0$ where $0 \leqslant x \leqslant 360$.

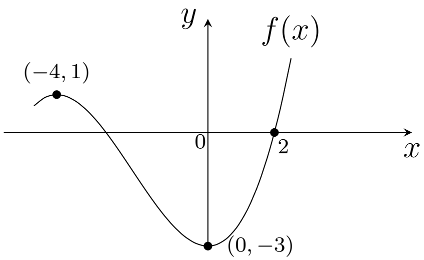
37. Solve $2\sin 2x^\circ + \sqrt{3} = 0$ where $0 \leqslant x \leqslant 360$.

38. Solve $\tan\left(x - \frac{\pi}{6}\right) + 1 = 0$ where $0 \leqslant x \leqslant 2\pi$.

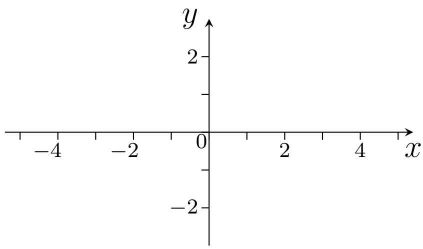
39. Solve $2\cos\left(2x + \frac{\pi}{3}\right) + \sqrt{3} = 0$ where $0 \leqslant x \leqslant 2\pi$.

7 Graph Transformations

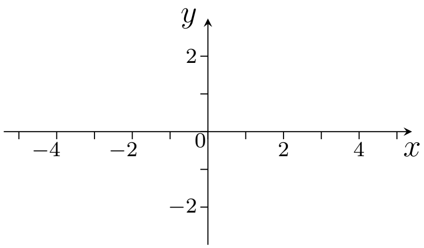
40. Given $y = f(x)$...



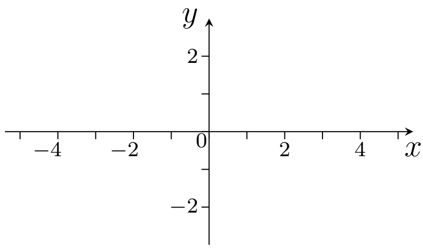
(a) Sketch $y = f(x-2)+1$



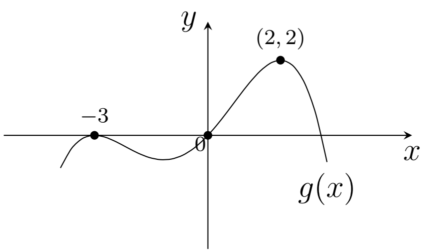
(b) Sketch $y = -f(x)-1$



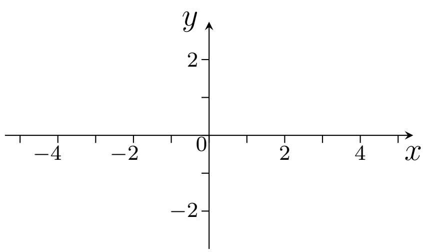
(c) Sketch $y = f(-x)+2$



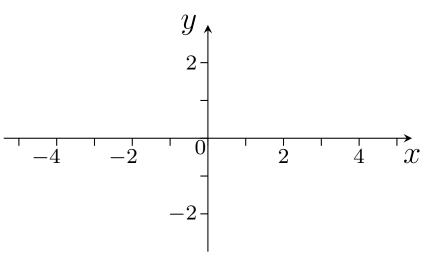
41. Given $y = g(x)$...



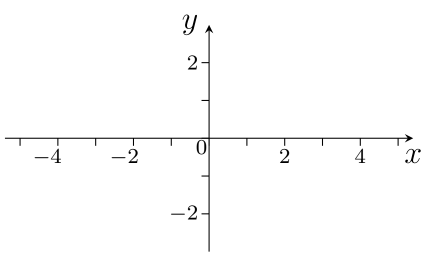
(a) Sketch $y = g(x+1)-2$



(b) Sketch $y = 2 - g(x)$



(c) Sketch $y = \frac{1}{2}g(x)+3$



8 Vectors

42. (a) Shows that points P$(-3, 5, -7)$, Q$(1, 2, -6)$ and R$(9, -5, -4)$ are collinear.

    (b) State the ratio in which the points Q divides the lines PR.

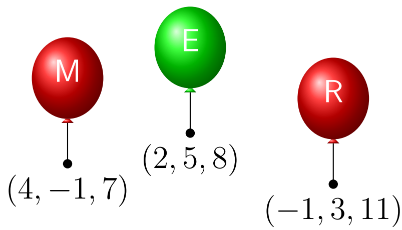
43. D and F have coordinates $(8, -7, 4)$ and $(3, 3, 4)$ respectively.
    Find the coordinates of point E which divides DF in the ratio $3 : 2$.

44. The first of three festive balloons spelling M, E and R are attached to pegs in a room whose positions can be described by the coordinates $(4, -1, 7)$, $(2, 5, 8)$ and $(-1, 3, 11)$ respectively.

    (a) Express $\overrightarrow{EM}$ and $\overrightarrow{ER}$ in component form.

    (b) Calculate $\overrightarrow{EM} \cdot \overrightarrow{ER}$.

    (c) Calculate the size of angle MER.

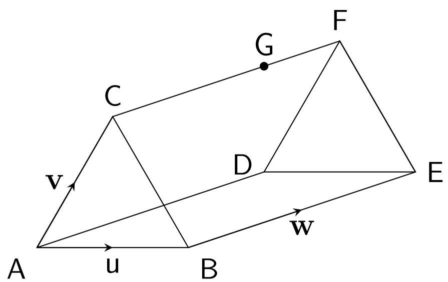


45. Given $\mathbf{u} = 2\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$:

    (a) Calculate $|\mathbf{u}|$.

    (b) Hence find the components of the unit vector $\mathbf{a}$ parallel to $\mathbf{u}$.

46. The prism below has an equilateral triangle as its cross-section.
    $|\mathbf{u}| = 3$ and G divides CF in the ratio 2:1.



    (a) Express vector $\overrightarrow{BF}$ in terms of $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$.

    (b) Express vector $\overrightarrow{GB}$ in terms of $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$.

    (c) Evaluate: (i) $\mathbf{u} \cdot (\mathbf{u} - \mathbf{v})$ (ii) $\mathbf{v} \cdot (\mathbf{u} + \mathbf{w})$

9 Differentiation II

47. Find the range of values of x for which $f(x) = \frac{3}{2}x^2 + 6x - 7$ is increasing.

48. Determine the range of values of x for which the curve with equation $y = x^3 - 3x^2 - 24x - 1$ is decreasing.

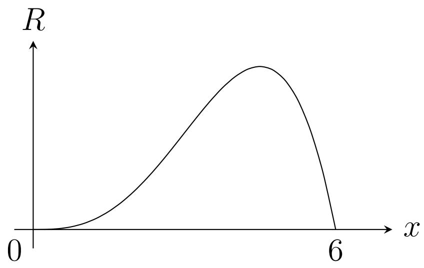
49. A function f is defined on $\mathbb{R}$ by $f(x) = \frac{1}{3}x^3 + x^2 - 15x + 7$.
Determine the x-coordinates of the stationary points of $f(x)$.

50. Find the coordinates of the stationary points on the curve with equation $y = 2x^3 - 3x^2 - 36x + 5$ and determine their nature.

51. A function g, defined by $g(x) = x^3 - x^2 + 2x - 1$ is strictly increasing for $-1 \leqslant x \leqslant 2$. Determine the greatest and least values of g.

52. Find the value of x which minimises $f(x) = x^2 + \dfrac{54}{x}$ for $x > 0$.

53. A company has determined that a mathematical equation can model the amount of revenue in thousands of pounds, R, which may be earned by setting the price for a new product as x pounds.

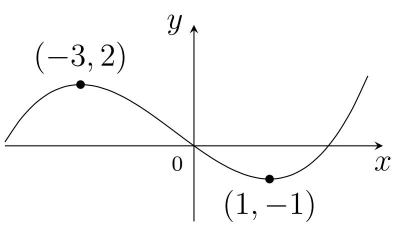


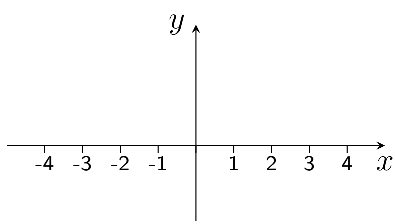
The diagram illustrates the model used by the company, which is:

$$R(x) = 6x^3 - x^4 \text{ for } 0 < x \leqslant 6$$

Find the value of x which gives the maximum revenue for the product.

54. Part of the graph of $y = f(x)$ for the cubic function $f(x)$ is below.





Sketch $y = f'(x)$ using the set of axes to the right as a guide.

10 Polynomials

55. Determine the remainder when $2x^3 - 5x^2 + 7x - 3$ is divided by $(x - 2)$.

56. (a) Show that $(x + 1)$ is a factor of $x^3 - x^2 - 10x - 8$.

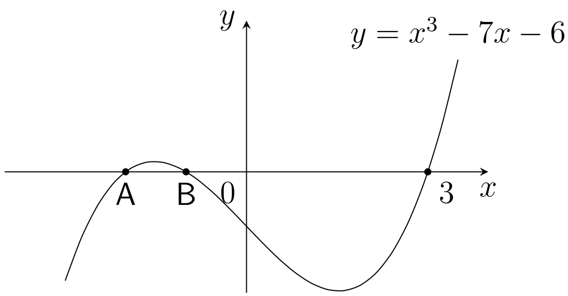
    (b) Hence, or otherwise, factorise $x^3 - x^2 - 10x - 8$ fully.

57. A function f is defined on a suitable domain by $f(x) = 2x^3 + x^2 - 7x - 6$.

    (a) Show that $(x - 2)$ is a factor of $f(x)$.

    (b) Solve $f(x) = 0$.

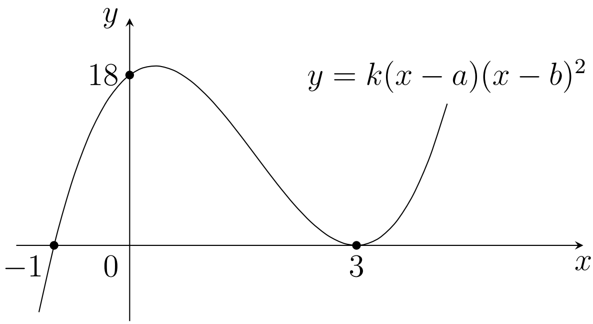
58. Part of the graph of $y = x^3 - 7x - 6$ is shown below.



Determine the coordinates of points A and B.

59. Given $(x + 2)$ is a factor of $2x^3 + kx^2 - 14x + 8$, find k.

60. Part of the graph of $y = k(x - a)(x - b)^2$ is shown below.



Determine the values of k, a and b.

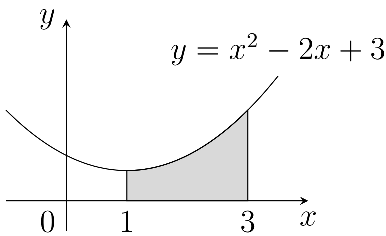
11 Integration

61. Calculate: $\int \left(x^3 - 6x^2 + 10x - 7\right)$.

62. Find: $\int (12x^3 + 6\sqrt{x})dx$, where $x \geqslant 0$.

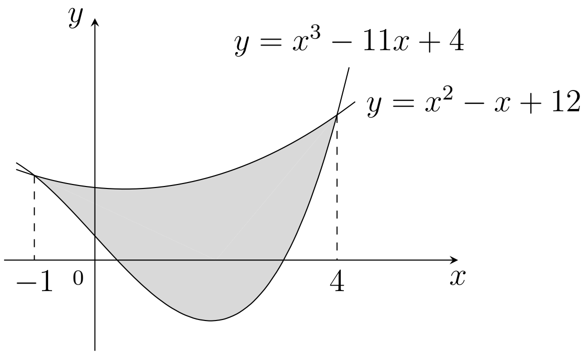
63. Calculate: $\int_1^2 \left(3x^2 + \frac{1}{x^2}\right) dx$, $x \neq 0$.

64. The diagram shows part of the graph of $y = x^2 - 2x + 3$.



Calculate the shaded area.

65. In the diagram below, the region enclosed by two curves is shaded.

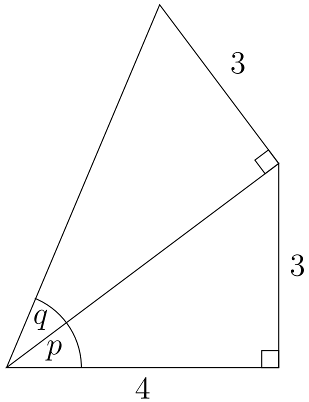


Calculate the area of the shaded region. 🖩

66. Given that function f is defined such that $f'(x) = 3x^2 - 8x + 7$ and that the graph of $y = f(x)$ passes through $(2, 10)$, find an expression for $f(x)$.

12 Addition Formulae

67. The diagram below shows two right-angled triangles placed together.



(a) Determine the value of:

  i. $\sin p$

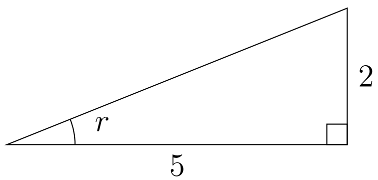
  ii. $\sin q$

(b) Hence determine the value of:

  i. $\sin(p+q)$

  ii. $\cos(p+q)$

68. The right-angled triangle in the diagram below is such that $0 < r < \dfrac{\pi}{4}$.



Determine the value of:

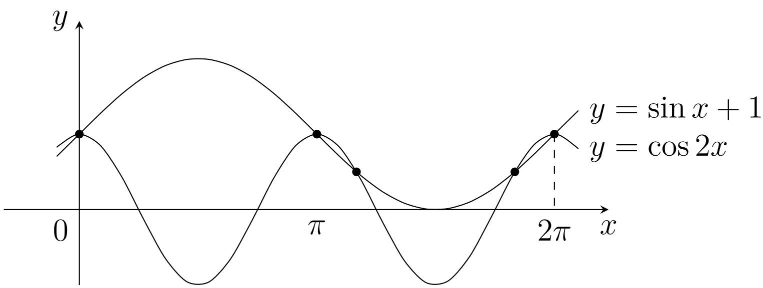
(a) $\cos r$

(b) $\cos 2r$

69. Solve the equation $\sin 2x^\circ - \cos x^\circ = 0$ for $0 \leqslant x \leqslant 360$.

70. Solve the equation $\cos 2x - \cos x = 0$ for $0 \leqslant x \leqslant 2\pi$.

71. Part of the graphs of $y = \cos 2x$ and $y = \sin x + 1$ are shown below.



Determine the x-coordinates of the five points of intersection shown.

13 The Circle

72. Determine the radius and centre for each of the following circles:

    (a) $(x-6)^2 + (y+1)^2 = 20$

    (b) $x^2 + y^2 - 8x - 6y + 21 = 0$

73. C_1 is the circle with equation $x^2 + y^2 + 10x - 4y - 7 = 0$.

    (a) Find the centre of circle C_1.

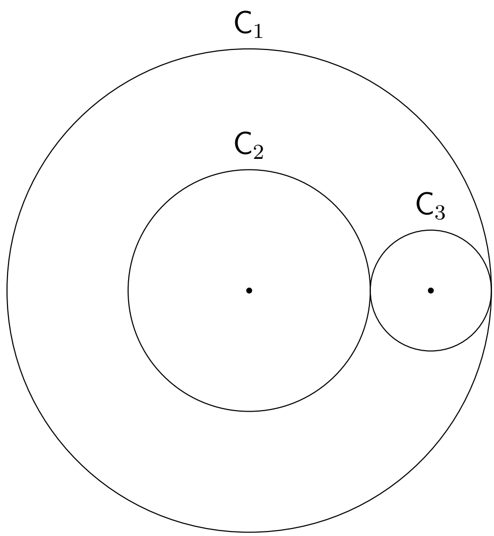
    (b) Find the radius of circle C_1.

    C_2 is a circle with a diameter half that of C_1, and centre $(-7, 1)$.

    (c) Determine the equation of C_2

74. The diagram below shows two concentric circles which each touch a smaller circle.

    - The largest circle C_1 has equation $x^2 + y^2 + 12x - 8y - 12 = 0$
    - The smallest circle C_3 has equation $x^2 + (y-4)^2 = 4$



    (a) Determine the equation of the remaining circle, C_2 .

    (b) Find the coordinates of the point where circles C_2 and C_3 touch.

Answers

Answers: 1 Straight Line

1. $4y = 3x - 6$ or $y = \frac{3}{4}x - \frac{3}{2}$

2. $2y = -x - 5$ or $y = -\frac{1}{2}x - \frac{5}{2}$

3. $y = -x - 2$

4. $m_{PQ} = m_{QR} = -3$, valid statement

5. (a) $y = -3x - 5$

   (b) $2y = -x - 5$ or $y = -\frac{1}{2}x - \frac{5}{2}$

   (c) $(3, -4)$

Answers 2 Recurrence Relations

6. (a) 11

   (b) i. $-1 < \frac{1}{3} < 1$, valid statement

      ii. 9

7. (a) -4

   (b) $-2 > 1$, valid statement

8. (a) $6k - 4$

   (b) $\frac{1}{2}$

9. (a) $a = 0.88, b = 30$

   (b) $-1 < 0.88 < 1$, valid statement, limit$= 250$

10. $p = -\frac{1}{2}, q = 2$

Answers 3 Differentiation I

11. -19

12. $\frac{dy}{dx} = 2x^2 + 2x^{-3}$

13. 9

14. $y = 6x - 14$

15. 2

16. $k = -\frac{1}{2}$

Answers 4 Quadratic Theory

17. $3(x-4)^2 + 9$

18. $-2(x-2)^2 + 7$

19. $x < -5, x > 1$

20. $-4 \leqslant m \leqslant 5$

21. $p = \pm 6$

22. $q < -\frac{6}{5}$

23. $k < -2, k > 6$

24. $(-2, -1)$ and $(4, 5)$

25. $(x-2)$ is a repeated root, $(2, 5)$

Answers 5 Sets and Functions

26. $h^{-1}(x) = 3x + 4$

27. $f^{-1}(x) = (\frac{x+3}{2})^3$

28. $x \neq \frac{1}{2}$

29. $x < 3$

30. $2x - 1$

31. (a) x

    (b) f and g are inverse to each other

32. (a) $x^2 - 6x + 8$

    (b) $k(x) \geqslant -1$

    (c) $x = 2, x = 4$

Answers 6 Trigonometry

33. $x = \frac{\pi}{6}, \frac{5\pi}{6}$

34. $x = \frac{5\pi}{6}, \frac{11\pi}{6}$

35. (a) $(\pi, -4)$

    (b) Q$(\frac{\pi}{3}, 2)$,R$(\frac{5\pi}{3}, 2)$

36. $x = 30, 150, 210, 330$

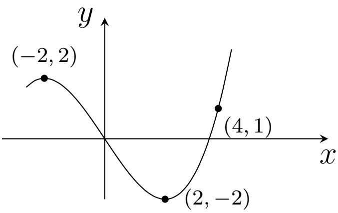
37. $x = 120, 150, 300, 330$

38. $x = \frac{11}{12}, \frac{23\pi}{12}$

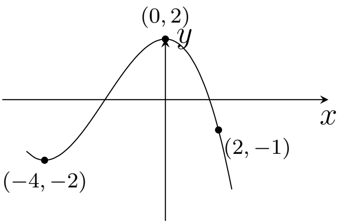
39. $x = \frac{2\pi}{3}, \pi, \frac{8\pi}{3}, 3\pi$

Answers 7 Graph Transformations

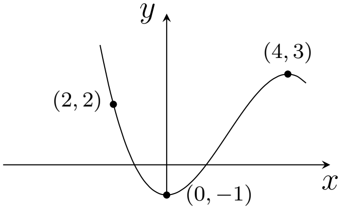
40. (a) $y = f(x-2)+1$



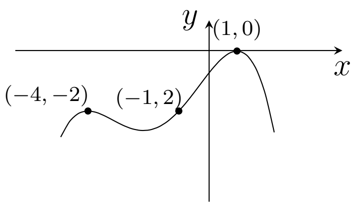
(b) $y = -f(x) - 1$



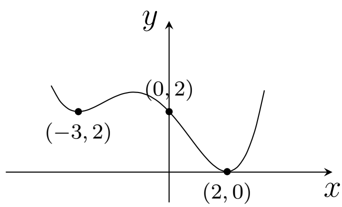
(c) $y = f(-x) + 2$



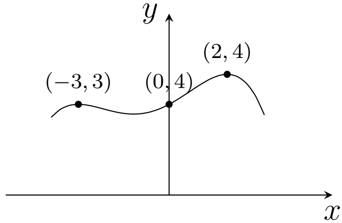
41. (a) $y = g(x+1) - 2$



(b) $y = 2 - g(x)$



(c) $y = \frac{1}{2}g(x) + 3$



Answers 8 Vectors

42. (a) $2\overrightarrow{PQ} = \overrightarrow{QR}$ or equivalent, valid statement

    (b) $1:2$

43. $(5, -1, 4)$

44. (a) $\overrightarrow{EM} = \begin{pmatrix} 2 \\ -6 \\ -1 \end{pmatrix}$ and $\overrightarrow{ER} = \begin{pmatrix} -3 \\ -2 \\ 3 \end{pmatrix}$

    (b) 3

    (c) 84.3°

45. (a) $\sqrt{21}$

    (b) $\begin{pmatrix} 1/3 \\ -2/3 \\ 2/3 \end{pmatrix}$

46. (a) $\overrightarrow{BF} = -\mathbf{u} + \mathbf{v} + \mathbf{w}$

    (b) $\overrightarrow{GB} = -\frac{2}{3}\mathbf{w} - \mathbf{v} + \mathbf{u}$

    (c) i. $\dfrac{9}{2}$

        ii. $\dfrac{9}{2}$

Answers 9 Differentiation II

47. $x > -2$

48. $-2 < x < 4$

49. $x = -5, x = 3$

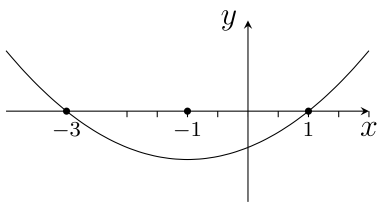
50. maximum turning point at $(-2, 49)$, minimum turning point at $(3, -76)$

51. minimum value $= -5$, maximum value $= 7$

52. $x = 3$

53. $x = 4.5$

54. $y = f'(x)$ must be a parabola with turning point when $x = -1$



Answers 10 Polynomials

55. 7

56. (a) Remainder$= 0$ therefore $(x + 1)$ is a factor

    (b) $(x + 1)(x + 2)(x - 4)$

57. (a) Remainder$= 0$ therefore $(x - 2)$ is a factor

    (b) $x = -\frac{3}{2}, x = -1, x = 2$

58. A$(-2, 0)$ and B$(-1, 0)$

59. $k = -7$

60. $a = -1, b = 3, k = 2$

Answers 11 Integration

61. $\frac{1}{4}x^4 - 2x^3 + 5x^2 - 7x + C$

62. $3x^4 + 4x^{\frac{3}{2}} + C$

63. $\frac{15}{2}$

64. $\frac{20}{3}$ square units

65. $\frac{875}{12}$

66. $f(x) = x^3 - 4x^2 + 7x + 4$

Answers 12 Addition Formulae

67. (a) i. $\frac{3}{5}$
    ii. $\frac{3}{\sqrt{34}}$
    (b) i. $\frac{3}{5\sqrt{34}}$
    ii. $\frac{11}{5\sqrt{34}}$

68. (a) $\frac{5}{\sqrt{29}}$
    (b) $\frac{21}{29}$

69. $x = 30, 90, 150, 270$

70. $x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi$
    item $x = 0, \frac{7\pi}{6}, \pi, \frac{11\pi}{6}, 2\pi$

Answers 13 The Circle

71. (a) radius$= \sqrt{20}$, centre$= (6, -1)$

    (b) radius$= 2$, centre$= (4, 3)$

72. (a) centre$= (-5, 2)$

    (b) radius$= 6$

    (c) $(x + 7)^2 + (x - 1)^2 = 9$

73. (a) $= (x + 6)^2 + (y - 4)^2 = 16$

    (b) $(-2, 4)$