FORMULAE LIST

Circle

$x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$, radius $\sqrt{g^2 + f^2 - c}$.

$(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta, \text{ where } \theta \text{ is the angle between } \mathbf{a} \text{ and } \mathbf{b}$$

or

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 \text{ where } \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
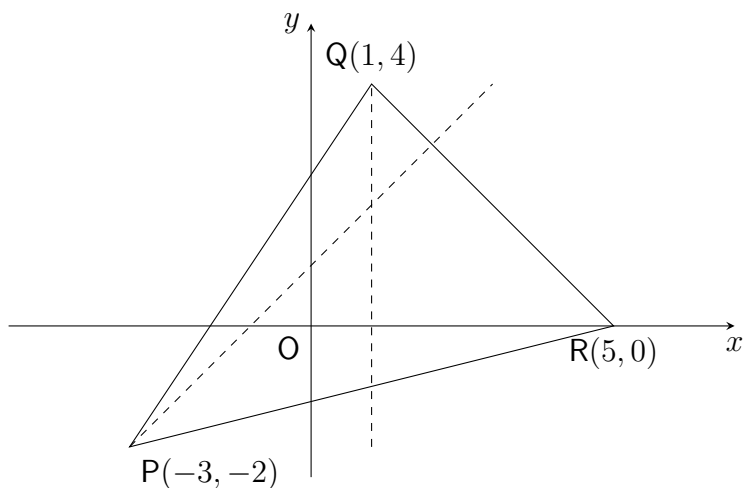$$= 2 \cos^2 A - 1$$
$$= 1 - 2 \sin^2 A$$

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1. Find $\int (8x^5 - 12x^2) \, dx$ (2)

2. Vectors $\mathbf{u} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\mathbf{v} = k\mathbf{i} + 6\mathbf{k}$ are perpendicular. Find the value of k. (2)

3. A curve has equation $y = 2x^3 - 4x^2 + 7x - 2$. (4)
   Find the equation of the tangent to this curve at the point where $x = 1$.

4. Triangle PQR has vertices P$(-3, -2)$, Q$(1, 4)$ and R$(5, 0)$.

(a) Find the equation of the altitude through P. (3)

(b) Find the equation of the median through Q. (3)

(c) Find the coordinates of the point of intersection of the altitude through P and the median through Q. (2)

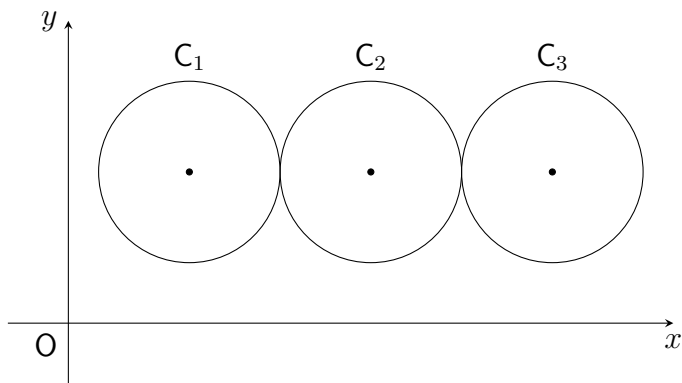5. Part of the graph of $y = f(x)$ is shown below, where $f(x)$ is a cubic function. (3)
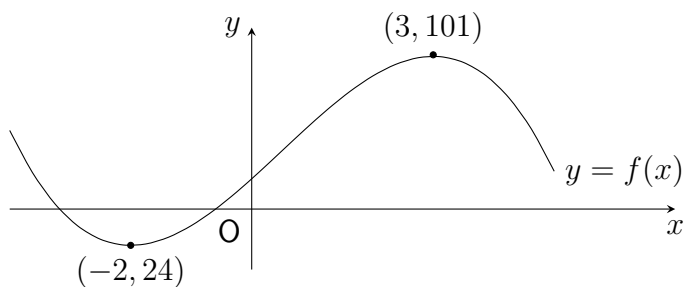
Sketch the graph of $y = f'(x)$.

6. Given that $\cos p = \dfrac{\sqrt{7}}{4}$, where $0 < p < \dfrac{\pi}{2}$, find the exact value of:

(a) $\sin p$ (1)

(b) $\sin 2p$ (2)

7. Three congruent circles are aligned horizontally and touch as shown below.

Circle C_1 has equation $x^2 + y^2 - 8x - 10y + 32 = 0$.

(a) Find the centre and radius of circle C_1 (2)

(b) Determine the equation of circle C_2.   (3)

(c) Find the coordinates of the point at which circles C_2 and C_3 touch.   (2)

8. The diagram shows the line with equation $y = 2x + 3$ intersecting with the parabola with equation $y = x^2 - 3x - 3$ at the point $(6, 15)$.   (5)
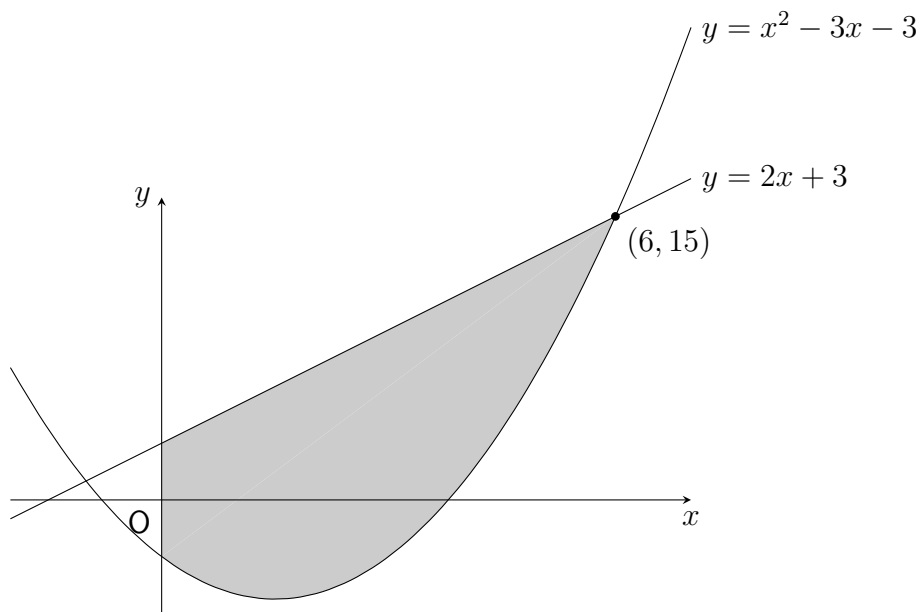
Calculate the shaded area.

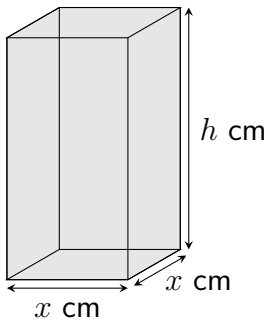9. Solve the equation $2\sin 2x° + 3\cos x° = 0$, where $0 < x < 360$. (4)

10. The coordinates of points A, B and C are given by $A(-4, 1, 5)$, $B(3, 2, 0)$ and $C(1, -5, 6)$.

   (a) Express $\overrightarrow{CA}$ and $\overrightarrow{CB}$ in component form. (2)

   (b) Hence calculate the size of angle ACB. (5)

11. Determine the range of values of x for which $y = x^3 - 6x^2 - 15x - 4$ is increasing. (4)

12. A solid cuboid measures x cm by x cm by h cm. Its volume is 125 cm³. (8)

h cm

x cm

x cm

   (a) Show that the surface area of the cuboid can be given by:

$$A(x) = 2x^2 + \frac{500}{x}$$

   (b) Find the value of x such that the surface area is minimised.

13. Given $\mathbf{u} = \begin{pmatrix} 2\cos x \\ 3 \\ -1 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} \sin x \\ 0 \\ \frac{1}{2} \end{pmatrix}$ are perpendicular, and $\frac{\pi}{4} < x < \frac{\pi}{2}$, find x. (6)

Question	Topic	Marks Available	Marks Awarded
1	Integration	2	
2	Vectors	2	
3	Differentiation	4	
4	Straight Lines	8	
5	Differentiation	3	
6	Addition Formulae	3	
7	The Circle	7	
8	Integration	5	
9	Addition Formulae	4	
10	Vectors	7	
11	Differentiation II	4	
12	Differentiation II	8	
12	Addition Formulae	6	
Total		65	

ANSWERS - Practice Exam A Paper 2

1. $\dfrac{4x^6}{3} - 4x^3 + C$

2. $k = -4$
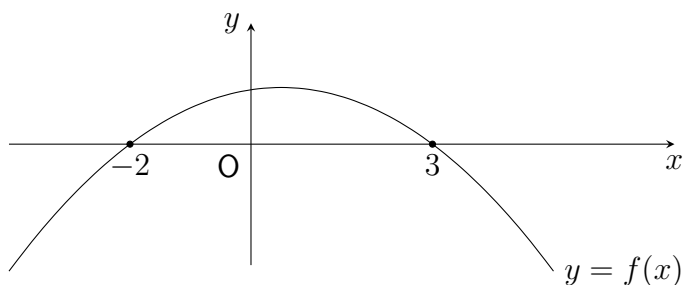
3. $y = 5x - 2$

4.   (a) $y = x + 1$
     (b) $x = 1$
     (c) $(1, 2)$

5.

6.   (a) $\sin p = \dfrac{3}{4}$

    (b) $\sin 2p = \dfrac{3\sqrt{7}}{8}$

7.   (a) Centre $(4, 5)$, radius$= 3$
     (b) $(x - 10)^2 + (y - 5)^2 = 9$
     (c) $(13, 5)$

8. Area$= 54$ square units

9. $x = 90, 228.6, 270, 311.4$

10.   (a) $\overrightarrow{CA} = \begin{pmatrix} -5 \\ 6 \\ -1 \end{pmatrix}$ and $\overrightarrow{CB} = \begin{pmatrix} 2 \\ 7 \\ -6 \end{pmatrix}$

    (b) $59.2°$

11. $x < -1, x > 5$

12. $x = 5$

13. $x = \dfrac{5\pi}{12}$