H National Qualifications

Practice Exam C
1 hour 30 mins
Total marks: 65

Mathematics
Paper 2
Calculator

FORMULAE LIST

Circle

$x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$, radius $\sqrt{g^2 + f^2 - c}$.

$(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta, \text{ where } \theta \text{ is the angle between } \mathbf{a} \text{ and } \mathbf{b}$$

or

$$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 \text{ where } \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae

$$\begin{aligned} \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin 2A &= 2\sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

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1. Express $-2x^2 + 12x - 11$ in the form $p(x+q)^2 + r$. (3)

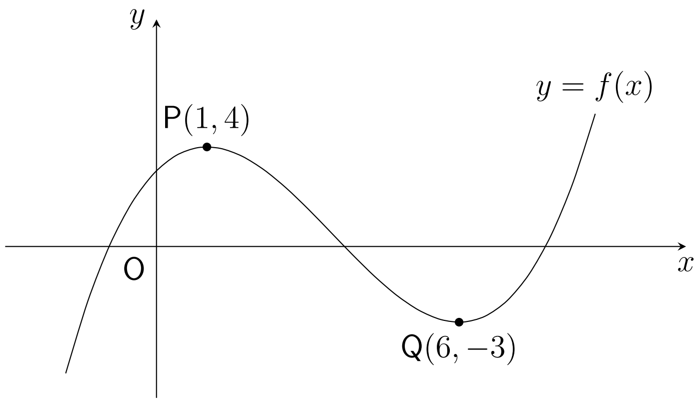
2. (a) A curve has equation $y = 5x^2 + 4x + 2$. (3)
   Find the equation of the tangent to this curve at the point $(-1, 3)$.

   (b) Points A and B are given by A$(-1, 12)$ and B$(1, 8)$. (3)
   Find the equation of the perpendicular bisector of AB.

   (c) Determine the coordinates of the point of intersection of these two lines. (2)

3. The diagram below shows part of the graph of $y = f(x)$.

The curve has two stationary points, at P$(1, 4)$ and Q$(6, -3)$.



(a) Sketch the graph of $y = 2f(x) - 1$. (3)

(b) Function g is defined as $g(x) = f(x + k)$, where $k > 0$. (1)

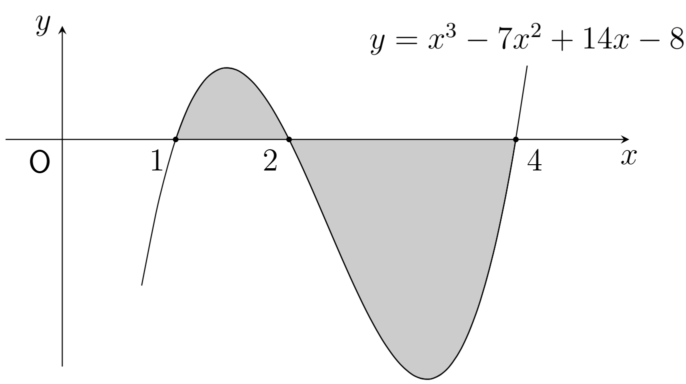
Given that $g'(0) < 0$, find the range of possible values for k.

4. (a) Show that the P$(5, -2, 3)$, Q$(3, 0, 2)$ and R$(-3, 6, -1)$ are collinear. (3)

(b) State the ratio in which Q divides PR. (1)

5. Solve the equation $7\sin(2x - 10)^\circ + 6 = 0$ where $0 < x < 360$. (4)

6. The diagram shows the curve with equation $y = x^3 - 7x^2 + 14x - 8$. (7)



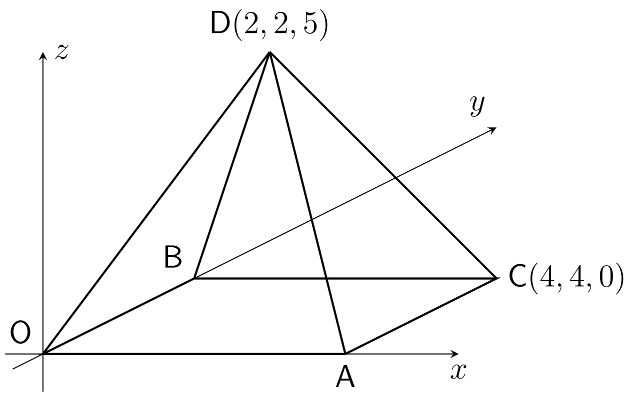
Calculate the shaded area.

7. A function f is defined for $x \in \mathbb{R}$ such that $f'(x) = x(x-4)$.

(a) Find the x-coordinates of the stationary points on the curve $y = f(x)$. (2)

(b) Determine the nature of each stationary point. (2)

8. A square-based pyramid with height 5 units has vertices O, A, B, C and D, as shown.



(a) State the coordinates of B. (1)

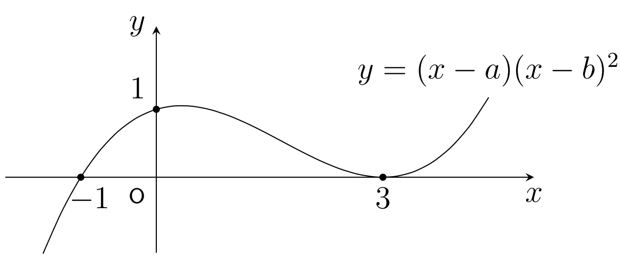
(b) Find the components of vectors $\overrightarrow{CB}$ and $\overrightarrow{CD}$. (2)

(c) Calculate the size of angle BCD. (5)

9. (a) Show that $(x-2)$ is a factor of $f(x) = 2x^3 + x^2 - 13x + 6$. (2)

(b) Hence, or otherwise, solve $f(x) = 0$. (3)

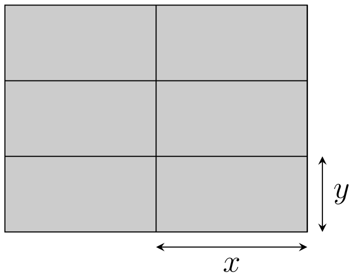
10. Part of the graph of $y = (x-a)(x-b)^2$ is shown in the diagram below. (3)



Find the values of k, a and b.

11. Given $\mathbf{u} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, find $\mathbf{u} \cdot \mathbf{u}$.

12. A sports facility manager wishes to use 360 metres of fencing to create six individual courts, each measuring x metres by y metres as shown in the diagram below.



(a) Show that the total area of the six courts is given by: (3)

$$A(x) = 240x - \frac{16}{3}x^2$$

(b) Find the value of x which gives the maximum area. (5)

(c) Determine the maximum area. (1)

13. Functions f and g are defined on suitable domains by:

- $f(x) = \dfrac{2}{x^2 - 4}$
- $g(x) = x + 1$

Function h is defined as $f(g(x))$.

(a) Find an expression for $k(x)$. (2)

(b) Determine the domain of $k(x)$. (2)

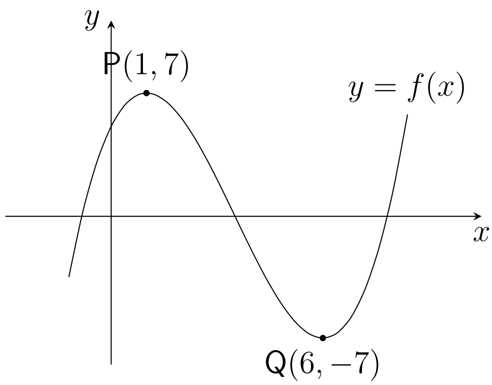
Question	Topic	Marks Available	Marks Awarded
1	Quadratic Theory	3	
2	Differentiation I / Straight Line	8	
3	Graph Transformations	4	
4	Vectors	4	
5	Trigonometry	4	
6	Integration	7	
7	Differentiation II	4	
8	Vectors	8	
9	Polynomials	5	
10	Polynomials	3	
11	Vectors	2	
12	Differentiation II	9	
13	Sets and Functions	4	
Total		**65**	

ANSWERS - Practice Exam C Paper 2

1. $-2(x-3)^2+7$

2. (a) $y=-6x-3$
   (b) $y=\frac{1}{2}x+10$
   (c) $(-2,9)$

3. (a) Sketch of $y=2f(x)-1$.



   (b) $1<k<6$

4. (a) $3\overrightarrow{PQ}=\overrightarrow{QR}$ therefore the lines are parallel,
   and they share a common point Q
   therefore points P,Q,R are collinear.
   (b) $1:3$

5. $x=124.5, 155.5, 304.5, 335.5$

6. Area $= \frac{37}{12}$ square units

7. (a) $x = 0, x = 4$

   (b) Maximum turning point when $x = 0$, minimum turning point when $x = 4$

8. (a) B$(0, 4, 0)$

   (b) $\overrightarrow{\mathsf{CB}} = \begin{pmatrix} -4 \\ 0 \\ 0 \end{pmatrix}$ and $\overrightarrow{\mathsf{CD}} = \begin{pmatrix} -2 \\ -2 \\ 5 \end{pmatrix}$

   (c) $69.6°$

9. (a) Remainder $= 0$ therefore $(x - 2)$ is a factor

   (b) $x = -3, x = \frac{1}{2}, x = 2$

10. $k = \frac{1}{9}, a = -1, b = 3$

11. 14

12. (a) Step-by-step working towards the required formula, likely using $A = 2x \times 3y$ and $360 = 8x + 9y$

    (b) $x = 22.5$

    (c) Area $= 2700$ square metres

13. (a) $k(x) = \dfrac{2}{x^2 + 2x - 3}$

    (b) $x \neq -3, x \neq 1$