

FORMULAE LIST**Circle**

$x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$, radius $\sqrt{g^2 + f^2 - c}$.

$(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar product

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b}

or

$$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 \text{ where } \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

1. A curve has equation $y = 2x^3 - 3x + 7$. (4)

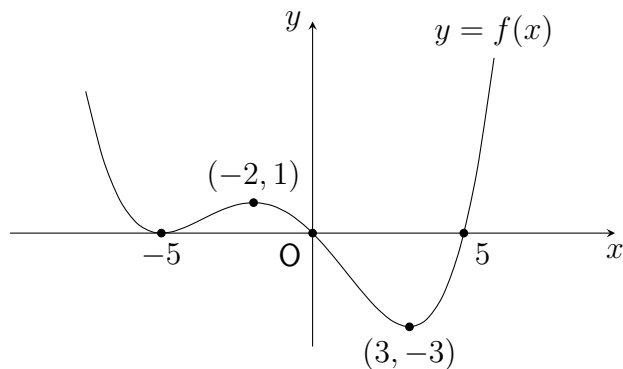
Find the equation of the tangent to the curve at the point where $x = 1$.

2. Triangle PQR has vertices $P(-3, 2)$, $Q(0, 5)$ and $R(4, -1)$. (3)

Determine the equation of the altitude through P.

3. Solve $2 \cos\left(2x - \frac{\pi}{6}\right) = 1$ where $0 < x < 2\pi$. (4)

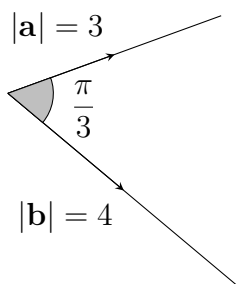
4. The diagram below shows part of the graph of $y = f(x)$.



(a) Sketch the graph of $y = 1 - f(x)$. (3)

(b) Sketch the graph of $y = f'(x)$. (3)

5. The angle between \mathbf{a} and \mathbf{b} is $\frac{\pi}{3}$ as shown in the diagram below.



(a) Determine the value of $\mathbf{a} \cdot (\mathbf{b} - \mathbf{a})$. (3)

(b) State the value of k such that $k\mathbf{a}$ is a unit vector. (1)

6. A sequenced generated by $u_{n+1} = pu_n + 5$ has consecutive terms 3 and 4.

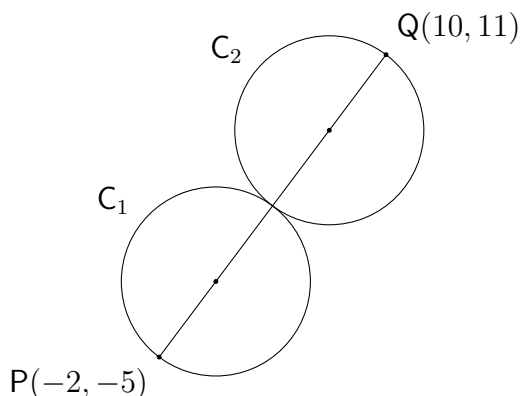
(a) Determine the value of p . (2)

(b) Explain whether the sequence converges to a limit as $x \rightarrow \infty$. (1)

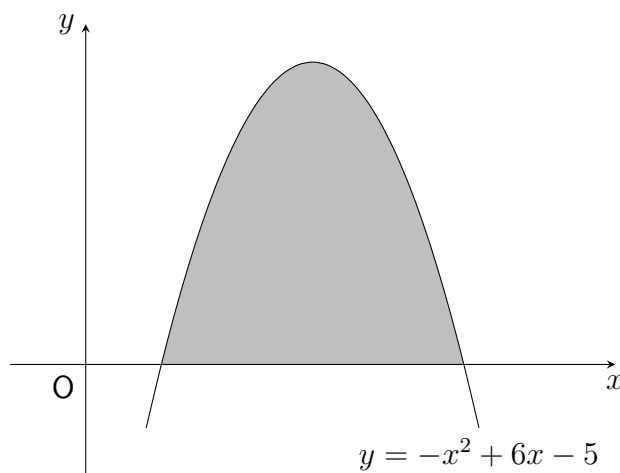
7. (a) Express $-3x^2 - 6x - 4$ in the form $p(x + q)^2 + r$. (3)

(b) Hence show that $f(x) = -x^3 - 3x^2 - 4x + 7$ is strictly decreasing for all $x \in \mathbb{R}$. (3)

8. Congruent circles C_1 and C_2 touch and points $P(-2, -5)$ and $Q(10, 11)$ lie on the circumferences of C_1 and C_2 respectively. Line PQ passes through the centres of C_1 and C_2 , as shown in the diagram.



- (a) Determine the coordinates of the point where the circles touch. (1)
- (b) Find the equation of circle C_1 . (4)
9. Show that the line with equation $y = 2x - 2$ meets the curve with equation $y = x^2 - 4x + 7$ at a tangent, and find the coordinates of the point of tangency. (5)
10. The shaded region in the diagram below is enclosed by the x -axis and the curve with equation $y = -x^2 + 6x - 5$. (6)



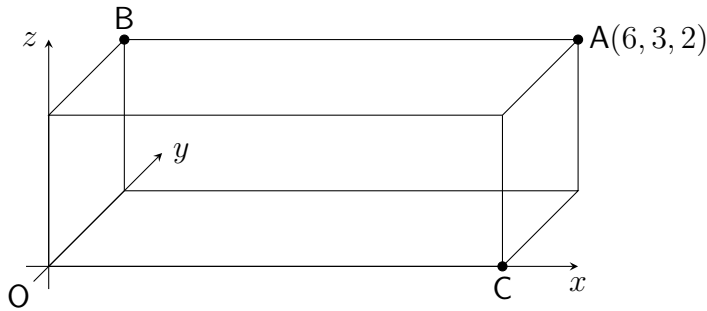
Calculate the area of the shaded region.

11. A function f is defined on a suitable domain such that: (4)

- $f'(x) = 4x + 5$
- The curve with equation $y = f(x)$ passes through the point $(-2, 7)$.

Determine an expression for $f(x)$.

12. A cuboid is placed along the coordinate axes as shown below.

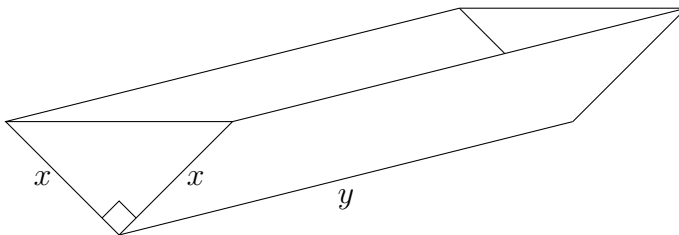


(a) Express \vec{CA} and \vec{CB} in component form. (2)

(b) Calculate the size of angle ACB. (5)

13. A trough is to be made from sheets of plastic in the shape of a triangular prism of length y with an open top, as shown in the diagram below. Each end of the prism is a right-angled, isosceles triangle with its shorter sides having length x .

The total area of plastic required is 12 square metres.



(a) Show that the volume of the trough, V , is given by: (3)

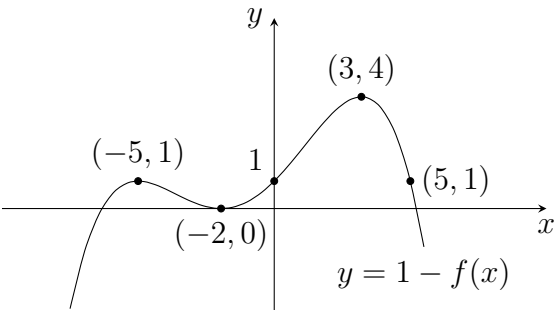
$$V(x) = 3x - \frac{1}{4}x^3$$

(b) Calculate the value of x for which the volume of the trough is maximised. (5)

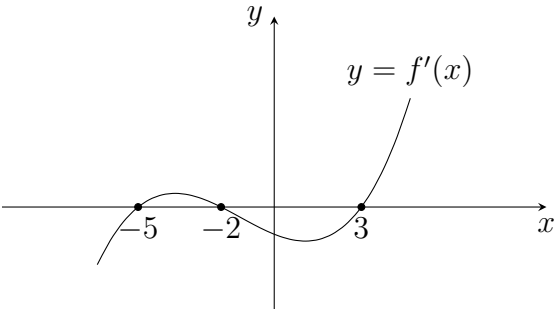
Question	Topic	Marks Available	Marks Awarded
1	Differentiation I	4	
2	Straight Line	3	
3	Trigonometry	4	
4	Graph Transformations/Differentiation II	6	
5	Vectors	4	
6	Recurrence Relations	3	
7	Quadratics/Differentiation II	6	
8	The Circle	5	
9	Quadratics	5	
10	Integration	6	
11	Integration	4	
12	Vectors	7	
13	Differentiation II	8	
Total		65	

ANSWERS - Practice Exam B Paper 2

- $y = 3x + 3$
- $y = \frac{2}{3}x + 4$ or $3y = 2x + 12$
- $x = \frac{\pi}{4}, \frac{11\pi}{12}, \frac{5\pi}{4}, \frac{23\pi}{12}$
- (a) Sketch of $y = 1 - f(x)$ with five points shown:



- (b) Sketch of $y = f'(x)$ with roots shown:



5. (a) -3
(b) $k = \frac{1}{3}$
6. (a) $p = -\frac{1}{3}$
(b) Since $-1 < -\frac{1}{3} < 1$, the sequence converges towards a limit.
7. (a) $-3(x+1)^2 - 1$
(b) Maximum value of $f'(x)$ is -1 , and since $-1 < 0$ then $f'(x) < 0$ for all x .
Hence, $f(x)$ is strictly decreasing for all $x \in \mathbb{R}$.
8. (a) $(4, 3)$
(b) $(x-1)^2 + (y+1)^2 = 25$
9. Repeated factor therefore the line meets the parabola at a tangent, at $(3, 4)$.
10. $\frac{32}{3}$ square units
11. $f(x) = 2x^2 + 5x + 9$
12. (a) $\overrightarrow{CA} = \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}$ and $\overrightarrow{CB} = \begin{pmatrix} -6 \\ 3 \\ 2 \end{pmatrix}$
(b) $\angle ACB = 59.0^\circ$
13. (a) Step-by-step proof, likely beginning with $V = \frac{1}{2}x^2y$ and using $12 = x^2 + xy$
(b) $x = 2$