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Chapter 23 Trig Equations

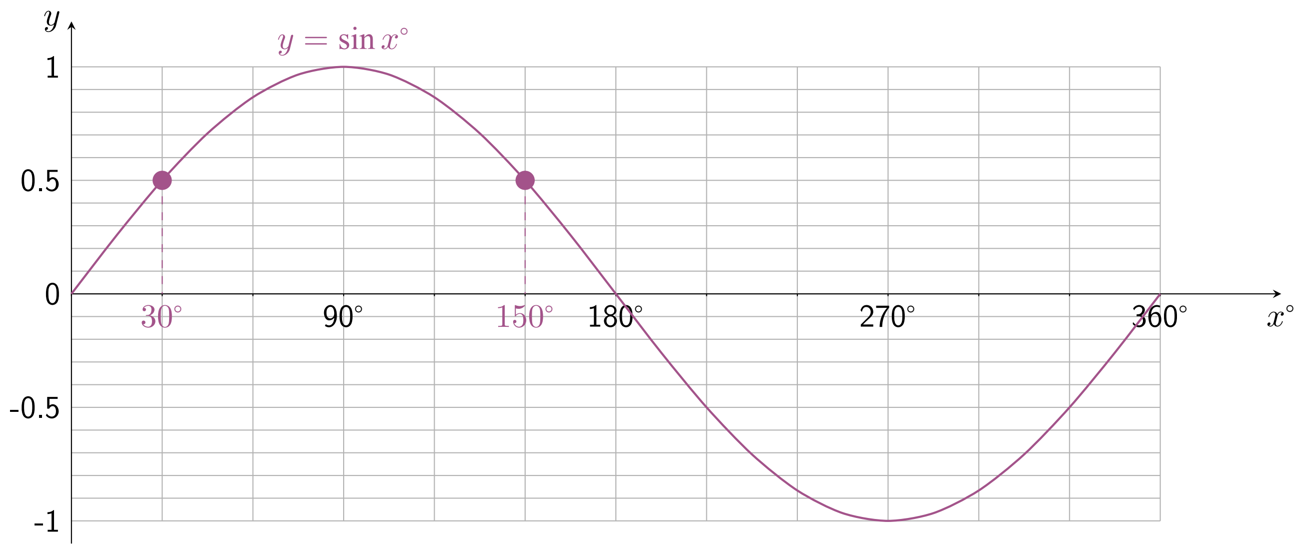
Different types of equations (such as *quadratic* or *linear*) require different methods in order to solve them.

Trigonometric equations are those which include $\sin x^\circ$, $\cos x^\circ$ or $\tan x^\circ$. Since these are some of the more complicated operations encountered so far, the best way to begin thinking about how trig equations may be solved is to consider a *graphical* interpretation:

Equation: $\sin x^\circ = 0.5$

Interpretation: *"For which value(s) of x is the graph $y = \sin x$ at a height of 0.5?"*

Plotting the graph of $y = \sin x^\circ$ and marking the points at which its height is 0.5 reveals that, between 0° and 360°, there are **two solutions**:



Hence the solutions to the equation $\sin x^\circ = 0.5$ where $0 < x < 360$ are $x^\circ = 30^\circ$ and $x^\circ = 150^\circ$.

Note that $\sin^{-1}(0.5)$ gives the *acute* solution 30°, and $180^\circ - 30^\circ = 150^\circ$ using the graph's symmetry.

The same above graph can similarly be used to solve a number of equations in $\sin x^\circ$:

Equation: $\sin x^\circ = -1$

Interpretation: *"For which value(s) of x is the graph $y = \sin x$ at a height of -1?"*

Solution: $x^\circ = 270^\circ$

The graph also reveals that equations of the form $\sin x^\circ = k$ only have solutions for $-1 \leqslant k \leqslant 1$:

Equation: $\sin x^\circ = 2$

Interpretation: *"For which value(s) of x is the graph $y = \sin x$ at a height of 2?"*

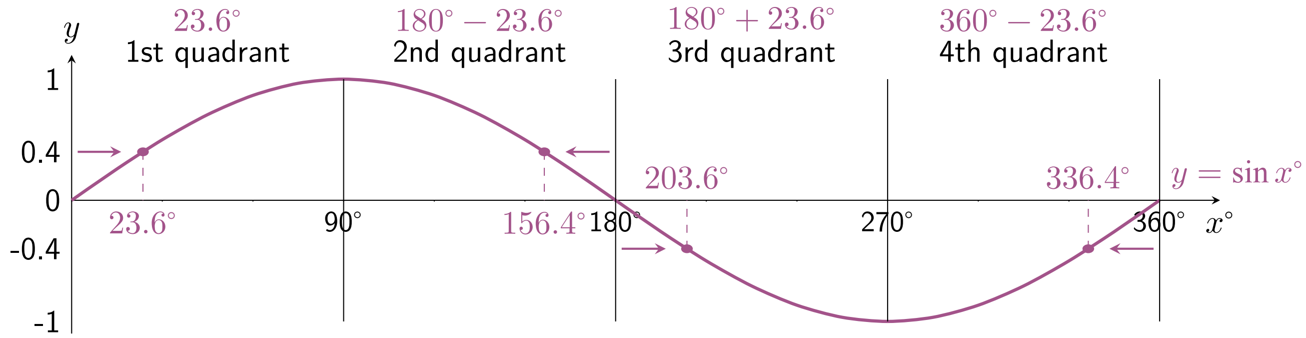
Solution: **No solutions.**

This chapter will introduce a method for solving trig equations without a graph, and explore contexts in which the ability to solve trig equations may be useful in the real world.

23.1 Solving Trig Equations Graphically

When considering $\sin x^\circ = 0.4$, calculating $\sin^{-1}(0.4)$ gives the **acute angle** 23.6° (to 1 decimal place).

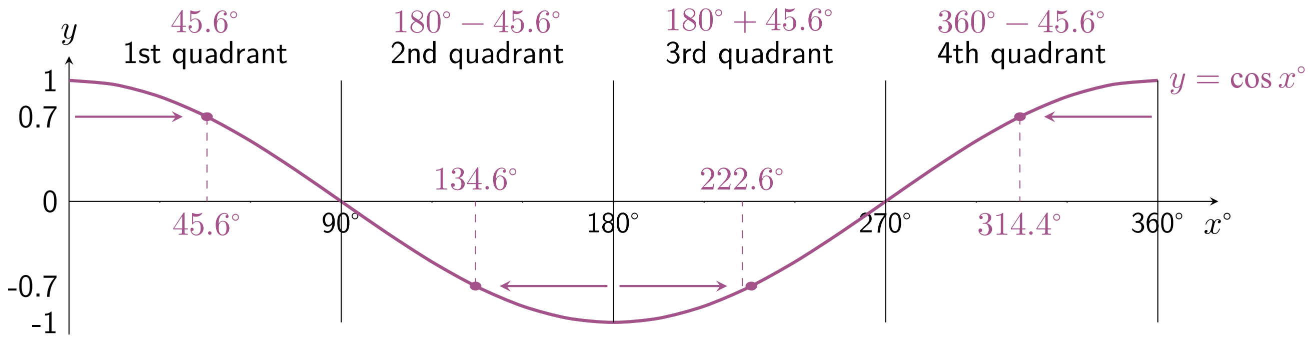
This acute angle and the symmetries of $y = \sin x^\circ$ allow $\sin x^\circ = 0.4$ and $\sin x^\circ = -0.4$ to be solved:



Equation: $\sin x^\circ = 0.4$
Solutions: $x^\circ = 23.6^\circ, 156.4^\circ$

Equation: $\sin x^\circ = -0.4$
Solutions: $x^\circ = 203.6^\circ, 336.4^\circ$

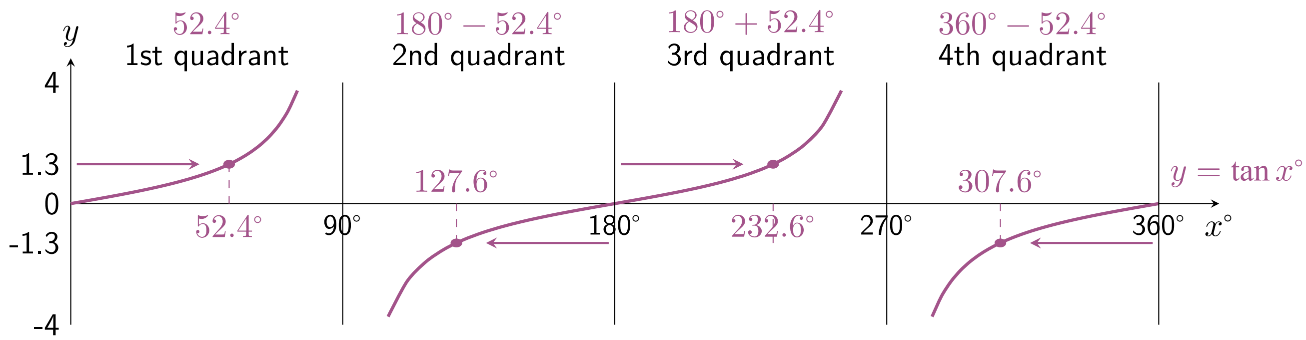
Similarly, $\cos x^\circ = 0.7$ and $\cos x^\circ = -0.7$ can be solved using **acute angle** $\cos^{-1}(0.7) = 45.6^\circ$ (1dp):



Equation: $\cos x^\circ = 0.7$
Solutions: $x^\circ = 45.6^\circ, 314.4^\circ$

Equation: $\cos x^\circ = -0.7$
Solutions: $x^\circ = 134.4^\circ, 225.6^\circ$

To solve $\tan x^\circ = 1.3$ and $\tan x^\circ = -1.3$, again the **acute angle** $\tan^{-1}(1.3) = 52.4^\circ$ (1dp) is helpful:

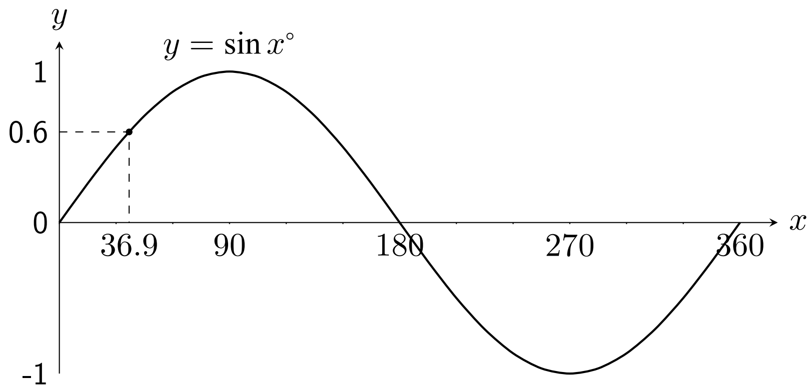


Equation: $\tan x^\circ = 1.3$
Solutions: $x^\circ = 52.4^\circ, 232.4^\circ$

Equation: $\tan x^\circ = -1.3$
Solutions: $x^\circ = 127.6^\circ, 307.6^\circ$

Exercise 23.1

1. Use the graph of $y = \sin x°$ below to solve the following equations for $0 \leqslant x \leqslant 360$.



(a) $\sin x° = 1$

(b) $\sin x° = 0$

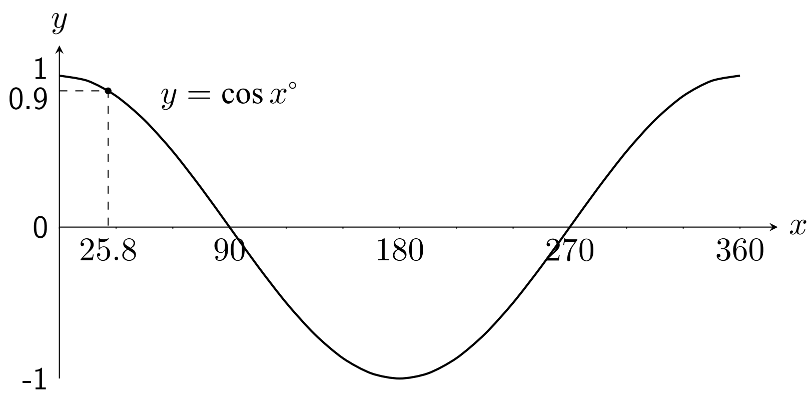
(c) $\sin x° = -1$

(d) $\sin x° = 0.6$

(e) $\sin x° = -0.6$

(f) $\sin x° = 1.1$

2. Use the graph of $y = \cos x°$ below to solve the following equations for $0 \leqslant x \leqslant 360$.



(a) $\cos x° = 1$

(b) $\cos x° = 0$

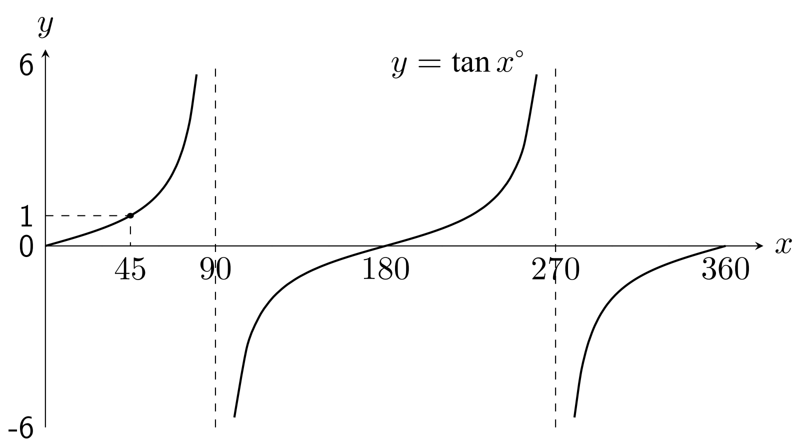
(c) $\cos x° = -1$

(d) $\cos x° = 0.9$

(e) $\cos x° = -0.9$

(f) $\cos x° = 1.1$

3. Use the graph of $y = \tan x°$ below to solve the following equations for $0 \leqslant x \leqslant 360$.



(a) $\tan x° = 1$

(b) $\tan x° = 0$

(c) $\tan x° = -1$

(d) $\tan x° = 3$

(e) $\tan x° = -3$

(f) $\tan x° = 17$

4. Given that $\sin^{-1}\left(\frac{1}{3}\right) = 19.5°$, to one decimal place, solve $\sin x° = \frac{1}{3}$ for $0 \leqslant x \leqslant 360$.

5. Given that $\cos^{-1}\left(\frac{3}{4}\right) = 41.4°$, to one decimal place, solve $4\cos x° = 3$ for $0 \leqslant x \leqslant 360$.

6. Given that $\sin^{-1}\left(\frac{2}{3}\right) = 41.8°$, to one decimal place, solve $3\sin x° + 2 = 0$ for $0 \leqslant x \leqslant 360$.

7. Solve $5\cos x° + 3 = 0$ for $0 \leqslant x \leqslant 720$.

23.2 Using the ASTC Diagram

Exercise 23.2

23.3 Points of Intersection

Exercise 23.3

23.4 Trig Functions in Context

Exercise 23.4

23.5 Trig Identities

Exercise 23.5

Review Exercise

Answers

Challenge Problems

The following problems **do not** represent the kind of question expected to feature in a Higher Mathematics exam, either in the way they are presented or the level of difficulty. Instead, they aim to encourage a flexible approach towards problem-solving and an understanding that the skills covered in the course have applications beyond those featured in any typical exam. *Some questions may be solveable without using the skills covered in this chapter, and some questions may be unreleated to this chapter.*