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Introduction

Intro

Chapter 15 Trigonometric Equations

Each part of an operation of the form a^n can be described using the following terminology:

"base" $\longrightarrow a^n \longleftarrow$ "power" or "exponent"

Power functions take the form $f(x) = x^n$, with x as the *base* and a constant power, n.

$$\text{e.g.} \quad f(x) = x^3$$

Exponential functions take the form $f(x) = a^x$, with x as the *exponent* and a constant base, $a > 0, a \neq 1$.

$$\text{e.g.} \quad f(x) = 3^x$$

An example of an exponential function in everyday life is that of something *appreciating* by a percentage of its value, such as an antique vase of value £4000 increasing by 20% each year. Its value after 0 years, 1 years, 2 years, and so on, can be calculated as follows:

$$\begin{aligned}
4000 \times 1.2^0 &= 4000 && \longleftarrow \text{After 0 years} \\
4000 \times 1.2^1 &= 4800 && \longleftarrow \text{After 1 year} \\
4000 \times 1.2^2 &= 5760 && \longleftarrow \text{After 2 years} \\
4000 \times 1.2^3 &= 6912 && \longleftarrow \text{After 3 years} \\
4000 \times 1.2^4 &= 8294.40 && \longleftarrow \text{After 4 years}
\end{aligned}$$

The function to describe its value V after x years is given by: $V(x) = 4000 \times 1.2^x$

One advantage of defining this function is the ability to calculate the value at times other that after whole years. For example, the value after three and a half years can be calculated as:

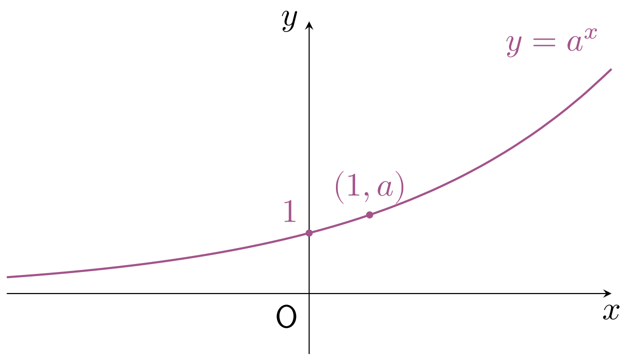
$$V(3.5) = 4000 \times 1.2^{3.5} = 7571.72$$

This chapter will introduce a range of skills required when working with exponential functions.

1.1 Graphs of Exponential Functions

Where $a > 1$:

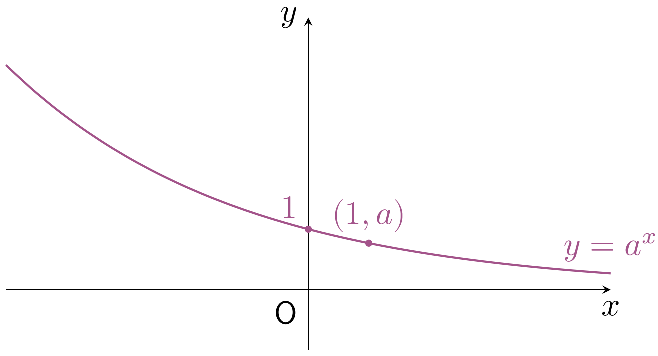
$y = a^x$ is *strictly increasing* on $x \in \mathbb{R}$:



This describes **exponential growth**.

Where $0 < a < 1$:

$y = a^x$ is *strictly decreasing* on $x \in \mathbb{R}$:



This describes **exponential decay**.

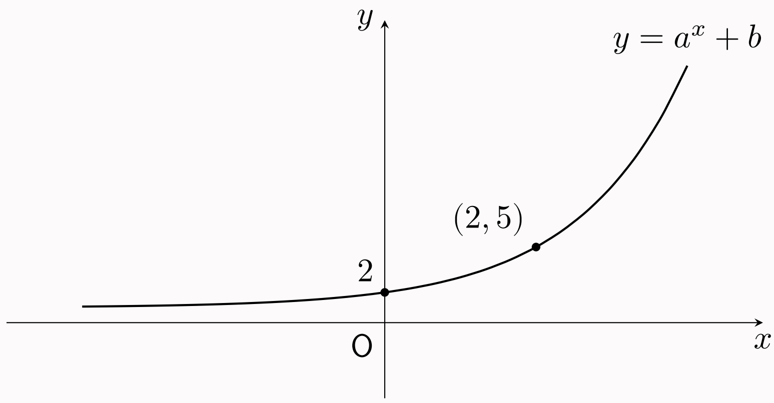
Since $a^0 = 1$, any graph of the form $y = a^x$ will pass through the point $(0, 1)$ for all $a \neq 0$.

Since $a^1 = a$, any graph of the form $y = a^x$ will pass through the point $(1, a)$ for all a.

Determining the equation of the graph of an exponential function can typically be achieved using substitution or consideration of graph transformations, along with knowledge of points $(0, 1)$ and $(1, a)$.

Example

The graph of $y = a^x + b$ is shown below.



$2 = a^0 + b$ $\longleftarrow$ Substitute $(0, 2)$

$2 = 1 + b$

$1 = b$ $\longleftarrow$ Solve to obtain b

$5 = a^2 + 1$ $\longleftarrow$ Substitute $(2, 5)$ and $b = 1$

$4 = a^2$

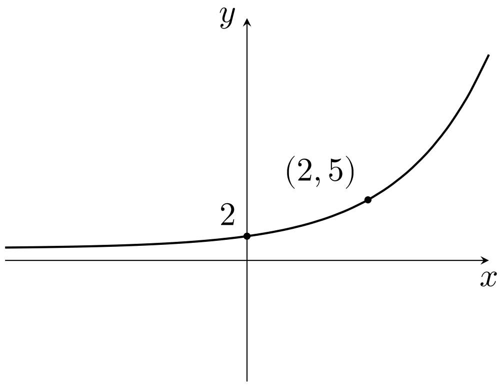
$2 = a$ $\longleftarrow$ Solve to obtain a

$y = 2^x + 1$ $\longleftarrow$ State equation

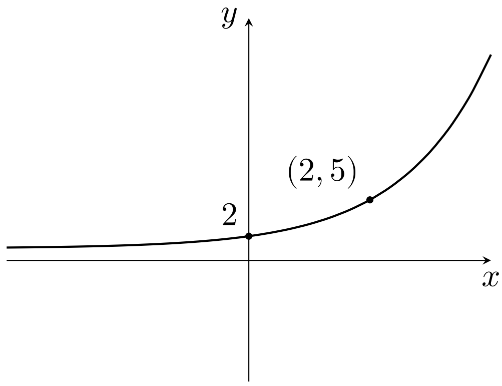
Exercise 15.1

1. Find the equation of each exponential graphs using the form given.

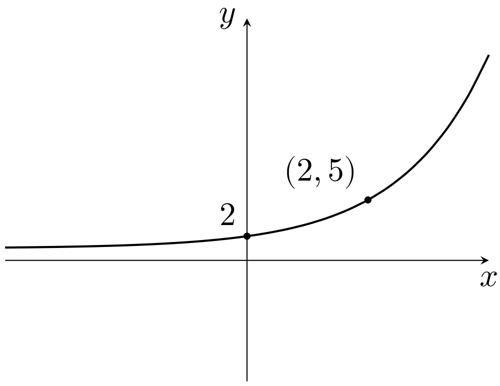
(a) $y = a^x$



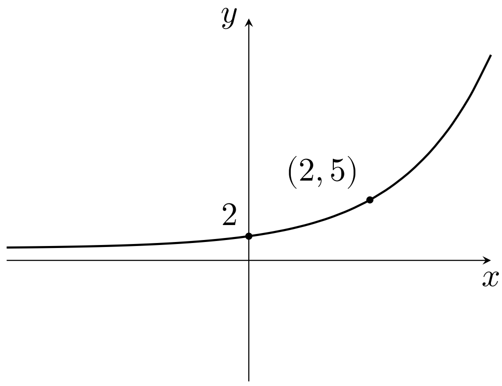
(b) $y = a^x$



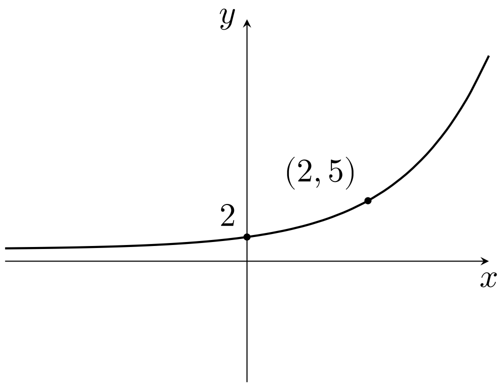
(c) $y = a^x$



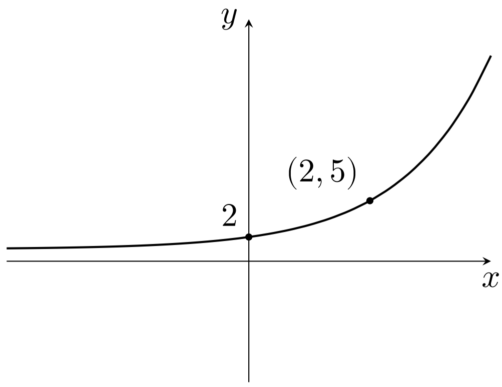
(d) $y = a^x$



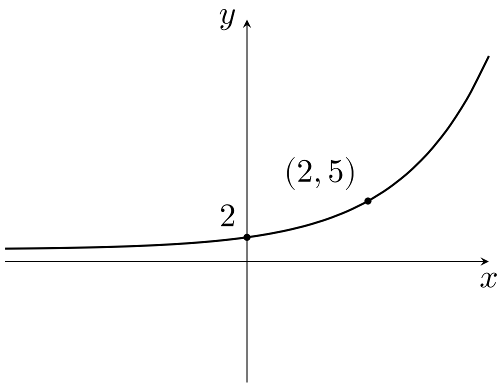
(e) $y = a^x$



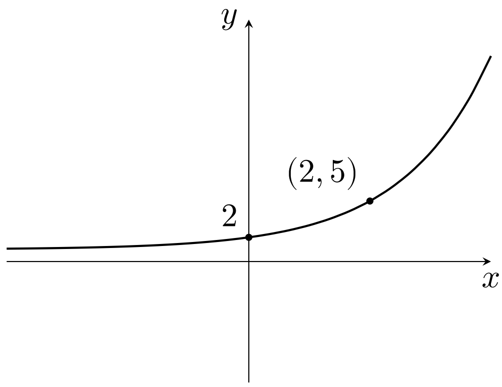
(f) $y = a^x$



(g) $y = a^x$



(h) $y = a^x$



1.2 Logarithms

Answers

Challenge Problems

The following problems **do not** represent the kind of question expected to feature in a Higher Mathematics exam, either in the way they are presented or the level of difficulty. Instead, they aim to encourage a flexible approach towards problem-solving and an understanding that the skills covered in the course have applications beyond those featured in any typical exam. *Some questions may be solveable without using the skills covered in this chapter, and some questions may be unreleated to this chapter.*