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# Chapter 23 Trig Equations

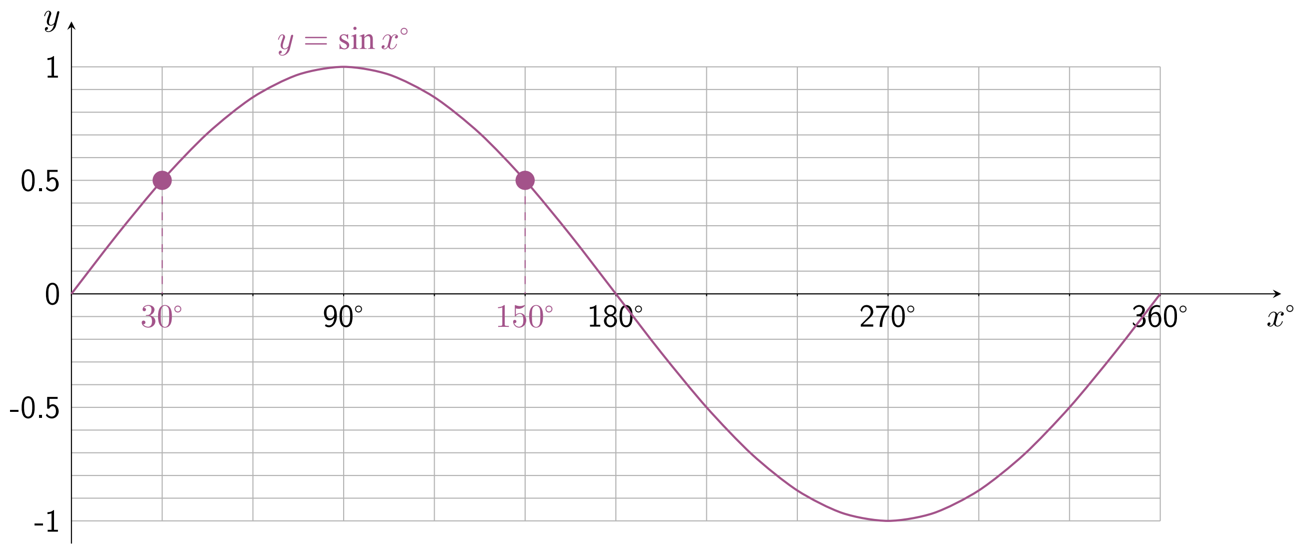
Different types of equations (such as *quadratic* or *linear*) require different methods in order to solve them.

*Trigonometric equations* are those which include $\sin x°$, $\cos x°$ or $\tan x°$. Since these are some of the more complicated operations encountered so far, the best way to begin thinking about how trig equations may be solved is to consider a *graphical* interpretation:

Equation: $\sin x° = 0.5$

Interpretation: *"For which value(s) of $x$ is the graph $y = \sin x$ at a height of $0.5$?"*

Plotting the graph of $y = \sin x°$ and marking the points at which its height is $0.5$ reveals that, between $0°$ and $360°$, there are **two solutions**:




Hence the solutions to the equation $\sin x° = 0.5$ where $0 < x < 360$ are $x° = 30°$ and $x° = 150°$.

Note that $\sin^{-1}(0.5)$ gives the *acute* solution $30°$, and $180° - 30° = 150°$ using the graph's symmetry.

The same above graph can similarly be used to solve a number of equations in $\sin x°$:

Equation: $\sin x° = -1$

Interpretation: *"For which value(s) of $x$ is the graph $y = \sin x$ at a height of $-1$?"*

Solution: $x° = 270°$

The graph also reveals that equations of the form $\sin x° = k$ only have solutions for $-1 \leqslant k \leqslant 1$:

Equation: $\sin x° = 2$

Interpretation: *"For which value(s) of $x$ is the graph $y = \sin x$ at a height of $2$?"*

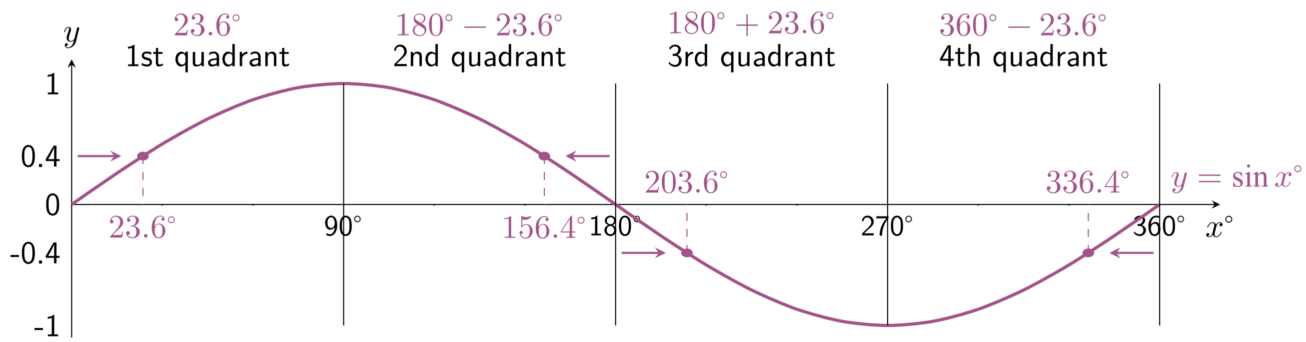
Solution: **No solutions.**

This chapter will introduce a method for solving trig equations without a graph, and explore contexts in which the ability to solve trig equations may be useful in the real world.

## 23.1 Solving Trig Equations Graphically

When considering $\sin x^\circ = 0.4$, calculating $\sin^{-1}(0.4)$ gives the **acute angle** $23.6^\circ$ (to 1 decimal place).

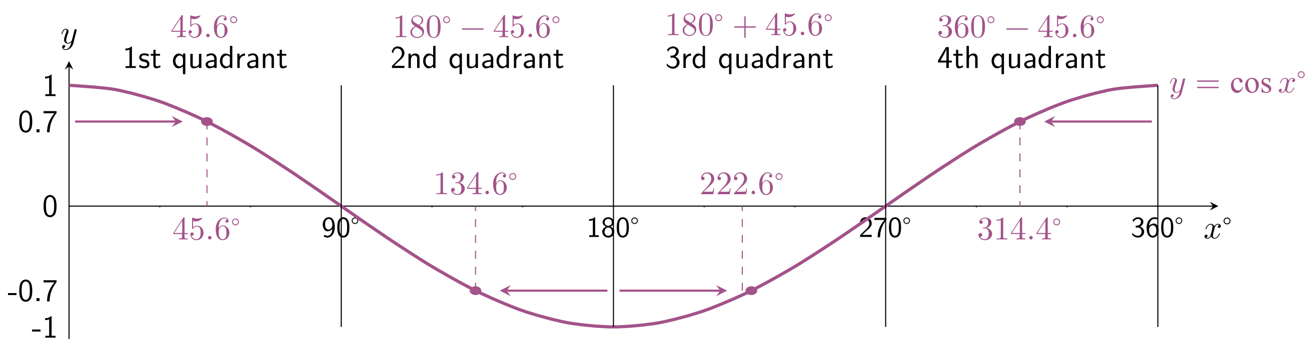
This acute angle and the symmetries of $y = \sin x^\circ$ allow $\sin x^\circ = 0.4$ and $\sin x^\circ = -0.4$ to be solved:




Equation: $\sin x^\circ = 0.4$
Solutions: $x^\circ = 23.6^\circ, 156.4^\circ$

Equation: $\sin x^\circ = -0.4$
Solutions: $x^\circ = 203.6^\circ, 336.4^\circ$

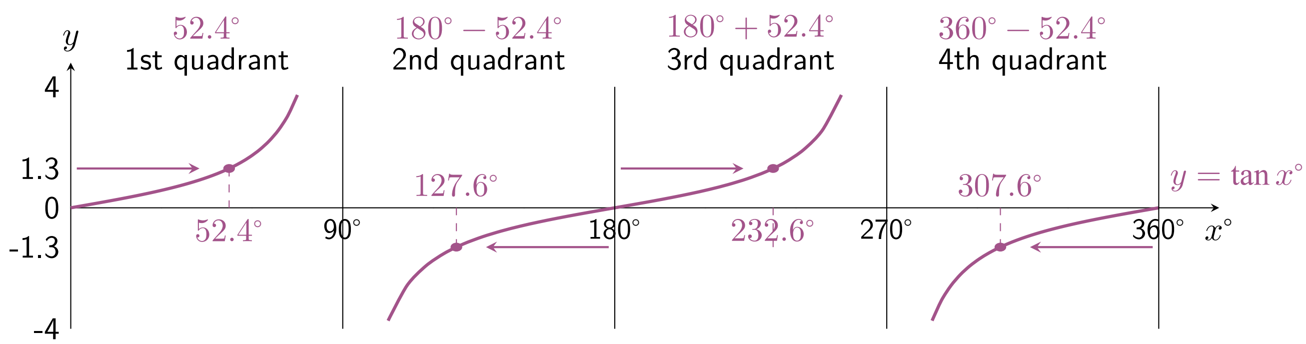
Similarly, $\cos x^\circ = 0.7$ and $\cos x^\circ = -0.7$ can be solved using **acute angle** $\cos^{-1}(0.7) = 45.6^\circ$ (1dp):




Equation: $\cos x^\circ = 0.7$
Solutions: $x^\circ = 45.6^\circ, 314.4^\circ$

Equation: $\cos x^\circ = -0.7$
Solutions: $x^\circ = 134.4^\circ, 225.6^\circ$

To solve $\tan x^\circ = 1.3$ and $\tan x^\circ = -1.3$, again the **acute angle** $\tan^{-1}(1.3) = 52.4^\circ$ (1dp) is helpful:

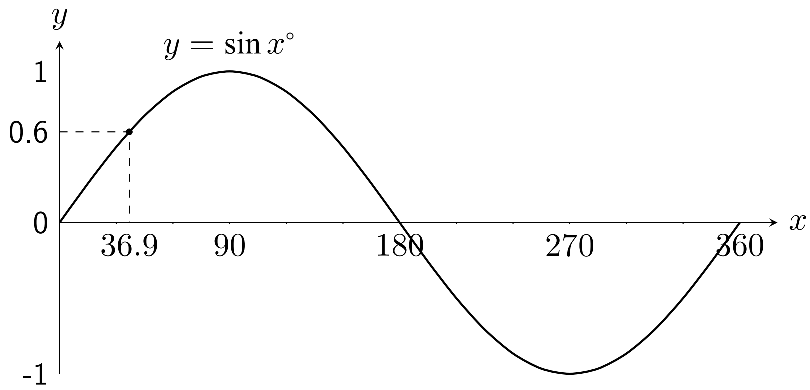



Equation: $\tan x^\circ = 1.3$
Solutions: $x^\circ = 52.4^\circ, 232.4^\circ$

Equation: $\tan x^\circ = -1.3$
Solutions: $x^\circ = 127.6^\circ, 307.6^\circ$

## Exercise 23.1

1. Use the graph of $y = \sin x^\circ$ below to solve the following equations for $0 \leqslant x \leqslant 360$.




(a) $\sin x^\circ = 1$

(b) $\sin x^\circ = 0$

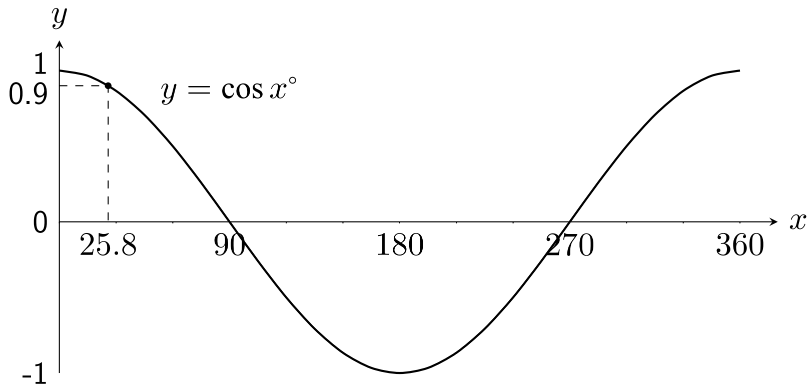
(c) $\sin x^\circ = -1$

(d) $\sin x^\circ = 0.6$

(e) $\sin x^\circ = -0.6$

(f) $\sin x^\circ = 1.1$

2. Use the graph of $y = \cos x^\circ$ below to solve the following equations for $0 \leqslant x \leqslant 360$.




(a) $\cos x^\circ = 1$

(b) $\cos x^\circ = 0$

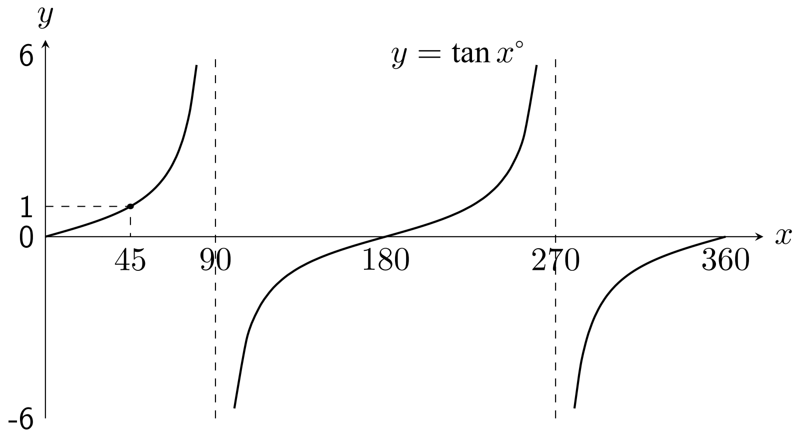
(c) $\cos x^\circ = -1$

(d) $\cos x^\circ = 0.9$

(e) $\cos x^\circ = -0.9$

(f) $\cos x^\circ = 1.1$

3. Use the graph of $y = \tan x^\circ$ below to solve the following equations for $0 \leqslant x \leqslant 360$.




(a) $\tan x^\circ = 1$

(b) $\tan x^\circ = 0$

(c) $\tan x^\circ = -1$

(d) $\tan x^\circ = 3$

(e) $\tan x^\circ = -3$

(f) $\tan x^\circ = 17$

4. Given that $\sin^{-1}\left(\frac{1}{3}\right) = 19.5^\circ$, to one decimal place, solve $\sin x^\circ = \frac{1}{3}$ for $0 \leqslant x \leqslant 360$.

5. Given that $\cos^{-1}\left(\frac{3}{4}\right) = 41.4^\circ$, to one decimal place, solve $4\cos x^\circ = 3$ for $0 \leqslant x \leqslant 360$.

6. Given that $\sin^{-1}\left(\frac{2}{3}\right) = 41.8^\circ$, to one decimal place, solve $3\sin x^\circ + 2 = 0$ for $0 \leqslant x \leqslant 360$.

7. Solve $5\cos x^\circ + 3 = 0$ for $0 \leqslant x \leqslant 720$.

## 23.2 Using the ASTC Diagram

## Exercise 23.2

## 23.3 Points of Intersection

## Exercise 23.3

## 23.4 Trig Functions in Context

## Exercise 23.4

## 23.5 Trig Identities

## Exercise 23.5

## Review Exercise

# Answers

# Challenge Problems

The following problems **do not** represent the kind of question expected to feature in a Higher Mathematics exam, either in the way they are presented or the level of difficulty. Instead, they aim to encourage a flexible approach towards problem-solving and an understanding that the skills covered in the course have applications beyond those featured in any typical exam. *Some questions may be solveable without using the skills covered in this chapter, and some questions may be unreleated to this chapter.*