

# Formatted manual of singularcurve.lib

---

---

# 1 Singular libraries

## 1.1 singularcurve\_lib

-----BEGIN OF PART WHICH IS INCLUDED IN MANUAL-----

**Library:** singularcurve.lib

**Purpose:** targets calculation of Zeta-function of a curve with quotient singularity over a finite field

**Authors:** Sebastian Mitterle, sebastianmitterle@mathume.com

**Procedures:**

### 1.1.0.1 sumintvec

Procedure from library `singularcurve.lib` (see Section 1.1 [`singularcurve_lib`], page 1).

**Usage:** `sumintvec(e); e intvec`

**Assume:** none

**Return:** int: the sum of the entries of e

**Note:** none

**Example:**

```
LIB "singularcurve.lib";
intvec e = 1,2,3;
int six = sumintvec(e);
six;
↪ 6
```

### 1.1.0.2 list2ideal

Procedure from library `singularcurve.lib` (see Section 1.1 [`singularcurve_lib`], page 1).

**Usage:** `list2ideal(polylist); list polylist`

**Assume:** none

**Return:** ideal: the ideal  $I = \text{polylist}[1], \dots, \text{polylist}[n]$ ;

**Note:** this procedure can be used to sum ideals avoiding simplification

**Example:**

```
LIB "singularcurve.lib";
ring r = 0, x, dp;
ideal I1 = x;
ideal I2 = 1;
list IL;
for(int i = 1; i<=size(I1); i++){
IL = insert(IL, I1[i]);
}
for(int i = 1; i<=size(I2); i++){
IL = insert(IL, I2[i]);
}
↪ // ** redefining i **
```

```

ideal I3 = list2ideal(IL);
I1 + I2;
 $\mapsto$   $_{[1]}=1$ 
I3;
 $\mapsto$  I3[1]=1
 $\mapsto$  I3[2]=x

```

### 1.1.0.3 multinomial

Procedure from library `singularcurve.lib` (see Section 1.1 [singularcurve.lib], page 1).

**Usage:** `multinomial(N, iv):` int N, intvec iv

**Assume:** none

**Return:** bigint multinomial coefficient  $N!/(iv[1]! \cdots iv[n]!)$

**Note:** needs `general.lib`; doesn't check  $N = \text{sumintvec}(iv)$

**Example:**

```

LIB "singularcurve.lib";
LIB "general.lib";
intvec iv = 1,2,3;
bigint mulCoef = multinomial(6, iv);
mulCoef;
 $\mapsto$  60

```

### 1.1.0.4 ceilquot

Procedure from library `singularcurve.lib` (see Section 1.1 [singularcurve.lib], page 1).

**Usage:** `ceilquot(n1, n2):` int n1, int n2

**Assume:** none

**Return:** `ceil(decimal(n1/n2))`

**Note:** none

**Example:**

```

LIB "singularcurve.lib";
ceilquot(1,2);
 $\mapsto$  1
ceilquot(2,2);
 $\mapsto$  1
ceilquot(-1,2);
 $\mapsto$  0

```

### 1.1.0.5 padicOrder

Procedure from library `singularcurve.lib` (see Section 1.1 [singularcurve.lib], page 1).

**Usage:** `padicOrder(n, p):` number n, bigint p

**Assume:** basering is of characteristic 0

**Return:** returns the padic order of n: if  $n = p^k \cdot a/b$  then returns  $\text{order}(a) - \text{order}(b)$ ;  
returns 'Inf' if  $n == 0$

**Note:** needs `poly.lib`

**Example:**

```

LIB "singularcurve.lib";
ring r = 0, x, dp;
LIB "poly.lib";
number n1 = number(1/2);
number n2 = number(98/3);
number n3 = number(0);
padicOrder(n1, 7);
  ↦ 0
padicOrder(n2, 7);
  ↦ 2
padicOrder(n3, 7);
  ↦ Inf

```

**1.1.0.6 padicApprox**

Procedure from library `singularcurve.lib` (see Section 1.1 [`singularcurve.lib`], page 1).

**Usage:** `padicApprox(quot, p, N): number(quot), int p, int N`

**Assume:** basering is of characteristic 0

**Return:** returns the p-adic approximation of a rational number up to certain precision

**Note:**  $a$  is approximation of  $b$  to precision  $N \iff a - b = 0 \pmod{p^N}$ , i.e.  $a_i = 0$ ,  $i \geq N$

**Example:**

```

LIB "singularcurve.lib";
LIB "poly.lib";
ring r = 0, x, dp;
number n1 = number(0);
number n2 = number(1/2);
number n3 = 7*3 + 49*4;
padicApprox(n1, 7, 0);
  ↦ 0
padicApprox(n2, 7, 7);
  ↦ 411772
padicApprox(n2, 7, 4);
  ↦ 1201
padicApprox(n3, 7, 3);
  ↦ 217

```

**1.1.0.7 reduceStep**

Procedure from library `singularcurve.lib` (see Section 1.1 [`singularcurve.lib`], page 1).

**Usage:** `reduceStep(nom, jf, s): poly nom, ideal jf, int s`

**Assume:**  $\text{nom} = 0 \pmod{jf}$ , basering of characteristic 0

**Return:** the polynomial  $f$  s.t.  $f/F^{(s-1)} \Omega = \text{nom}/F^{(s)} \Omega$

**Note:** none

**Example:**

```

LIB "singularcurve.lib";
ring r = 0, (x,y,z), dp;
poly f = x^2 + y^2 + z^2;

```

```

ideal jf = jacob(ideal(f));
"2s-2:0->2::-2,0,2";
↪ 2s-2:0->2::-2,0,2
"2s-2:3::4::s==3";
↪ 2s-2:3::4::s==3
poly nom = x^2*y^2;
def reduced = reduceStep(nom, jf, 3);
reduced;
↪ 1/4x2

```

### 1.1.0.8 reduceAll

Procedure from library `singularcurve.lib` (see Section 1.1 [`singularcurve.lib`], page 1).

**Usage:** `reduceAll(nom, jf, s):` poly nom, ideal jf, int s

**Assume:** `nom == 0 mod jf`

**Return:** poly expression that expresses nom in terms of a base of  $R/jf$

**Note:** s is the degree of the denominator  $F_k^s$

**Example:**

```

LIB "singularcurve.lib";
ring r = 0, (x,y,z), dp;
poly f = x2z3 + y2z3 + x5 + y5 + 7z5;
ideal jf = jacob(f);
poly nom = x^2 + y^2;
nom = nom*(f^10);
int s = (deg(nom)+3)/5;
reduceAll(nom, jf, s);
↪ x2+y2

```

### 1.1.0.9 extractIdeal

Procedure from library `singularcurve.lib` (see Section 1.1 [`singularcurve.lib`], page 1).

**Usage:** `extractIdeal(base, degrees):` ideal base, intvec degrees

**Assume:** none

**Return:** the ideal containing only those generators of the input ideal of certain degrees

**Note:** none

**Example:**

```

LIB "singularcurve.lib";
LIB "general.lib";
ring r = 0, (x,y,z), dp;
poly f = x2z3 + y2z3 + x5 + y5 + 7z5;
ideal jf = jacob(f);
ideal base = kbase(std(jf));
size(base);
↪ 64
base = extractIdeal(base, intvec(2,7));
↪ // ** redefining j **
↪ // ** redefining j **
↪ // ** redefining j **
↪ // ** redefining j **
↪ // ** redefining j **

```

[illegible]

```

base;
⇒ base[1]=z2
⇒ base[2]=yz
⇒ base[3]=xz
⇒ base[4]=y2
⇒ base[5]=xy
⇒ base[6]=x2
⇒ base[7]=z7
⇒ base[8]=yz6
⇒ base[9]=xz6
⇒ base[10]=xyz5
⇒ base[11]=xy3z3
⇒ base[12]=x3y3z

```

### 1.1.0.10 getMatrix

Procedure from library `singularcurve.lib` (see Section 1.1 [`singularcurve.lib`], page 1).

**Usage:** `getMatrix(base, image)`: ideal base, list image

**Assume:** the generators of base are base vectors of a module; image contains the image of each generator represented by a linear combination of the generators of base

**Return:** the matrix representing the map  $\text{base}[i] \rightarrow \text{image}[i]$

**Example:**

```

LIB "singularcurve.lib";
LIB "general.lib";
ring r = 0, (x,y,z), dp;
ideal bf = x2,y2,z2, xy,xz,yz;
bf = sort(bf)[1];
matrix m1[6][6] = 1,2,3,4,5,6,7,8,9,10,11,12,1,2,3,4,5,6,7,8,9,10,11,12,1,2,3,4,5,6,7,8,9,10,11,12;
list image = list();
for(int col = 6; col>=1; col--){
  poly tmpoly = 0;
  for(int row = 1; row<=6; row++){
    tmpoly = tmpoly + m1[row, col]*bf[row];
  }
  image = insert(image, tmpoly);
}
⇒ // ** redefining tmpoly **
⇒ // ** redefining row **
⇒ // ** redefining tmpoly **
⇒ // ** redefining row **
⇒ // ** redefining tmpoly **
⇒ // ** redefining row **
⇒ // ** redefining tmpoly **
⇒ // ** redefining row **
⇒ // ** redefining tmpoly **
⇒ // ** redefining row **
def m2 = getMatrix(bf, image);
⇒ // ** redefining row **
⇒ // ** redefining row **
⇒ // ** redefining row **
⇒ // ** redefining row **
⇒ // ** redefining row **
⇒ // ** redefining bindex **
⇒ // ** redefining row **
⇒ // ** redefining row **

```

```

⇒ // ** redefining row **
⇒ // ** redefining row **
⇒ // ** redefining row **
⇒ // ** redefining row **
⇒ // ** redefining bindex **
⇒ // ** redefining row **
⇒ // ** redefining row **
⇒ // ** redefining row **
⇒ // ** redefining row **
⇒ // ** redefining row **
⇒ // ** redefining row **
⇒ // ** redefining bindex **
⇒ // ** redefining row **
⇒ // ** redefining row **
⇒ // ** redefining row **
⇒ // ** redefining row **
⇒ // ** redefining row **
⇒ // ** redefining bindex **
⇒ // ** redefining row **
⇒ // ** redefining row **
⇒ // ** redefining row **
⇒ // ** redefining row **
⇒ // ** redefining row **
⇒ // ** redefining row **
⇒ // ** redefining bindex **
⇒ // ** redefining row **
⇒ // ** redefining row **
⇒ // ** redefining row **
⇒ // ** redefining row **
⇒ // ** redefining row **
m2;
⇒ m2[1,1]=1
⇒ m2[1,2]=2
⇒ m2[1,3]=3
⇒ m2[1,4]=4
⇒ m2[1,5]=5
⇒ m2[1,6]=6
⇒ m2[2,1]=7
⇒ m2[2,2]=8
⇒ m2[2,3]=9
⇒ m2[2,4]=10
⇒ m2[2,5]=11
⇒ m2[2,6]=12
⇒ m2[3,1]=1
⇒ m2[3,2]=2
⇒ m2[3,3]=3
⇒ m2[3,4]=4
⇒ m2[3,5]=5
⇒ m2[3,6]=6
⇒ m2[4,1]=7
⇒ m2[4,2]=8
⇒ m2[4,3]=9
⇒ m2[4,4]=10
⇒ m2[4,5]=11
⇒ m2[4,6]=12
⇒ m2[5,1]=1

```



```
↦ m2[5,2]=2
↦ m2[5,3]=3
↦ m2[5,4]=4
↦ m2[5,5]=5
↦ m2[5,6]=6
↦ m2[6,1]=7
↦ m2[6,2]=8
↦ m2[6,3]=9
↦ m2[6,4]=10
↦ m2[6,5]=11
↦ m2[6,6]=12
```

## 2 Index

### C

ceilquot ..... 2

### E

extractIdeal ..... 4

### G

getMatrix ..... 6

### L

list2ideal ..... 1

### M

multinomial ..... 2

### P

padicApprox ..... 3

padicOrder ..... 2

### R

reduceAll ..... 4

reduceStep ..... 3

### S

singularcurve.lib ..... 1

singularcurve\_lib ..... 1

sumintvec ..... 1