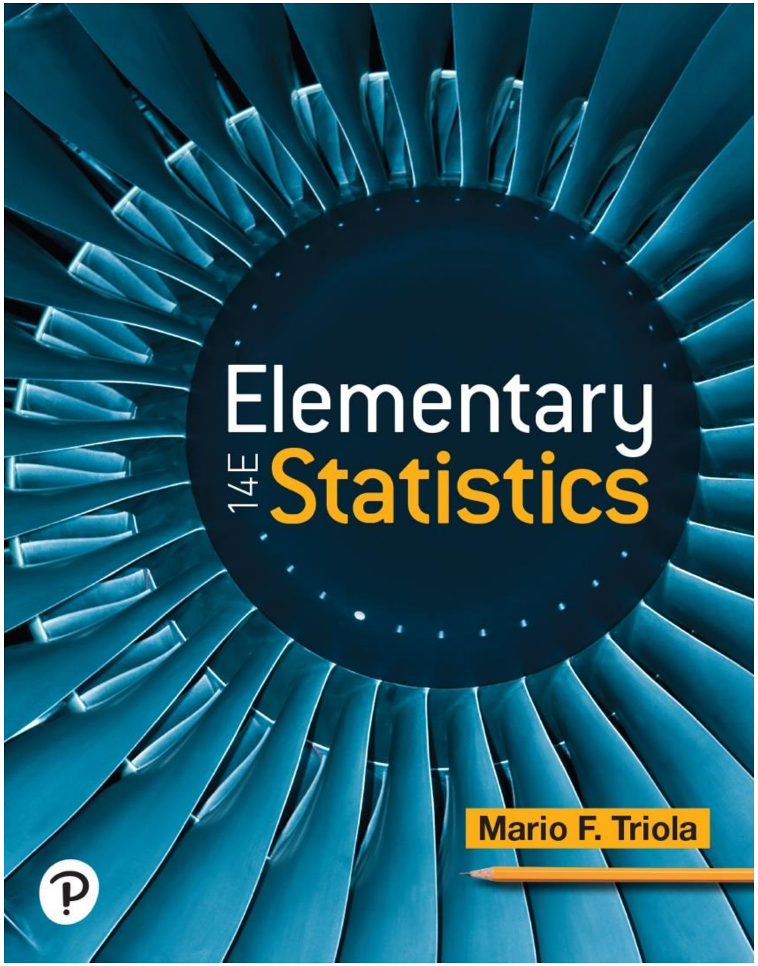
Elementary Statistics

Fourteenth Edition



Chapter 4

Probability

Probability

4-1 Basic Concepts of Probability

4-2 Addition Rule and Multiplication Rule

4-3 Complements and Conditional Probability

4-4 Counting

Key Concept

The single most important objective of this section is to learn how to **interpret** probability values, which are expressed as values between 0 and 1. A small probability, such as 0.001, corresponds to an event that rarely occurs.

Basics of Probability

- An **event** is any collection of results or outcomes of a procedure.
- A **simple event** is an outcome or an event that cannot be further broken down into simpler components.
- The **sample space** for a procedure consists of all possible **simple** events. That is, the sample space consists of all outcomes that cannot be broken down any further.

Example: Simple Events and Sample Spaces (1 of 5)

In the following display, we use “b” to denote a baby boy and “g” to denote a baby girl.

Procedure	Example of Event	Sample Space: Complete List of Simple Events
Single birth	1 girl (simple event)	{b, g}
3 births	2 boys and 1 girl (bbg, bgb, and gbb are all simple events resulting in 2 boys and 1 girl)	{bbb, bbg, bgb, gbb, bgg, gbg, ggb, ggg}

Example: Simple Events and Sample Spaces (2 of 5)

Solution

Simple Events:

- With one birth, the result of 1 girl is a **simple event** and the result of 1 boy is another simple event. They are individual simple events because they cannot be broken down any further.

Example: Simple Events and Sample Spaces (3 of 5)

Solution

Simple Events:

- With three births, the result of 2 girls followed by a boy (ggb) is a simple event.
- When rolling a single die, the outcome of 5 is a simple event, but the outcome of an even number is not a simple event.

Example: Simple Events and Sample Spaces (4 of 5)

Solution

Not a Simple Event: With three births, the event of “2 girls and 1 boy” is **not a simple event** because it can occur with these different simple events: ggb, gbg, bgg.

Example: Simple Events and Sample Spaces (5 of 5)

Solution

Sample Space: With three births, the **sample space** consists of the eight different simple events listed in the table.

Procedure	Example of Event	Sample Space: Complete List of Simple Events
Single birth	1 girl (simple event)	{b, g}
3 births	2 boys and 1 girl (bbg, bgb, and gbb are all simple events resulting in 2 boys and 1 girl)	{bbb, bbg, bgb, gbb, bgg, gbg, ggb, ggg}

Three Common Approaches to Finding the Probability of an Event (1 of 6)

Notation for Probabilities

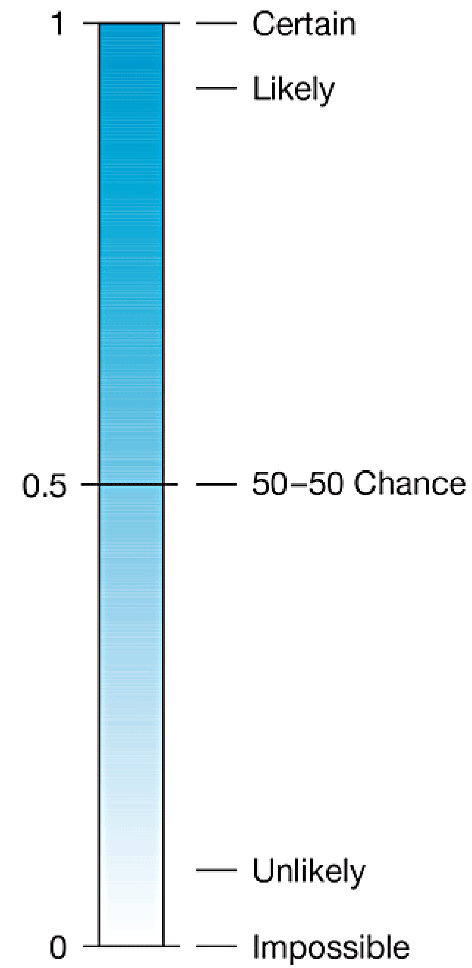
P denotes a probability.

A, B, and *C* denote specific events.

P(*A*) denotes the “probability of event *A* occurring.”

Three Common Approaches to Finding the Probability of an Event (2 of 6)

Possible values of probabilities and the more familiar and common expressions of likelihood



Three Common Approaches to Finding the Probability of an Event (3 of 6)

The following three approaches for finding probabilities result in values between 0 and 1: $0 \leq P(A) \leq 1$.

Three Common Approaches to Finding the Probability of an Event

1. **Relative Frequency Approximation of Probability**
   Conduct (or observe) a procedure and count the number of times that event *A* occurs. *P*(*A*) is then **approximated** as follows:

$$P(A) = \frac{\text{number of times A occurred}}{\text{number of times the procedure was repeated}}$$

Three Common Approaches to Finding the Probability of an Event (5 of 6)

2. **Classical Approach to Probability (Requires Equally Likely Outcomes)** If a procedure has *n* different simple events that are **equally likely**, and if event *A* can occur in *s* different ways, then

$$P(A) = \frac{\text{number of ways A occurs}}{\text{number of different simple events}} = \frac{s}{n}$$

Caution When using the classical approach, always confirm that the outcomes are **equally likely**.

Three Common Approaches to Finding the Probability of an Event (6 of 6)

3. **Subjective Probabilities** $P(A)$, the probability of event A, is **estimated** by using knowledge of the relevant circumstances.

Rounding Probabilities

When expressing the value of a probability, either give the **exact** fraction or decimal or round off final decimal results to **three** significant digits.

(**Suggestion:** When a probability is not a simple fraction such as $\frac{2}{3}$ or $\frac{5}{9}$, express it as a decimal so that the number can be better understood.)

Law of Large Numbers (1 of 2)

Law of Large Numbers

As a procedure is repeated again and again, the relative frequency probability of an event tends to approach the actual probability.

Law of Large Numbers (2 of 2)

- **Law of Large Numbers**

CAUTIONS

1. The law of large numbers applies to behavior over a large number of trials, and it does not apply to any one individual outcome.
2. If we know nothing about the likelihood of different possible outcomes, we should not assume that they are equally likely.

Example: Relative Frequency Probability: Airline Crashes (1 of 3)

Find the probability that a commercial airliner will crash on any given flight.

Example: Relative Frequency Probability: Airline Crashes (2 of 3)

Solution

In a recent year, there were about 39 million commercial airline flights and 16 of them crashed. We use the relative frequency approach as follows:

$$P(\text{airline crash}) = \frac{\text{number of airline crashes}}{\text{total number of airline flights}}$$

$$= \frac{16}{39{,}000{,}000} = 0.000000410$$

Example: Relative Frequency Probability: Airline Crashes (3 of 3)

Solution

Here the classical approach cannot be used because the two outcomes (crashing, not crashing) are not equally likely.

A subjective probability can be estimated in the absence of historical data.

Example: Ghosts! (1 of 4)

In a Pew Research Center survey, randomly selected adults were asked if they have seen or have been in the presence of a ghost. 366 of the respondents answered “yes,” and 1637 of the respondents answered “no”. Based on these results, find the probability that a randomly selected adult says that they have seen or been in the presence of a ghost.

Example: Ghosts! (2 of 4)

Solution

Instead of trying to determine an answer directly from the given statement, first summarize the information in a format that allows clear understanding, such as this format:

366	responses of "yes"
1637	responses of "no"
2003	total number of responses

Example: Ghosts! (3 of 4)

Solution

We can now use the relative frequency approach as follows:

$$P(\text{response of "yes"}) = \frac{\text{number of "yes" responses}}{\text{total number of responses}}$$

$$= \frac{366}{2003} = 0.183$$

Example: Ghosts! (4 of 4)

Interpretation

There is a 0.183 probability that a randomly selected adult says that they have seen or been in the presence of a ghost.

Complementary Events

- **Complement**
  - The **complement** of event A, denoted by $\overline{A}$, consists of all outcomes in which event A does **not** occur.

Example: Complement of Internet Users (1 of 2)

In a Pew Research Center survey of 2002 randomly selected adults, 1782 of those respondents said that they use the Internet. Find the probability that a randomly selected adult does *not* use the Internet.

Example: Complement of Internet Users (2 of 2)

Solution

Among 2002 survey subjects, 1782 use the Internet, so it follows that the other 220 do *not* use the Internet. We get the following

$$P(\text{not using the Internet}) = \frac{220}{2002} = 0.110$$

The probability of randomly selecting an adult who does *not* use the Internet is 0.110.

Identifying Significant Results with Probabilities (1 of 3)

The Rare Event Rule for Inferential Statistics

If, under a given assumption, the probability of a particular observed event is very small and the observed event occurs **significantly less than or significantly greater than** what we typically expect with that assumption, we conclude that the assumption is probably not correct.

Identifying Significant Results with Probabilities (2 of 3)

Using Probabilities to Determine When Results Are Significantly High or Significantly Low

- **Significantly high number of successes:** x successes among n trials is a **significantly high** number of successes if the probability of x or more successes is unlikely with a probability of 0.05 or less. That is, x is a significantly high number of successes if $P(x \text{ or more}) \leq 0.05$*.

*The value 0.05 is not absolutely rigid.

Identifying Significant Results with Probabilities (3 of 3)

Using Probabilities to Determine When Results Are Significantly High or Significantly Low

- **Significantly low number of successes:** *x* successes among *n* trials is a **significantly low** number of successes if the probability of *x* or fewer successes is unlikely with a probability of 0.05 or less. That is, *x* is a significantly low number of successes if $P(x \text{ or fewer}) \le 0.05$*.

*The value 0.05 is not absolutely rigid.

Probability Review

- The probability of an event is a fraction or decimal number between 0 and 1 inclusive.
- The probability of an impossible event is 0.
- The probability of an event that is certain to occur is 1.
- Notation: $P(A)$ = the probability of event A.
- Notation: $P(\bar{A})$ = the probability that event A does **not** occur.