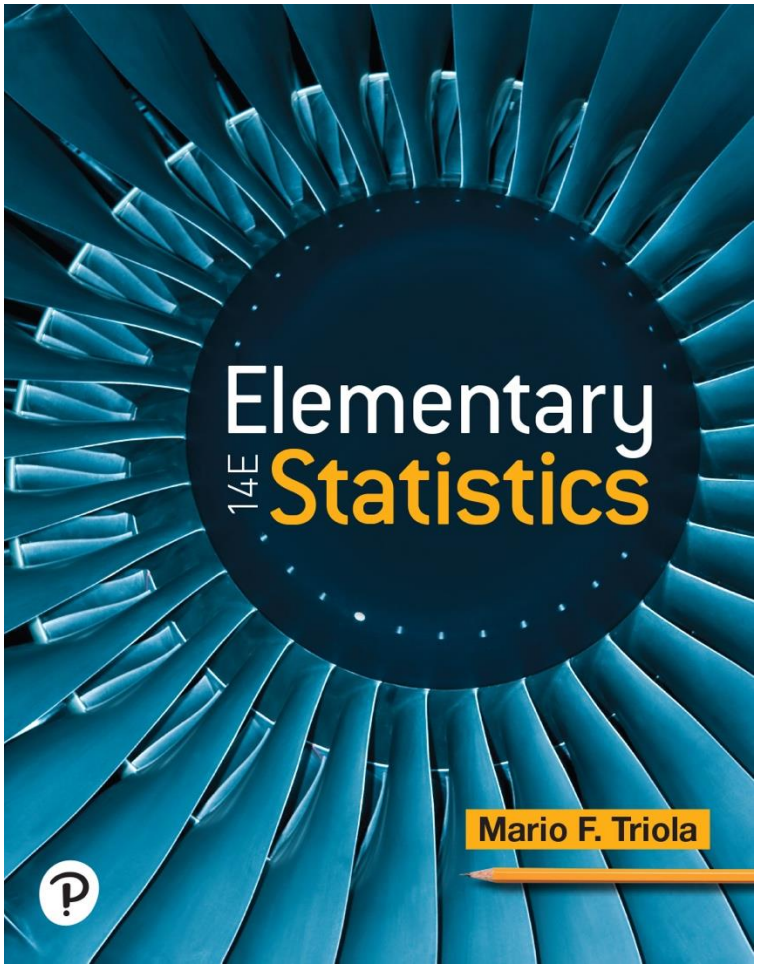
Elementary Statistics

Fourteenth Edition



Chapter 3

Describing, Exploring, and Comparing Data

Describing, Exploring, and Comparing Data

3-1 Measures of Center

3-2 Measures of Variation

3-3 Measures of Relative Standing

Key Concept

This section introduces measures of relative standing, which are numbers showing the location of data values relative to the other values within the same data set.

The most important concept in this section is the ***z* score**.

z Scores

- *z* Score
  - A ***z* score** (or **standard score** or **standardized value**) is the number of standard deviations that a given value *x* is above or below the mean. The *z* score is calculated by using one of the following:

Sample

$$z = \frac{x - \bar{x}}{s}$$

or

Population

$$z = \frac{x - \mu}{\sigma}$$

Round-off Rule for *z* Scores

Round *z* scores to two decimal places (such as 2.31).

Important Properties of *z* Scores

1. A *z* score is the number of standard deviations that a given value *x* is above or below the mean.
2. *z* scores are expressed as numbers with no units of measurement.
3. A data value is **significantly low** if its *z* score is less than or equal to −2 or the value is **significantly high** if its *z* score is greater than or equal to +2.
4. If an individual data value is less than the mean, its corresponding *z* score is a negative number.

Example: Comparing a Quarter's Weight and Adult Body Temperature
(1 of 3)

Which of the following two data values is more extreme relative to the data set from which it came?

- The 99°F temperature of an adult (among 106 adults with sample mean $\bar{x}$ = 98.20°F and sample standard deviation s = 0.62°F)
- The 5.7790 g weight of a quarter (among 40 quarters with sample mean $\bar{x}$ = 5.63930 g and sample standard deviation s = 0.06194 g)

Example: Comparing a Quarter's Weight and Adult Body Temperature

(2 of 3)

Solution

The 99°F body temperature and the 5.7790 g weight of a quarter can be standardized by converting each of them to *z* scores as shown below.

- 99°F body temperature:

$$z = \frac{x - \bar{x}}{s} = \frac{99°\text{F} - 98.20°\text{F}}{0.62°\text{F}} = 1.29$$

- 5.7790 g weight of a quarter:

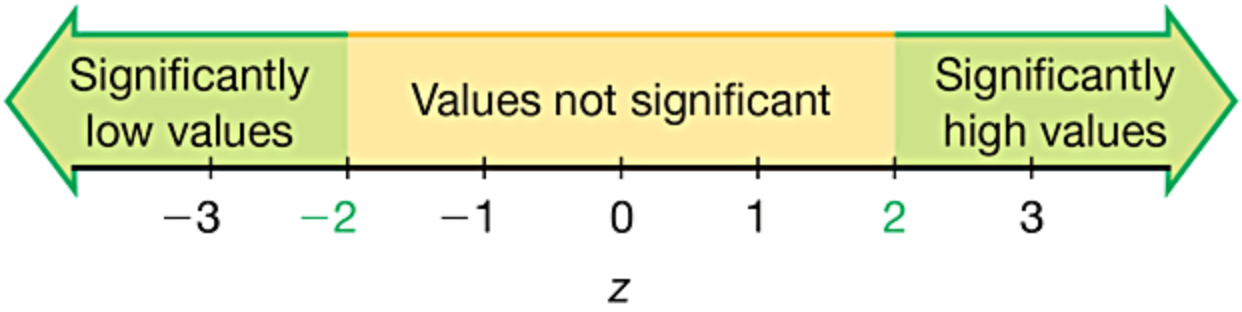
$$z = \frac{x - \bar{x}}{s} = \frac{5.7790\ g\ - 5.63930\ g}{0.06194\ g} = 2.26$$

Example: Comparing a Quarter's Weight and Adult Body Temperature
(3 of 3)

Interpretation

- The *z* scores show that the 99°F body temperature is 1.29 standard deviations above the mean, and the 5.7790 g weight of the quarter is 2.26 standard deviations above the mean.
- Because the weight of the quarter is farther above the mean, it is the more extreme value. A weight of 5.7790 g of a quarter is more extreme than a 99°F body temperature.

Using *z* Scores to Identify Significant Values



- Significantly low values: $z \leq -2$
- Significantly high values: $z \geq 2$
- Values not significant: $-2 < z < 2$

Example: Is an Earthquake Magnitude of 4.01 Significantly High? (1 of 2)

Among the earthquakes listed in Data Set 24 "Earthquakes," one of the stronger earthquakes had a magnitude of 4.01. The magnitudes are measured on the Richter scale, and only earthquakes of magnitude 1.00 or higher are included. The 600 magnitudes in the data set have a mean of 2.572 and a standard deviation of 0.651. For this data set, is the magnitude of 4.01 significantly high?

Example: Is an Earthquake Magnitude of 4.01 Significantly High? (2 of 2)

Solution

The magnitude of 4.01 is converted to a z score:

$$z = \frac{x - \bar{x}}{s} = \frac{4.01 - 2.572}{0.651} = 2.21$$

Interpretation

The magnitude of 4.01 converts to the z score of 2.21. Because the z score of 2.21 is greater than or equal to +2, that magnitude is significantly high.