Module 5

9.4 Backward Chaining

This algorithm work backward from the goal, chaining through rules to find known facts that support the proof. It is called with a list of goals containing the original query, and returns the set of all substitutions satisfying the query. The algorithm takes the first goal in the list and finds every clause in the knowledge base whose **head,** unifies with the goal. Each such clause creates a new recursive call in which **body,** of the clause is added to the goal stack . Remember that facts are clauses with a head but no body, so when a goal unifies with a known fact, no new sub goals are added to the stack and the goal is solved. The algorithm for backward chaining and proof tree for finding criminal (West) using backward chaining are given below.

```
function FOL-BC-ASK(KB, query) returns a generator of substitutions
  return FOL-BC-OR(KB, query, { })

generator FOL-BC-OR(KB, goal, θ) yields a substitution
  for each rule (lhs ⇒ rhs) in FETCH-RULES-FOR-GOAL(KB, goal) do
    (lhs, rhs) ← STANDARDIZE-VARIABLES((lhs, rhs))
    for each θ′ in FOL-BC-AND(KB, lhs, UNIFY(rhs, goal, θ)) do
      yield θ′

generator FOL-BC-AND(KB, goals, θ) yields a substitution
  if θ = failure then return
  else if LENGTH(goals) = 0 then yield θ
  else do
    first, rest ← FIRST(goals), REST(goals)
    for each θ′ in FOL-BC-OR(KB, SUBST(θ, first), θ) do
      for each θ″ in FOL-BC-AND(KB, rest, θ′) do
        yield θ″
```

Figure 9.6 A simple backward-chaining algorithm for first-order knowledge bases.

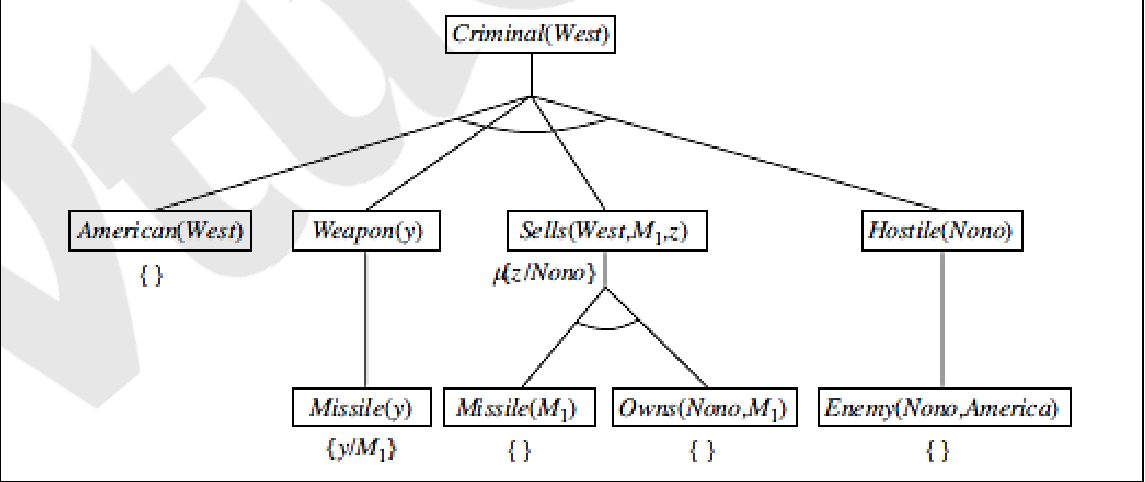


Figure 9.7 Proof tree constructed by backward chaining to prove that West is a criminal. The tree should be read depth first, left to right. To prove *Criminal*(*West*), we have to prove the four conjuncts below it. Some of these are in the knowledge base, and others require further backward chaining. Bindings for each successful unification are shown next to the corresponding subgoal. Note that once one subgoal in a conjunction succeeds, its substitution is applied to subsequent subgoals. Thus, by the time FOL-BC-ASK gets to the last conjunct, originally *Hostile*(z), z is already bound to *Nono*.

Logic programming:

- **Prolog** is by far the most widely used logic programming language.
- Prolog programs are sets of definite clauses written in a notation different from standard first-order logic.
- Prolog uses uppercase letters for variables and lowercase for constants.
- Clauses are written with the head preceding the body; " : -" is used for left implication, commas separate literals in the body, and a period marks the end of a sentence .

```
criminal(X) :- american(X), weapon(Y), sells(X,Y,Z), hostile(Z)
```

- Prolog includes "syntactic sugar" for list notation and arithmetic. Prolog program for append (X, Y, Z), which succeeds if list Z is the result of appending lists x and Y.

```
append([],Y,Y).
append([A|X],Y,[A|Z]) :- append(X,Y,Z)
```

- For example, we can ask the query append (A, B, [1, 2]): what two lists can be appended to give [1, 2]? We get back the solutions

```
A=[]      B=[1,2]
A=[1]     B=[2]
A=[1,2]   B=[]
```

- The execution of Prolog programs is done via depth-first backward chaining
- Prolog allows a form of negation called **negation as failure.** A negated goal not P is considered proved if the system fails to prove p. Thus, the sentence
- **Alive (X) : - not dead(X)** can be read as "Everyone is alive if not provably dead."
- Prolog has an equality operator, =, but it lacks the full power of logical equality. An equality goal succeeds if the two terms are *unifiable* and fails otherwise. So X+Y=2+3 succeeds with x bound to *2* and Y bound to 3, but Morningstar=evening star fails.
- The occur check is omitted from Prolog's unification algorithm.

Efficient implementation of logic programs:

- The execution of a Prolog program can happen in two modes: interpreted and compiled.
- Interpretation essentially amounts to running the FOL-BC-ASK algorithm, with the program as the knowledge base. These are designed to maximize speed.
- First, instead of constructing the list of all possible answers for each sub goal before continuing to the next, Prolog interpreters generate one answer and a "promise" to generate the rest when the current answer has been fully explored. This promise is called a **choice point.**FOL-BC-ASK spends a good deal of time in generating and composing substitutions when a path in search fails. Prolog will backup to previous choice point and unbind some variables. This is called ―TRAIL‖. So, new variable is bound by UNIFY-VAR and it is pushed on to trail.
- Prolog Compilers compile into an intermediate language i.e., Warren Abstract Machine or WAM named after David. H. D. Warren who is one of the implementers of first prolog compiler. So, WAM is an abstract instruction set that is suitable for prolog and can be either translated or interpreted into machine language.

Continuations are used to implement choice point'scontinuation as packaging up a procedure and a list of arguments that together define what should be done next whenever the current goal succeeds.

Parallelization can also provide substantial speedup. There are two principal sources of parallelism

1. The first, called **OR-parallelism,** comes from the possibility of a goal unifying with many different clauses in the knowledge base. Each gives rise to an independent branch in the search space that can lead to a potential solution, and all such branches can be solved in parallel.

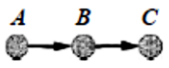
2. The second, called **AND-parallelism,** comes from the possibility of solving each conjunct in the body of an implication in parallel. AND-parallelism is more difficult to achieve, because solutions for the whole conjunction require consistent bindings for all the variables.

Redundant inference and infinite loops:

Consider the following logic program that decides if a path exists between two points on a directed graph.

```
path(X,Z) :- link(X,Z).
path(X,Z) :- path(X,Y), link(Y,Z)
```

A simple three-node graph, described by the facts link (a, b) and link (b, c)



It generates the query path (a, c)

Hence each node is connected to two random successors in the next layer.

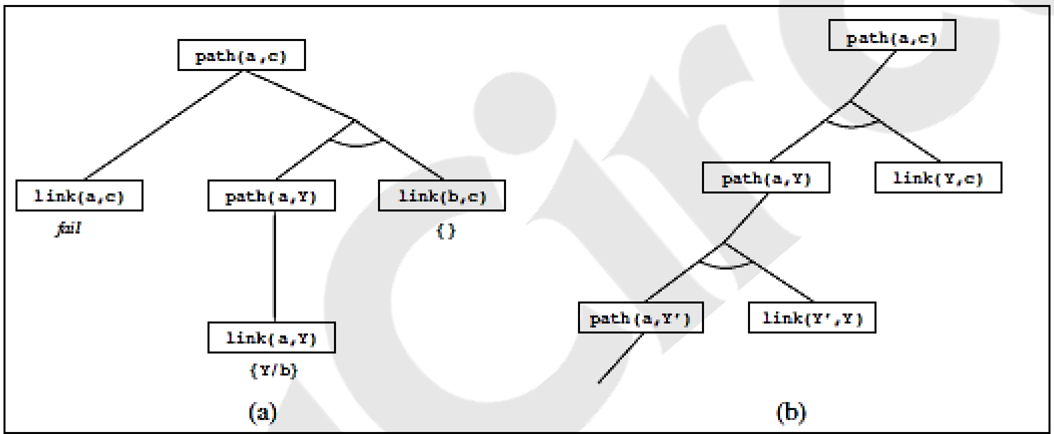


Figure 9.10 (a) Proof that a path exists from A to C. (b) Infinite proof tree generated when the clauses are in the "wrong" order.

Constraint logic programming:

The Constraint Satisfaction problem can be solved in prolog as same like backtracking algorithm. Because it works only for finite domain CSP's in prolog terms there must be finite number of solutions for any goal with unbound variables.

```
triangle(X,Y,Z) :-
    X>=0, Y>=0, Z>=0, X+Y>=Z, Y+Z>=X, X+Z>=Y.
```

- If we have a query, triangle (3, 4, and 5) works fine but the query like, triangle (3, 4, Z) no solution.
- The difficulty is variable in prolog can be in one of two states i.e., Unbound or bound.
- Binding a variable to a particular term can be viewed as an extreme form of constraint namely "equality".CLP allows variables to be constrained rather than bound.
- The solution to triangle (3, 4, Z) is Constraint 7>=Z>=1.

9.5 RESOLUTION

As in the propositional case, first-order resolution requires that sentences be in **conjunctive normal form** (CNF) that is, a conjunction of clauses, where each clause is a disjunction of literals.

Literals can contain variables, which are assumed to be universally quantified. Every sentence of first-order logic can be converted into an inferentially equivalent CNF sentence. We will illustrate the procedure by translating the sentence

"Everyone who loves all animals is loved by someone," or

$$\forall x\ [\forall y\ Animal(y) \Rightarrow Loves(x,y)] \Rightarrow [\exists y\ Loves(y,x)]$$

The steps are as follows:

- Eliminate implications:

$$ti\ x\ [\neg\forall y\ \neg Animal(y) \lor Loves(x,y)]\ v\ [\exists y\ Loves(y,x)]$$

- Move Negation inwards: In addition to the usual rules for negated connectives, we need rules for negated quantifiers. Thus, we have

$$\neg\forall x\ p \quad \text{becomes} \quad 3\,x\ \neg p$$
$$\neg\exists x\ p \quad \text{becomes} \quad \forall x\ \neg p$$

Our sentence goes through the following transformations.

$$\forall x\ [\exists y\ \neg(\neg Animal(y) \lor Loves(x,y))] \lor [\exists y\ Loves(y,x)]\,.$$
$$\forall x\ [\exists y\ \neg\neg Animal(y) \land \neg Loves(x,y)] \lor [\exists y\ Loves(y,x)].$$
$$\forall x\ [\exists y\ Animal(y) \land \neg Loves(x,y)]\ v\ [\exists y\ Loves(y,x)]\,.$$

Standardize variables: For sentences like $(\forall x\ \ P(x))\ v\ (3x\ \ Q(x))$

use the same variable name twice, change the name of one of the variables. This avoids confusion later when we drop the quantifiers. Thus, we have

$$\forall x\ [\exists y\ Animal(y) \land \neg Loves(x,y)] \lor [\exists z\ Loves(z,x)]$$

Skolemize: Skolemization is the process of removing existential quantifiers by elimination. Translate *3 x P(x)* into *P(A),* where *A* is a new constant. If we apply this rule to our sample sentence, however, we obtain

$$\forall x\ [Animal(A) \land \neg Loves(x,A)] \lor Loves(B,x)$$

Which has the wrong meaning entirely: it says that everyone either fails to love a particular animal *A* or is loved by some particular entity *B.* In fact, our original sentence allows each person to fail to love a different animal or to be loved by a different person.

Thus, we want the Skolem entities to depend on *x:*

$$ti\ x\ [Animal(F(x)) \land \neg Loves(x,F(x))]\ v\ Loves(G(x),x)$$

Here *F* and *G* are Skolem functions. The general rule is that the arguments of the Skolem function are all the universally quantified variables in whose scope the existential quantifier appears.

- Drop universal quantifiers: At this point, all remaining variables must be universally quantified. Moreover, the sentence is equivalent to one in which all the universal quantifiers have been moved to the left. We can therefore drop the universal quantifiers

$$[Animal(F(x))\ A\ \neg Loves(x,F(x))] \lor Loves(G(x),x)$$

- Distribute V over A

$$[Animal(F(x)) \lor Loves(G(x),x)] \land [\neg Loves(x,F(x)) \lor Loves(G(x),x)].$$

This is the CNF form of given sentence.

The resolution inference rule:

The resolution rule for first-order clauses is simply a lifted version of the propositional resolution rule. Propositional literals are complementary if one is the negation of the other; first-order literals are complementary if one ***unifies with*** the negation of the other. Thus we have

$$\frac{\ell_1 \lor \cdots \lor \ell_k, \qquad m_1 \lor \cdots \lor m_n}{\textsc{Subst}(\theta, \ell_1 \lor \cdots \lor \ell_{i-1} \lor \ell_{i+1} \lor \cdots \lor \ell_k \lor m_1 \lor \cdots \lor m_{j-1} \lor m_{j+1} \lor \cdots \lor m_n)}$$

Where UNIFY (li, m j*)* == θ.

For example, we can resolve the two clauses

$$[Animal(F(x)) \lor Loves(G(x),x)] \quad \text{and} \quad [\neg Loves(u,v) \lor \neg Kills(u,v)]$$

By eliminating the complementary literals *Loves (G(x), x)* and ¬*Loves (u, v),* with unifier

θ = *{u/G(x), v/x),* to produce the resolvent clause

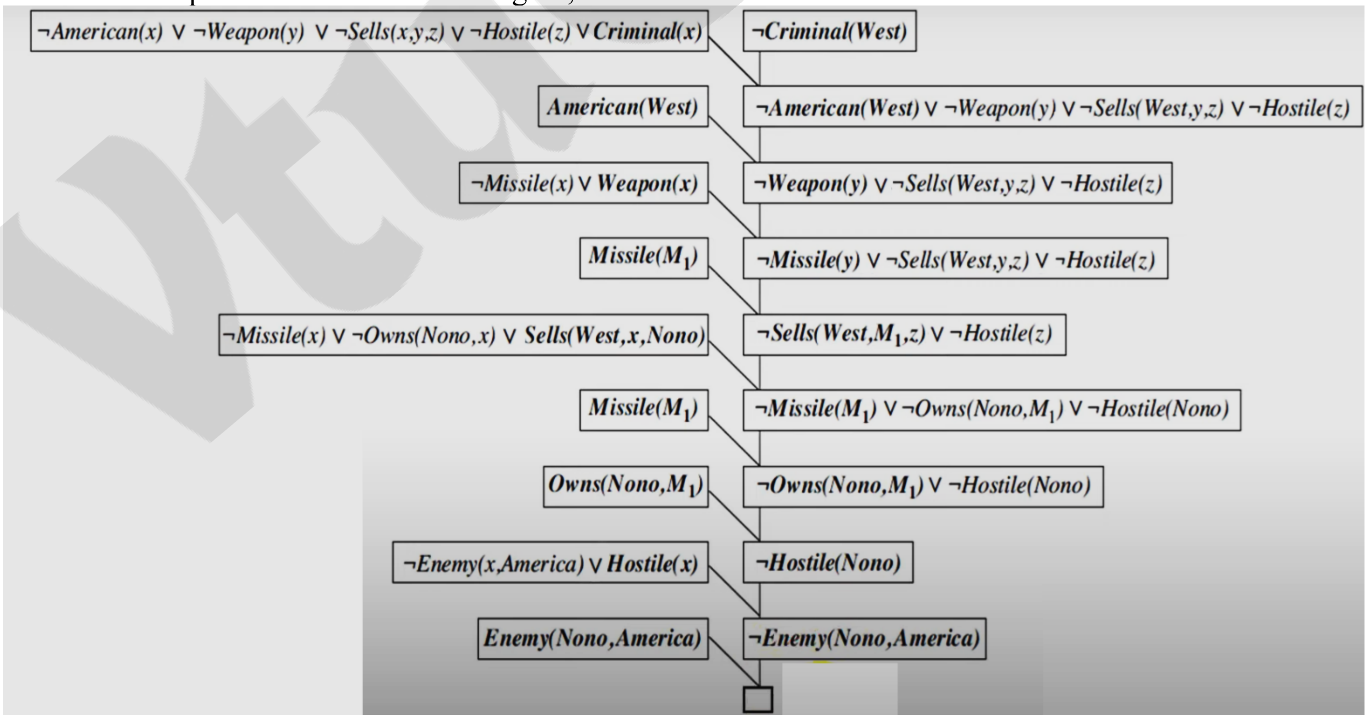
$$[Animal(F(x)) \lor \neg Kills(G(x),x)] .$$

Example proofs:

Resolution proves that KB /= *a* by proving *KB* A *la* unsatisfiable, i.e., by deriving the empty clause. The sentences in CNF are

$$\neg American(x) \lor \neg Weapon(y) \lor \neg Sells(x,y,z) \lor \neg Hostile(z) \lor Criminal(x)$$
$$\neg Missile(x) \lor \neg Owns(Nono,x) \lor Sells(West,x,Nono) .$$
$$\neg Enemy(x, America) \lor Hostile(x) .$$
$$\neg Missile(x) \lor Weapon(x) .$$
$$Owns(Nono, M_1) . \qquad Missile(M_1) .$$
$$American(West) . \qquad Enemy(Nono, America) .$$

The resolution proof is shown in below figure;


¬American(x) ∨ ¬Weapon(y) ∨ ¬Sells(x,y,z) ∨ ¬Hostile(z) ∨ Criminal(x)
¬Criminal(West)
American(West)
¬American(West) ∨ ¬Weapon(y) ∨ ¬Sells(West,y,z) ∨ ¬Hostile(z)
¬Missile(x) ∨ Weapon(x)
¬Weapon(y) ∨ ¬Sells(West,y,z) ∨ ¬Hostile(z)
Missile(M1)
¬Missile(y) ∨ ¬Sells(West,y,z) ∨ ¬Hostile(z)
¬Missile(x) ∨ ¬Owns(Nono,x) ∨ Sells(West,x,Nono)
¬Sells(West,M1,z) ∨ ¬Hostile(z)
Missile(M1)
¬Missile(M1) ∨ ¬Owns(Nono,M1) ∨ ¬Hostile(Nono)
Owns(Nono,M1)
¬Owns(Nono,M1) ∨ ¬Hostile(Nono)
¬Enemy(x,America) ∨ Hostile(x)
¬Hostile(Nono)
Enemy(Nono,America)
¬Enemy(Nono,America)

Notice the structure: single "spine" beginning with the goal clause, resolving against clauses from the knowledge base until the empty clause is generated. Backward chaining is really just a special case of resolution with a particular control strategy to decide which resolution to perform next.

10.1 CLASSICAL PLANNING

Planning Classical Planning: AI as the study of rational action, which means that planning—devising a plan of action to achieve one's goals—is a critical part of AI. We have seen two examples of planning agents so far the search-based problem-solving agent.

DEFINITION OF CLASSICAL PLANNING: The problem-solving agent can find sequences of actions that result in a goal state. But it deals with atomic representations of states and thus needs good domain-specific heuristics to perform well. The hybrid propositional logical agent can find plans without domain-specific heuristics because it uses domain-independent heuristics based on the logical structure of the problem but it relies on ground (variable-free) propositional inference, which means that it may be swamped when there are many actions and states. For example, in the world, the simple action of moving astep forward had to be repeated for all four agent orientations, *T* time steps, and *n*2 current locations.

In response to this, planning researchers have settled on a **factored representation**— one in which a state of the world is represented by a collection of variables. We use a language called **PDDL**, the Planning Domain Definition Language that allows us to express all 4*Tn*2 actions with one action schema. There have been several versions of PDDL.we select a simple version and alter its syntax to be consistent with the rest of the book. We now show how PDDL describes the four things we need to define a search problem: the initial state, the actions that are available in a state, the result of applying an action, and the goal test.

Each state is represented as a conjunction of flaunts that are ground, functionless atoms. For example, Poor ∧ Unknown might represent the state of a hapless agent, and a state in a package delivery problem might be At(Truck 1, Melbourne) ∧ At(Truck 2, Sydney). Database semantics is used: the closed-world assumption means that any flaunts that are not mentioned are false, and the unique names assumption means that Truck 1 and Truck 2 are distinct.

A set of ground (variable-free) actions can be represented by a single action schema. The schema is a lifted representation—it lifts the level of reasoning from propositional logic to a restricted subset of first-order logic. For example, here is an action schema for flying a plane from one location to another:

Action(Fly (p, from, to),

PRECOND:At(p, from) ∧ Plane(p) ∧ Airport (from) ∧ Airport (to)

EFFECT:¬At(p, from) ∧ At(p, to))

The schema consists of the action name, a list of all the variables used in the schema, a precondition and an effect.

A set of action schemas serves as a definition of a planning domain. A specific problem within the domain is defined with the addition of an initial state and a goal.

state is a conjunction of ground atoms. (As with all states, the closed-world assumption is used, which means that any atoms that are not mentioned are false.) The goal is just like a precondition: a conjunction of literals (positive or negative) that may contain variables, such as At(p, SFO) ∧ Plane(p). Any variables are treated as existentially quantified, so this goal is to have any plane at SFO. The problem is solved when we can find a sequence of actions that end in a states that entails the goal.

Example: Air cargo transport

An air cargo transport problem involving loading and unloading cargo and flying it from place to place. The problem can be defined with three actions: Load , Unload , and Fly . The actions affect two predicates: In(c, p) means that cargo c is inside plane p, and At(x, a) means that object x (either plane or cargo) is at airport a. Note that some care must be taken to make sure the At predicates are maintained properly. When a plane flies from one airport to another, all the cargo inside the plane goes with it. In first-order logic it would be easy to quantify over all objects that are inside the plane. But basic PDDL does not have a universal quantifier, so we need a different solution. The approach we use is to say that a piece of cargo ceases to be At anywhere when it is In a plane; the cargo only becomes At the new airport when it is unloaded. So At really means "available for use at a given location."

The complexity of classical planning :

We consider the theoretical complexity of planning and distinguish two decision problems. PlanSAT is the question of whether there exists any plan that solves a planning problem. Bounded PlanSAT asks whether there is a solution of length k or less; this can be used to find an optimal plan.

The first result is that both decision problems are decidable for classical planning. The proof follows from the fact that the number of states is finite. But if we add function symbols to the language, then the number of states becomes infinite, and PlanSAT becomes only semi decidable: an algorithm exists that will terminate with the correct answer for any solvable problem, but may not terminate on unsolvable problems. The Bounded PlanSAT problem remains decidable even in the presence of function symbols.

Both PlanSAT and Bounded PlanSAT are in the complexity class PSPACE, a class that is larger (and hence more difficult) than NP and refers to problems that can be solved by a deterministic Turing machine with a polynomial amount of space. Even if we make some rather severe restrictions, the problems remain quite difficult.

10.2 ALGORITHMS FOR PLANNING AS STATE-SPACE SEARCH

Forward (progression) state-space search:

Now that we have shown how a planning problem maps into a search problem, we can solve planning problems with any of the heuristic search algorithms from Chapter 3 or a local search algorithm from Chapter 4 (provided we keep track of the actions used to reach the goal). From the earliest days of planning research (around 1961) until around 1998 it was assumed that forward state-space search was too inefficient to be practical. It is not hard to come up with reasons why .

First, forward search is prone to exploring irrelevant actions. Consider the noble task of buying a copy of AI: A Modern Approach from an online bookseller. Suppose there is an action schema Buy(isbn) with effect Own(isbn). ISBNs are 10 digits, so this action schema represents 10 billion ground actions. An uninformed forward-search algorithm would have to start enumerating these 10 billion actions to find one that leads to the goal.

Second, planning problems often have large state spaces. Consider an air cargo problem with 10 airports, where each airport has 5 planes and 20 pieces of cargo. The goal is to move all the cargo at airport A to airport B. There is a simple solution to the problem: load the 20 pieces of cargo into one of the planes at A, fly the plane to B, and unload the cargo. Finding the solution can be difficult because the average branching factor is huge: each of the 50 planes can fly to 9 other airports, and each of the 200 packages can be either unloaded (if it is loaded) or loaded into any plane at its airport (if it is unloaded). So in any state there is a minimum of 450 actions (when all the packages are at airports with no planes) and a maximum of 10,450 (when all packages and planes are at the same airport). On average, let's say there are about 2000 possible actions per state, so the search graph up to the depth of the obvious solution has about 2000 nodes.

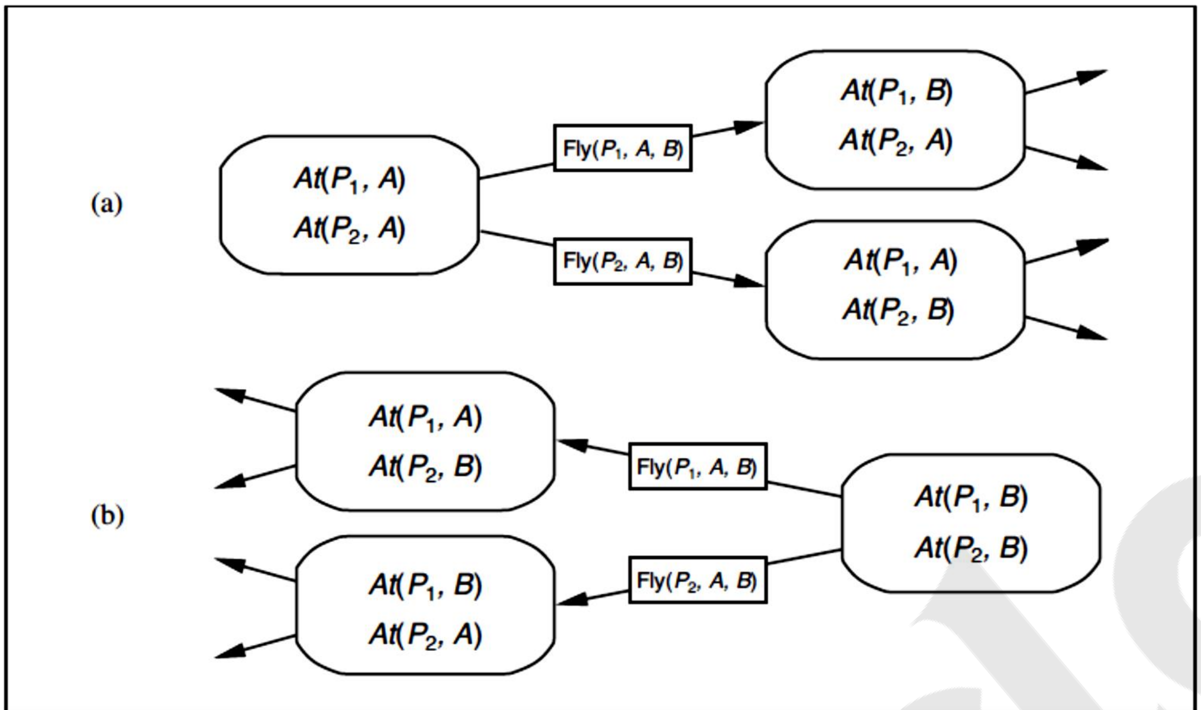


Figure 10.5 Two approaches to searching for a plan. (a) Forward (progression) search through the space of states, starting in the initial state and using the problem's actions to search forward for a member of the set of goal states. (b) Backward (regression) search through sets of relevant states, starting at the set of states representing the goal and using the inverse of the actions to search backward for the initial state.

Backward (regression) relevant-states search:

In regression search we start at the goal and apply the actions backward until we find a sequence of steps that reaches the initial state. It is called relevant-states search because we only consider actions that are relevant to the goal (or current state). As in belief-state search (Section 4.4), there is a set of relevant states to consider at each step, not just a single state.

We start with the goal, which is a conjunction of literals forming a description of a set of states—for example, the goal ¬Poor ∧ Famous describes those states in which Poor is false, Famous is true, and any other fluent can have any value. If there are n ground flaunts in a domain, then there are 2n ground states (each fluent can be true or false), but 3n descriptions of sets of goal states (each fluent can be positive, negative, or not mentioned).

In general, backward search works only when we know how to regress from a state description to the predecessor state description. For example, it is hard to search backwards for a solution to the n-queens.

problem because there is no easy way to describe the states that are one move away from the goal. Happily, the PDDL representation was designed to make it easy to regress actions—if a domain can be expressed in PDDL, then we can do regression search on it.

The final issue is deciding which actions are candidates to regress over. In the forward direction we chose actions that were applicable—those actions that could be the next step in the plan. In backward search we want actions that are relevant—those actions that could be the last step in a plan leading up to the current goal state.

Heuristics for planning:

Neither forward nor backward search is efficient without a good heuristic function. Recall from Chapter 3 that a heuristic function h(s) estimates the distance from a state s to the goal and that if we can derive an admissible heuristic for this distance—one that does not overestimate—then we can use A∗ search to find optimal solutions. An admissible heuristic can be derived by defining a relaxed problem that is easier to solve. The exact cost of a solution to this easier problem then becomes the heuristic for the original problem.

By definition, there is no way to analyze an atomic state, and thus it it requires some ingenuity by a human analyst to define good domain-specific heuristics for search problems with atomic states. Planning uses a factored representation for states and action schemas. That makes it possible to define good domain-independent heuristics and for programs to automatically apply a good domain-independent heuristic for a given problem.

10.3 Planning Graphs:

All of the heuristics we have suggested can suffer from inaccuracies. This section shows how a special data structure called a planning graph can be used to give better heuristic estimates. These heuristics can be applied to any of the search techniques we have seen so far. Alternatively, we can search for a solution over the space formed by the planning graph, using an algorithm called GRAPHPLAN.

$Init(Have(Cake))$
$Goal(Have(Cake) \wedge Eaten(Cake))$
$Action(Eat(Cake)$
  PRECOND: $Have(Cake)$
  EFFECT: $\neg Have(Cake) \wedge Eaten(Cake))$
$Action(Bake(Cake)$
  PRECOND: $\neg Have(Cake)$
  EFFECT: $Have(Cake))$

Figure 10.7 The “have cake and eat cake too” problem.

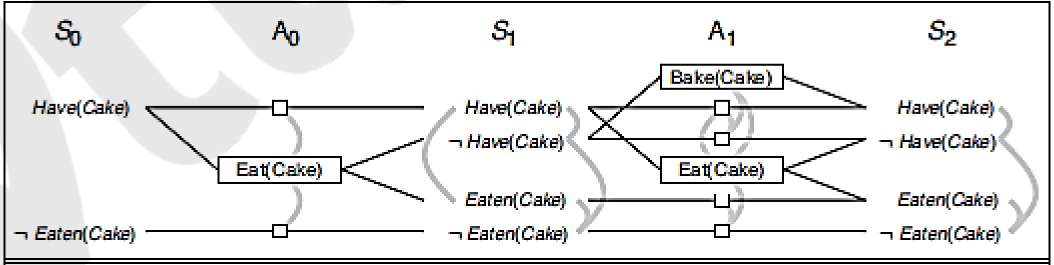


Figure 10.8 The planning graph for the “have cake and eat cake too” problem up to level S_2. Rectangles indicate actions (small squares indicate persistence actions), and straight lines indicate preconditions and effects. Mutex links are shown as curved gray lines. Not all mutex links are shown, because the graph would be too cluttered. In general, if two literals are mutex at S_i, then the persistence actions for those literals will be mutex at A_i and we need not draw that mutex link.

Figure 10.7 shows a simple planning problem, and Figure 10.8 shows its planning graph. Each action at level Ai is connected to its preconditions at Si and its effects at Si+1. So a literal appears because an action caused it, but we also want to say that a literal can persist if no action negates it. This is represented by a persistence action (sometimes called a no-op). For every literal C, we add to the problem a persistence action with precondition C and effect C. Level A0 in Figure 10.8 shows one "real" action, Eat (Cake), along with two persistence actions drawn as small square boxes.

```
function GRAPHPLAN(problem) returns solution or failure
   graph ← INITIAL-PLANNING-GRAPH(problem)
   goals ← CONJUNCTS(problem.GOAL)
   nogoods ← an empty hash table
   for tl = 0 to ∞ do
      if goals all non-mutex in $S_t$ of graph then
         solution ← EXTRACT-SOLUTION(graph, goals, NUMLEVELS(graph), nogoods)
         if solution ≠ failure then return solution
      if graph and nogoods have both leveled off then return failure
      graph ← EXPAND-GRAPH(graph, problem)
```

Figure 10.9 The GRAPHPLAN algorithm. GRAPHPLAN calls EXPAND-GRAPH to add a level until either a solution is found by EXTRACT-SOLUTION, or no solution is possible.

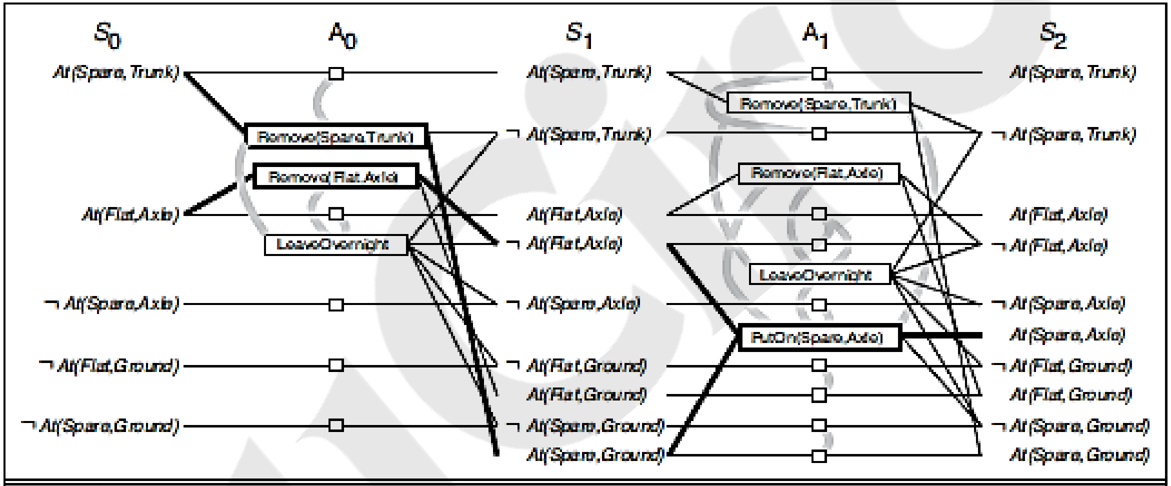


Figure 10.10 The planning graph for the spare tire problem after expansion to level S_2. Mutex links are shown as gray lines. Not all links are shown, because the graph would be too cluttered if we showed them all. The solution is indicated by bold lines and outlines.

- *Interference:* $Remove(Flat, Axle)$ is mutex with $LeaveOvernight$ because one has the precondition $At(Flat, Axle)$ and the other has its negation as an effect.
- *Competing needs:* $PutOn(Spare, Axle)$ is mutex with $Remove(Flat, Axle)$ because one has $At(Flat, Axle)$ as a precondition and the other has its negation.
- *Inconsistent support:* $At(Spare, Axle)$ is mutex with $At(Flat, Axle)$ in S_2 because the only way of achieving $At(Spare, Axle)$ is by $PutOn(Spare, Axle)$, and that is mutex with the persistence action that is the only way of achieving $At(Flat, Axle)$. Thus, the mutex relations detect the immediate conflict that arises from trying to put two objects in the same place at the same time.

A planning problem asks if we can reach a goal state from the initial state. Suppose we are given a tree of all possible actions from the initial state to successor states, and their successors, and so on. If we indexed this tree appropriately, we could answer the planning question "can we reach state G from state S0" immediately, just by looking it up. Of course, the tree is of exponential size, so this approach is impractical. A planning graph is polynomial- size approximation to this tree that can be constructed quickly. The planning graph can't answer definitively whether G is reachable from S0, but it can estimate how many steps it takes to reach G. The estimate is always correct when it reports the goal is not reachable, and it never overestimates the number of steps, so it is an admissible heuristic.

A planning graph is a directed graph organized into levels: first a level S0 for the initial state, consisting of nodes representing each fluent that holds in S0; then a level A0 consisting of nodes for each ground action that might be applicable in S0; then alternating levels Si followed by Ai; until we reach a termination condition (to be discussed later).

Roughly speaking, Si contains all the literals that could hold at time i, depending on the actions executed at preceding time steps. If it is possible that either P or ¬P could hold, then both will be represented in Si. Also roughly speaking, Ai contains all the actions that could have their preconditions satisfied at time i. We say "roughly speaking" because the planning graph records only a restricted subset of the possible negative interactions among actions; therefore, a literal might show up at level Sj when actually it could not be true until a later level, if at all. (A literal will never show up too late.) Despite the possible error, the level j at which a literal first appears is a good estimate of how difficult it is to achieve the literal from the initial state.

We now define mutex links for both actions and literals. A mutex relation holds between two actions at a given level if any of the following three conditions holds:

• Inconsistent effects: one action negates an effect of the other. For example, Eat(Cake) and the persistence of Have(Cake) have inconsistent effects because they disagree on the effect Have(Cake).

• Interference: one of the effects of one action is the negation of a precondition of the other. For example Eat(Cake) interferes with the persistence of Have(Cake) by its precondition.

• Competing needs: one of the preconditions of one action is mutually exclusive with a precondition of the other. For example, Bake(Cake) and Eat(Cake) are mutex because they compete on the value of the Have(Cake) precondition.

A mutex relation holds between two literals at the same level if one is the negation of the other or if each possible pair of actions that could achieve the two literals is mutually exclusive. This condition is called inconsistent support. For example, Have(Cake) and Eaten(Cake) are mutex in S1 because the only way of achieving Have(Cake), the persistence action, is mutex with the only way of achieving Eaten(Cake), namely Eat(Cake). In S2 the two literals are not mutex, because there are new ways of achieving them, such as Bake(Cake) and the persistence of Eaten(Cake), that are not mutex. other Classical Planning Approaches:

Currently the most popular and effective approaches to fully automated planning are:

• Translating to a Boolean satisfiability (SAT) problem

• Forward state-space search with carefully crafted heuristics

• Search using a planning graph (Section 10.3)

These three approaches are not the only ones tried in the 40-year history of automated planning. Figure 10.11 shows some of the top systems in the International Planning Competitions, which have been held every even year since 1998. In this section we first describe the translation to a satisfiability problem and then describe three other influential approaches: planning as first-order logical deduction; as constraint satisfaction; and as plan refinement.

Classical planning as Boolean satisfiability :

we saw how SATPLAN solves planning problems that are expressed in propositional logic. Here we show how to translate a PDDL description into a form that can be processed by SATPLAN. The translation is a series of straightforward steps:

• Proposition Alize the actions: replace each action schema with a set of ground actions formed by substituting constants for each of the variables. These ground actions are not part of the translation, but will be used in subsequent steps.

• Define the initial state: assert F 0 for every fluent F in the problem's initial state, and ¬F for every fluent not mentioned in the initial state.

• Proposition Alize the goal: for every variable in the goal, replace the literals that contain the variable with a disjunction over constants. For example, the goal of having block A on another block, On(A, x) ∧ Block (x) in a world with objects A, B and C, would be replaced by the goal

(On(A, A) ∧ Block (A)) ∨ (On(A, B) ∧ Block (B)) ∨ (On(A, C) ∧ Block (C)) .

• Add successor-state axioms: For each fluent F , add an axiom of the form

F t+1 ⇔ ActionCausesF t ∨ (F t ∧ ¬ActionCausesNotF t) ,

where Action CausesF is a disjunction of all the ground actions that have F in their add list, and Action CausesNotF is a disjunction of all the ground actions that have F in their delete list.

Analysis of Planning approaches:

Planning combines the two major areas of AI we have covered so far: search and logic. A planner can be seen either as a program that searches for a solution or as one that (constructively) proves the existence of a solution. The cross-fertilization of ideas from the two areas has led both to improvements in performance amounting to several orders of magnitude in the last decade and to an increased use of planners in industrial applications. Unfortunately, we do not yet have a clear understanding of which techniques work best on which kinds of problems. Quite possibly, new techniques will emerge that dominate existing methods.

Planning is foremost an exercise in controlling combinatorial explosion. If there are n propositions in a domain, then there are 2n states. As we have seen, planning is PSPACE-hard. Against such pessimism, the identification of independent sub problems can be a powerful weapon. In the best case—full decomposability of the problem—we get an exponential speedup.

Decomposability is destroyed, however, by negative interactions between actions. GRAPHPLAN records mutexes to point out where the difficult interactions are. SATPLAN rep- resents a similar range of mutex relations, but does so by using the general CNF form rather than a specific data structure. Forward search addresses the problem heuristically by trying to find patterns (subsets of propositions) that cover the independent sub problems. Since this approach is heuristic, it can work even when the sub problems are not completely independent.

Sometimes it is possible to solve a problem efficiently by recognizing that negative interactions can be ruled out. We say that a problem has serializable sub goals if there exists an order of sub goals such that the planner can achieve them in that order without having to undo any of the previously achieved sub goals. For example, in the blocks world, if the goal is to build a tower (e.g., A on B, which in turn is on C, which in turn is on the Table, as in Figure 10.4 on page 371), then the sub goals are serializable bottom to top: if we first achieve C on Table, we will never have to undo it while we are achieving the other sub goals. Planners such as GRAPHPLAN, SATPLAN, and FF have moved the field of planning forward, by raising the level of performance of planning systems.

Planning and Acting in the Real World:

This allows human experts to communicate to the planner what they know about how to solve the problem. Hierarchy also lends itself to efficient plan construction because the planner can solve a problem at an abstract level before delving into details. Presents agent architectures that can handle uncertain environments and interleave deliberation with execution, and gives some examples of real-world systems.

Time, Schedules, and Resources:

The classical planning representation talks about what to do, and in what order, but the representation cannot talk about time: how long an action takes and when it occurs. For example, the planners of Chapter 10 could produce a schedule for an airline that says which planes are assigned to which flights, but we really need to know departure and arrival times as well. This is the subject matter of scheduling. The real world also imposes many resource constraints; for example, an airline has a limited number of staff—and staff who are on one flight cannot be on another at the same time. This section covers methods for representing and solving planning problems that include temporal and resource constraints.

The approach we take in this section is “plan first, schedule later”: that is, we divide the overall problem into a planning phase in which actions are selected, with some ordering constraints, to meet the goals of the problem, and a later scheduling phase, in which temporal information is added to the plan to ensure that it meets resource and deadline constraints.

$Jobs(\{AddEngine1 \prec AddWheels1 \prec Inspect1\},$
$\quad \{AddEngine2 \prec AddWheels2 \prec Inspect2\})$

$Resources(EngineHoists(1), WheelStations(1), Inspectors(2), LugNuts(500))$

$Action(AddEngine1, \text{DURATION}{:}30,$
$\quad \text{USE}{:}EngineHoists(1))$
$Action(AddEngine2, \text{DURATION}{:}60,$
$\quad \text{USE}{:}EngineHoists(1))$
$Action(AddWheels1, \text{DURATION}{:}30,$
$\quad \text{CONSUME}{:}LugNuts(20), \text{USE}{:} WheelStations(1))$
$Action(AddWheels2, \text{DURATION}{:}15,$
$\quad \text{CONSUME}{:}LugNuts(20), \text{USE}{:} WheelStations(1))$
$Action(Inspect_i, \text{DURATION}{:}10,$
$\quad \text{USE}{:}Inspectors(1))$

Figure 11.1 A job-shop scheduling problem for assembling two cars, with resource constraints. The notation $A \prec B$ means that action A must precede action B.

This approach is common in real-world manufacturing and logistical settings, where the planning phase is often performed by human experts. The automated methods of Chapter 10 can also be used for the planning phase, provided that they produce plans with just the minimal ordering constraints required for correctness. GRAPHPLAN (Section 10.3), SATPLAN (Section 10.4.1), and partial-order planners (Section 10.4.4) can do this; search-based methods (Section 10.2) produce totally ordered plans, but these can easily be converted to plans with minimal ordering constraints.

Termination of GRAPHPLAN
So far, we have skated over the question of termination. Here we show that GRAPHPLAN will in fact terminate and return failure when there is no solution. The first thing to understand is why we can't stop expanding the graph as soon as it has leveled off. Consider an air cargo domain with one plane and n pieces of cargo at airport A, all of which have airport B as their destination. In this version of the problem, only one piece of cargo can fit in the plane at a time. The graph will level off at level 4, reflecting the fact that for any single piece of cargo, we can load it, fly it, and unload it at the destination in three steps. But that does not mean that a solution can be extracted from the graph at level 4; in fact a solution will require 4n − 1 steps: for each piece of cargo we load, fly, and unload, and for all but the last piece we need to fly back to airport A to get the next piece.
How long do we have to keep expanding after the graph has leveled off? If the function EXTRACT-SOLUTION fails to find a solution, then there must have been at least one set of goals that were not achievable and were marked as a no-good. So if it is possible that there might be fewer no-goods in the next level, then we should continue. As soon as the graph itself and the no-goods have both leveled off, with no solution found, we can terminate with failure because there is no possibility of a subsequent change that could add a solution.

The properties are as follows:

- *Literals increase monotonically:* Once a literal appears at a given level, it will appear at all subsequent levels. This is because of the persistence actions; once a literal shows up, persistence actions cause it to stay forever.
- *Actions increase monotonically:* Once an action appears at a given level, it will appear at all subsequent levels. This is a consequence of the monotonic increase of literals; if the preconditions of an action appear at one level, they will appear at subsequent levels, and thus so will the action.
- *Mutexes decrease monotonically:* If two actions are mutex at a given level A_i, then they will also be mutex for all *previous* levels at which they both appear. The same holds for mutexes between literals. It might not always appear that way in the figures, because the figures have a simplification: they display neither literals that cannot hold at level S_i nor actions that cannot be executed at level A_i. We can see that "mutexes decrease monotonically" is true if you consider that these invisible literals and actions are mutex with everything.

  The proof can be handled by cases: if actions A and B are mutex at level A_i, it must be because of one of the three types of mutex. The first two, inconsistent effects and interference, are properties of the actions themselves, so if the actions are mutex at A_i, they will be mutex at every level. The third case, competing needs, depends on conditions at level S_i: that level must contain a precondition of A that is mutex with a precondition of B. Now, these two preconditions can be mutex if they are negations of each other (in which case they would be mutex in every level) or if all actions for achieving one are mutex with all actions for achieving the other. But we already know that the available actions are increasing monotonically, so, by induction, the mutexes must be decreasing.
- *No-goods decrease monotonically:* If a set of goals is not achievable at a given level, then they are not achievable in any *previous* level. The proof is by contradiction: if they were achievable at some previous level, then we could just add persistence actions to make them achievable at a subsequent level.

Because the actions and literals increase monotonically and because there are only a finite number of actions and literals, there must come a level that has the same number of actions and literals as the previous level. Because mutexes and no-goods decrease, and because there can never be fewer than zero mutexes or no-goods, there must come a level that has the same number of mutexes and no-goods as the previous level. Once a graph has reached this state, then if one of the goals is missing or is mutex with another goal, then we can stop the GRAPHPLAN algorithm and return failure. That concludes a sketch of the proof; for more details see Ghallab *et al.* (2004).