Yohan Lee Lab 5

Examples

Simpon's method

In[214]:=
```
f[x_] := 1 + 0 * x;
a = 0;
b = 1;
n = 10;
```

In[218]:=
```
width := (b - a) / n
```

In[219]:=
```
width / 3 *
 Dot[Table[f[a + i * width], {i, 0, n}], Flatten[{1, 3 + Table[(-1) ^i, {i, 0, n - 2}], 1}]]
```

Out[219]= 1

Midpoint method

In[8]:=
```
f[x_] := 1 + 0 * x;
a = 0;
b = 1;
n = 10;
width := (b - a) / n;
```

In[13]:=
```
width * Sum[f[a + (i + 1 / 2) * width], {i, 0, n - 1}]
```

Out[13]= 1

Trapezoid method

In[14]:=
```
f[x_] := 1 + 0 * x;
a = 0;
b = 1;
n = 10;
width := (b - a) / n;
```

In[19]:=
```
width * Sum[(f[a + i * width] + f[a + (i + 1) * width]) / 2, {i, 0, n - 1}]
```

Out[19]= 1

Question 1

In[278]:=
```
Clear[f];
a = 0;
b = 1;
n = 1;
width := (b - a) / n;
```

Compare the Midpoint and Trapezoid methods for integrals of x^p using only one piece (N=1).

Midpoint

In[283]:=
```
Clear[f];
f[x_] := 1 + 0 * x;
width * Sum[f[a + (i + 1 / 2) * width], {i, 0, n - 1}]
```
Out[285]= 1

In[233]:=
```
Integrate[f[x], {x, a, b}]
```
Out[233]= 1

In[286]:=
```
Clear[f];
f[x_] := x;
width * Sum[f[a + (i + 1 / 2) * width], {i, 0, n - 1}]
```
Out[288]= $\frac{1}{2}$

In[242]:=
```
Integrate[f[x], {x, a, b}]
```
Out[242]= $\frac{1}{2}$

In[289]:=
```
Clear[f];
f[x_] := x^2;
width * Sum[f[a + (i + 1 / 2) * width], {i, 0, n - 1}]
```
Out[291]= $\frac{1}{4}$

In[236]:=
```
Integrate[f[x], {x, a, b}]
```
Out[236]= $\frac{1}{3}$

Trapezoid

In[292]:=
```
Clear[f];
f[x_] := 1 + 0 * x;
width * Sum[(f[a + i * width] + f[a + (i + 1) * width]) / 2, {i, 0, n - 1}]
```
Out[294]= 1

In[295]:=
```
Clear[f];
f[x_] := x;
width * Sum[(f[a + i * width] + f[a + (i + 1) * width]) / 2, {i, 0, n - 1}]
```
Out[297]= $\frac{1}{2}$

In[298]:=
```
Clear[f];
f[x_] := x^2;
width * Sum[(f[a + i * width] + f[a + (i + 1) * width]) / 2, {i, 0, n - 1}]
```
Out[300]= $\frac{1}{2}$

Midpoint method and Trapezoid method are both exact for order 1. In other words they both diverge from exact for p>1.

Question 2

In[42]:=
```
Clear[f];
f[x_] := Exp[x] * Cos[2 * Pi * x];
Integrate[f[x], {x, 0, 5}] // N
```

Out[44]= 3.64177

Simpson's with n=52

In[204]:=
```
a = 0;
b = 5;
n = 52;
width := (b - a) / n;
width / 3 * Dot[Table[f[a + i * width], {i, 0, n}],
   Flatten[{1, 3 + Table[(-1) ^i, {i, 0, n - 2}], 1}]] // N
```

Out[208]= 3.63295

Trapezoid with n=180

In[184]:=
```
a = 0;
b = 5;
n = 180;
width := (b - a) / n;
width * Sum[(f[a + i * width] + f[a + (i + 1) * width]) / 2, {i, 0, n - 1}] // N
```

Out[188]= 3.65126

Simpson's converged more quickly as it required fewer (52 vs. 180) pieces.

In[301]:=
```
3.64177 - 3.6329511609553253`
```

Out[301]= 0.00881884

In[302]:=
```
3.64177 - 3.6512648426577234`
```

Out[302]= -0.00949484