Yohan Lee 102L Lab 2

In[8]:=

Question 1:

Define function

In[9]:=
```
f[x_] := x^3 - 5 x
```

In[10]:= Function[x, x^3 - 5 x]

Out[10]= Function[x, x^3 - 5 x]

Compute using Newton's method with 10 iterations

In[11]:=
```
iterations = 10;
z = {4.8};
For[j = 1, j ≤ iterations, j++, z = Append[z, z[[j]] - f[z[[j]]] / f'[z[[j]]]]]
```

In[14]:= 10

Out[14]= 10

Estimate of root is last element in the list

In[15]:=
```
Print[z]
```

{4.8, 3.44953, 2.67426, 2.32457, 2.24088, 2.23608, 2.23607, 2.23607, 2.23607, 2.23607, 2.23607}

Results is as expected from calculating value the value of Sqrt[5]

In[16]:=
```
N[Sqrt[5]]
```

Out[16]= 2.23607

Question 2:

Run again with new starting value

In[17]:=
```
Clear[z]
```

In[18]:=
```
z = {1}
```

Out[18]= {1}

In[19]:=
```
For[j = 1, j ≤ iterations, j++, z = Append[z, z[[j]] - f[z[[j]]] / f'[z[[j]]]]]
```

In[20]:=
```
Print[z]
```

{1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1}

In this case Newton's method does not converge but instead cycles between 1 and -1.

Question 3:

Run again with new starting value

In[21]:=
```
Clear[z]
```

In[22]:=
```
z = {.5}
```

Out[22]=
```
{0.5}
```

In[23]:=
```
For[j = 1, j ≤ iterations, j++, z = Append[z, z[[j]] - f[z[[j]]] / f'[z[[j]]]]]
```

In[24]:=
```
Print[z]
```

```
{0.5, -0.0588235, 0.000081586, -2.17224×10^-13, 0., 0., 0., 0., 0., 0., 0.}
```

Newton's method converges to the root = 0 since the initial value was chosen close to that root

Question 4:

Define a new function as f

In[25]:=
```
Clear[f]; Clear[z];
```

In[26]:=
```
f[x_] := Sin[Pi * x]
```

In[27]:=
```
Function[x, Sin[π x]]
```

Out[27]=
```
Function[x, Sin[π x]]
```

In[28]:=
```
z = {.45}
```

Out[28]=
```
{0.45}
```

In[29]:=
```
For[j = 1, j ≤ iterations, j++, z = Append[z, z[[j]] - f[z[[j]]] / f'[z[[j]]]]]
```

In[30]:=
```
Print[z]
```

```
{0.45, -1.55973, -3.23611, -2.94444, -3.00057, -3., -3., -3., -3., -3., -3.}
```

Result converges to -3.
This root is not the closest to the initial guess.

Try again with new initial value

In[31]:=
```
Clear[z]
```

In[32]:=
```
z = {.49}
```

Out[32]=
```
{0.49}
```

In[33]:=
```
For[j = 1, j ≤ iterations, j++, z = Append[z, z[[j]] - f[z[[j]]] / f'[z[[j]]]]]
```

In[34]:=
```
Print[z]
```

{0.49, -9.63878, -10.322, -9.81342, -10.0248, -9.99995, -10., -10., -10., -10., -10.}

Result now converges to -10.

The result varies significantly for these initial values even though they are close in value. This is not a desirable property.

Question 5:

Use the previous function with an initial guess of x=0.5

In[35]:=
```
Clear[z]
```

In[36]:=
```
z = {.5}
```

Out[36]= {0.5}

In[37]:=
```
For[j = 1, j ≤ iterations, j++, z = Append[z, z[[j]] - f[z[[j]]] / f'[z[[j]]]]]
```

In[38]:=
```
Print[z]
```

$\{0.5, -5.19839\times10^{15}, -5.19839\times10^{15}, -5.19839\times10^{15}, -5.19839\times10^{15}, -5.19839\times10^{15}, -5.19839\times10^{15}, -5.19839\times10^{15}, -5.19839\times10^{15}, -5.19839\times10^{15}, -5.19839\times10^{15}\}$

The result converges to a large negative value. This value is not close to the initial guess. This value is also very different than the result from an initial value of .49.

Question 6:

The choice of initial value has a strong effect on the convergence of Newton’s method. An initial value close to the final value can lead to convergence in fewer iterations.

Some choices of initial value can lead to a situation where the values fail to converge. This can happen if the choice results in f’[a] close to zero.

The case of Question 5 shows how a small difference in choices for initial value can lead to large differences in final result for the same function. This can be expected in the case of Sin[Pi*x] which has all the integers as roots. The following are the values for the first 2 steps of Question 5.

In[39]:=
```
f[.5]
```

Out[39]= 1.

In[40]:= `f'[.5]`

Out[40]= 1.92367×10^{-16}

In[41]:= `.5 - f[.5] / f'[.5]`

Out[41]= -5.19839×10^{15}

In[44]:= `f[-5.198394939755862`*^15]`

Out[44]= -0.140327

In[45]:= `f'[-5.198394939755862`*^15]`

Out[45]= 3.11051

In[43]:= `-5.198394939755862`*^15 - f[-5.198394939755862`*^15] / f'[-5.198394939755862`*^15]`

Out[43]= -5.19839×10^{15}