Lab10-1

```
f[x_] := 1/Sqrt[x^4 + x^3 + 1]
```

Function$\left[x, \dfrac{1}{\sqrt{x^4 + x^3 + 1}}\right]$

Function$\left[x, \dfrac{1}{\sqrt{x^4 + x^3 + 1}}\right]$

```
T3[x_] = Series[f[x], {x, -1, 3}] // Normal
```

$$1 + \frac{1+x}{2} - \frac{9}{8}(1+x)^2 - \frac{7}{16}(1+x)^3$$

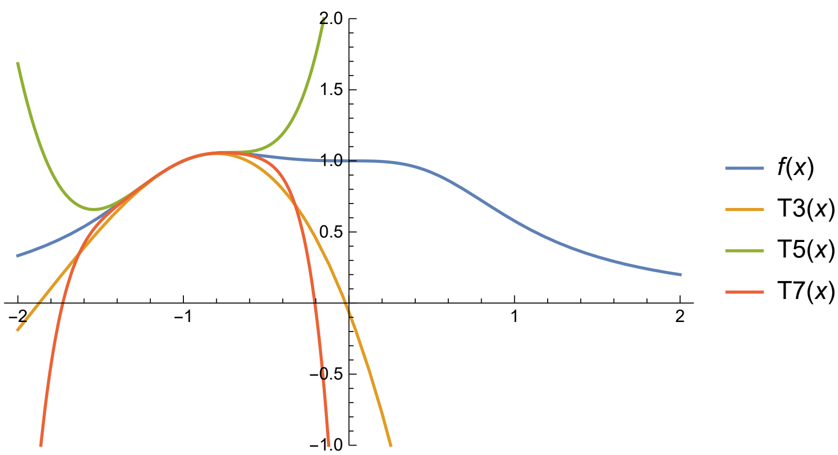
```
T5[x_] = Series[f[x], {x, -1, 5}] // Normal
```

$$1 + \frac{1+x}{2} - \frac{9}{8}(1+x)^2 - \frac{7}{16}(1+x)^3 + \frac{331}{128}(1+x)^4 + \frac{183}{256}(1+x)^5$$

```
T7[x_] = Series[f[x], {x, -1, 7}] // Normal
```

$$1 + \frac{1+x}{2} - \frac{9}{8}(1+x)^2 - \frac{7}{16}(1+x)^3 + \frac{331}{128}(1+x)^4 + \frac{183}{256}(1+x)^5 - \frac{6189\,(1+x)^6}{1024} - \frac{2135\,(1+x)^7}{2048}$$

```
Plot[{f[x], T3[x], T5[x], T7[x]}, {x, -2, 2},
 PlotLegends → "Expressions", PlotRange → {-1, 2}]
```



Question 1

```
Clear[f]
```

```
f[x_] := Cos[1 - Exp[x]]
```

```
Function[x, Cos[1 - Exp[x]]]
```

Function[x, Cos[1 - Exp[x]]]

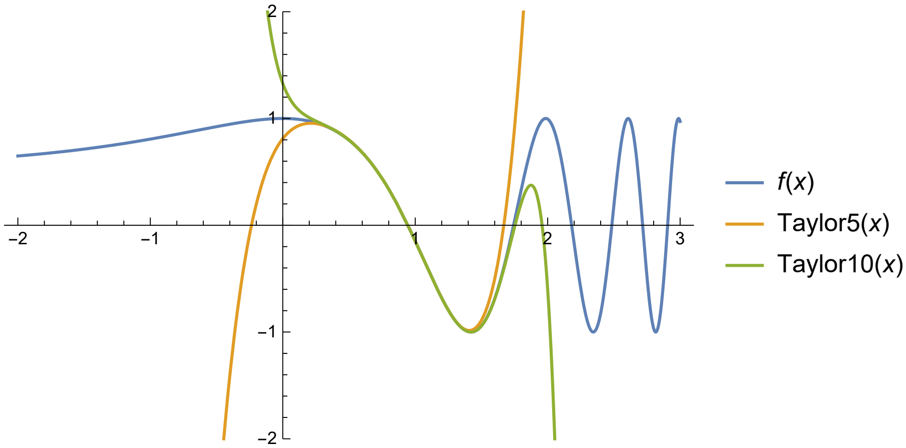
```
Taylor5[x_] = Series[f[x], {x, 1, 5}] // Normal
```

$$\cos[1-e] + e\,(-1+x)\,\sin[1-e] + \frac{1}{2}(-1+x)^2\left(-e^2\cos[1-e] + e\sin[1-e]\right) + \frac{1}{24}(-1+x)^4\left(-7e^2\cos[1-e] + e^4\cos[1-e] + e\sin[1-e] - 6e^3\sin[1-e]\right) + \frac{1}{6}(-1+x)^3\left(-3e^2\cos[1-e] + e\sin[1-e] - e^3\sin[1-e]\right) + \frac{1}{120}(-1+x)^5\left(-15e^2\cos[1-e] + 10e^4\cos[1-e] + e\sin[1-e] - 25e^3\sin[1-e] + e^5\sin[1-e]\right)$$

```
Taylor10[x_] = Series[f[x], {x, 1, 10}] // Normal
```

$$\cos[1-e] + e\,(-1+x)\,\sin[1-e] + \frac{1}{2}(-1+x)^2\left(-e^2\cos[1-e] + e\sin[1-e]\right) + \frac{1}{6}(-1+x)^3\left(-3e^2\cos[1-e] + e\sin[1-e] - e^3\sin[1-e]\right) + (-1+x)^4\left(-\frac{7}{24}e^2\cos[1-e] + \frac{1}{24}e^4\cos[1-e] + \frac{1}{24}e\sin[1-e] - \frac{1}{4}e^3\sin[1-e]\right) + (-1+x)^5\left(-\frac{1}{8}e^2\cos[1-e] + \frac{1}{12}e^4\cos[1-e] + \frac{1}{120}e\sin[1-e] - \frac{5}{24}e^3\sin[1-e] + \frac{1}{120}e^5\sin[1-e]\right) + (-1+x)^6\left(-\frac{31}{720}e^2\cos[1-e] + \frac{13}{144}e^4\cos[1-e] - \frac{1}{720}e^6\cos[1-e] + \frac{1}{720}e\sin[1-e] - \frac{1}{8}e^3\sin[1-e] + \frac{1}{48}e^5\sin[1-e]\right) + (-1+x)^8\left(-\frac{127e^2\cos[1-e]}{40320} + \frac{27}{640}e^4\cos[1-e] - \frac{19e^6\cos[1-e]}{2880} + \frac{e^8\cos[1-e]}{40320} + \frac{e\sin[1-e]}{40320} - \frac{23}{960}e^3\sin[1-e] + \frac{5}{192}e^5\sin[1-e] - \frac{e^7\sin[1-e]}{1440}\right) + (-1+x)^7\left(-\frac{1}{80}e^2\cos[1-e] + \frac{5}{72}e^4\cos[1-e] - \frac{1}{240}e^6\cos[1-e] + \frac{e\sin[1-e]}{5040} - \frac{43}{720}e^3\sin[1-e] + \frac{1}{36}e^5\sin[1-e] - \frac{e^7\sin[1-e]}{5040}\right) + (-1+x)^9\left(-\frac{17e^2\cos[1-e]}{24192} + \frac{37e^4\cos[1-e]}{1728} - \frac{7}{960}e^6\cos[1-e] + \frac{e^8\cos[1-e]}{10080} + \frac{e\sin[1-e]}{362880} - \frac{605e^3\sin[1-e]}{72576} + \frac{331e^5\sin[1-e]}{17280} - \frac{11e^7\sin[1-e]}{8640} + \frac{e^9\sin[1-e]}{362880}\right) + (-1+x)^{10}\left(-\frac{73e^2\cos[1-e]}{518400} + \frac{6821e^4\cos[1-e]}{725760} - \frac{1087e^6\cos[1-e]}{172800} + \frac{5e^8\cos[1-e]}{24192} - \frac{e^{10}\cos[1-e]}{3628800} + \frac{e\sin[1-e]}{3628800} - \frac{311e^3\sin[1-e]}{120960} + \frac{3}{256}e^5\sin[1-e] - \frac{7e^7\sin[1-e]}{4320} + \frac{e^9\sin[1-e]}{80640}\right)$$

```
Plot[{f[x], Taylor5[x], Taylor10[x]}, {x, -2, 3},
 PlotLegends → "Expressions", PlotRange → {-2, 2}]
```




```
ClearAll
```

```
ClearAll
```

```
f[x_] := x * Exp[-2 x]
```

```
Function[x, x Exp[-2 x]]
```

```
Function[x, x Exp[-2 x]]
```

```
TaylorList = Table[SeriesCoefficient[f[x], {x, 0, n}], {n, 0, 10}]
```

$$\left\{0, 1, -2, 2, -\frac{4}{3}, \frac{2}{3}, -\frac{4}{15}, \frac{4}{45}, -\frac{8}{315}, \frac{2}{315}, -\frac{4}{2835}\right\}$$

```
a[n_] = FindSequenceFunction[TaylorList, n]
```

$$\frac{(-1)^n 2^{-2+n}}{\text{Pochhammer}[1, -2+n]}$$

```
SumConvergence[a[n] (x - 0)^n, n]
```

```
True
```

Question 2

In[17]:= `ClearAll`

Out[17]= `ClearAll`

In[18]:= `f[x_] := (1 - 3 x)^-5`

In[19]:= $\text{Function}\left[x, \frac{1}{(1-3x)^5}\right]$

Out[19]= $\text{Function}\left[x, \frac{1}{(1-3x)^5}\right]$

In[20]:= `Taylor3a[x_] = Series[f[x], {x, 1/6, 3}] // Normal`

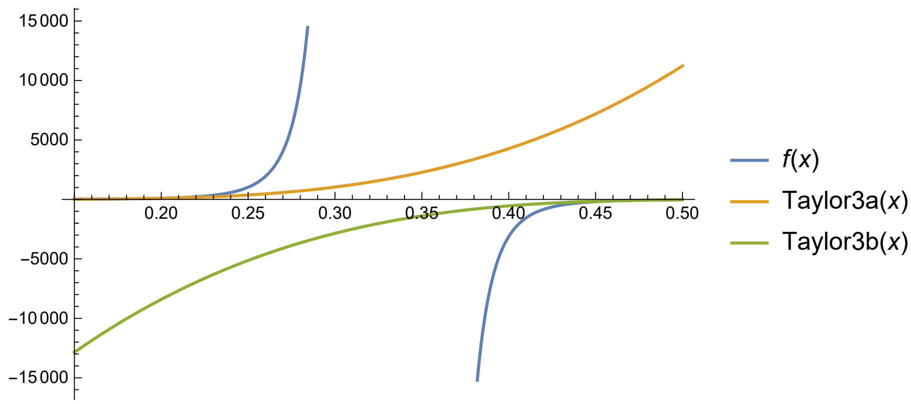
Out[20]= $32 + 960 \left(-\frac{1}{6} + x\right) + 17280 \left(-\frac{1}{6} + x\right)^2 + 241920 \left(-\frac{1}{6} + x\right)^3$

In[21]:= `Taylor3b[x_] = Series[f[x], {x, 1/2, 3}] // Normal`

Out[21]= $-32 + 960 \left(-\frac{1}{2} + x\right) - 17280 \left(-\frac{1}{2} + x\right)^2 + 241920 \left(-\frac{1}{2} + x\right)^3$

In[22]:= `Plot[{f[x], Taylor3a[x], Taylor3b[x]}, {x, 0.15, 0.5}, PlotLegends → "Expressions"]`

Out[22]=



`TaylorList = Table[SeriesCoefficient[f[x], {x, 0, n}], {n, 0, 10}]`

{1, 15, 135, 945, 5670, 30618, 153090, 721710, 3247695, 14073345, 59108049}

`a[n_] = FindSequenceFunction[TaylorList, n]`

$\frac{1}{8} \times 3^{-2+n}\, n\, (1+n)\, (2+n)\, (3+n)$

`SumConvergence[a[n] (x - 1/6)^n, n]`

$\text{Abs}\left[-\frac{1}{2} + 3x\right] < 1$

`SumConvergence[a[n] (x - 1/2)^n, n]`

$\text{Abs}\left[-\frac{3}{2} + 3x\right] < 1$