# Lab 10-2

## Approximate Sin[0.3] by 3 degree Taylor series centered at x=0

In[2]:=
```
T3[x_] = Series[Sin[x], {x, 0, 3}] // Normal
```

Out[2]= $x - \frac{x^3}{6}$

In[3]:=
```
T3[0.3]
```

Out[3]= 0.2955

## Q1: Approximate e^2 by 5th order Taylor polynomial centered at x=1

In[12]:=
```
Q1T[x_] = Series[Exp[2 x], {x, 1, 5}]
```

Out[12]= $e^2 + 2\,e^2\,(x-1) + 2\,e^2\,(x-1)^2 + \frac{4}{3}\,e^2\,(x-1)^3 + \frac{2}{3}\,e^2\,(x-1)^4 + \frac{4}{15}\,e^2\,(x-1)^5 + O[x-1]^6$

In[17]:=
```
Q1T[1]
```

Out[17]= $e^2$

## Error Estimate of Cos[x]

In[18]:=
```
Table[{n, ((Pi/4.)^(n+1))/((n+1)!)}, {n, 4, 8}] // TableForm // N
```

Out[18]//TableForm=

| | |
|---|---|
| 4. | 0.00249039 |
| 5. | 0.000325992 |
| 6. | 0.0000365762 |
| 7. | $3.59086 \times 10^{-6}$ |
| 8. | $3.13362 \times 10^{-7}$ |

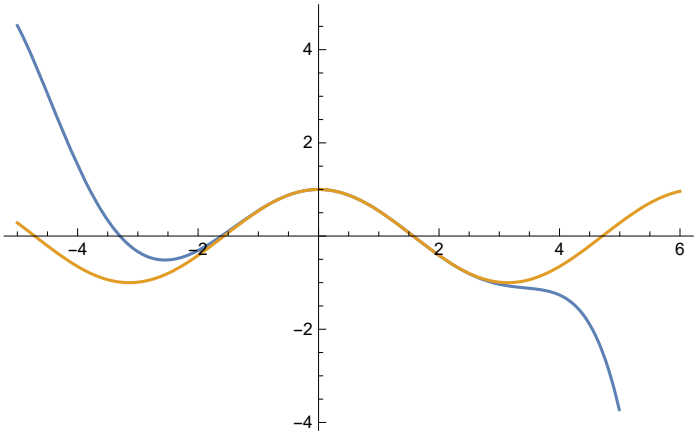
Choose n to be at least 6

In[19]:=
```
T6[x_] = Series[Cos[x], {x, Pi/4, 6}] // Normal
```

Out[19]= $\frac{1}{\sqrt{2}} - \frac{-\frac{\pi}{4}+x}{\sqrt{2}} - \frac{\left(-\frac{\pi}{4}+x\right)^2}{2\sqrt{2}} + \frac{\left(-\frac{\pi}{4}+x\right)^3}{6\sqrt{2}} + \frac{\left(-\frac{\pi}{4}+x\right)^4}{24\sqrt{2}} - \frac{\left(-\frac{\pi}{4}+x\right)^5}{120\sqrt{2}} - \frac{\left(-\frac{\pi}{4}+x\right)^6}{720\sqrt{2}}$

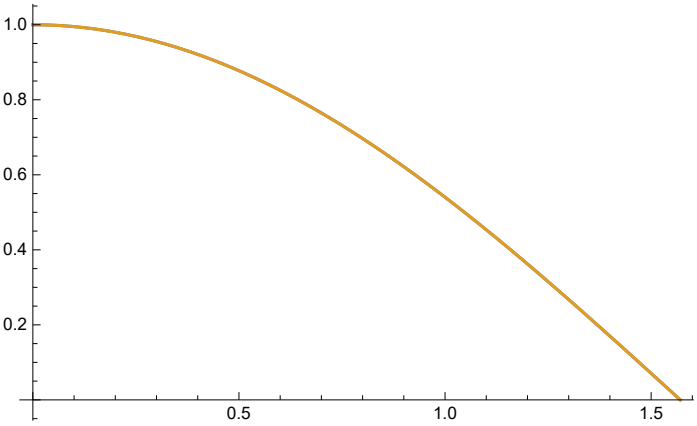
In[20]:=
```
Plot[{T6[x], Cos[x]}, {x, -5, 6}]
```

Out[20]=




In[21]:=
```
Plot[{T6[x], Cos[x]}, {x, 0, Pi / 2}]
```

Out[21]=




## Q3: Find polynomial approximation to f(x)=e^x centered at x=0 in [-2,2] with error less than 0.005

In[23]:=
```
M = Exp[2]
```

Out[23]= $e^2$

In[25]:=
```
Table[{n, (Exp[2] * (2) ^ (n + 1)) / ((n + 1) !)}, {n, 8, 12}] // TableForm // N
```

Out[25]//TableForm=

| | |
|---|---|
| 8. | 0.0104255 |
| 9. | 0.0020851 |
| 10. | 0.000379108 |
| 11. | 0.0000631847 |
| 12. | $9.72072 \times 10^{-6}$ |

Choose n to be at least 10

In[36]:=
```
Q3T[x_] = Series[Exp[x], {x, 0, 10}] // Normal
```

Out[36]= $1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720} + \frac{x^7}{5040} + \frac{x^8}{40\,320} + \frac{x^9}{362\,880} + \frac{x^{10}}{3\,628\,800}$

In[38]:=
```
Plot[{Q3T[x], Exp[x]}, {x, -10, 10}]
```

Out[38]=