Yohan Lee Lab 8

Examples

```
s[n_] := Sum[1/k, {k, 1, n}]
```

$$\text{Function}\left[n, \sum_{k=1}^{n} \frac{1}{k}\right]$$

$$\text{Function}\left[n, \sum_{k=1}^{n} \frac{1}{k}\right]$$

```
vals = Table[{n, s[n]}, {n, 1, 101}];
```

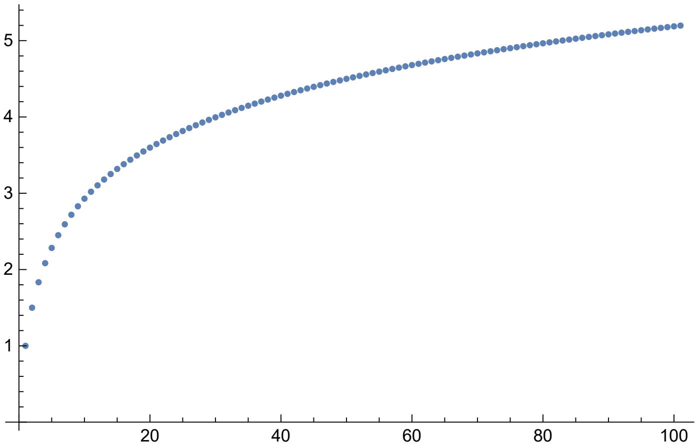
```
vals[[ ;;  ;; 10]] // N // TableForm
```

1.	1.
11.	3.01988
21.	3.64536
31.	4.02725
41.	4.30293
51.	4.51881
61.	4.69626
71.	4.84692
81.	4.97782
91.	5.09356
101.	5.19728

```
vals2 = Table[{n, s[n]}, {n, 1, 101, 10}] // N // TableForm
```

1.	1.
11.	3.01988
21.	3.64536
31.	4.02725
41.	4.30293
51.	4.51881
61.	4.69626
71.	4.84692
81.	4.97782
91.	5.09356
101.	5.19728

```
ListPlot[vals]
```



```
s[Infinity]
```

Sum: Sum does not converge.

$$\sum_{k=1}^{\infty} \frac{1}{k}$$

Question 1

a)

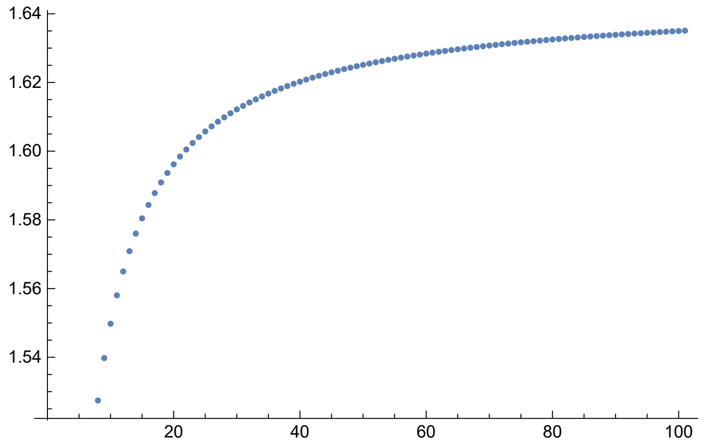
```
In[47]:= Clear[s, vals];
s[n_] := Sum[1/k^2, {k, 1, n}];
vals = Table[{n, s[n]}, {n, 1, 101}];
vals[[ ;;  ;; 10]] // N // TableForm
```

Out[50]//TableForm=

1.	1.
11.	1.55803
21.	1.59843
31.	1.61319
41.	1.62084
51.	1.62552
61.	1.62867
71.	1.63095
81.	1.63266
91.	1.63401
101.	1.63508

In[51]:=
```
ListPlot[vals]
```

Out[51]=



In[52]:=
```
s[Infinity]
```

Out[52]= $\frac{\pi^2}{6}$

Converges to

In[53]:=
```
s[Infinity] // N
```

Out[53]= 1.64493

b)

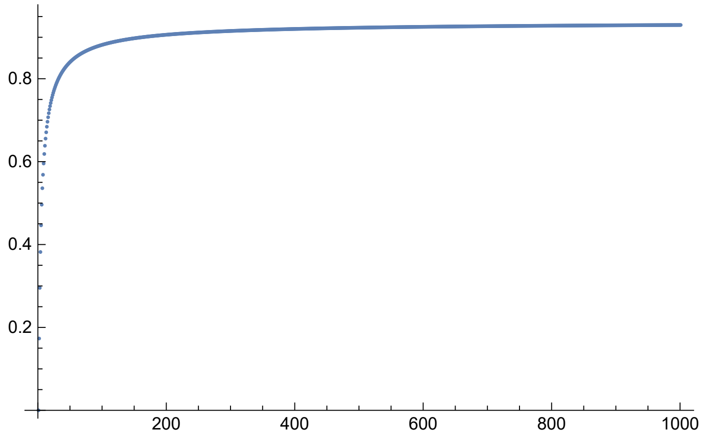
In[68]:=
```
Clear[s, vals];
s[n_] := Sum[Log[k] / k^2, {k, 1, n}];
vals = Table[{n, s[n]}, {n, 1, 1001}];
vals[[ ;;  ;; 100]] // N // TableForm
```

Out[71]//TableForm=

1.	0.
101.	0.882179
201.	0.906254
301.	0.915297
401.	0.920126
501.	0.923156
601.	0.925247
701.	0.926781
801.	0.927958
901.	0.928892
1001.	0.929651

In[72]:=
```
ListPlot[vals]
```

Out[72]=



In[59]:=
```
s[Infinity]
```

Out[59]= $-\frac{1}{6}\pi^2$ (EulerGamma + Log[2] − 12 Log[Glaisher] + Log[π])

Converges to

In[60]:=
```
s[Infinity] // N
```

Out[60]=
```
0.937548
```

c)

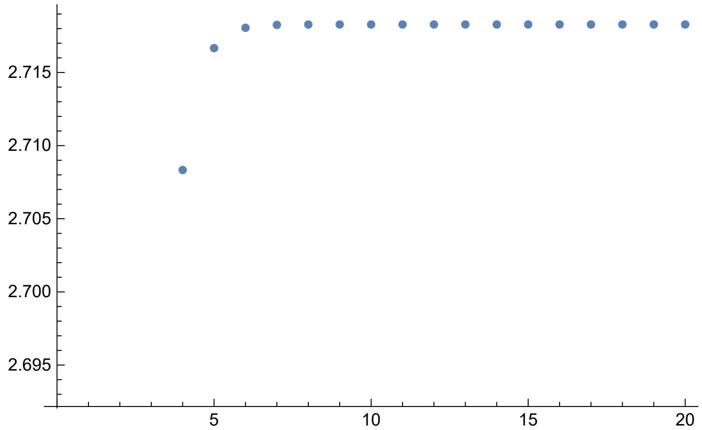
In[61]:=
```
Clear[s, vals];
s[n_] := Sum[1 / Factorial[k], {k, 0, n}];
vals = Table[{n, s[n]}, {n, 1, 20}];
vals[[ ;;  ;; 2]] // N // TableForm
```

Out[64]//TableForm=

1.	2.
3.	2.66667
5.	2.71667
7.	2.71825
9.	2.71828
11.	2.71828
13.	2.71828
15.	2.71828
17.	2.71828
19.	2.71828

In[65]:=
```
ListPlot[vals]
```

Out[65]=



In[66]:=
```
s[Infinity]
```

Out[66]= e

Converges to

In[67]:=
```
s[Infinity] // N
```

Out[67]= 2.71828

Question 2

```
t[n_] := Sum[1/Factorial[k], {k, 0, n}]
```

$$\text{Function}\left[n, \sum_{k=0}^{n} \frac{1}{k!}\right]$$

$$\text{Function}\left[n, \sum_{k=0}^{n} \frac{1}{k!}\right]$$

```
errors = {}
```

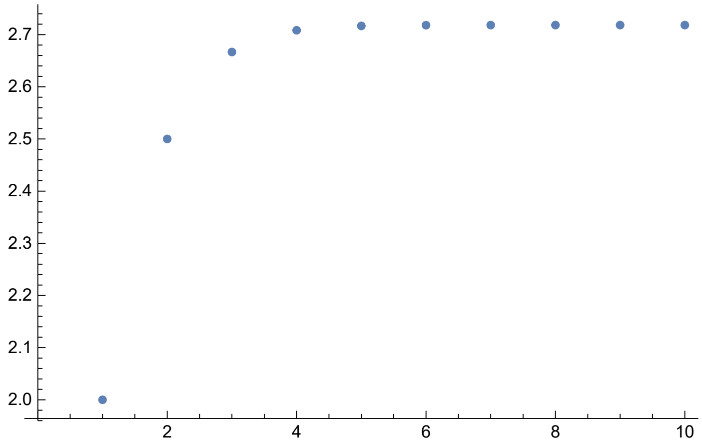
{}

```
exact = Exp[1]
```

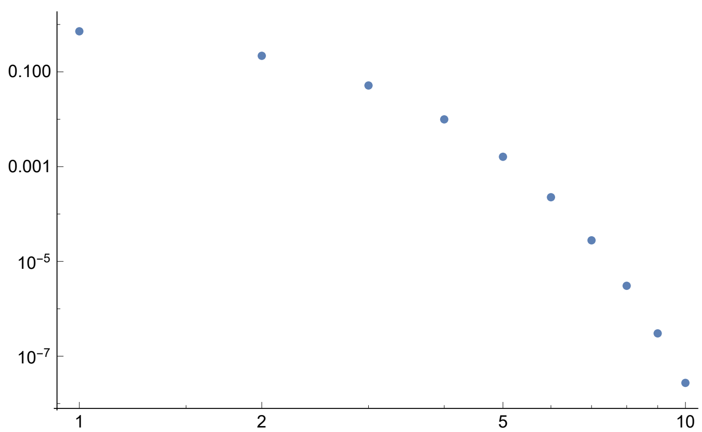
e

```
For[m = 1, m ≤ 10, m++, errors = Append[errors, Abs[t[m] - exact]]]
```

```
ListPlot[Table[{n, t[n]}, {n, 1, 10}]]
```



```
ListLogLogPlot[errors]
```



Observations

The error decreases with subsequent values of the partial sum t[n]. The partial sums are a good approximation of e.

Question3

```
f[n_] := Sum[1/k^2, {k, 1, n}]
```

$$\text{Function}\left[n, \sum_{k=1}^{n} \frac{1}{k^2}\right]$$

$$\text{Function}\left[n, \sum_{k=1}^{n} \frac{1}{k^2}\right]$$

```
errs = {}
```

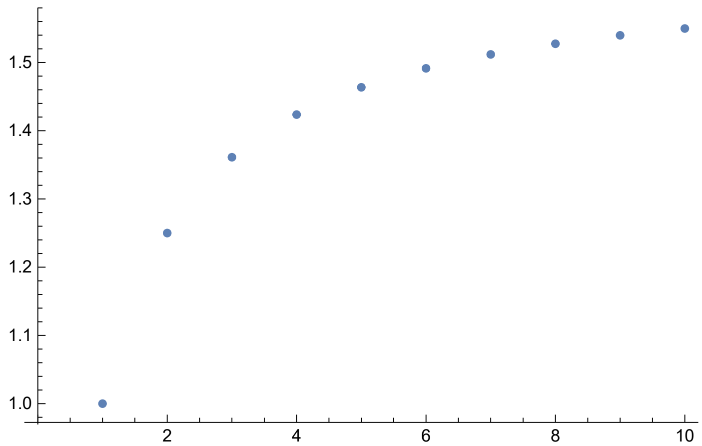
{}

```
exact = Pi^2/6
```

$$\frac{\pi^2}{6}$$

```
For[m = 1, m ≤ 10, m++, errs = Append[errs, Abs[f[m] - exact]]]
```

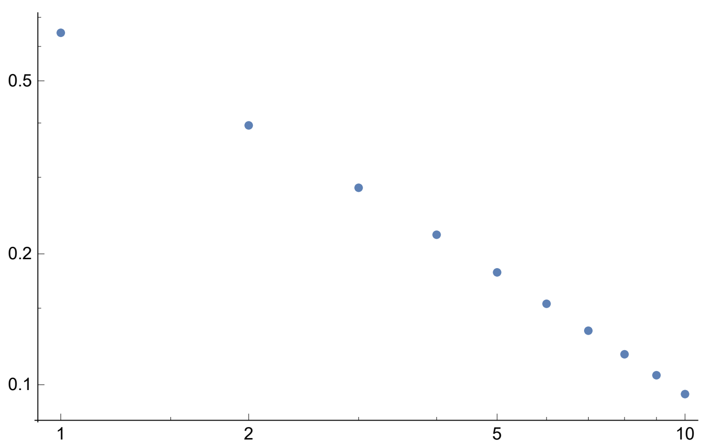
```
ListPlot[Table[{n, f[n]}, {n, 1, 10}]]
```




```
exact // N
```

1.64493

```
ListLogLogPlot[errs]
```



Observations

The error decreases with subsequent values of the partial sum f[n]. The partial sums are a good approximation of $\frac{\pi^2}{6}$.