Yohan Lee Lab 4

Example 1 - Left endpoint

```
In[1]:= Clear[a, b, n]

In[16]:= a = 3;

In[17]:= b = 8.5;

In[18]:= n = 5;

In[5]:= width = (b - a) / n;

In[6]:= Clear[f];

In[7]:= f[x_] := x^2 - 3

In[8]:= Function[x, x^2 - 3]

Out[8]= Function[x, x^2 - 3]

In[11]:= width * Sum[f[a + i * width], {i, 0, n - 1}]

Out[11]= 145.53
```

Example 2 - Right endpoint

```
In[12]:= width * Sum[f[a + (i + 1) * width], {i, 0, n - 1}]

Out[12]= 215.105
```

Example 3 - Midpoint

```
In[13]:= width * Sum[f[a + (i + 1 / 2) * width], {i, 0, n - 1}]

Out[13]= 178.654

In[14]:= Integrate[f[x], {x, a, b}]

Out[14]= 179.208
```

Example 4 - Trapezoid

```
In[19]:= width * Sum[(f[a + i * width] + f[a + (i + 1) * width]) / 2, {i, 0, n - 1}]

Out[19]= 180.318
```

Question 1

```
In[20]:= Clear[a, b, n]

In[21]:= a = 0;

In[22]:= b = 5;

In[23]:= n = 10;
```

```
In[25]:= width = (b - a) / n;

In[26]:= width
Out[26]= 1/2

In[29]:= Clear[f]

In[30]:= f[x_] := x^2

In[31]:= Function[x, x^2]
Out[31]= Function[x, x^2]

In[32]:= width * Sum[f[a + i * width], {i, 0, n - 1}]
Out[32]= 285/8

In[36]:= width * Sum[f[a + i * width], {i, 0, n - 1}] // N
Out[36]= 35.625
```

Question 2

```
In[33]:= width * Sum[f[a + (i + 1) * width], {i, 0, n - 1}]
Out[33]= 385/8

In[37]:= width * Sum[f[a + (i + 1) * width], {i, 0, n - 1}] // N
Out[37]= 48.125

In[34]:= Integrate[f[x], {x, a, b}]
Out[34]= 125/3

In[35]:= Integrate[f[x], {x, a, b}] // N
Out[35]= 41.6667
```

a) Approx 48.125 vs Exact 41.6667

b) The left rectangles underestimate the area. The right rectangles overestimate the area.

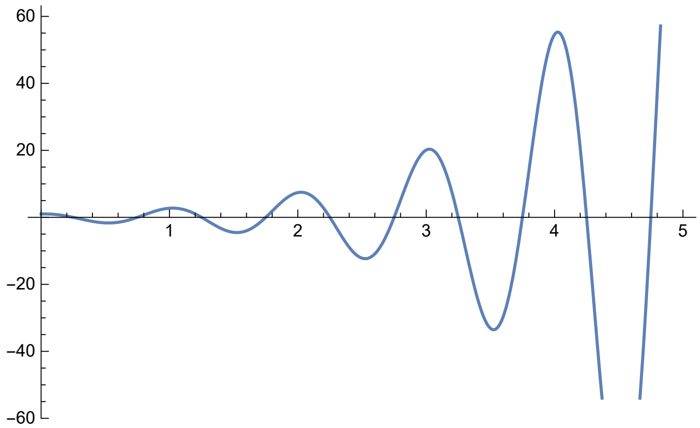
Question 3

```
In[38]:= Clear[f]

In[39]:= f[x_] := Exp[x] * Cos[2 * Pi * x]

In[40]:= Function[x, Exp[x] Cos[2 π x]]
Out[40]= Function[x, Exp[x] Cos[2 π x]]
```

In[71]:=
```
Clear[a, b, n];
a = 0;
b = 5;
```

In[74]:=
```
Plot[f[x], {x, 0, 5}]
```

Out[74]=



In[75]:=
```
Integrate[f[x], {x, a, b}]
```

Out[75]= $\frac{-1+e^5}{1+4\pi^2}$

In[76]:=
```
Integrate[f[x], {x, a, b}] // N
```

Out[76]= 3.64177

In[77]:=
```
n = 10;
width = (b - a) / n;
width
```

Out[79]= $\frac{1}{2}$

In[80]:=
```
width * Sum[f[a + (i + 1 / 2) * width], {i, 0, n - 1}] // N
```

Out[80]= 0.

In[81]:=
```
n = 50;
width = (b - a) / n;
width
```

Out[83]= $\frac{1}{10}$

In[84]:=
```
width * Sum[f[a + (i + 1 / 2) * width], {i, 0, n - 1}] // N
```

Out[84]= 3.57821

In[85]:=
```
n = 100;
width = (b - a) / n;
width
```

Out[87]= $\frac{1}{20}$

In[88]:=
```
width * Sum[f[a + (i + 1 / 2) * width], {i, 0, n - 1}] // N
```
Out[88]= 3.62628

In[89]:=
```
n = 5;
width = (b - a) / n;
width
```
Out[91]= 1

In[92]:=
```
width * Sum[f[a + (i + 1 / 2) * width], {i, 0, n - 1}] // N
```
Out[92]= -141.445

a) A small number of intervals will incorporate more negative portions of the function than positive portions.
b) 10% of the exact answer (3.64177) is .364177. We seek an estimate within this error. An estimate with n=23 has error of .34119.

In[97]:=
```
3.64177 - .36417
```
Out[97]= 3.2776

In[126]:=
```
n = 23;
width = (b - a) / n;
width
```
Out[128]= $\frac{5}{23}$

In[129]:=
```
width * Sum[f[a + (i + 1 / 2) * width], {i, 0, n - 1}] // N
```
Out[129]= 3.30058

In[130]:=
```
3.64177 - 3.30058
```
Out[130]= 0.34119

Question 4

In[131]:=
```
Clear[a, b, n];
a = 0;
b = 5;
```

In[134]:=
```
n = 10;
width = (b - a) / n;
width
```
Out[136]= $\frac{1}{2}$

In[138]:=
```
width * Sum[(f[a + i * width] + f[a + (i + 1) * width]) / 2, {i, 0, n - 1}] // N
```
Out[138]= 9.02606

a) The trapezoid method overestimates the area.

In[139]:=
```
n = 23;
width = (b - a) / n;
width
```

Out[141]= $\frac{5}{23}$

In[142]:=
```
width * Sum[(f[a + i * width] + f[a + (i + 1) * width]) / 2, {i, 0, n - 1}] // N
```

Out[142]= 4.28007

In[160]:=
```
n = 30;
width = (b - a) / n;
width
```

Out[162]= $\frac{1}{6}$

In[163]:=
```
width * Sum[(f[a + i * width] + f[a + (i + 1) * width]) / 2, {i, 0, n - 1}] // N
```

Out[163]= 4.00236

b) The estimate must be less than 4.00594 to be within .36417 of 3.64177. This requires requires n=30.

In[147]:=
```
3.64177 + .36417
```

Out[147]= 4.00594

Question 5

The midpoint method was more accurate as the same choice of n for midpoint will yield a closer answer. The midpoint estimate is less sensitive to the positive a negative fluctuations of the function while the trapezoid is skewed toward the positive values.