Yohan Lee Lab 7

Examples

In[1]:=
```
Table[Sqrt[i], {i, 1, 10}] // N
```

Out[1]=
```
{1., 1.41421, 1.73205, 2., 2.23607, 2.44949, 2.64575, 2.82843, 3., 3.16228}
```

In[2]:=
```
Table[Sqrt[i], {i, 1, 10}] // TableForm // N
```

Out[2]//TableForm=
```
1.
1.41421
1.73205
2.
2.23607
2.44949
2.64575
2.82843
3.
3.16228
```

In[3]:=
```
a[n_] := (3 n^2 - (-1)^n + n) / (7 n^2 - 6 n + 4);
```

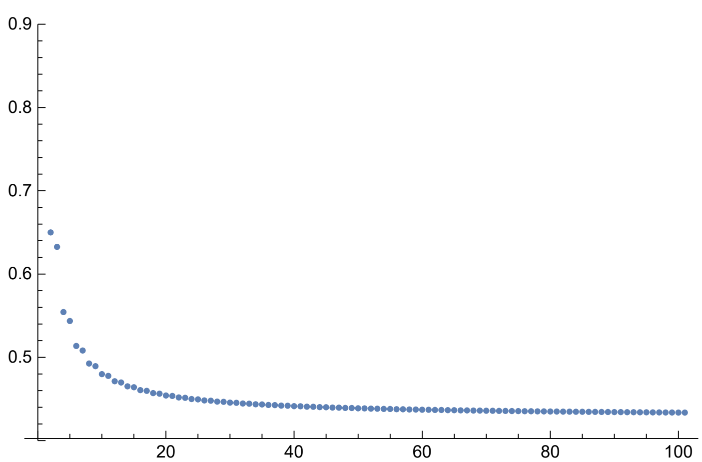
In[7]:=
```
seq = Table[{n, a[n]}, {n, 1, 101}] // N;
seq[[ ;;  ;; 10]] // TableForm
```

Out[8]//TableForm=

1.	1.
11.	0.477707
21.	0.453626
31.	0.445378
41.	0.441215
51.	0.438704
61.	0.437026
71.	0.435824
81.	0.434921
91.	0.434219
101.	0.433656

In[9]:=
```
ListPlot[seq, PlotRange → {.4, .9}]
```

Out[9]=



In[10]:=
```
Limit[a[n], n → Infinity] // N
```
Out[10]= 0.428571

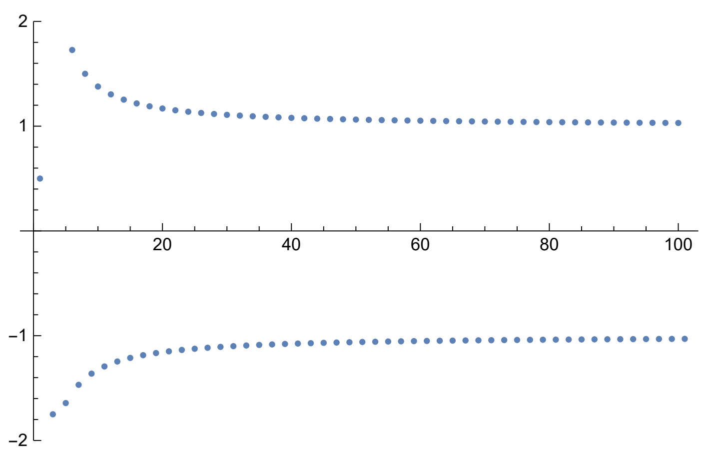
Question 1

In[11]:=
```
b[n_] = (2 + (-1)^n * n^2) / (n^2 - 3 n + 4);
```
In[12]:=
```
b[n]
```
Out[12]= $\frac{2 + (-1)^n n^2}{4 - 3 n + n^2}$

In[18]:=
```
seq = Table[{n, b[n]}, {n, 1, 101}] // N;
seq[[ ;;  ;; 10]] // TableForm
```
Out[19]//TableForm=

1.	0.5
11.	-1.29348
21.	-1.14921
31.	-1.09977
41.	-1.0749
51.	-1.05995
61.	-1.04997
71.	-1.04284
81.	-1.03749
91.	-1.03333
101.	-1.02999

In[24]:=
```
ListPlot[seq, PlotRange → {-2, 2}]
```
Out[24]=



In[25]:=
```
Limit[b[n], n → Infinity] // N
```
Out[25]= $2.71828^{(0.+2.\,i)\ \mathrm{Interval}[\{-2.22507\times10^{-308},3.14159\}]}$

Limit DNE because elements of seq approach a number but alternate in sign.

Question 2

In[26]:=
```
c[n_] := Log[n] / n^(1/3)
```

In[27]:=
$$\text{Function}\left[n, \frac{\text{Log}[n]}{n^{1/3}}\right]$$

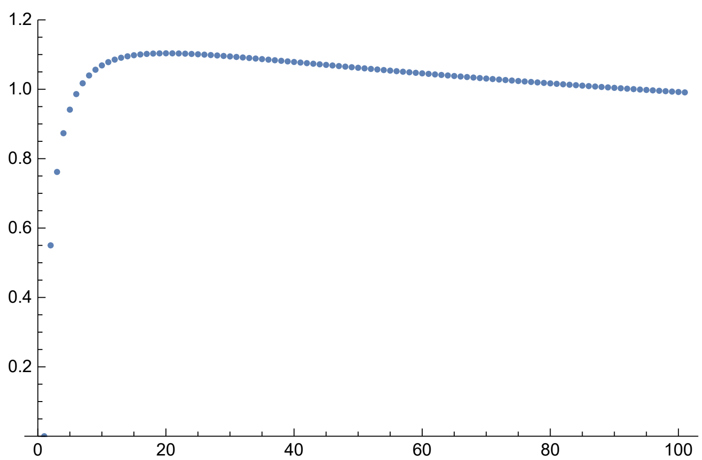
Out[27]=
$$\text{Function}\left[n, \frac{\text{Log}[n]}{n^{1/3}}\right]$$

In[30]:=
```
seq = Table[{n, c[n]}, {n, 1, 101}] // N;
seq[[ ;;  ;; 5]] // TableForm
```

Out[31]//TableForm=

1.	0.
6.	0.986043
11.	1.0782
16.	1.1003
21.	1.10352
26.	1.09978
31.	1.09315
36.	1.08528
41.	1.07695
46.	1.06854
51.	1.06024
56.	1.05214
61.	1.0443
66.	1.03673
71.	1.02943
76.	1.02241
81.	1.01565
86.	1.00914
91.	1.00287
96.	0.996831
101.	0.991005

In[32]:=
```
ListPlot[seq, PlotRange → {0, 1.2}]
```

Out[32]=


In[33]:=
```
Limit[c[n], n → Infinity] // N
```

Out[33]= 0.

Limit exists as the sequence converges to 0.

Question 3

In[37]:=
```
d[n_] := Sin[Tan[Sqrt[n^2 - n]]]
```

In[38]:=
```
Function[n, Sin[Tan[Sqrt[n^2 - n]]]]
```
$\text{Function}\left[n, \text{Sin}\left[\text{Tan}\left[\sqrt{n^2 - n}\right]\right]\right]$

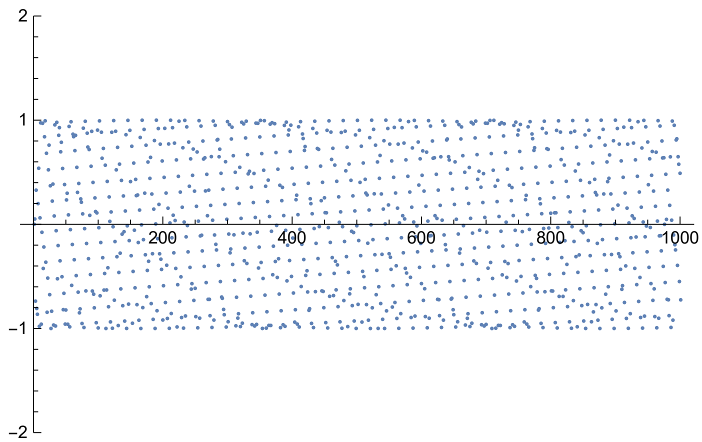
Out[38]=
$\text{Function}\left[n, \text{Sin}\left[\text{Tan}\left[\sqrt{n^2 - n}\right]\right]\right]$

In[46]:=
```
seq = Table[{n, d[n]}, {n, 1, 1001}] // N;
seq[[ ;;  ;; 71]] // TableForm
```

Out[47]//TableForm=

1.	0.
72.	-0.811884
143.	0.860248
214.	-0.129242
285.	0.814655
356.	0.519175
427.	-0.810237
498.	0.858423
569.	-0.128843
640.	0.819297
711.	0.519403
782.	-0.810056
853.	0.858045
924.	-0.128727
995.	0.820974

In[50]:=
```
ListPlot[seq, PlotRange → {2, -2}]
```

Out[50]=


In[51]:=
```
Limit[d[n], n → Infinity] // N
```

Out[51]= $\text{Limit}\left[\text{Sin}\left[\text{Tan}\left[\sqrt{-1.\, n + n^2}\right]\right], n \to \infty\right]$

The sequence does not converge to a number so the limit DNE.

Question 4

In[52]:=
```
f[n_] := ((-1)^(n + 1) * Factorial[2 n + 1]) / (4^(2 n + 3) * Factorial[n + 2] * Factorial[n])
```

In[53]:= **Function**$\left[n, \frac{(-1)^{n+1}\,(2n+1)!}{4^{2n+3}\,(n+2)!\,n!}\right]$

Out[53]= Function$\left[n, \frac{(-1)^{n+1}\,(2n+1)!}{4^{2n+3}\,(n+2)!\,n!}\right]$

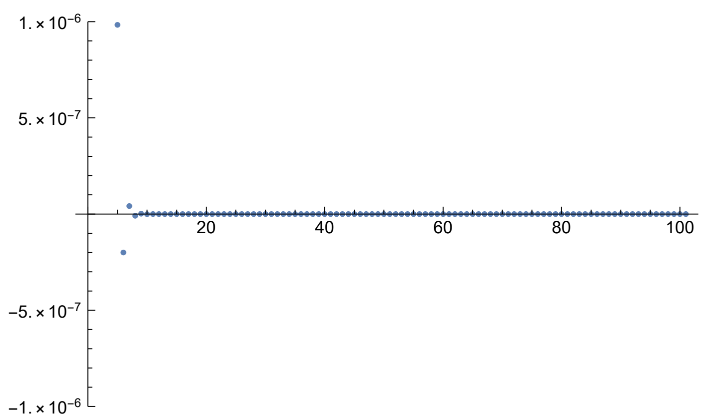
In[58]:=
```
seq = Table[{n, f[n]}, {n, 1, 101}] // N;
seq[[ ;;  ;; 7]] // TableForm
```

Out[59]//TableForm=

1.	0.000976563
8.	-8.84393×10^{-9}
15.	2.39593×10^{-13}
22.	-8.66004×10^{-18}
29.	3.5876×10^{-22}
36.	-1.60977×10^{-26}
43.	7.61242×10^{-31}
50.	-3.73614×10^{-35}
57.	1.88518×10^{-39}
64.	-9.71851×10^{-44}
71.	5.09651×10^{-48}
78.	-2.71022×10^{-52}
85.	1.45806×10^{-56}
92.	-7.92142×10^{-61}
99.	4.33982×10^{-65}

In[63]:=
```
ListPlot[seq, PlotRange → {-.000001, .000001}]
```

Out[63]=



In[64]:=
```
Limit[f[n], n → Infinity] // N
```

Out[64]= 0.

Limit exists as the sequence converges to 0.

In[67]:=
```
13/8 + Sum[f[n], {n, 0, Infinity}]
```

Out[67]= $\frac{13}{8} + \frac{1}{8}\left(-9 + 4\sqrt{5}\right)$

```
In[68]:= 13/8 + Sum[f[n], {n, 0, Infinity}] // N
Out[68]= 1.61803
```

The above expression has a finite sum as expected as the sequence converges to 0.