

# Exam 3 review problems

MAT 121, SPRING 2016

## Sets

Define the sets  $A = \{3, 6, 9, 12\}$  and  $B = \{2, 4, 6, 8, 10, 12\}$ , and  $C = \{-2, -1\}$ . Write the following sets.

1.  $A \cup B$
2.  $A \cap B$
3.  $B \cap C$
4.  $A \cap B'$

## Basic probability

5. Suppose we draw one card from a deck.
  - (a) What is the probability that we draw a black card?
  - (b) What is the probability that we draw a queen?
  - (c) What is the probability that we draw a black queen?
  - (d) What is the probability that we draw a card that is black or a queen?
  - (e) What is the probability that we draw a card that is neither black nor a queen?
6. We roll two dice.
  - (a) What is the probability that the first die shows an even number?
  - (b) What is the probability that the sum of the two dice is 10?
  - (c) What is the probability that the first die shows an even number and the sum of the two dice is 10?
  - (d) What is the probability that the first die is even or the sum of the two dice is 10?
7. A computer chip is made by connecting one circuit from factory A with a second circuit from factory B. The circuits from factory A have a 25% defect rate, and factory B has a 10% defect rate. What is the probability that a computer chip chosen at random has at least one defect?

## Permutations and combinations

8. In a class of 9 children, 3 children will be chosen for the roles of president, vice-president, and treasurer. In how many different ways can these roles be assigned?
9. From a team of 10 people, we are going to choose 3 people to fly to Australia. In how many ways can this group be chosen?

10. The letters A, B, C, and D are put in a random order to make a secret code.
- (a) How many different codes are there?
  - (b) How many of these codes begin with the letter A?
  - (c) If we choose a code at random, what is the probability that it begins with the letter B?
  - (d) What is the probability that a code begins with a letter other than B?
11. Five cards will be dealt from a deck to make a poker hand.
- (a) How many different five-card hands are there?
  - (b) How many different hands contain all clubs?
  - (c) How many different hands contain 3 aces and 2 kings?
  - (d) Find the probabilities for these two hands. Which one is more likely?
  - (e) What is the probability that a hand contains at least one card that is not a club?

### **Conditional probability and Bayes' formula**

12. We have a jar with 1 blue chip and 4 white chips. We draw two chips from the jar.
- (a) What is the probability that the first chip is blue? What is the probability that the first chip is white?
  - (b) If the first chip drawn is blue, what is the probability that the second chip is also blue?
  - (c) If the first chip is white, what is the probability that the second chip is blue?
  - (d) What is the probability of drawing two white chips?
  - (e) What is the probability of drawing two blue chips?
  - (f) What is the probability of drawing a white chip first and then a blue chip?
  - (g) What is the probability that the second chip is blue?
  - (h) What is the probability that the second chip is white?
  - (i) If the second chip is blue, what is the probability that the first chip was white?
  - (j) If the second chip is white, what is the probability that the first chip was blue?
13. First, we flip a coin. If the coin shows heads, we draw a card from the deck at random. If the coin shows tails, we pull out the queen of hearts.
- (a) If the coin shows heads, what is the probability of getting a heart?
  - (b) If the coin shows tails, what is the probability of getting a heart?
  - (c) What is the probability of flipping heads and getting a heart?
  - (d) What is the probability of flipping tails and getting a heart?
  - (e) What is the probability of getting a heart?
  - (f) If we have a heart, what is the probability that we flipped a tail?
  - (g) If we have a heart, what is the probability that we flipped a head?

14. It is estimated that 3 out of every 10,000 people have a certain blood disease. A new test for this disease has been developed that is 98% accurate. If a randomly-chosen patient gets a positive test result, what is the probability that he has the disease?

**Expected value**

15. An urn contains 10 green chips and 5 red chips. You draw one chip. If the chip is red, you win \$8. If the chip is green, you pay \$5. What is the expected value of drawing a chip?
16. To play a card game you must pay \$5. You draw a card from a deck, and if it is a king you win \$50. What is the expected value for one play of the game?
17. The cost of developing a new product is \$2.5 million. There is a 12% chance that this product will be successful. If it does succeed, the company will receive \$30 million. Would you recommend that the company develop the product?