Integral equations

1. $y' + 110y + 1000 \int_0^t y(\tau)d\tau = 90 - 90\,\mathcal{U}(t-1),\ y(0) = 0$

2. $y' + 150y + \dfrac{1}{20000} \int_0^t y(\tau)d\tau = f(t),\ y(0) = 0,$ where $f(t) = \begin{cases} 100t & \text{if } 0 \le t < 1 \\ 0 & \text{if } t \ge 1 \end{cases}$

ANSWERS

1. $y(t) = \begin{cases} e^{-10t} - e^{-100t} & \text{if } t < 1 \\ e^{-10t} - e^{-100t} - e^{-10(t-1)} + e^{-100(t-1)} & \text{if } t \geq 1 \end{cases}$

2. $y(t) = \dfrac{1}{50} \left[\left(1 - e^{-50t}\right)^2 - \mathcal{U}(t-1)\left(1 + 98e^{-50(t-1)} - 99e^{-100(t-1)}\right) \right]$