# Eigenvalues and eigenvectors

MAT 265, Spring 2017

Find all eigenvalues of the following matrices. Find one eigenvector corresponding to each eigenvalue.

1. $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$
2. $B = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$
3. $C = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$
4. $D = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$
5. $A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$

Find the solution to each system of differential equations or IVP.

6. $\dfrac{dx}{dt} = y$
   $\dfrac{dy}{dt} = 4x - 3y$
7. $\dfrac{dx}{dt} = y$
   $\dfrac{dy}{dt} = 6x - 7y$
   $x(0) = 2,\ y(0) = -1$

ANSWERS

1. $\lambda_1 = 1,\ \lambda_2 = 9$ with possible eigenvectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ \mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

2. $\lambda_1 = 1,\ \lambda_2 = 3$ with possible eigenvectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ \mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

3. $\lambda_1 = 0,\ \lambda_2 = 5$ with possible eigenvectors $\mathbf{v}_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix},\ \mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

4. $\lambda_1 = i,\ \lambda_2 = -i$ with possible eigenvectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ i \end{bmatrix},\ \mathbf{v}_2 = \begin{bmatrix} 1 \\ -i \end{bmatrix}$

5. $\lambda_1 = 0, \lambda_2 = 1, \lambda_3 = 3$ with possible eigenvectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

6. $x(t) = C_1 e^t + C_2 e^{-4t}$
   $y(t) = C_1 e^{2t} - 4C_2 e^{-4t}$

7. $x(t) = 3e^{2t} - e^{-5t}$
   $y(t) = 2e^{2t} - 3e^{-5t}$