Quiz 5

Name:

For a population of fish in a lake, we assume the growth rate is proportional to the number of fish present. Initially, the population is 10 fish. After four months, there are 50 fish in the lake.

1. Write a differential equation to describe this population.

2. Based on your model, how many fish do we expect to see in 10 months? in 18 months? in 10 years?

3. Consider your answers to the last question. Under what circumstances is this differential equation a good model for the population we are discussing?

Next we assume that there is a maximum population M that can be supported by the resources available in the environment. This leads to the equation

$$\frac{dP}{dt} = kP(M - P),$$

called the *logistic equation*. For the fish model, we suppose that the maximum population of the lake is 1000 fish. Use the same growth constant k that you found in for the last model.

4. Identify any equilibrium solutions to this equation.

5. Plot the phase portrait and sketch a set of typical solution curves.

6. Suppose our initial population of fish is 1050. What will happen in the long term? What if the initial population is 800?

7. Find an explicit general solution to the differential equation. Then apply the initial condition $P(0) = 10$.

8. Using the model we created in the last question, how many fish do we expect to see in 10 months? in 18 months? in 10 years?