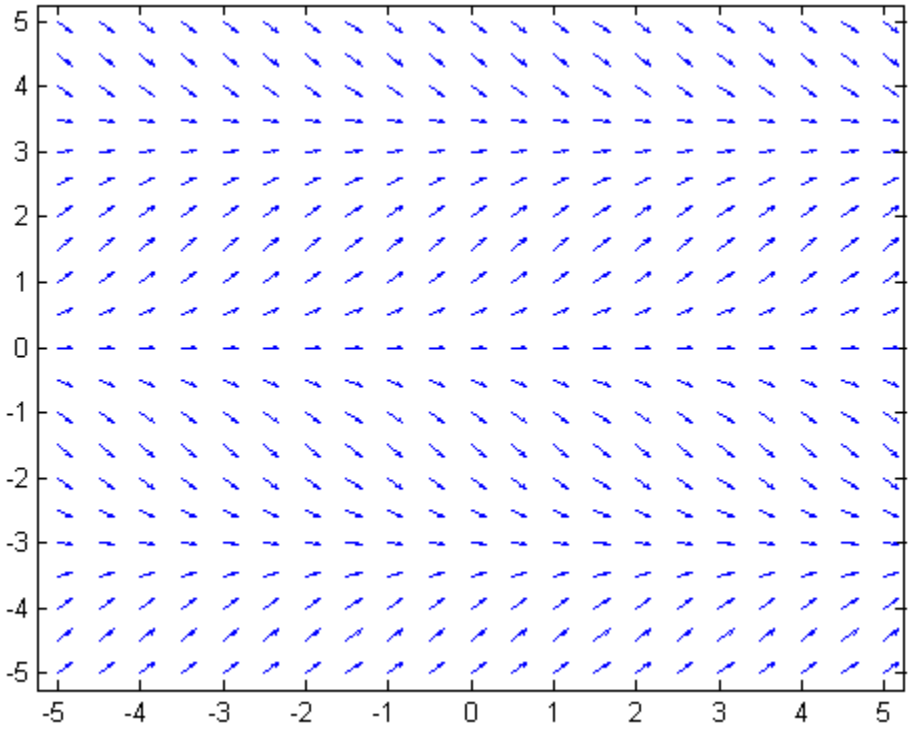
Our goal is to describe the solution to the IVP

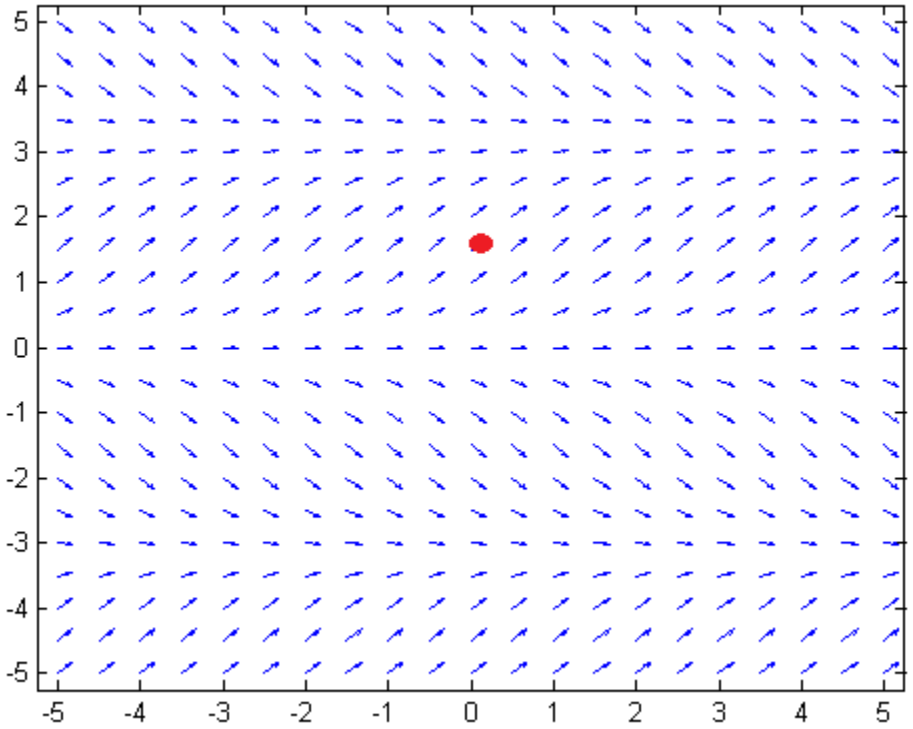
$$\frac{dy}{dt} = \sin y\,, \qquad y(0) = \frac{3}{2}.$$

We begin by evaluating $\frac{dy}{dt}$ at a grid of (x, t) values. The value of the derivative tells us the slope of the solution curve at that point, and we mark the slope with a little arrow of the correct steepness.

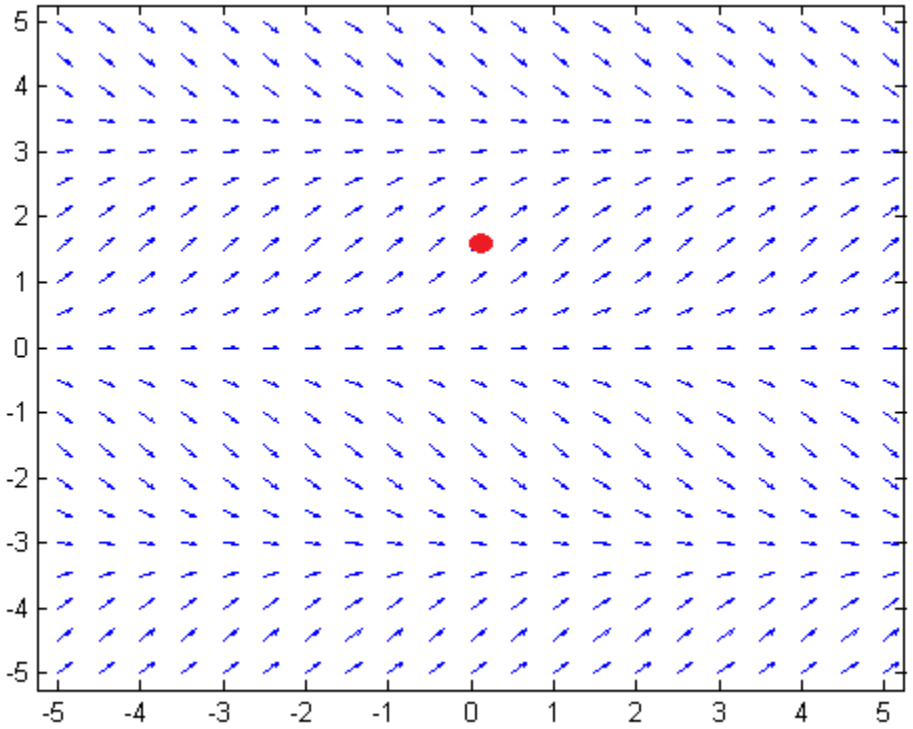
Here is the slope field for this particular problem.



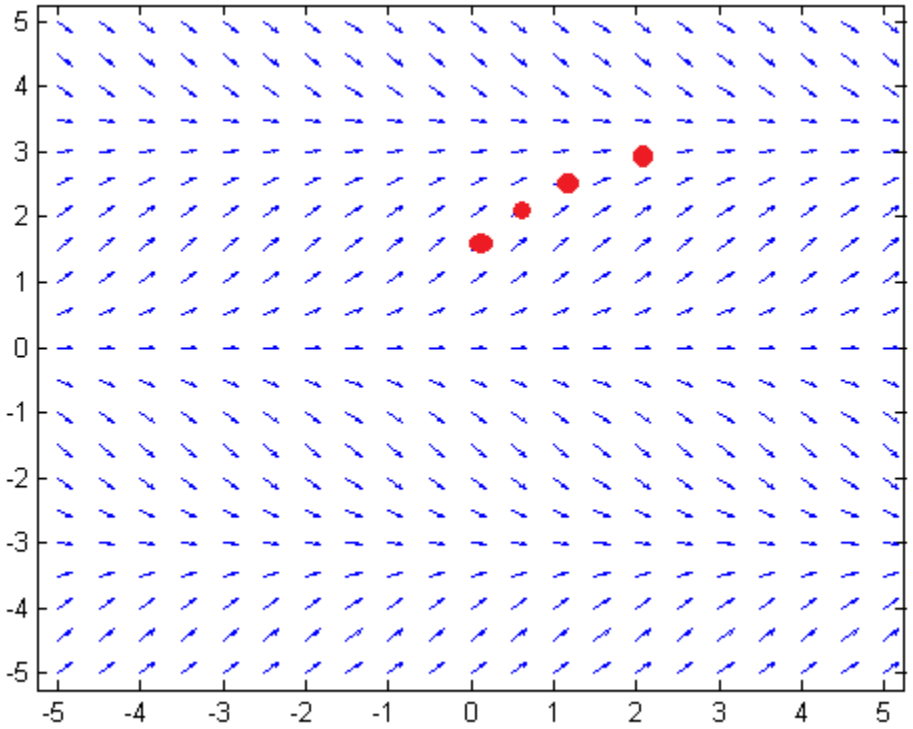
We begin constructing a solution curve by plotting the value of the initial condition, $(t, y) = \left(0, \frac{3}{2}\right)$.

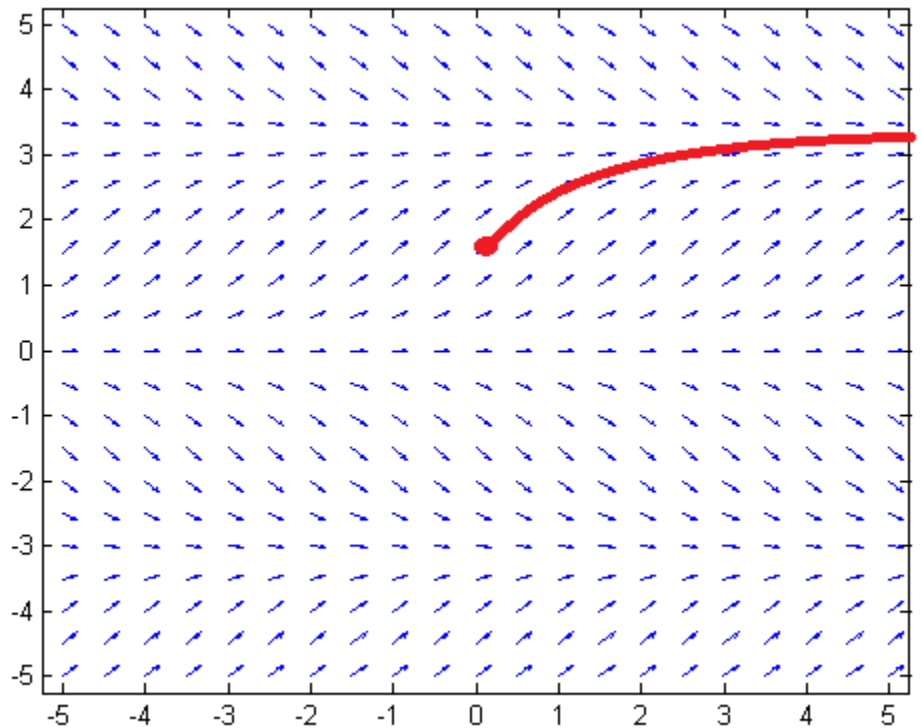


The little arrows tell us the slope of the solution at this initial point.
Follow the indicated direction until we hit another arrow.

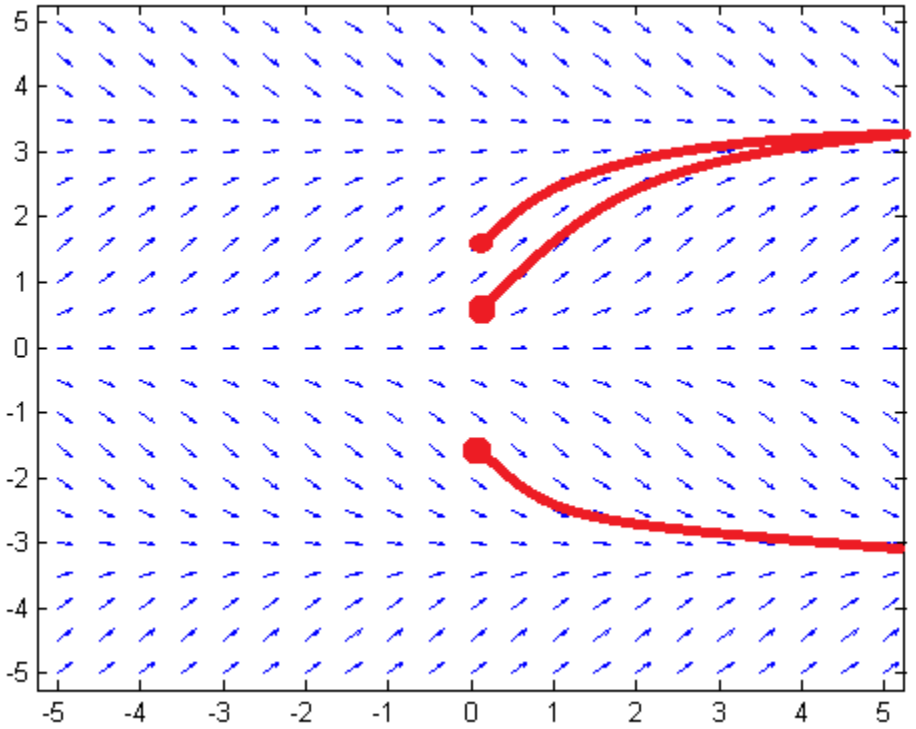


Continue the process until we have enough sample points to see the shape of the solution curve.




5
4
3
2
1
0
-1
-2
-3
-4
-5
-5
-4
-3
-2
-1
0
1
2
3
4
5

Different initial conditions lead to solutions with different long-term behavior.



Solutions that are unchanging in time are called equilibrium solutions.