Laplace transforms with piecewise functions

Use the Laplace transform to solve the following problems.

1. $x'' + 4x = f(t),\ x(0) = 0,\ x'(0) = 0$, where
$$f(t) = \begin{cases} \cos 2t & \text{if } 0 \le t < 2\pi \\ 0 & \text{if } t \ge 2\pi. \end{cases}$$

2. $x'' + 4x' + 4x = f(t),\ x(0) = 0,\ x'(0) = 0$, where
$$f(t) = \begin{cases} t & \text{if } 0 \le t < 2 \\ 0 & \text{if } t \ge 2. \end{cases}$$

ANSWERS

1. $f(t) = \begin{cases} \frac{1}{4} t \sin 2t & \text{if } 0 \le t < 2\pi \\ \frac{1}{2} \pi \sin 2t & \text{if } t \ge 2\pi \end{cases}$

2. $f(t) = \begin{cases} -\frac{1}{4} + \frac{t}{4} + \frac{t+1}{4} e^{-2t} & \text{if } 0 \le t < 2 \\ \frac{t+1}{4} e^{-2t} + \frac{3t-5}{4} e^{-2(t-2)} & \text{if } t \ge 2 \end{cases}$