Exam 2

MAT 265, Spring 2017

1. (20 points) A tank initially contains 1000 liters of pure water. A solution containing 0.05 kilograms of salt per liter is pumped into the tank at a rate of 5 liters per minute. A second solution containing 0.04 kilograms of salt per liter is pumped into the tank at a rate of 10 liters per minute. The well-mixed solution is drained from the tank at a rate of 15 liters per minute. How many kilograms of salt are in the tank after one hour?

2. (5 points) On the interval $I = (0, \infty)$, two solutions of the differential equation

$$x^2y'' - xy' + y = 0$$

are the functions

$$y_1(x) = x, \qquad y_2(x) = x \ln x.$$

(a) Use the Wronskian to show that these solutions are linearly independent on I.

(b) Give the general solution to the differential equation.

3. (20 points) Find the general solution for the following homogeneous problems.

(a) $4y'' + 12y' + 9y = 0$

(b) $y'' + 6y = 0$

(c) $2y'' + 2y' + y = 0$

(d) $y'' - 3y' - 10y = 0$

4. Find the solution to each differential equation or IVP.

   (a) (10 points) $y'' - 3y' = 6e^x,\ y(0) = 0, y'(0) = 0$

(b) (20 points) $y'' - 2y' + y = e^t \ln t$

(c) (10 points) $y'' - 11y' + 30y = -3e^{5x}$

5. (20 points) A mass of 1 kilogram is attached to a spring with a spring constant of 20 newtons per meter. The system is submerged in a liquid that imparts a damping force with a magnitude equal to 8 times the velocity of the mass. At time $t = 0$, the mass is pulled 0.5 meters below its equilibrium position and given an initial speed of 1 meter per second in the downward direction. Find the equation of motion.