Conditional probability

MAT 123, Summer 2016

We use the formula for conditional probability in two ways.

$$P(A\,|B) = \frac{P(A \cap B)}{P(B)}$$
$$P(A \cap B) = P(B)P(A\,|B)$$

1. We have a jar with 5 blue chips and 6 yellow chips. We draw two chips from the jar.
   (a) What is the probability that the first chip is blue? What is the probability that the first chip is yellow?
   (b) If the first chip drawn is blue, what is the probability that the second chip is also blue?
   (c) If the first chip is yellow, what is the probability that the second chip is blue?
   (d) What is the probability of drawing two blue chips?
   (e) What is the probability of drawing a yellow chip first and then a blue chip?
   (f) What is the probability that the second chip is blue?
   (g) What is the probability that the second chip is yellow?
2. We roll two dice.
   (a) What is the probability that the first die is a six?
   (b) What is the probability that the sum of the two dice is greater than or equal to 10?
   (c) If the first die is a six, what is the probability that the sum of the two dice is greater than or equal to 10?
   (d) If the sum of the two dice is greater than 10, what is the probability that the first die is a six?
   (e) What is the probability that the first die is six and the sum of the two dice is greater than or equal to 10?
3. We flip a coin. If the coin shows heads, we roll a die. If the coin shows tails, we turn the die so that it reads 6.
   (a) What is the probability of getting heads on the coin?
   (b) If the coin is heads, what is the probability of getting a 6 on the die?
   (c) If the coin shows tails, what is the probability of getting a 6?
   (d) What is the probability of getting heads and rolling a 6?
   (e) What is the probability of getting tails and rolling a 6?
   (f) What is the probability of getting a 6?

4. Two factories make parts for an automaker. A part is equally likely to come from either of the factories. Factory A has a defect rate of 1%, and factory B has a defect rate of 4%.
   (a) What is the probability that a part was made in factory A?
   (b) If a part was made in factory A, what is the probability that it is defective?
   (c) If a part was made in factory B, what is the probability that it is defective?
   (d) What is the probability that a part is defective and is made in factory A?
   (e) What is the probability that a part is defective and is made in factory B?
   (f) What is the probability that a part is defective?
   (g) What is the probability that a part is not defective?

5. An urn contains 5 blue chips, 3 white chips, and 7 red chips. Two chips are drawn.
   (a) What is the probability that the first chip is blue? White? Red?
   (b) If the first chip is blue, what is the probability that the second chip is blue?
   (c) If the first chip is blue, what is the probability that the second chip is not blue?
   (d) What is the probability that both chips are blue?
   (e) What is the probability that both chips are the same color?
   (f) What is the probability that the two chips are different colors?

ANSWERS

1. (a) $5/11$, $6/11$
   (b) $2/5$
   (c) $1/2$
   (d) $2/11$
   (e) $3/11$
   (f) $5/11$
   (g) $6/11$
2. (a) $1/6$
   (b) $1/6$
   (c) $1/2$
   (d) $1/2$
   (e) $1/12$
3. (a) $1/2$
   (b) $1/6$
   (c) 1
   (d) $1/12$
   (e) $1/2$
   (f) $7/12$
4. (a) $1/2$
   (b) 1% or $1/100$
   (c) 4% or $1/25$
   (d) $1/200$
   (e) $1/50$
   (f) $1/40$
   (g) $39/40$
5. (a) $1/3$, $1/5$, $7/15$
   (b) $2/7$
   (c) $5/7$
   (d) $2/21$
   (e) $34/105$
   (f) $71/105$