

# Final exam

MAT 265, SPRING 2017

1. (10 points) Find the general solution of the equation.

(a)  $\frac{dy}{dt} = ty^2 + t$

(b)  $y'' - 6y' + 9y = 0$

2. (12 points) Initially, a tank contains 5000 liters of pure water. A solution containing 2 kilograms of salt per liter is pumped into the tank at a rate of 15 liters per minute. A well-stirred mixture is drained from the tank at the same rate, 15 liters per minute. How many kilograms of salt are in the tank after one hour?

3. (22 points) A mass of 3 kilograms is connected to spring with constant 6 newtons per meter. The system is free from damping and friction. The mass experiences a driving force of strength  $\cos 2t$ , where  $t$  is time in seconds. This force is directed upward at time  $t = 0$ . At the same time ( $t = 0$ ), the mass is in its equilibrium position with a velocity of 1 meter per second downward. Find the equation of motion for the system.

4. (10 points) Find the Laplace transform of each function.

(a)  $f(t) = te^{-4t}$

(b)  $g(t) = 3 \sin \frac{t}{2}$

(c)  $h(t) = \begin{cases} 3t & \text{if } t < 2 \\ 6 & \text{if } t \geq 2 \end{cases}$

5. (16 points) Find the inverse Laplace transform of each function.

(a)  $H(s) = \frac{10}{s + 12}$

(b)  $F(s) = \frac{s + 2}{s^2 + 2s + 5}$

(c)  $Y(s) = \frac{1}{s^2 + 4}e^{-5\pi s}$

6. (14 points) Use the power series method to find the general solution to

$$(\cos t) y' + y = 0.$$

Write the first four nonzero terms of your answer.

7. (16 points) Find the general solution to the system.

$$\begin{aligned}\frac{dx}{dt} &= x - 8y \\ \frac{dy}{dt} &= x - 3y\end{aligned}$$

8. Find the solution to the IVP (10 points bonus)

$$\frac{dx}{dt} = 2x + 2y$$

$$x(0) = 0$$

$$\frac{dy}{dt} = x + 3y$$

$$y(0) = 1$$