1. Consider the initial value problem

$$y' + y = x + 1,\ y(0) = 0$$

Is this problem guaranteed to have a unique solution?

2. Consider the initial value problem

$$y'x = 2y,\ y(0) = 1.$$

Is this problem guaranteed to have a unique solution?

3. Consider the initial value problem

$$y' - y^2 - 1 = 0,\ y(0) = 0$$

Is this problem guaranteed to have a unique solution?

4. Consider the initial value problem

$$y' = 2\sqrt{|y|},\ y(0) = 0.$$

   (a) Show that the function $y(x) = 0$ is a solution to the IVP.

   (b) Show that the function $y(x) = x|x|$ is a solution to the IVP.

   (c) The previous two parts show that this problem does not have a unique solution. (It has two.) Explain why this result is consistent with the existence and uniqueness theorem that we discussed in class.