Final exam review

Here are some review questions on the topics that we discussed during the last two weeks of class. You should also study the exam review materials that we used for Exams 1 to 3.
Riemann sums Indefinite integral Definite integral u-substitution Average value of a function Integral as a function

Integration by substitution

1. $\int (1-x^2)^5 x \, dx$

2. $\int_1^2 x^2 e^{x^3} \, dx$

3. $\int \dfrac{x \, dx}{(x^2+1)^3}$

4. $\int_0^{\pi/2} \cos^5 \theta \sin \theta \, d\theta$

5. $\int e^x \sin e^x \, dx$

6. $\int_1^e \dfrac{(\ln x)^3}{x} dx$

7. $\int x^4 \sin x^5 dx$

8. $\int \dfrac{dx}{\sqrt{2x+5}}$

9. $\int \dfrac{dx}{x^2+25}$

Basic properties of integration

Suppose we know that $\int_0^3 f(t) \, dt = 2$, $\int_3^6 f(t) \, dt = -5$, and $\int_3^6 g(t) = 1$. Evaluate the following expressions.

10. $\int_0^3 5f(t) \, dt$

11. $\int_6^3 f(u) \, du$

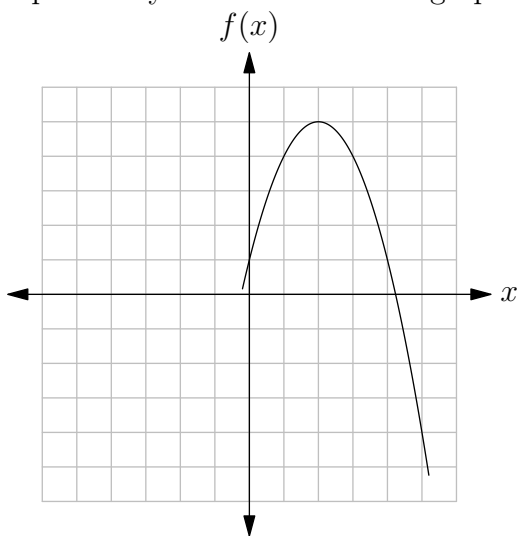12. $\int_3^6 (f(t) - 2g(t)) \, dt$

13. $\int_3^3 g(t) \, dt$

14. $\int_0^6 f(t) \, dt$

Riemann sums

15. The graph of $f(x) = -x^2 + 4x + 1$ is shown in the figure. Use a Riemann sum to estimate the area under this curve over the interval $[1, 5]$. Use $n = 4$ subintervals to calculate the sum. Use the left endpoint, right endpoint, or midpoint method. Draw the rectangles that correspond to your method on the graph.

$f(x)$

Average of a function

Find the average value of each function on the indicated integral.

16. $f(x) = \dfrac{1}{x^2 + 1}$ on $[-1, 1]$

17. $g(\theta) = \cos 2\theta$ on $[-\pi/4, \pi/4]$

Functions defined as definite integrals

Use the fundamental theorem of calculus to find the derivative of each function given below.

18. $f(x) = \displaystyle\int_3^x \left(t^2 + t + 2\right) dt$

19. $g(x) = \displaystyle\int_x^0 \dfrac{dp}{p^2 + 1}$

20. $h(x) = \displaystyle\int_1^{e^x} \dfrac{dx}{x}$