Exam 2 review problems

MAT 265, Spring 2017

1. Consider the initial value problem

$$x^2y'' - 2xy' + 2y = 0,\ y(1) = 3,\ y'(1) = 1.$$

   (a) Use the theorem from class to show that this IVP is guaranteed to have a unique solution. Find the largest interval I where this solution is guaranteed to exist.

   (b) Verify that the function $y_1 = x$ is a solution to the differential equation in this problem.

   (c) Show the same thing for the function $y_2 = x^2$.

   (d) Use the Wronskian to show that y_1 and y_2 are linearly independent.

   (e) Assemble the general solution to this differential equation and then find the solution to the IVP.

2. Describe the solutions to each boundary value problem.

   (a) $y'' + 4y = 0,\ y(0) = 0, y\left(\frac{3\pi}{4}\right) = 0$

   (b) $y'' + 4y = 0,\ y(0) = 0, y(\pi) = 0$

3. A mass of 2 kilograms stretches a spring by a distance of 39.2 centimeters. The system is suspended in a liquid that provides 12 newtons of resistance for every meter per second of velocity. At time 0, the weight is in its equilibrium position. It is struck with a hammer that gives it an instantaneous upward velocity of 8 meters per second. Find the equation of motion for the mass.

4. A mass of 1 kilogram is attached to a spring with spring constant $k = 5$ newtons per meter. The system is subject to damping of 3 newtons for every meter per second of velocity. The mass is driven by a motor that gives a force $f(t) = -4\cos 5t$ at t seconds. When $t = 0$, the mass is at rest in its equilibrium position. Find the equation of motion for the mass. Identify the transient and the steady-state solutions.