Exam 2

MAT 255, Fall 2016

Please start each question at the top of a new page.

1. (7 points) Find the determinant of the matrix $\begin{bmatrix} 1-x & 1 & 0 \\ 1 & 1-x & 1 \\ 0 & 1 & 1-x \end{bmatrix}$.

2. (5 points) Find all values of k such that the matrix $\begin{bmatrix} k^2 & k \\ 1 & k \end{bmatrix}$ is invertible.

3. (10 points) Let H be the set consisting of all vectors in $\mathbf{R}^2$ that have the form $\begin{bmatrix} 2t \\ -t \end{bmatrix}$ where t is a **non-negative** real number. Is H is a subspace of $\mathbf{R}^2$? Justify your answer.

4. (15 points) Determine if each of the following sets is a basis for $\mathbf{R}^2$. Explain the reasons for your conclusions.
   (a) $\left\{ \begin{bmatrix} 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$
   (b) $\left\{ \begin{bmatrix} 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix} \right\}$
   (c) $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 11 \end{bmatrix} \right\}$
   (d) $\left\{ \begin{bmatrix} 15 \\ 5 \end{bmatrix}, \begin{bmatrix} -6 \\ -2 \end{bmatrix} \right\}$

5. (10 points) Are the polynomials

$$2+4t^3 \qquad 3+6t+6t^3$$
$$4+7t+8t^3 \qquad 1+9t^2+2t^3$$

   linearly independent?

6. (18 points) Find a basis set for the nullspace, columnspace, and rowspace of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 6 & 13 & 20 & 27 \\ 9 & 26 & 44 & 62 \end{bmatrix}.$$

   What is the rank of the matrix A?

7. (15 points) Suppose that A is a 4×7 matrix with four pivot columns. What is the rank of this matrix? Give the dimensions for the nullspace, rowspace, and columnspace. Is Null(A) the same as the entire space $\mathbf{R}^m$ for some integer m? Is Col(A) $= \mathbf{R}^n$ for some integer n? Does the equation

$$A\vec{x} = \vec{b}$$

have a solution for every possible right-hand side vector $\vec{b}$? Does the equation

$$A\vec{x} = \vec{0}$$

have a solution other than the trivial solution $\vec{x} = \vec{0}$?

8. (20 points) Let V be a vector space, and let $\vec{u}$ be any vector in V. Prove that the additive inverse $-\vec{u}$ is unique. You will be graded on both the clarity of your writing and the correctness of your argument.