Exam 3 review problems

MAT 265, Spring 2017

Solve the equation using the power series method. Write out the first three nonzero terms of each linearly independent solution.

1. $y'' + x^2 y' + x^2 y = 0$

2. $y'' + e^{-x} y = 0$

Use the table to find the Laplace transform of each function.

3. $f(t) = 3e^{6t}$

4. $g(t) = 3\cos t + \sin(3t)$

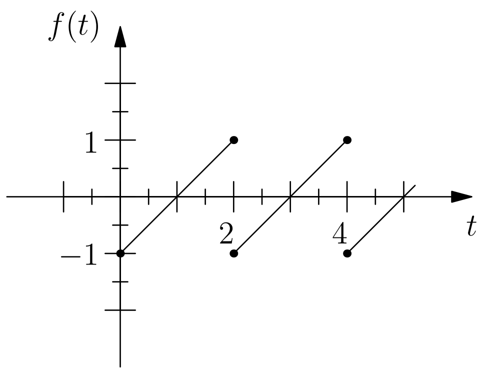
5. $h(t) = (t-1)^2$

6. $f(t) = e^{2t}\,\mathcal{U}(t-3)$

7. $g(t) = \begin{cases} \cos 2t & \text{if } 0 \le t < 2\pi \\ 0 & \text{if } t \ge 2\pi \end{cases}$

8. $h(t) = \displaystyle\int_0^t e^{\tau}\,\sin(t-\tau)\,d\tau$
   Do not evaluate the integral before finding the transform.

9. $f(t)$ is the periodic function shown in the figure below.



Find the inverse Laplace transform of each function.

10. $F(s) = \dfrac{5}{s-6} - \dfrac{6s}{s^2+9}$

11. $G(s) = \dfrac{s^2+9s+2}{(s-1)^2(s+3)}$

12. $H(s) = \dfrac{3s+2}{s^2+2s+10}$

13. $F(s) = \left(\dfrac{s+1}{s^2+4}\right)e^{-s}$

SOLUTIONS

1. $y(x) = c_0\left(1 - \frac{x^4}{12} + \frac{x^7}{126} + \dots\right) + c_1\left(x - \frac{x^4}{12} - \frac{x^5}{20} + \dots\right)$

2. $y(x) = c_0\left(1 - \frac{x^2}{2} + \frac{x^3}{6} + \dots\right) + c_1\left(x - \frac{x^3}{6} - \frac{x^4}{12} + \dots\right)$

3. $\frac{3}{s-6}$

4. $\frac{3s}{s^2+1} + \frac{3}{s^2+9}$

5. $\frac{s^2-2s+2}{s^3}$

6. $\frac{e^{-3(s-2)}}{s-2}$

7. $\frac{s}{s^2+4} - \frac{e^{-2\pi s}}{s^2+4}$

8. $\frac{1}{(s-1)(s^2+1)}$

9. $\frac{1}{1-e^{-2s}}\left(\frac{1}{s^2} - \frac{1}{s} - \frac{e^{-2s}}{s} - \frac{e^{-2s}}{s^2}\right)$

10. $5e^{6t} - 6\cos 3t$

11. $2e^t + 3te^t - e^{-3t}$

12. $3e^{-t/2}\cos\left(\frac{\sqrt{39}}{2}t\right) + \frac{1}{\sqrt{39}}e^{-t/2}\sin\left(\frac{\sqrt{39}}{2}t\right)$

13. $\begin{cases} 0 & \text{if } t < 1 \\ \cos(2t-2) + \frac{1}{2}\sin(2t-2) & \text{if } t \geq 1 \end{cases}$