Homework 5b

MAT 255, Fall 2016

1. Let A be an $\ell \times m$ matrix, and let B, C be $m \times n$ matrices. Show that $A(B+C) = AB+AC$.
2. Let A be an $\ell \times m$ matrix, and let B be an $m \times n$ matrix. Show that $(AB)^T = B^T A^T$.
3. Let A, B, C be matrices. Suppose that $AB = AC$ and that the matrix A is invertible. What assumptions are necessary about the sizes of the matrices in order for these statements to hold? Show the steps we use to conclude that $B = C$.
4. Let A be a 2×2 matrix with entries
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$
Prove that A has an inverse if and only if $ad - bc \neq 0$. You will use the following ideas in your proof.
   (a) If $ad - bc = 0$, then the equation $A\mathbf{x} = \mathbf{0}$ has more than one solution. First, consider the case when $a = b = 0$. Next, look at the case where a and b are not both 0.
   (b) If $ad - bc \neq 0$, then the matrix
$$\frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$
satisfies the definition of an inverse.
5. Suppose $T : \mathbb{R}^n \to \mathbb{R}^n$ is a linear transformation with the property that there are distinct vectors $\mathbf{u}, \mathbf{v}$ in $\mathbb{R}^n$ such that $T(\mathbf{u}) = T(\mathbf{v})$. Is T an onto transformation? Justify your answer.