Exam 2 review problems

MAT 123, Summer 2016

Basic systems of equations

Solve each system.

1. $3x + 4y = 7$
   $-5x + 2y = 10$

2. $x - 4y = 9$
   $2x + 5y = 5$

3. $x + 2y = 1$
   $3x - 6y = 4$

Gaussian elimination

Write the system in matrix form.

4. $x - 2y + 3z = 12$
   $2x - 4z = 8$
   $3y + z = y$

5. $-x_1 + 2x_2 + 2x_3 - 2x_4 = -10$
   $x_1 + x_5 = 25$

Solve the system of equations using Gaussian elimination. Clearly indicate your row operations at each step.

6. $4x + 2y = 11$
   $3x - y = 2$

7. $2x + y = 1$
   $3x - 6y = 4$

The following matrices are in reduced row echelon form. Write the solutions for the corresponding system of equations. Use free variables where needed.

8. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 17 \end{array}\right]$

9. $\left[\begin{array}{cccc|c} 1 & 0 & 3 & 4 & 10 \\ 0 & 1 & -2 & -1 & 15 \end{array}\right]$

10. $\left[\begin{array}{ccccc|c} 1 & -4 & 0 & 0 & 0 & -16 \\ 0 & 0 & 1 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$

Matrices

Define the matrices $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & -2 \\ -2 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 & 6 \\ 1 & -2 & 0 \\ 0 & 1 & -3 \end{bmatrix}$, $C = \begin{bmatrix} -2 & 0 \\ 1 & 3 \end{bmatrix}$, and $D = \begin{bmatrix} 0 & 1 \\ -1 & 0 \\ 0 & 1 \end{bmatrix}$. Evaluate each of the following expressions, or state that the answer does not exist.

10. $A + B$
11. $-A$
12. $B - C$
13. BA
14. DC
15. CD

Find the inverse of the matrix, or show that the matrix is not invertible.

10. $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$
11. $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
12. $C = \begin{bmatrix} 6 & 9 \\ 4 & 6 \end{bmatrix}$
13. $D = \begin{bmatrix} 4 & -3 \\ 1 & -2 \end{bmatrix}$

Applications

14. An investor has \$15,000 to put into two different accounts. The first account pays 4% interest per year, and the second account pays 6% per year. In the first year, the total interest earned is \$690. Find the amount invested in each account.

15. A coffee shop creates a blend of two different types of coffee. Dark roast sells for \$12 per pound, and medium roast sells for \$9.50 per pound. How much of each type should be used to create 20 pounds valued at \$10.50 per pound?

16. A company manufactures and sells radios. They have found that the supply of radios is described by the equation
$$y = 90 + 30x,$$
where y is the price in dollars, and x is thousands of radios. The demand for the same item is given by
$$y = 200 - 25x.$$
If the price of radios is currently \$144, what do you predict will happen to this price in the future?

17. A car service wants to rent a fleet of 12 vans. In total, the vans need to transport 220 passengers at one time. There are three sizes available that can carry 10, 15, and 20 passengers. How many of each size should the company order?

Linear inequalities

Graph the solution to each inequality or system of inequalities. For the systems, locate the coordinates of any corner points.

18. $y < x - 3$

19. $3x + 4y \geq 12$

20. $x - y \leq 3$
    $x + y \leq 5$

21. $y \leq 3x - 4$
    $x \geq 0$

22. $2x + y \geq 4$
    $3x - 5y \leq 15$

23. $x \leq 5$
    $y \geq 1$