Exam 1, part 2

MAT 255, Fall 2016

Name:

You can use a calculator on the following problems.

3. (10 points) (a) Applying Gaussian elimination to the linear system

$$\begin{aligned} 3x_1 + 4x_2 + x_3 &= 9 \\ 6x_1 + 5x_2 + 2x_3 + x_4 &= 9 \end{aligned}$$

produces the reduced row echelon form

$$\left[\begin{array}{cccc|c} 1 & 0 & 1/3 & 4/9 & -1 \\ 0 & 1 & 0 & -1/3 & 3 \end{array}\right].$$

What are the solutions to the linear system? Use parameters for any free variables.

(b) Applying Gaussian elimination to the linear system

$$\begin{aligned} x_1 + 4x_2 + 3x_3 + 4x_4 &= 16 \\ 2x_1 + 8x_2 + 6x_3 + 8x_4 &= 8 \end{aligned}$$

produces the reduced row echelon form

$$\left[\begin{array}{cccc|c} 1 & 4 & 3 & 4 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right].$$

What are the solutions to the linear system? Use parameters for any free variables.

4. (20 points) (a) Describe the shape made by the span of the vectors $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$.

(b) Does the vector $\vec{v} = \begin{bmatrix} 4 \\ 2 \\ -2 \end{bmatrix}$ lie in Span $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \right\}$?

(c) For what values of the parameter k does the vector $\vec{v} = \begin{bmatrix} 0 \\ 4 \\ k \end{bmatrix}$ lie in Span $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \right\}$?

5. (18 points) Determine if the given vectors form a linearly independent set. Explain the reasons that support your conclusion.

(a) $\vec{u} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \vec{v} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, \vec{w} = \begin{bmatrix} 2 \\ 6 \\ 6 \end{bmatrix}$

(b) $\vec{u} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \vec{v} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, \vec{w} = \begin{bmatrix} 2 \\ 6 \\ 8 \end{bmatrix}$

(c) $\vec{u} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

6. (15 points) Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the following linear transformation. First the plane is rotated 90° in a counterclockwise (positive) direction. Next, the plane is reflected across the y-axis. Finally, the plane is rotated 45° in a clockwise (negative) direction.

   (a) Draw pictures that show the locations of the unit coordinate vectors $\vec{e_1}$ and $\vec{e_2}$ before and after the transformation T.

   (b) Write the matrix for this transformation.

   (c) Find the image of the vector $\vec{v} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$ under the transformation T.

7. (12 points) (a) Let A be a square matrix of size $n \times n$. Give the definition of the inverse matrix of A.

(b) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$, or show that the inverse does not exist.