Exam 1 review problems

MAT 255, Fall 2016

Be prepared to do the following problems without a calculator.

1. Solve the system of linear equations. Use a parameter to stand in for any free variable in the solution.

   (a)

$$\begin{aligned} 2y + 3z &= 3 \\ x + y + z &= 4 \\ 4x + 8y - 3z &= 35 \end{aligned}$$

   (b)

$$\begin{aligned} x_1 - 3x_2 + 2x_3 - x_4 + 2x_5 &= 2 \\ 3x_1 - 9x_2 + 7x_3 - x_4 + 3x_5 &= 7 \\ 2x_1 - 6x_2 + 7x_3 + 4x_4 - 5x_5 &= 7 \end{aligned}$$

   (c)

$$\begin{aligned} x_1 + 2x_2 - 3x_3 + 4x_4 &= 2 \\ 2x_1 + 5x_2 - 2x_3 + x_4 &= 1 \\ 5x_1 + 12x_2 - 7x_3 + 6x_4 &= 3 \end{aligned}$$

2. Define the matrices $A = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 & 4 \\ 3 & -2 & 6 \end{bmatrix}$.

   (a) Find AB.

   (b) Find BA.

3. Find the inverse of the matrix if it exists.

   (a) $A = \begin{bmatrix} 2 & -3 \\ 1 & 3 \end{bmatrix}$

   (b) $A = \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$

You may use a calculator on the following problems if you find it helpful.

3. For the system

$$\begin{aligned} x + 2y + z &= 3 \\ ay + 5z &= 10 \\ 2x + 7y + az &= b, \end{aligned}$$

   (a) Find the values of the pair (a, b) for which the system has a unique solution.
   (b) Find the values of the pair (a, b) for which the system has no solution.
   (c) Find the values of the pair (a, b) for which the system has infinite solutions.

4. Describe the shape made by span of the vector $\vec{v} = \begin{bmatrix} 1 \\ 0 \\ -5 \end{bmatrix}$.

5. Does the vector $\vec{v} = \begin{bmatrix} 3 \\ 0 \\ -4 \end{bmatrix}$ lie in Span $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} \right\}$?

6. Determine whether the given vectors form a linearly independent set.
   (a) $\vec{u} = \begin{bmatrix} 3 \\ -5 \end{bmatrix}, \vec{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$
   (b) $\vec{u} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \vec{v} = \begin{bmatrix} -2 \\ 6 \end{bmatrix}$
   (c) $\vec{u} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \vec{v} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} \; \vec{w} = \begin{bmatrix} 4 \\ 4 \\ 5 \end{bmatrix}$

7. Consider the following linear transformation T that acts on $\mathbf{R}^2$. First the plane is rotated $225°$ in a counterclockwise direction. Next, the plane is reflected across the y-axis. Finally, the plane is stretched by a factor of 3 in the vertical direction.
   (a) Draw two pictures that show the locations of the unit coordinate vectors $\vec{e_1}$ and $\vec{e_2}$ before and after the transformation T.
   (b) Write the matrix for this transformation.

8. Give one example of a linear transformation from $\mathbf{R}^2$ to $\mathbf{R}^2$ that is one-to-one. Give an example of a transformation that is not one-to-one. Here I am looking for a description of the geometry of a transformation and not the matrix.

9. Suppose that two $n \times n$ matrices A and B are both invertible. Show that their product AB is invertible.