Neighborhoods and limits

1. Let $f(x) = \frac{1}{2}x - 3$, and consider $\lim_{x \to 4} f(x) = -1$. Find a neighborhood of $x = 4$ that guarantees y is in the neighborhood $(-2, 0)$.

2. Let $g(x) = 3x + \frac{1}{2}$, and consider $\lim_{x \to -2} f(x) = -\frac{11}{2}$. Find a constant ε so $|x - (-2)| < \varepsilon$ guarantees that $\left| f(x) - \left(-\frac{11}{2} \right) \right| < \frac{1}{2}$.

3. Challenge: let $h(x) = x^2$, and consider $\lim_{x \to 3} f(x) = 9$. Find a constant ε so $|x - 3| < \varepsilon$ guarantees that $|f(x) - 9| < \dfrac{1}{2}$.