Final exam review

MAT 265, Spring 2017

First order problems

You should be able to use the following techniques to solve a first order equation.

Separation of variables
Integrating factors
Substitutions, including the Bernoulli method

Also be prepared to write the differential equation that corresponds to a written description. The specific applications that we worked with were population growth, radioactive decay, and mixtures of solutions. You should also be able to use a phase line to discuss the solutions of an autonomous problem.

Second order problems

Use the following techniques to solve a second order problem.

Method of undetermined coefficients
Variation of parameters

You will need to set up an equation for a mass-spring system. This includes understanding how the terms of the differential equation corresponds to the parameters of the physical system.

Laplace transforms and power series methods

Find the transform of a simple function using the definition, and use a table of transforms to solve a more complicate problem. Understand how to solve equations using unit step functions or periodic functions. You will also need to incorporate an impulse (like a hammer strike) into a mass-spring model.

Use the power series method to solve a differential equation with variable coefficients, similar to the question on Exam 3.

Systems of differential equations

Use the eigenvalue method to solve a system of differential equations. You should be prepared for problems with real distinct eigenvalues and problems with complex conjugate eigenvalues.

Here are some practice problems on the last topic.

1. $\dfrac{dx}{dt} = -4x + 2y$

   $\dfrac{dy}{dt} = 2x - 4y$

2. $\dfrac{dx}{dt} = x + 2y$
   $\dfrac{dy}{dt} = -2x + y$

3. $\dfrac{dx}{dt} = 3x$
   $\dfrac{dy}{dt} = 6x - 3y$
   $x(0) = 2,\ y(0) = -5$

4. $\dfrac{dx}{dt} = 4x + 5y$
   $\dfrac{dy}{dt} = -5x - 4y$
   $x(0) = 0,\ y(0) = 3$

SOLUTIONS

1. $x(t) = C_1 e^{-6t} + C_2 e^{-2t}$
   $y(t) = -C_1 e^{-6t} + C_2 e^{-2t}$

2. $x(t) = C_1 e^{t} \sin 2t - C_2 e^{t} \cos 2t$
   $y(t) = C_1 e^{t} \cos 2t + C_2 e^{t} \sin 2t$

3. $x(t) = 2e^{3t}$
   $y(t) = -7e^{-3t} + 2e^{3t}$

4. $x(t) = 5 \sin 3t$
   $y(t) = 3 \cos 3t - 4 \sin 3t$