

Delta functions

1. A mass of 1 kg is attached to a spring with constant 4 N/m. The system is free from damping. The mass is released from rest at 3 m above its equilibrium position. At $t = 2\pi$ seconds, the mass is struck with a hammer that provides an impulse of 8 N·s. Find the equation of motion.
2. Consider a mass-spring system with $m = k = 1$, $b = 0$, and $x(0) = x'(0) = 0$. At each of the times $0, \pi, 2\pi, 3\pi, \dots$, the mass is struck with a hammer that imparts an impulse of 1. Describe the motion of the mass.

SOLUTIONS

1. $x(t) \approx \begin{cases} 3 \cos 2t & \text{if } t \leq 2\pi \\ 5 \cos (2t - 0.9273) & \text{if } t > 2\pi \end{cases}$

2. For $n\pi < t < (n+1)\pi$,

$$x(t) = \begin{cases} \sin t & \text{if } n \text{ is even,} \\ 0 & \text{if } n \text{ is odd.} \end{cases}$$

The resulting function is called a half-wave rectification of $\sin t$.