Bayes' formula MAT 123, SUMMER 2016

To use Bayes' formula, we start with the definition of conditional probability.

$$P(B\,|A) = \frac{P(A \cap B)}{P(A)}$$

Then we expand each part of the fraction, again using conditional probability.

$$P(A \cap B) = P(A\,|B)P(B)$$
$$P(A) = P(A\,|B)P(B) + P(A\,|B')P(B')$$

1. We have a jar with 5 blue chips and 6 yellow chips. We draw two chips from the jar.

   (a) If the second chip is blue, what is the probability that the first chip was blue?

   (b) If the second chip is blue, what is the probability that the first chip was yellow?

   (c) If the second chip is yellow, what is the probability that the first chip was blue?

2. We flip a coin. If the coin shows heads, we roll a die. If the coin shows tails, we turn the die so that it reads 6.

   (a) If the die shows 6, what is the probability that we flipped a tail?

   (b) If the die shows 6, what is the probability that we flipped a head?

   (c) If the die shows 3, what is the probability that we flipped a tail?

3. Two factories make parts for an automaker. A part is equally likely to come from either of the factories. Factory A has a defect rate of 1%, and factory B has a defect rate of 4%. If a part is defective, what is the probability that it was made in factory B?

4. An urn contains 5 blue chips, 7 red chips, and 3 white chips. Two chips are drawn.

   (a) If the second chip is blue, what is the probability that the first chip was blue?

   (b) If the second chip is blue, what is the probability that the two chips are different colors?

ANSWERS

1. (a) $2/5$

   (b) $3/5$

   (c) $1/2$

2. (a) $6/7$

   (b) $1/7$

   (c) 0

3. $4/5 = 80\ \%$

4. (a) $12/21$

   (b) $9/21$