Differential equations honors project

MAT 265, Spring 2017

Finishing the 1D model

We finish the one dimensional model by finding functions for position on both parts of the trip. For the trip up, you will need to integrate from v_0 to v instead of just v_0 to 0. Then solve to get a formula for v as a function of t. We know that this formula gives the height y of the projectile for $0 \leq t \leq t^\star$.

I found it helpful to simplify the algebra along the way. For example, I wrote the constant α in place of $\sqrt{\frac{gk}{m}}$. You should make similar substitutions as needed throughout your work.

The formula for y can be used to calculate $y_{\max}$, the maximum height of the potato.

The steps are similar for the trip down. Solve your equation for v and integrate to get an expression for y. A substitution such as

$$u = t - t^\star$$

is helpful here.

Two dimensional motion

We can use the 1D model for the motion of the potato in the y-direction. For simplicity, assume that we are launching the potato at a 45° angle, and use this fact to split the initial velocity v_0 into $x-$ and $y-$components.

Write and solve the differential equation for the motion of the potato in the x-direction.

Finally, write a formula for the range R of the potato. Recall that the range is the distance traveled in the x-direction before the projectile hits the ground.

Results

Now we will put some numbers into the model. We will just aim for one significant digit since this is our first estimate. So use $c_w = 0.2$ as the drag coefficient. The density of air is about 1 kg/m^3. You need to to estimate the cross-sectional area and mass of your projectile, since I am not sure what you were using. Finally, the magnitude of acceleration due gravity is 10 m/s^2.

With these parameters, we can consider R as a function of v_0, the initial velocity. Make a graph that compares R and R_0 for different values of v_0. Here R_0 is the range that you calculate for the given parameters based on a Physics I model that does not include air resistance. R is the range that you found above using your differential equations setup.

E-mail me if you have any questions or once you are done. Did you make any measurements last fall when you tested your cannon?