Exam 2 review problems

MAT 255, Fall 2016

The Determinant

1. Calculate the determinant of the following matrices.

   (a) $\begin{bmatrix} 9 & 1 & 9 & 9 & 9 \\ 9 & 0 & 9 & 9 & 2 \\ 4 & 0 & 0 & 5 & 0 \\ 9 & 0 & 3 & 9 & 0 \\ 6 & 0 & 0 & 7 & 0 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{bmatrix}$

2. Use a determinant to show that the matrix $\begin{bmatrix} 1 & 5 \\ 3 & 1 \end{bmatrix}$ is invertible.

3. Find the area of the parallelogram with vertices at $(0,0), (-1,3), (4,-5)$, and $(3,-2)$.

Axioms of a vector space

Prove the following properties for a *general* vector space V. This means that your proof cannot assume that vectors are columns of numbers.

4. Prove that for any vector $\vec{u}$ in V, the additive inverse $-\vec{u}$ is unique.

5. Prove that $c\vec{0} = \vec{0}$ for any scalar c.

6. Prove that $-\vec{u} = (-1)\vec{u}$.

Subspaces and basis sets

7. Let H be the set of all vectors in $\mathbf{R}^2$ that have the form $\begin{bmatrix} 3t \\ 2+5t \end{bmatrix}$. Determine if H is a subspace of $\mathbf{R}^2$.

8. Let $A = \begin{bmatrix} 10 & -8 & -2 & -2 \\ 0 & 2 & 2 & -2 \\ 1 & -1 & 6 & 0 \\ 1 & 1 & 0 & -2 \end{bmatrix}$ and let $\vec{w} = \begin{bmatrix} 2 \\ 2 \\ 0 \\ 2 \end{bmatrix}$. Does $\vec{w}$ lie in Col(A)? Does it lie in Null(A)?

9. Determine if each of the following sets is a basis for $\mathbf{R}^3$.

(a) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$

(b) $\left\{ \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -4 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix} \right\}$

10. Define the vectors $\vec{b_1} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \vec{b_2} = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$. Then the set $B = \left\{ \vec{b_1}, \vec{b_2} \right\}$ is a basis for $\mathbf{R}^2$. Find the coordinates of the vector $\vec{x} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ with regard to the basis B.

11. Are the polynomials

$$1 - 2t^2 - t^3 \qquad t + 2t^3 \qquad 1 + t - 2t^2$$

linearly independent?

12. Find a basis set for the nullspace, columnspace, and rowspace of the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 & -6 & 8 \\ 1 & -2 & -4 & 3 & -2 \\ -7 & 8 & 10 & 3 & -10 \\ 4 & -5 & -7 & 0 & 4 \end{bmatrix}.$$

What is the rank of this matrix?

13. Suppose that A is a 4×6 matrix and that the dimension of Null(A) is 3. What is the dimension of the columnspace of A? Is Col(A) $= \mathbf{R}^3$? What is the dimension of the rowspace of A? Does the equation

$$A\vec{x} = \vec{b}$$

have a solution for every possible right-hand side vector $\vec{b}$?