Final exam review problems

MAT 255, Fall 2016

These are review problems for material from the third part of class. Also look at the review questions for exams one and two, since the final is cumulative.

1. Determine if the following vectors are eigenvectors of the matrix $A = \begin{bmatrix} 3 & -4 \\ 2 & -6 \end{bmatrix}$. If so, give the corresponding eigenvalue.

   (a) $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$

   (b) $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$

   (c) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

2. Find the characteristic polynomial of the matrix $A = \begin{bmatrix} 7 & -3 \\ 5 & -2 \end{bmatrix}$.

3. One eigenvalue of the matrix $M = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$ is $\lambda = 5$. Find an eigenvector corresponding to this value.

4. Let $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$. Diagonalize this matrix. Then use the result to find A^5. Hint: you can use your calculator to find an inverse matrix and to multiply matrices.

5. Find the length of the vector $\vec{u} = \begin{bmatrix} 3 \\ -12 \\ 4 \end{bmatrix}$.

6. Find the distance between the vectors $\vec{v} = \begin{bmatrix} 1 \\ 1 \\ -3 \\ 4 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 3 \\ 1 \\ 4 \\ 7 \end{bmatrix}$.

7. Find a unit vector pointing in the direction of

   (a) $\vec{x} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$

   (b) $\vec{y} = \begin{bmatrix} 4 \\ -2 \\ 3 \\ 8 \end{bmatrix}$

8. Let $\vec{u} = \begin{bmatrix} 5 \\ 4 \\ 1 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix}$, and $\vec{w} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$. Which pairs of vectors are orthogonal?

9. Project the vector $\vec{v} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ to the vector $\vec{w} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

10. Project the vector $\vec{v} = \begin{bmatrix} 1 \\ -2 \\ 3 \\ -4 \end{bmatrix}$ to the vector $\vec{w} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}$.

11. Let $W = \left\{ \vec{v_1} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \vec{v_2} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 4 \end{bmatrix}, \vec{v_3} = \begin{bmatrix} 1 \\ 2 \\ -4 \\ -3 \end{bmatrix} \right\}$. Use the Gram-Schmidt process to find an orthonormal basis for Span W.