Final exam

MAT 255, Fall 2016

1. (10 points) Give precise definitions for **four** of the following five concepts.
   (a) linear independence of a set of vectors
   (b) the span of a set of vectors
   (c) the rank of a matrix
   (d) an eigenvector of a matrix
   (e) a pair of orthogonal vectors

2. (15 points) Define the matrices $A = \begin{bmatrix} 2 & k \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 13 & 12 & 11 \\ 1 & 2 & 3 \end{bmatrix}$.
   (a) Find the matrix products AB and BA, or state that the product is undefined.
   (b) Find A^{-1}, the inverse of the matrix A.

3. (10 points) Determine if each of the following sets is a basis for $\mathbf{R}^2$. Explain the reasons for your conclusions.
   (a) $\left\{ \begin{bmatrix} -3 \\ 19 \end{bmatrix}, \begin{bmatrix} 12 \\ -5 \end{bmatrix} \right\}$
   (b) $\left\{ \begin{bmatrix} 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 9 \\ 18 \end{bmatrix} \right\}$

4. (7 points) One eigenvalue of the matrix $M = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 6 & 3 \\ 4 & 8 & 4 \end{bmatrix}$ is $\lambda = 11$. Find an eigenvector corresponding to this value.

5. (22 points) Let $A = \begin{bmatrix} 7 & -3 \\ -2 & 6 \end{bmatrix}$.
   (a) Find all eigenvalues of A.
   (b) For every eigenvalue, find a corresponding eigenvector.
   (c) Use your answers to write A as the matrix product PDP^{-1}, where D is a diagonal matrix.

6. (5 points) Find the length of the vector $\vec{u} = \begin{bmatrix} -1 \\ 0 \\ 5 \\ 9 \end{bmatrix}$.

7. (5 points) Find the distance between the vectors $\vec{v} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$.

8. (8 points) Find a unit vector pointing in the direction of

(a) $\vec{x} = \begin{bmatrix} 5 \\ -9 \end{bmatrix}$ (b) $\vec{y} = \begin{bmatrix} 0 \\ 0 \\ 11 \end{bmatrix}$

9. (6 points) Let α be a real number. Give one example of a nonzero vector that is orthogonal to the vector $\vec{v} = \begin{bmatrix} 5 \\ 1 \\ \alpha \end{bmatrix}$.

10. (a) (12 points) Find a basis set for the columnspace and rowspace of the matrix

$$A = \begin{bmatrix} 5 & 2 & 7 \\ 17 & 5 & 22 \\ 13 & 7 & 20 \end{bmatrix}.$$

(b) 10 points extra credit:
Use the Gram-Schmidt process to find an orthonormal basis for each of the two spaces that you found in part (a).