

1. A rental car company has one location at DIA and a second location downtown. It is possible for a customer to rent a car in location and return it to the other. If a car is rented at DIA, the probability that it will be returned to DIA is 0.8. If a car is rented downtown, the probability that it will be returned downtown is 0.7. For our first simple model, we will assume that a car is rented and returned once per day.
 - (a) Draw a transition diagram for the state of a rental car.
 - (b) Make the transition matrix.
 - (c) Suppose that the company begins with half of their cars at DIA and half downtown. Write the state matrix corresponding to this setup.
 - (d) Find the next state matrix in the Markov chain. What percentage of the cars will be at DIA on the next day?
 - (e) Find the percentage of cars at DIA on day 2 and on day 10.
 - (f) Is the transition matrix for this system regular? If so, describe the way the rental fleet will be split between locations in the long term.
2. Based on census data, economists estimate that 95% of people who own their own home today will also own their home in four years. On the other hand, 15% of people who do not own their own home today will own a home in four years.
 - (a) Draw a transition diagram for the state of a customer's preference.
 - (b) Make the transition matrix.
 - (c) Suppose that currently 65.4% of people own their own home. Write the initial state matrix for this data.
 - (d) Find the next state matrix in the Markov chain. What percentage of people will own their home in four years?
 - (e) Find the percentage of homeowners in 20 years.
 - (f) Is the transition matrix for this system regular? If so, what percentage of people will own their own home in the long run?
3. Customers in a certain town can choose between McDonald's, Chipotle, and Pizza Hut. Every day, McDonald's loses 10% of its customers to Chipotle and 20% to Pizza Hut; Chipotle loses 15% of its customers to McDonald's and 10% to Pizza Hut; and Pizza Hut loses 5% of its customers to McDonald's and 5% to Pizza Hut.
 - (a) Draw a transition diagram for a consumer's preference.
 - (b) Make the transition matrix.
 - (c) Suppose that at the beginning of our study 40% of consumers prefer McDonald's, 30% prefer Chipotle, and 30% prefer Pizza Hut. Write the state matrix for this data.
 - (d) Find the next state matrix in the Markov chain, and describe the corresponding consumer preferences.
 - (e) How will customers be split between the restaurants in the long term?

ANSWERS

1. (b) $\begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix}$

(c) $\begin{bmatrix} 0.5 & 0.5 \end{bmatrix}$

(d) $\begin{bmatrix} 0.55 & 0.45 \end{bmatrix}$
55%

(e) 57.5%
60%

(f) The long-term split has 60% of the cars
at DIA and 40% downtown.

2. (b) $\begin{bmatrix} 0.95 & 0.05 \\ 0.15 & 0.85 \end{bmatrix}$

(c) $\begin{bmatrix} 0.654 & 0.346 \end{bmatrix}$

(d) $\begin{bmatrix} 0.6886 & 0.3114 \end{bmatrix}$
68.9%

(e) 71.9%

(f) 75%

3. (b) $\begin{bmatrix} 0.7 & 0.1 & .2 \\ 0.15 & 0.75 & 0.1 \\ 0.05 & 0.05 & 0.9 \end{bmatrix}$

(c) $\begin{bmatrix} 0.4 & 0.3 & 0.3 \end{bmatrix}$

(d) $\begin{bmatrix} 0.34 & 0.28 & 0.38 \end{bmatrix}$

The preferences are
34% McDonald's
28% Chipotle
38% Pizza Hut

(e) 20% McDonald's
20% Chipotle
60% Pizza Hut