

Series solutions at ordinary points

Find a power series solution to each differential equation. State the radius of convergence for each of your solutions.

1. $(x^2 - 1)y'' + 4xy' + 2y = 0$

2. $(x^2 - 3)y'' + 2xy' = 0$

3. $y'' + x^2y = 0$

4. $y'' + e^{-x}y = 0$

Find the first three terms of the solution by using a series for the coefficient function.

ANSWERS

$$1. \quad y(x) = c_0 \sum_{n=0}^{\infty} x^{2n} + c_1 \sum_{n=0}^{\infty} x^{2n+1}$$

$$2. \quad y(x) = c_0 + c_1 \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)3^n}$$

$$3. \quad y(x) = c_0 \left(1 - \frac{x^4}{12} + \frac{x^8}{672} - \dots \right) + c_1 \left(\frac{x}{20} - \frac{x^5}{1440} + \frac{x^9}{17280} - \dots \right)$$

$$4. \quad y(x) = c_0 \left(1 - \frac{x^2}{2} + \frac{x^3}{6} - \dots \right) + c_1 \left(x - \frac{x^3}{6} + \frac{x^4}{18} - \dots \right)$$