Determine whether a Function Satisfies the Wave Equation using MATLAB

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Problem definition

A given reference manual of unknown origin, concerning the vibration of strings, states that a particular function satisfies the wave equation. The function, $s(x,t)$, follows:

```
s =
```

$$\frac{\cos(x+c\,t)}{2}+\frac{\cos(x-c\,t)}{2}$$

Variable	Meaning	Domain
x	space	$-\infty < x < \infty$
t	time	$t > 0$
c	wave speed	$c > 0$

We might recall from d'Alembert that the superposition of a left traveling wave and right traveling wave with the same wave speed, c, should be a solution to the wave equation. Let's verify this result to certify that the source is accurate and document the derivation in a Live Script to share with other MATLAB users or export a .pdf report for archival purposes.

Solutions to separable equations, such as the wave equation, are often written in the form: $s(x,t) = X(x)\ \ T(t)$. Being able to re-write the function in these terms using the Simplify Live Editor Task will make the function easier to work with and gives a positive indication.

```
u =
```

$$\cos(c\,t)\cos(x)$$

Show that *u(x,t)* satisfies the wave equation

The one dimensional wave equation is given as follows:

$$\frac{\partial^2}{\partial t^2}u = c^2\frac{\partial^2}{\partial x^2}u$$

The left-hand side, substituting u:

```
lhs =
```

$$-c^2\cos(c\,t)\cos(x)$$

The right-hand side, substituting u:

```
rhs = −c^2 cos(c t) cos(x)
```

Does LHS=RHS given the assumptions? If so, s satisfies the wave equation.

```
ans = logical

   1
```

Plot the solution for a given wave speed

Select a numeric value for c and call it cn.

```
cn = 0.8000
```

Convert u to a MATLAB function for further plotting and numeric analysis

```
f = function_handle with value:

    @(t,x)cos(t.*(4.0./5.0)).*cos(x)
```