In[1]:=

```
<< "RISC`HolonomicFunctions`";
```

HolonomicFunctions Package version 1.7.3 (21-Mar-2017)
written by Christoph Koutschan
Copyright Research Institute for Symbolic Computation (RISC),
Johannes Kepler University, Linz, Austria

```
--> Type ?HolonomicFunctions for help.
```

In[2]:=

ann = Simplify [Annihilator [$\sum_{h=0}^{m-1} \frac{\text{Binomial}[n+h, h]}{(h+i)(i-1)!(1+h+i)}$, {S[i], S[n]}]]
化简

Out[2]=

$\{-i(1+i+m)\times(1+i-n) S_i - n(1+n) S_n + (i^2 + i m + n - i n + n^2),$
$(2+3n+n^2) S_n^2 + (-2-(4+m)n - 2n^2 + i(2+n)) S_n - (i-n)(1+m+n)\}$

In[3]:=

rec1 = ApplyOreOperator [Last [ann], $a_{n,i}$]
最后一个

Out[3]=

$-((i-n)(1+m+n) a_{n,i}) + (-2-(4+m)n-2n^2+i(2+n)) a_{1+n,i} + (2+3n+n^2) a_{2+n,i}$

In[4]:=

rec2 = ApplyOreOperator [First [ann], $a_{n,i}$]
第一个

Out[4]=

$(i^2 + i m + n - i n + n^2) a_{n,i} - i(1+i+m)\times(1+i-n) a_{n,1+i} - n(1+n) a_{1+n,i}$

In[5]:=

Simplify [Solve [(rec1 /. n → n - 1) == 0, $a_{n+1,i}$]]
化简 解方程

Out[5]=

$$\left\{\left\{a_{1+n,i} \to \frac{(1+i-n)(m+n) a_{-1+n,i} - (i+m+i n - m n - 2n^2) a_{n,i}}{n(1+n)}\right\}\right\}$$

In[6]:=

Simplify [Solve [(rec2 /. n → n - 1) == 0, $a_{n-1,i+1}$]]
化简 解方程

Out[6]=

$$\left\{\left\{a_{-1+n,1+i} \to \frac{(i^2 + i(1+m-n) + (-1+n)n) a_{-1+n,i} - (-1+n) n a_{n,i}}{i(1+i+m)\times(2+i-n)}\right\}\right\}$$

In[7]:=

ApplyOreOperator [FindRelation [ann, Support → {1, S[n] × S[i], S[i]}], $a_{n,i}$];

In[8]:=
```
Simplify [Solve [(% /. n → n - 1 /. i → i - 1) == 0, a_{n-1,i-1}]]
```
化简 解方程

Out[8]=
$$\left\{\left\{a_{-1+n,-1+i} \to \frac{(1+i-n) \times (-1+i+m+n)\, a_{-1+n,i} + (-1+n)\, n\, a_{n,i}}{-1+i+m}\right\}\right\}$$

In[9]:=
```
ApplyOreOperator [FindRelation [ann , Support → {1, S[n], S[i] × S[n]}], a[n, i]];
```

In[10]:=
```
Simplify [Solve [(% /. n → n - 1 ) == 0, a[n, i + 1]]]
```
化简 解方程

Out[10]=
$$\left\{\left\{a[n,\ 1+i] \to \frac{-((m+n)\, a[-1+n,\ i]) + (i+m+n)\, a[n,\ i]}{i\,(1+i+m)}\right\}\right\}$$

In[11]:=
$$X[n\_,\ i\_] := 1 - \frac{i+m}{2\,n} + \frac{m-1}{n+1} + \frac{\sqrt{i^2\,(1+n)^2 + 2\,i\,m\,(1+n)^2 + (m+2\,n-m\,n)^2}}{2\,n\,(1+n)};$$

In[12]:=
$$\text{Resolve}\left[\text{ForAll}\left[\{i,\ m\},\ i \geq 1\ \&\&\ m \geq 1,\ X[i+2,\ i] \leq \frac{i^2 + i\,(m+2) + 2}{(i+1)\,(i+2)}\right]\right]$$
解决 对于全部

Out[12]= True

In[13]:=
```
Resolve [ForAll [{n, i, m}, n ≥ i + 1 && i ≥ 2 && m ≥ 1, X[n, i] ≥ X[n + 1, i]]]
```
解决 对于全部

Out[13]= True

In[14]:=
$$Y[n\_,\ i\_] := 1 + \frac{i+2\,m}{2\,n} - \frac{1+2\,i+m}{2 \times (-1+n)} + \frac{1}{2 \times (-1+n)\,n} * \left(\sqrt{(i^2\,(1+n)^2 + (-2\,m-n+m\,n)^2 + 2\,i\,((-1+m) \times (-1+n)\,n + 2\,(m+n)))}\right);$$

In[15]:=
$$-\frac{(n-i-1)\,(m+n)}{(n\,(1+n))\,Y[n,\ i]} + \frac{2\,n^2 - i\,(n+1) + m\,(n-1)}{n\,(1+n)} \leq Y[n+1,\ i];$$

$$\text{Resolve}\left[\forall_{\{n,i,m\},n\geq i+1\&\&i\geq 1\&\&m\geq 1}\ \%\right]$$
解决

Out[16]= True

In[17]:=
$$\Delta_1[n\_,\ i\_] := (-1+n)^2\,n^2 \left((1+i^2-m+i\,m)^2 + 2 \times (-1+i+m) \times (-1+2\,i+i^2+m+i\,m)\,(n-i-2) + (-1+i+m)^2\,(n-i-2)^2\right);$$

In[18]:=

$$z2[n\_, i\_] := 1 - \frac{1 + i - m}{2\,n} + \frac{\sqrt{\Delta_1[n, i]}}{2\,(-1+n)^2\,n^2};$$

In[19]:=

$$\text{Resolve}\left[\forall_{\{n,i,m\},n\geq i+2\&\&i\geq 1\&\&m\geq 1}\; z2[n, i] \geq Y[n, i]\right]$$

解决

Out[19]=

True