Experimental Mathematics at Queens College

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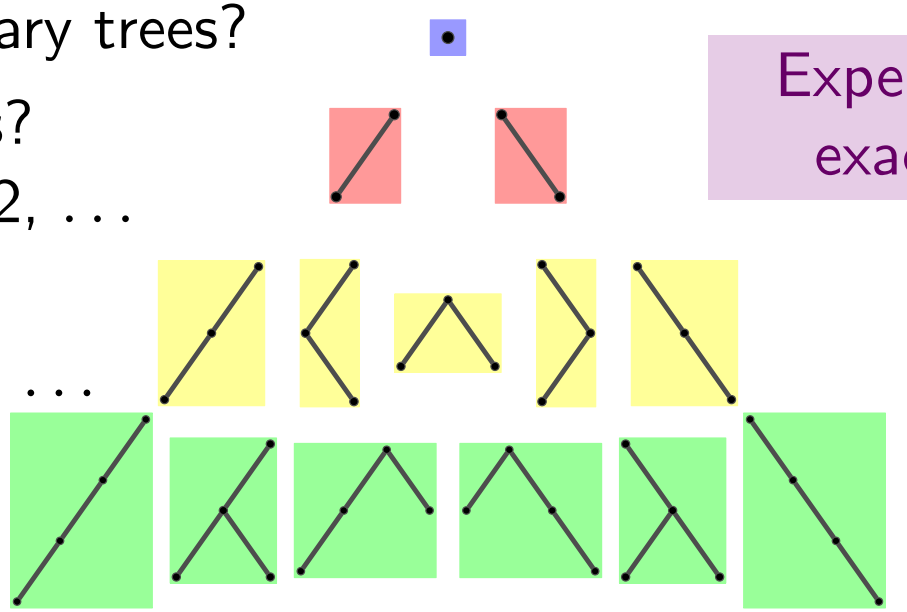
What is Experimental Mathematics?

I study discrete structures.

Q. How many binary trees?

- With n nodes?
  1, 2, 5, 14, 42, ...
- Of height h?
  1, 3, 21, 651, ...



Experiments generate exact, true results.

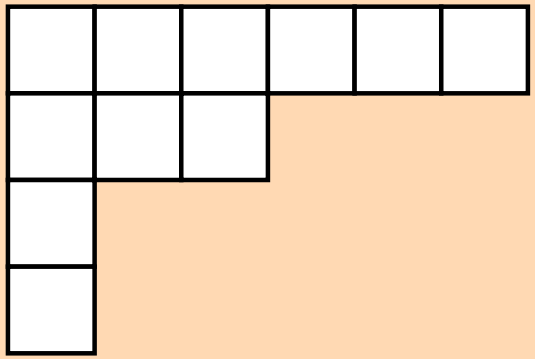
Experiment! Use a computer and mathematical software.

Generate all objects. Count. Filter. Match.

Learn something new. Prove it!

Experiments in core partitions

A **partition** breaks an integer into parts: $11 = 6 + 3 + 1 + 1$.



A t-core partition has no t-hooks

Show up in many areas of math.

Simultaneous (s, t)-core partitions **biject** with other discrete objects

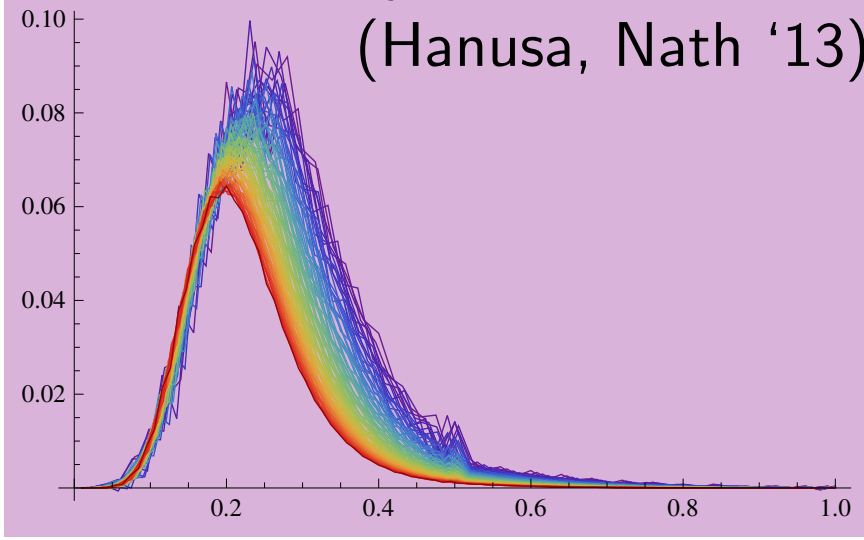
Stats in Cores $\overset{??}{\longleftrightarrow}$ Stats in Others

Conjecture: (Armstrong '11)
The average size of an (s, t)-core

$$(s + t + 1)(s - 1)(t - 1)/24$$

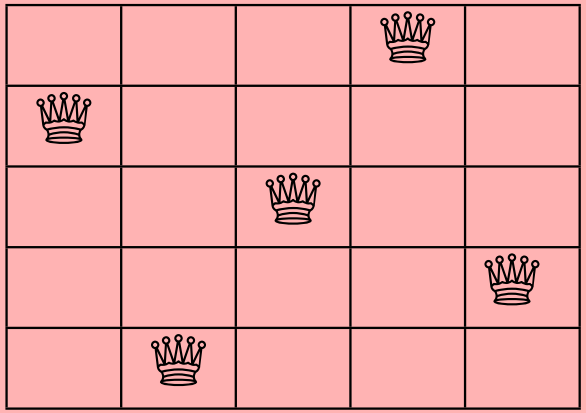
Generate, collect data $\rightsquigarrow$ Pattern.
Nice formula, hard to prove.

Structure of symmetric partitions (Hanusa, Nath '13)



Experiments in chess piece configurations

Q. Can you place n nonattacking queens on an $n \times n$ chessboard?



Q. In how many ways can you place n nonattacking queens?

n	1	2	3	4	5	6	7	8	9	10
#	1	0	0	2	10	4	40	92	352	724

Computer generated. $n = 100$?

Classic mathematical technique:
Solve a more general problem.

- Fixed # of pieces q
- Arbitrary polygonal board $\mathcal{B}$
- Arbitrary piece $\mathbb{P}$

Computers help:

- Calculations / simplifications
- Check formulas against data

Three-♕-config on $n \times n$ board:

$$f(n) = \frac{n^6}{6} - \frac{5n^5}{3} + \frac{79n^4}{12} - \frac{25n^3}{2} + 11n^2 - \frac{43n}{12} + \frac{1}{8} + (-1)^n\left\{\frac{n}{4} - \frac{1}{8}\right\}$$