Практическое занятие 2

Вычисление неопределенных интегралов методом подстановки.

Метод интегрирования по частям.

1. <u>Метод подстановки.</u>

$$\int f(\varphi(x))\cdot\varphi'(x)dx=\begin{bmatrix}\varphi(x)=t\\ \varphi'(x)dx=dt\end{bmatrix}=\int f(t)dt=F(t)+C=F\big(\varphi(x)\big)+C.$$

Этот же прием можно применять в виде:

$$\int f(\varphi(x))\cdot\varphi'(x)dx=\int f\big(\varphi(x)\big)\,d\varphi(x)=F\big(\varphi(x)\big)+C.$$

<u>Примеры</u>.

Вычислить неопределенный интеграл.

1. $\int\frac{dx}{5x+3}=\begin{bmatrix}5x+3=t\\ 5dx=dt\\ dx=\frac{1}{5}dt\end{bmatrix}=\int\frac{\frac{1}{5}dt}{t}=\frac{1}{5}ln|t|+C=\frac{1}{5}ln|5x+3|+C.$
2. $\int sin^2x\cdot cosxdx=\begin{bmatrix}sinx=t\\ cosxdx=dt\end{bmatrix}=\int t^2\,dt=\frac{1}{3}t^3+C=\frac{1}{3}sin^3x+C.$

   Или:

   $\int sin^2x\cdot cosxdx=\int sin^2x\,dsinx=\frac{1}{3}sin^3x+C.$
3. $\int e^{3x+2}\,dx=\frac{1}{3}\int e^{3x+2}\,d(3x+2)=\frac{1}{3}e^{3x+2}+C.$
4. $\int\frac{x}{\sqrt{2-3x^2}}dx=\begin{bmatrix}2-3x^2=t\\ -6xdx=dt\\ xdx=-\frac{1}{6}dt\end{bmatrix}=\int\frac{-\frac{1}{6}dt}{\sqrt{t}}=-\frac{1}{6}\cdot\frac{t^{\frac{1}{2}}}{\frac{1}{2}}+C=-\frac{1}{3}\sqrt{t}+C=$

   $=-\frac{1}{3}\sqrt{2-3x^2}+C.$
5. $\int ctgxdx=\int\frac{cosx}{sinx}dx=\int\frac{dsinx}{sinx}=ln|sinx|+C.$
6. $\int\frac{x^3dx}{x^8-4}=\begin{bmatrix}x^4=t\\ 4x^3dx=dt\\ x^3dx=\frac{1}{4}dt\end{bmatrix}=\int\frac{\frac{1}{4}dt}{t^2-4}=\frac{1}{4}\cdot\frac{1}{4}ln\left|\frac{t-2}{t+2}\right|+C=\frac{1}{16}ln\left|\frac{x^4-2}{x^4+2}\right|+C.$

7. $\int \sqrt{\frac{arcsinx}{1-x^2}}dx = \int \sqrt{arcsinx}\,\frac{dx}{\sqrt{1-x^2}} = \int (arcsinx)^{\frac{1}{2}}\,d(arcsinx) =$
$= \frac{2}{3}(arcsinx)^{\frac{3}{2}} + C.$

8. $\int \frac{dx}{chx} = 2\int \frac{dx}{e^x+e^{-x}} = 2\int \frac{e^x dx}{e^{2x}+1} = 2\int \frac{de^x}{e^{2x}+1} = 2arctg(e^x) + C.$

9. $\int \frac{dx}{sinx} = \int \frac{dx}{2sin\frac{x}{2}cos\frac{x}{2}} = \int \frac{cos\frac{x}{2}}{sin\frac{x}{2}} \cdot \frac{dx}{2cos^2\frac{x}{2}} = \int \frac{d\left(tg\frac{x}{2}\right)}{tg\frac{x}{2}} = ln\left|tg\,\frac{x}{2}\right| + C.$

10. $\int x \cdot sin(x^2)\,dx = \frac{1}{2}\int sin(x^2)\,d(x^2) = -\frac{1}{2}cos(x^2) + C.$

11. $\int x^2\,e^{-5x^3}dx = -\frac{1}{15}\int e^{-5x^3}\,d(-5x^3) = -\frac{1}{15}e^{-5x^3} + C.$

12. $f(x) = \frac{x}{16+x^4},\quad F(0) = 0,\quad F(2) = ?$

$\int \frac{x}{16+x^4}dx = \frac{1}{2}\int \frac{d(x^2)}{16+x^4} = \frac{1}{8}arctg\,\frac{x^2}{4} + C.$

$F(x) = \frac{1}{8}arctg\,\frac{x^2}{4} + C,\quad F(0) = C = 0,\quad F(2) = \frac{1}{8}arctg1 = \frac{1}{8}\cdot\frac{\pi}{4} = \frac{\pi}{32}.$

13. $f(x) = \frac{1}{xln^2x},\quad F(e) = \frac{1}{2},\quad F(e^2) = ?$

$\int \frac{dx}{xln^2x} = \int \frac{dlnx}{ln^2x} = -\frac{1}{lnx} + C.$

$F(x) = -\frac{1}{lnx} + C,\quad F(e) = -1 + C = \frac{1}{2},\quad C = \frac{3}{2},$

$F(e^2) = -\frac{1}{2} + \frac{3}{2} = 1.$

<u>Домашнее задание:</u> Типовой расчет, 2 семестр, задача 1.1, № 1-21.

<u>Примеры для самостоятельного решения.</u>

1. $f(x) = \frac{1}{2x+3},\ F(0) = ln\sqrt{3},\quad F(3) = ?$
2. $f(x) = \frac{1}{xlnx},\ F(e) = -ln2,\quad F(e^2) = ?$
3. $f(x) = x\sqrt{x^2+9},\ F(4) = \frac{2}{3},\quad F(0) = ?$
4. $f(x) = \frac{x^2}{\sqrt{5x^3+4}},\ F(1) = \frac{17}{15},\quad F(0) = ?$
5. $f(x) = \frac{arctgx}{1+x^2},\ F\left(\sqrt{3}\right) = -\frac{\pi^2}{18},\quad F(1) = ?$

6. $f(x) = \frac{x-3}{x^2-6x+10}$, $F(3) = ln\sqrt{5}$, $F(1) = ?$

7. $f(x) = \frac{2x+5}{x^2+1}$, $F(-1) = \frac{\pi}{2}$, $F(1) = ?$

8. $f(x) = \frac{\sqrt[3]{tgx}}{cos^2x}$, $F(0) = \frac{1}{4}$, $F\left(\frac{\pi}{4}\right) = ?$

9. $f(x) = \frac{e^{tgx}}{cos^2x}$, $F\left(\frac{\pi}{4}\right) = e$, $F(0) = ?$

10. $f(x) = sin\left(2x + \frac{\pi}{3}\right)$, $F\left(\frac{\pi}{3}\right) = \frac{5}{4}$, $F(0) = ?$

11. $f(x) = \frac{cosx}{3-2sinx}$, $F\left(\frac{\pi}{6}\right) = ln2$, $F\left(-\frac{\pi}{6}\right) = ?$

12. $f(x) = (3x-1)^8$, $F(0) = -\frac{1}{27}$, $F\left(\frac{1}{3}\right) = ?$

13. $f(x) = tgx$, $F(0) = 0$, $F\left(\frac{\pi}{3}\right) = ?$

14. $f(x) = \frac{1}{1+4x^2}$, $F(0) = \frac{\pi}{8}$, $F\left(\frac{1}{2}\right) = ?$

15. $f(x) = \frac{e^x}{2+e^x}$, $F(ln4) = 2ln6$, $F(0) = ?$

16. $f(x) = \frac{cosx}{sin^5x}$, $F\left(\frac{\pi}{2}\right) = \frac{3}{4}$, $F\left(\frac{\pi}{6}\right) = ?$

17. $f(x) = \frac{x^2-1}{x^4+3x^2+1}$, $F(2) = 5$, $F\left(\frac{1}{2}\right) = ?$

18. $f(x) = \frac{x}{x^2+5}$, $F(1) = ln\sqrt{6}$, $F(2) = ?$

19. $f(x) = \frac{e^{\frac{1}{x}}}{x^2}$, $F(1) = -e$, $F(\log_2 e) = ?$

20. $f(x) = \frac{\sin(lnx)}{x}$, $F(1) = 4$, $F\left(e^{\frac{\pi}{2}}\right) = ?$

2. <u>Интегрирование по частям</u>.

$$d(uv) = udv + vdu$$

$$\int udv = uv - \int vdu$$

<u>Примеры</u>.

Вычислить неопределенный интеграл.

1. $\int x \cdot sin2xdx = \begin{bmatrix} u=x, & dv=sin2xdx \\ du=dx, & v=-\frac{1}{2}cos2x \end{bmatrix} = -\frac{1}{2}xcos2x + \frac{1}{2}\int cos2xdx =$

$= -\frac{1}{2}xcos2x + \frac{1}{4}sin2x + C.$

2. $\int x^2 e^{5x}\,dx = \begin{bmatrix} u=x^2, & dv=e^{5x}dx \\ du=2xdx, & v=\frac{1}{5}e^{5x} \end{bmatrix} = \frac{1}{5}x^2e^{5x} - \frac{2}{5}\int xe^{5x}dx =$

$= \begin{bmatrix} u=x, & dv=e^{5x}dx \\ du=dx, & v=\frac{1}{5}e^{5x} \end{bmatrix} = \frac{1}{5}x^2e^{5x} - \frac{2}{5}\left(\frac{1}{5}xe^{5x} - \frac{1}{5}\int e^{5x}dx\right) =$

$= \frac{1}{5}x^2e^{5x} - \frac{2}{25}xe^{5x} + \frac{2}{125}e^{5x} + C\ .$

3. $\int x^2 lnx\,dx = \begin{bmatrix} u=lnx, & dv=x^2dx \\ du=\frac{dx}{x}, & v=\frac{1}{3}x^3 \end{bmatrix} = \frac{1}{3}x^3lnx - \frac{1}{3}\int x^2dx =$

$= \frac{1}{3}x^3lnx - \frac{1}{9}x^3 + C.$

4. $\int \frac{lnx}{\sqrt[3]{x}}dx = \begin{bmatrix} u=lnx, & dv=\frac{1}{\sqrt[3]{x}}dx=x^{-\frac{1}{3}}dx \\ du=\frac{dx}{x}, & v=\frac{3}{2}x^{\frac{2}{3}} \end{bmatrix} = \frac{3}{2}x^{\frac{2}{3}}lnx - \frac{3}{2}\int x^{-\frac{1}{3}}dx =$

$= \frac{3}{2}x^{\frac{2}{3}}lnx - \frac{9}{4}x^{\frac{2}{3}} + C.$

5. $\int \frac{arcsinx}{\sqrt{1-x}}dx = \begin{bmatrix} u=arcsinx, & dv=\frac{dx}{\sqrt{1-x}} \\ du=\frac{dx}{\sqrt{1-x^2}}, & v=-2\sqrt{1-x} \end{bmatrix} =$

$= -2\sqrt{1-x} \cdot arcsinx + 2\int \frac{\sqrt{1-x}}{\sqrt{1-x^2}}dx =$

$= -2\sqrt{1-x} \cdot arcsinx + 2\int \frac{dx}{\sqrt{1+x}} =$

$= -2\sqrt{1-x} \cdot arcsinx + 4\sqrt{1+x} + C\ .$

6. $\int \frac{x}{sin^2x}dx = \begin{bmatrix} u=x, & dv=\frac{dx}{sin^2x} \\ du=dx, & v=-ctgx \end{bmatrix} = -xctgx + \int ctgx =$

$= -xctgx + \int \frac{cosx}{sinx}dx = -\,xctgx + ln|sinx| + C.$

7. $\int \sqrt{1-x^2}\,dx = \begin{bmatrix} u=\sqrt{1-x^2}, & dv=dx \\ du=-\frac{x}{\sqrt{1-x^2}}dx, & v=x \end{bmatrix} = x\sqrt{1-x^2} + \int \frac{x^2}{\sqrt{1-x^2}}dx =$

$= x\sqrt{1-x^2} - \int \frac{1-x^2-1}{\sqrt{1-x^2}}dx =$

$= x\sqrt{1-x^2} - \int \frac{1-x^2}{\sqrt{1-x^2}}dx + \int \frac{1}{\sqrt{1-x^2}}dx =$

$= x\sqrt{1-x^2} - \int \sqrt{1-x^2}dx + arcsinx$

$2\int \sqrt{1-x^2}\,dx = x\sqrt{1-x^2} + arcsinx + 2C$

$\int \sqrt{1-x^2}\,dx = \frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}arcsinx + C$.

8. $\int x \cdot arcsinx\,dx = \begin{bmatrix} arcsinx=t \\ x=sint \\ dx=costdt \\ \sqrt{1-x^2}=cost \end{bmatrix} = \int t \cdot sint \cdot costdt = \frac{1}{2}\int t \cdot sin2tdt =$

$= \begin{bmatrix} u=t, & dv=sin2tdt \\ du=dt, & v=-\frac{1}{2}cos2t \end{bmatrix} = \frac{1}{2}\left(-\frac{t}{2}cos2t + \frac{1}{2}\int cos2tdt\right) =$

$= -\frac{t}{4}cos2t + \frac{1}{8}sin2t + C = -\frac{t}{4}(1-2sin^2t) + \frac{1}{8}2sintcost + C =$

$= -\frac{arcsinx}{4}(1-2x^2) + \frac{x}{4}\sqrt{1-x^2} + C =$

$= \frac{2x^2-1}{4}arcsinx + \frac{x}{4}\sqrt{1-x^2} + C.$

9. $\int x^2 arctgx\,dx = \begin{bmatrix} u=arctgx, & dv=x^2dx \\ du=\frac{dx}{1+x^2}, & v=\frac{1}{3}x^3 \end{bmatrix} = \frac{1}{3}x^3 \cdot arctgx - \frac{1}{3}\int \frac{x^3dx}{1+x^2} =$

$= \frac{x^3}{3}arctgx - \frac{1}{3}\int \frac{x^3+x-x}{1+x^2}dx = \frac{x^3}{3}arctgx - \frac{1}{3}\int\left(x - \frac{x}{1+x^2}\right)dx =$

$= \frac{x^3}{3}arctgx - \frac{x^2}{6} + \frac{1}{6}\ln(1+x^2) + C.$

10. $f(x) = \frac{lnx}{x^2}, \quad F(1) = -1, \quad F(e) = ?$

$\int f(x)\,dx = \int \frac{lnx}{x^2}dx = \begin{bmatrix} u=lnx, & dv=\frac{dx}{x^2} \\ du=\frac{dx}{x}, & v=-\frac{1}{x} \end{bmatrix} = -\frac{1}{x}lnx + \int \frac{dx}{x^2} =$

$= -\frac{1}{x}lnx - \frac{1}{x} + C.$

$F(1) = -1 + C = -1 \Rightarrow C = 0$

$F(e) = -\frac{1}{e} - \frac{1}{e} = -\frac{2}{e}.$

Домашнее задание: Типовой расчет, 2 семестр, задача 1.1, № 22-39.

Примеры для самостоятельного решения.

1. $f(x) = x \cdot cosx,\ F(0) = 3,\ \ F(\pi) = ?$
2. $f(x) = x \cdot e^{-3x},\ F(0) = -\frac{7}{72},\ \ F(ln2) = ?$
3. $f(x) = x \cdot lnx,\ F(1) = \frac{3}{4},\ \ F(2) = ?$
4. $f(x) = x \cdot sin2x,\ F(0) = \frac{3}{4},\ \ F\left(\frac{\pi}{4}\right) = ?$
5. $f(x) = arctgx,\ F(0) = ln\sqrt{2},\ \ F(1) = ?$
6. $f(x) = arcsinx,\ F(0) = 1,\ \ F(1) = ?$
7. $f(x) = \frac{x}{cos^2x},\ F(0) = ln\sqrt{2},\ \ F\left(\frac{\pi}{4}\right) = ?$
8. $f(x) = \sin(lnx),\ F(1) = -\frac{1}{2},\ \ F\left(e^{\frac{\pi}{2}}\right) = ?$
9. $f(x) = \frac{lnx}{\sqrt{x}},\ F(1) = 4,\ \ F(4) = ?$
10. $f(x) = \frac{arccosx}{\sqrt{1+x}},\ F(1) = 4,\ \ F(0) = ?$
11. $f(x) = x \cdot arctg3x,\ F(0) = \frac{1}{18},\ \ F\left(\frac{1}{3}\right) = ?$
12. $f(x) = e^{\sqrt{x}},\ F(0) = 3,\ \ F(1) = ?$
13. $f(x) = x^2 \cdot sinx,\ F(0) = 2,\ \ F\left(\frac{\pi}{2}\right) = ?$
14. $f(x) = x^2 \cdot e^{-x},\ F(0) = -2,\ \ F(-1) = ?$
15. $f(x) = e^{-3x} \cdot sin2x,\ F(0) = -\frac{2}{13},\ \ F(\pi) = ?$