

# NeuralNets 1

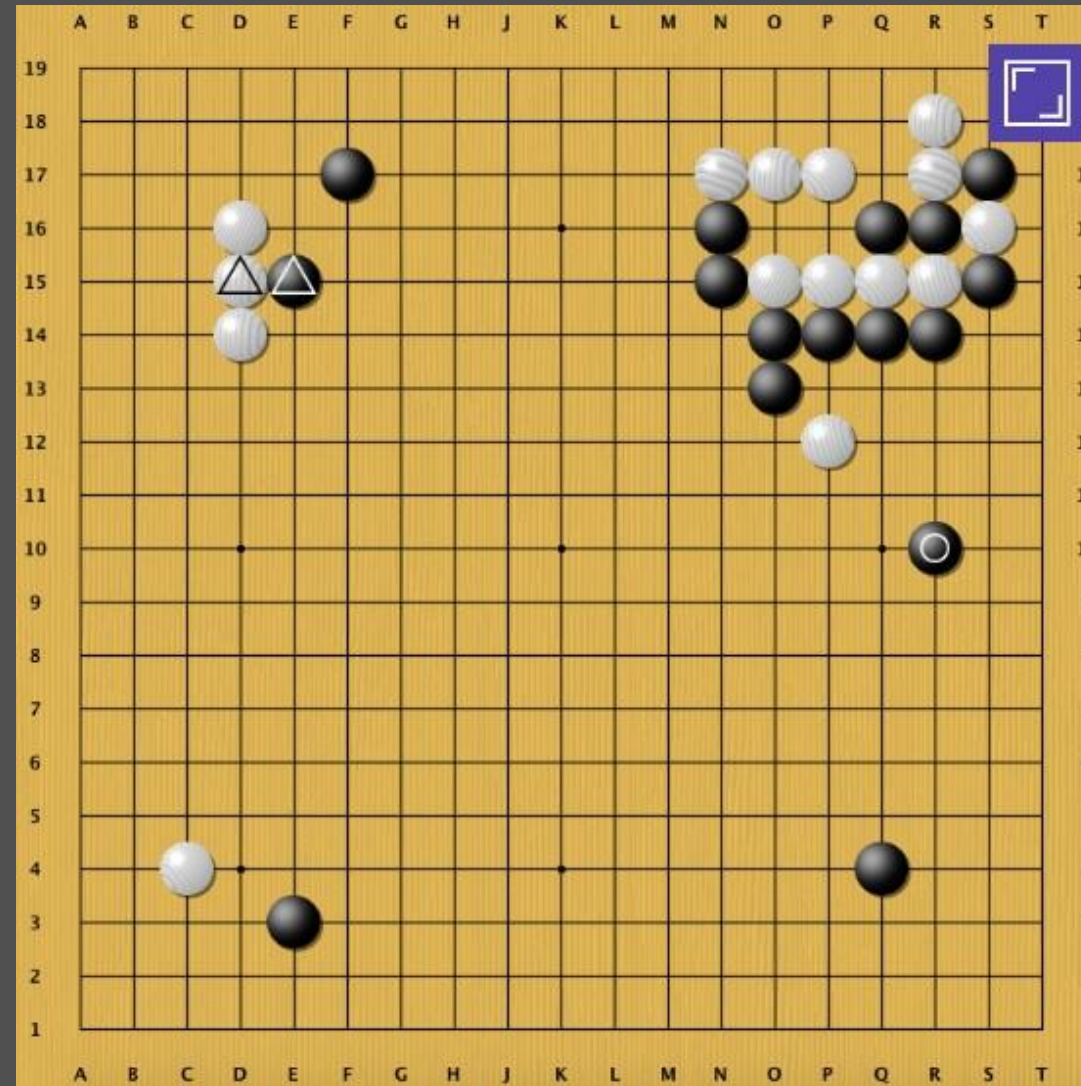
with Neural Nets  
MODALG SoSe18

# News & Motivation













# Robot teaches itself how to dress people









## Topics:

- Repetition Classification & Regression
- Learning Rate
- Perceptron & Training
- Multilayer Perceptron

# Machine Learning

**Supervised  
Learning**

**Classification**

**Regression**

**Reinforcement  
Learning**

**Unsupervised  
Learning**

**Clustering**

# Supervised Learning

## Classification

**Discrete:**  $y \in N$



## Regression

**Continuous:**  $y \in R$





# Data

| #Rooms/1 | Area/m <sup>2</sup> | Price/\$ |
|----------|---------------------|----------|
| 4        | 70                  | 1.000    |
| 3        | 65                  | 800      |
| 2        | 40                  | 400      |
| ...      | ...                 | ...      |

# Data

## Feature 1



| #Rooms/1 | Area/m <sup>2</sup> | Price/\$ |
|----------|---------------------|----------|
| 4        | 70                  | 1.000    |
| 3        | 65                  | 800      |
| 2        | 40                  | 400      |
| ...      | ...                 | ...      |

# Data

## Feature 2



| #Rooms/1 | Area/m <sup>2</sup> | Price/\$ |
|----------|---------------------|----------|
| 4        | 70                  | 1.000    |
| 3        | 65                  | 800      |
| 2        | 40                  | 400      |
| ...      | ...                 | ...      |



# Data

**Feature 3**

| #Rooms/1 | Area/m <sup>2</sup> | Price/\$ |
|----------|---------------------|----------|
| 4        | 70                  | 1.000    |
| 3        | 65                  | 800      |
| 2        | 40                  | 400      |
| ...      | ...                 | ...      |

# Data

**Example 1**

| #Rooms/1 | Area/m <sup>2</sup> | Price/\$ |
|----------|---------------------|----------|
| 4        | 70                  | 1.000    |
| 3        | 65                  | 800      |
| 2        | 40                  | 400      |
| ...      | ...                 | ...      |

# Data

**Example 2**

| #Rooms/1 | Area/m <sup>2</sup> | Price/\$ |
|----------|---------------------|----------|
| 4        | 70                  | 1.000    |
| 3        | 65                  | 800      |
| 2        | 40                  | 400      |
| ...      | ...                 | ...      |

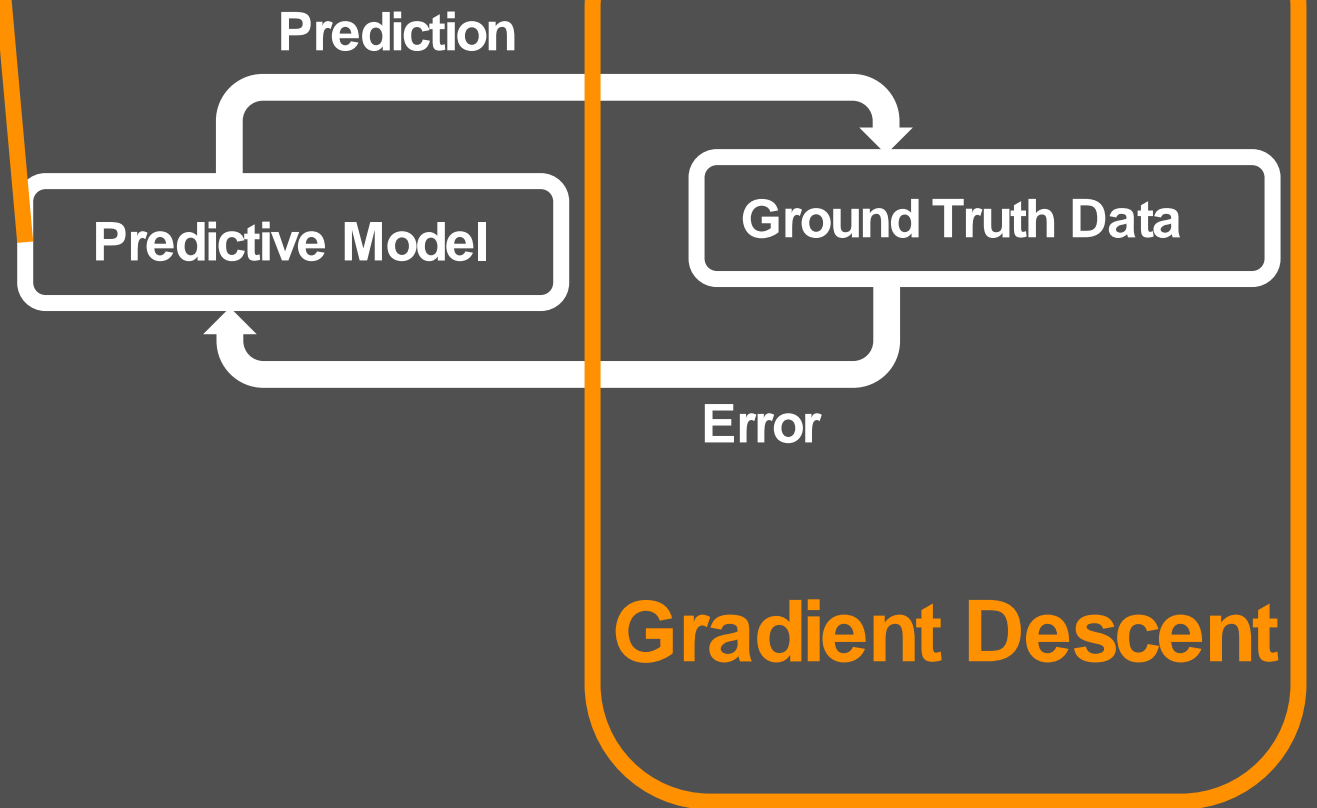
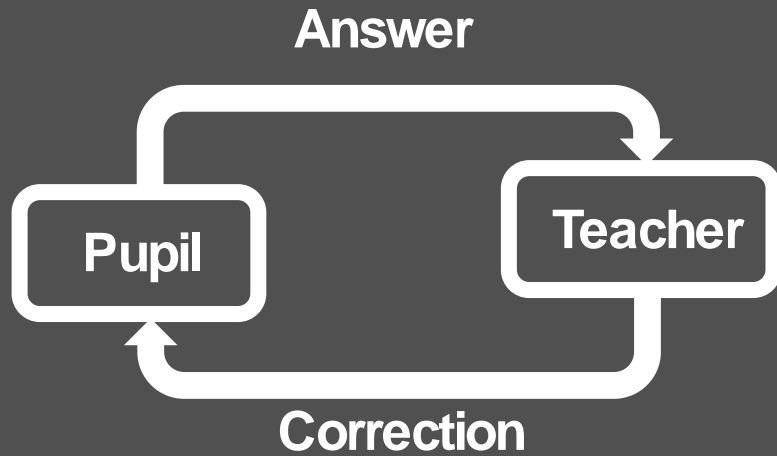


# Data

**Example 3**

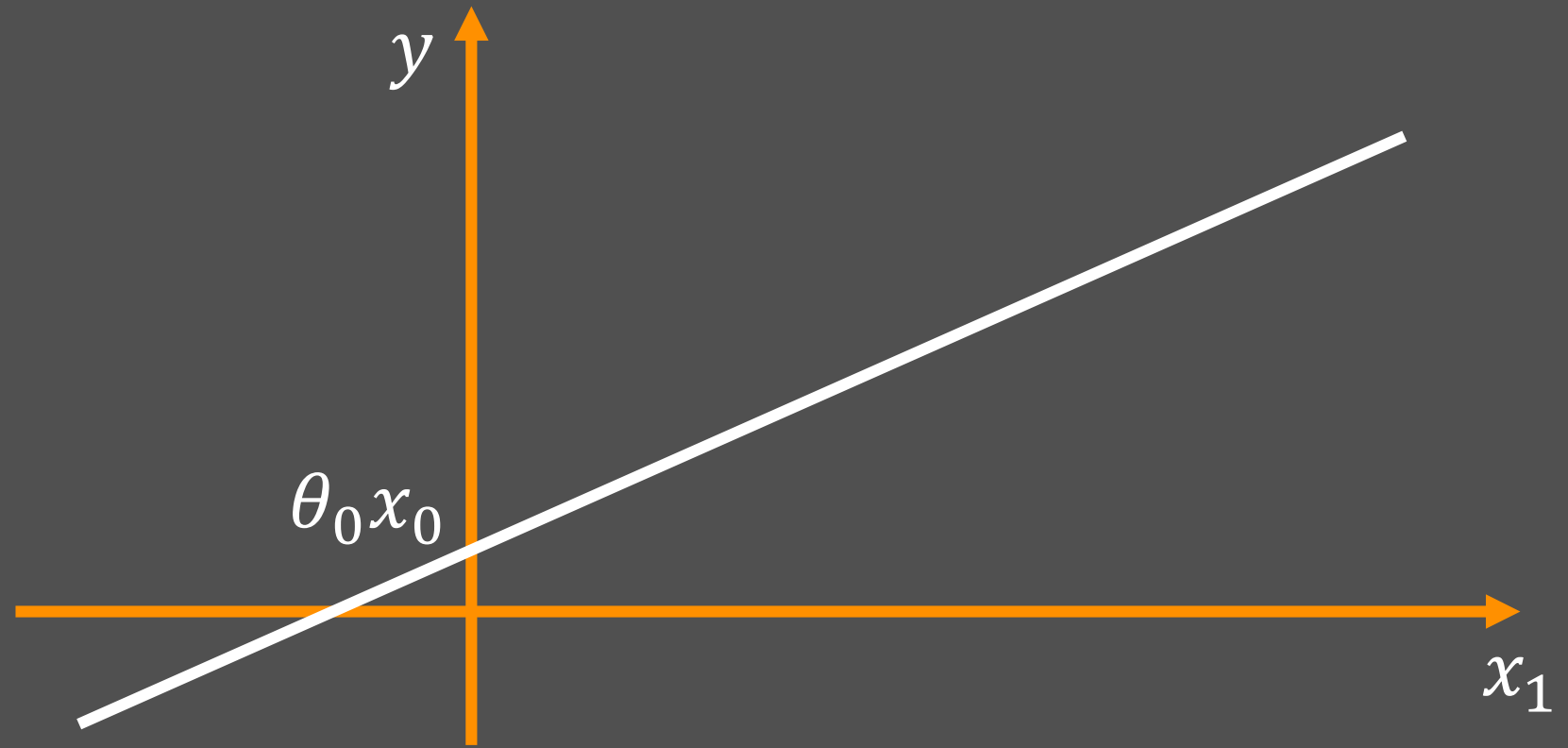
| #Rooms/1 | Area/m <sup>2</sup> | Price/\$ |
|----------|---------------------|----------|
| 4        | 70                  | 1.000    |
| 3        | 65                  | 800      |
| 2        | 40                  | 400      |
| ...      | ...                 | ...      |

## Linear Regression



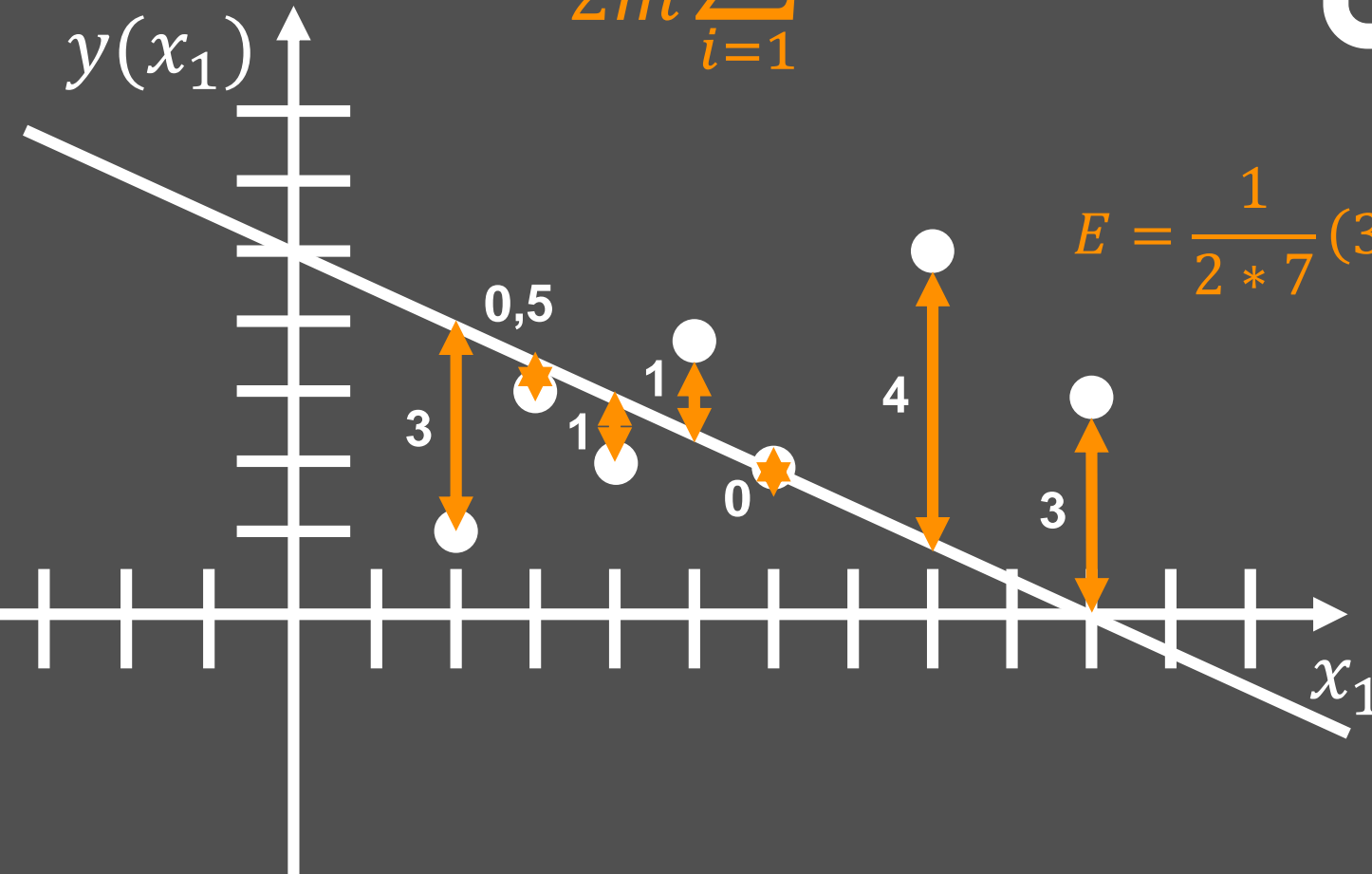
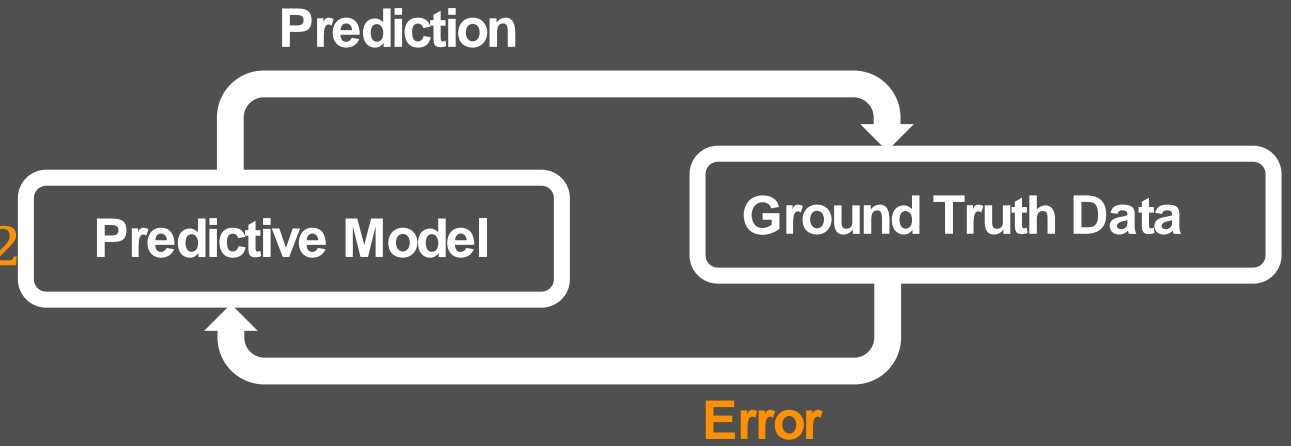
Slope of the line depends on weights  $\theta_0, \theta_1$

$$\hat{y} = \theta_1 x_1 + \theta_0 x_0$$





$$E = \frac{1}{2m} \sum_{i=1}^m (\hat{y}_i - y_i)^2$$

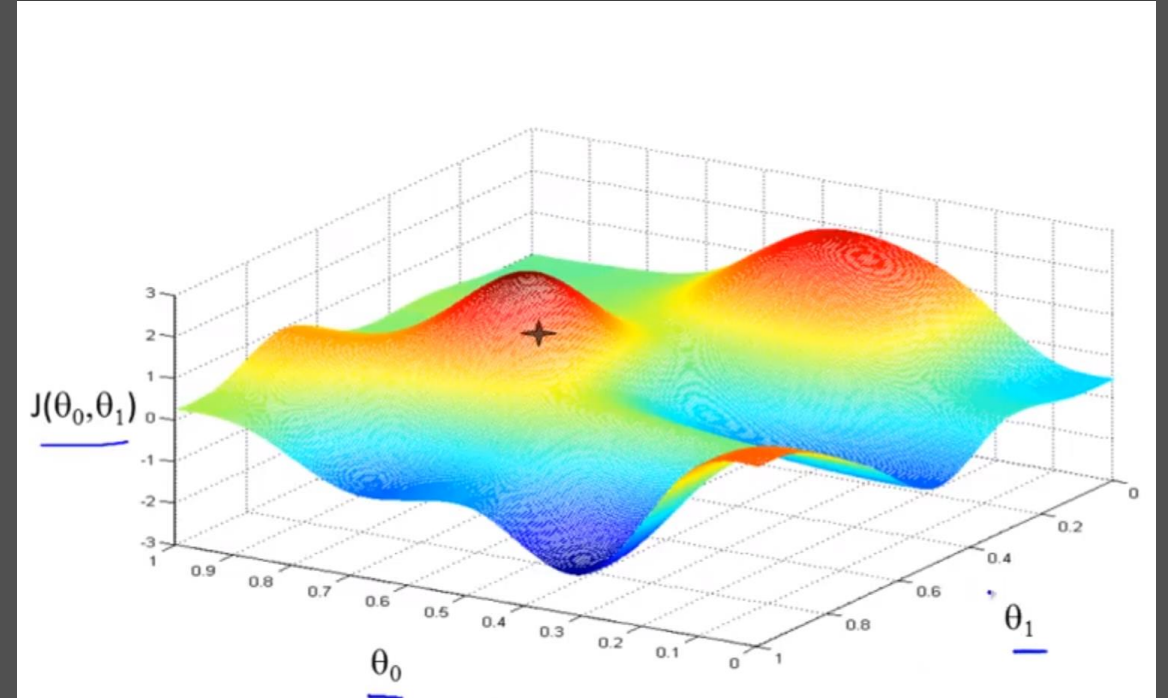
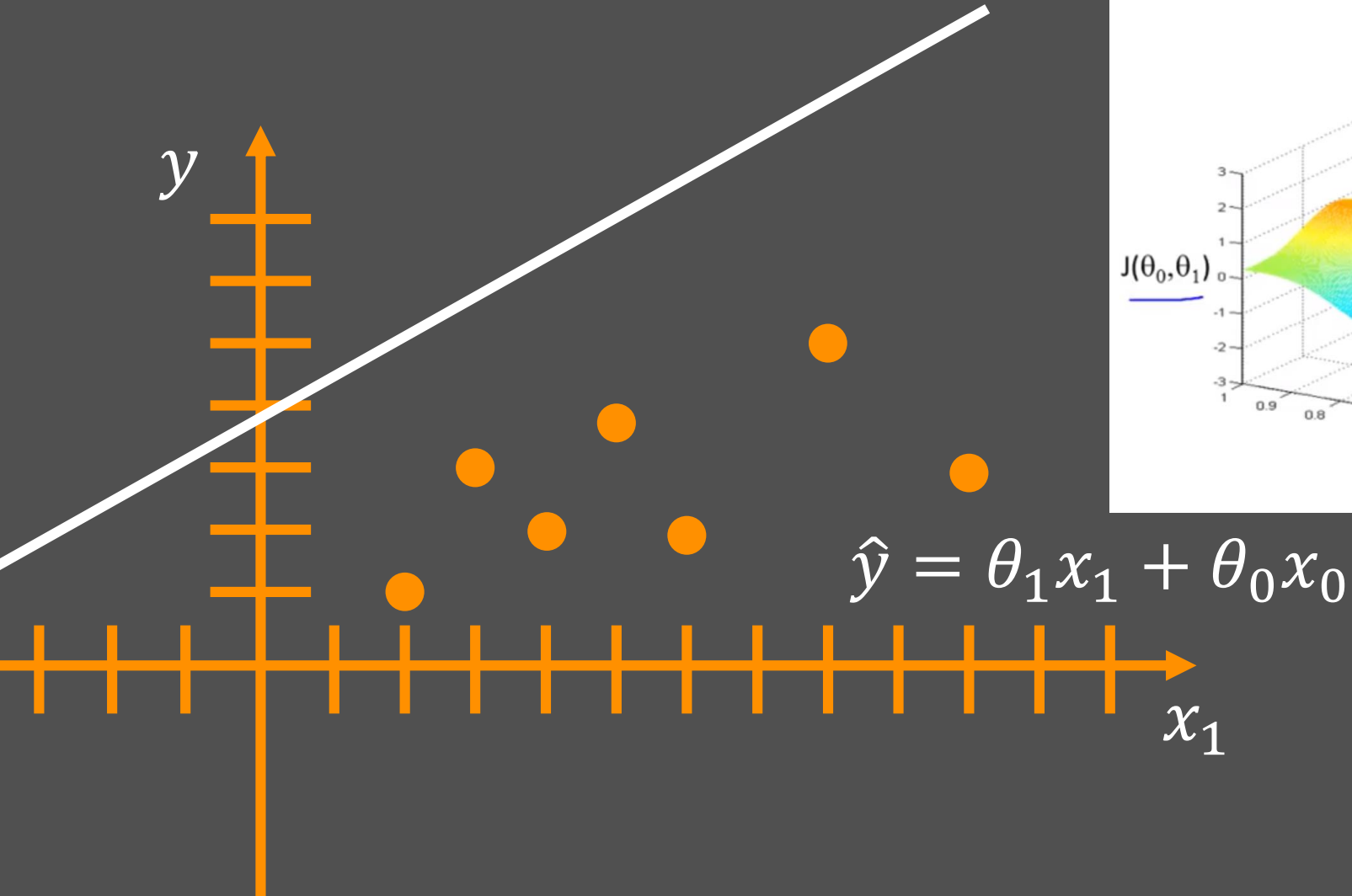


$$E = \frac{1}{2 * 7} (3^2 + 0,5^2 + 1^2 + 1^2 + 0^2 + 4^2 + 3^2)$$

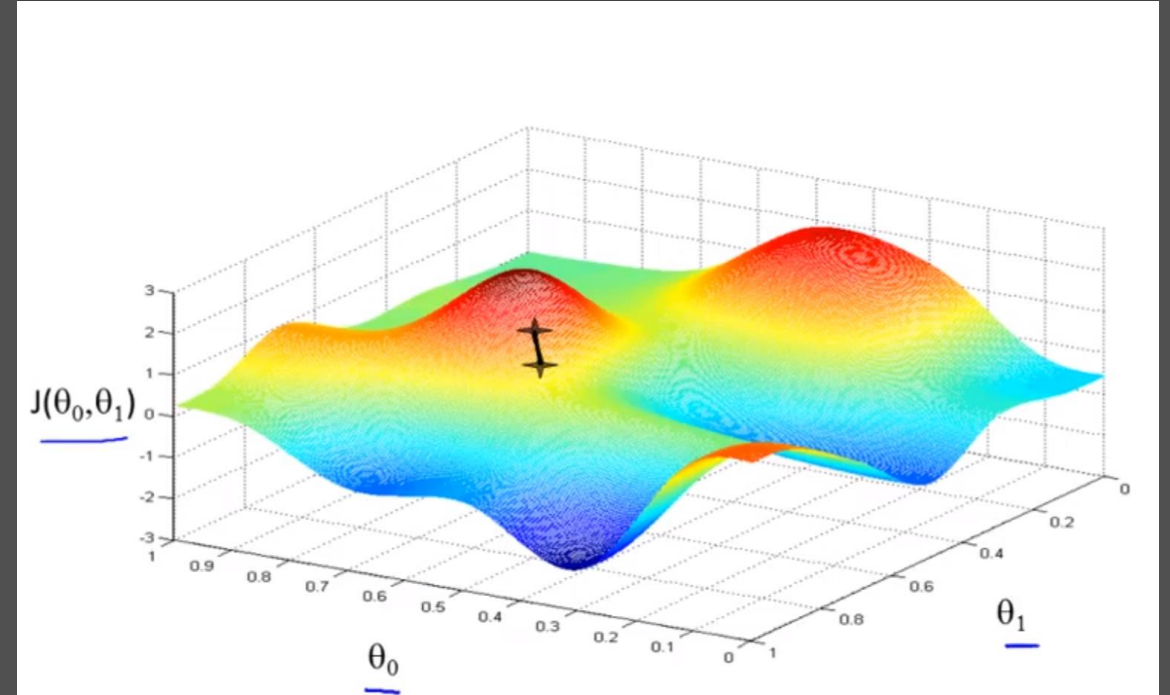
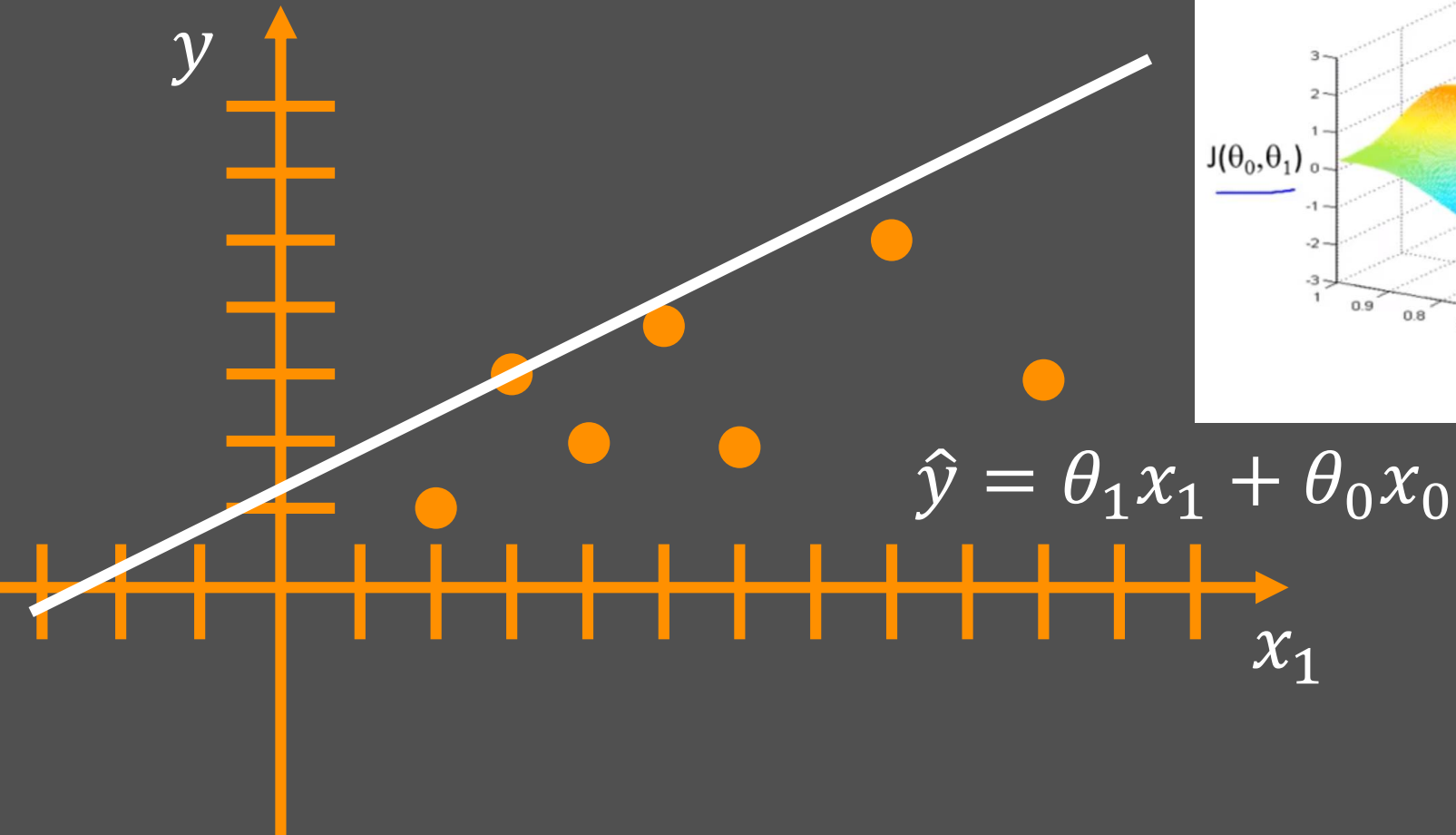
$$E = 2.859$$

$$m = 7$$

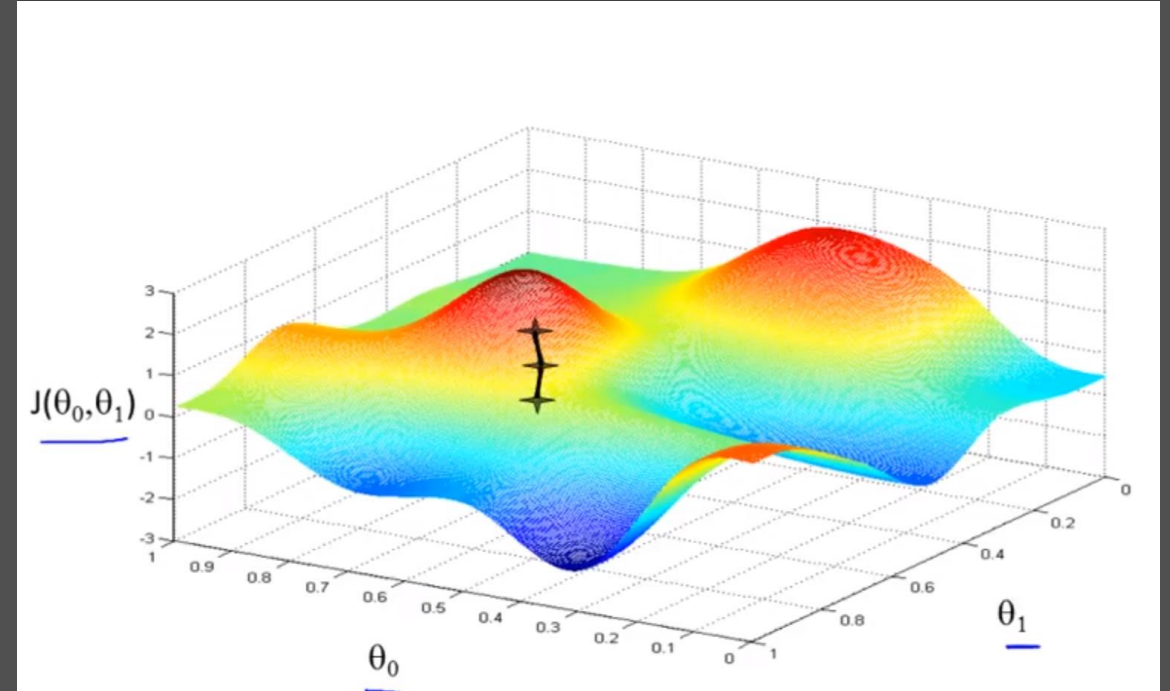
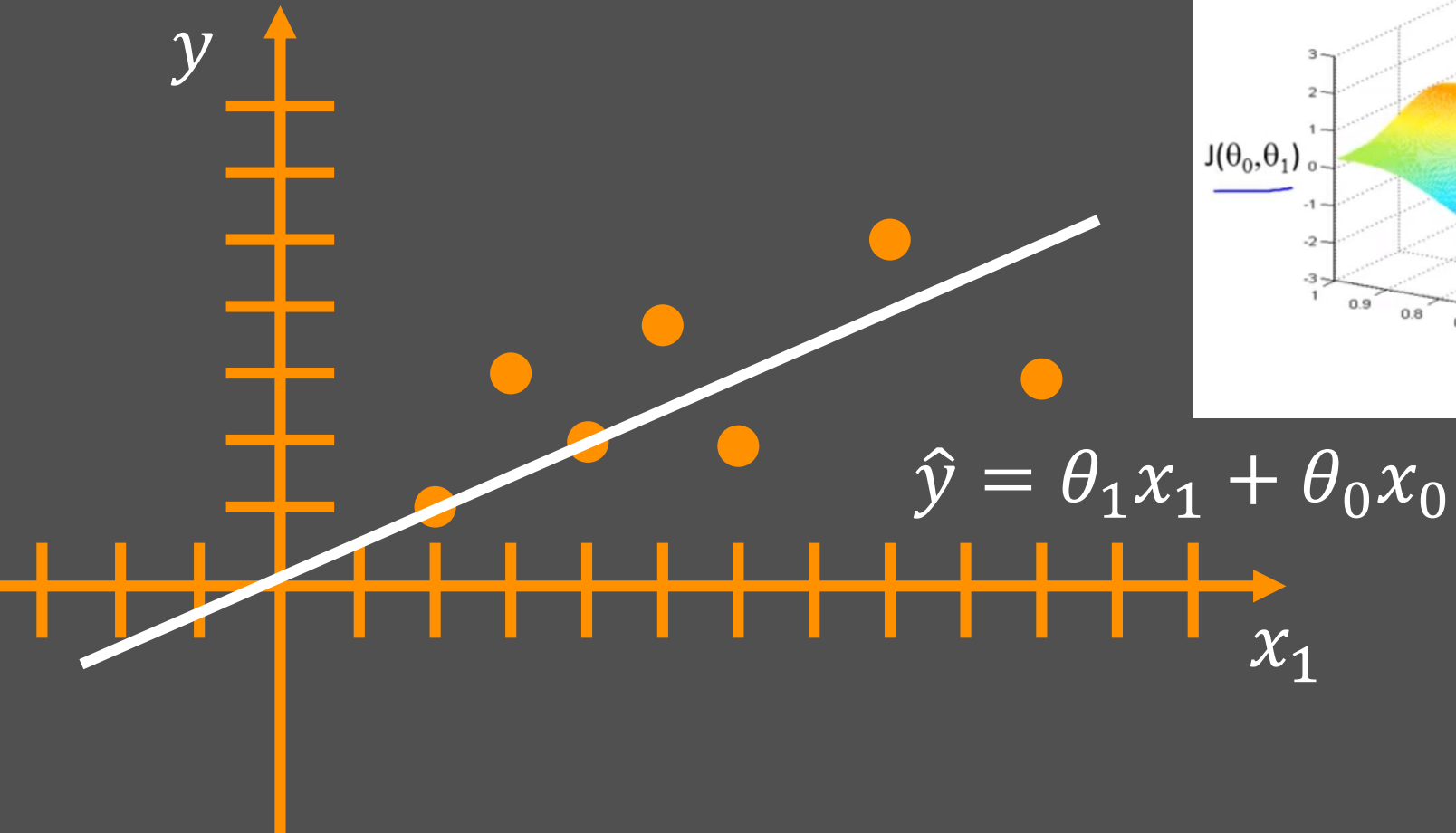
# Gradient Descent



# Gradient Descent



# Gradient Descent



# Learning Rate

Start with random  $\theta_0, \theta_1$

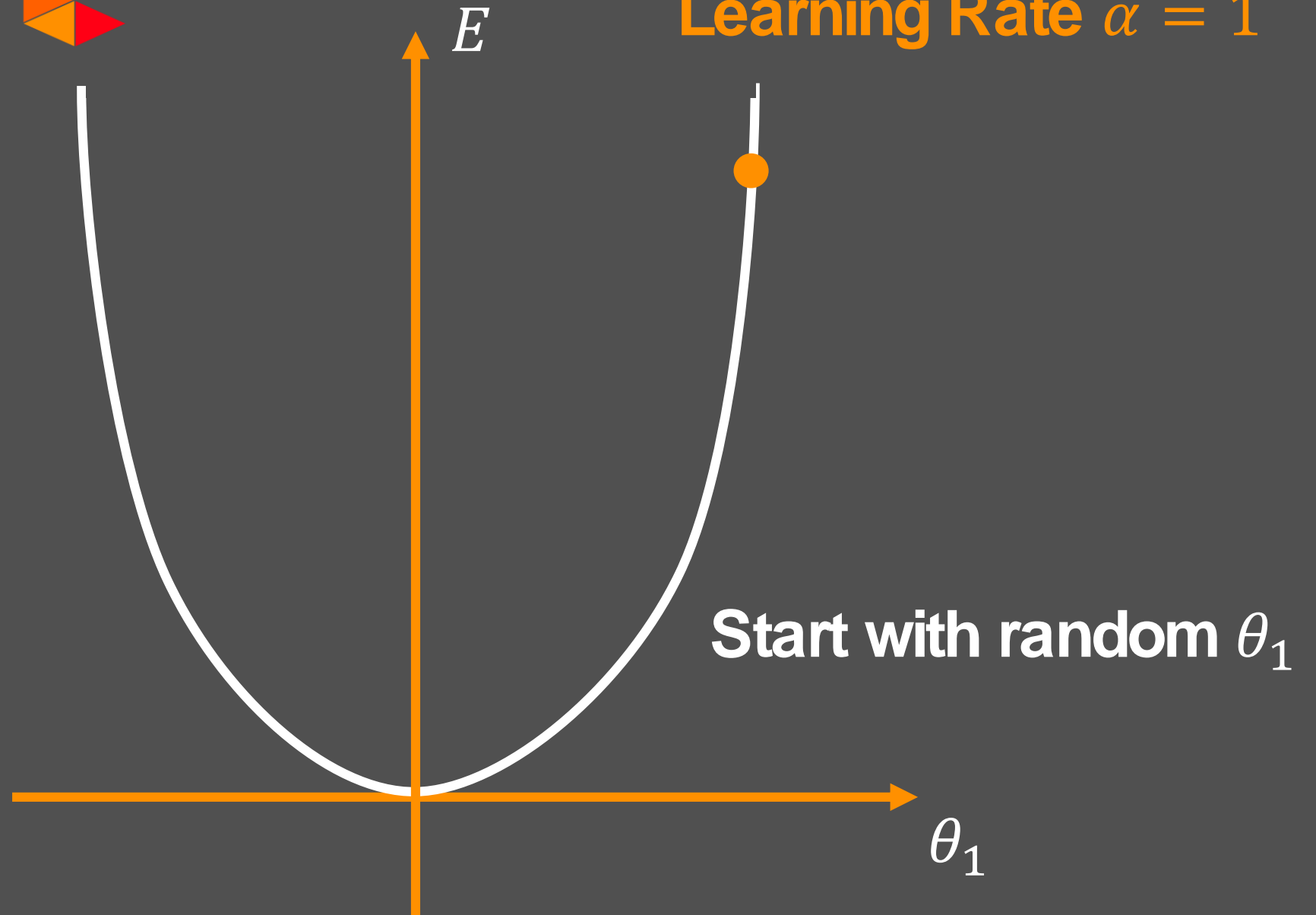
repeat until convergence{

$$\theta_0 := \theta_0 - \alpha * \frac{\partial E(\theta_0, \theta_1)}{\partial \theta_0}$$

$$\theta_1 := \theta_1 - \alpha * \frac{\partial E(\theta_0, \theta_1)}{\partial \theta_1}$$

}





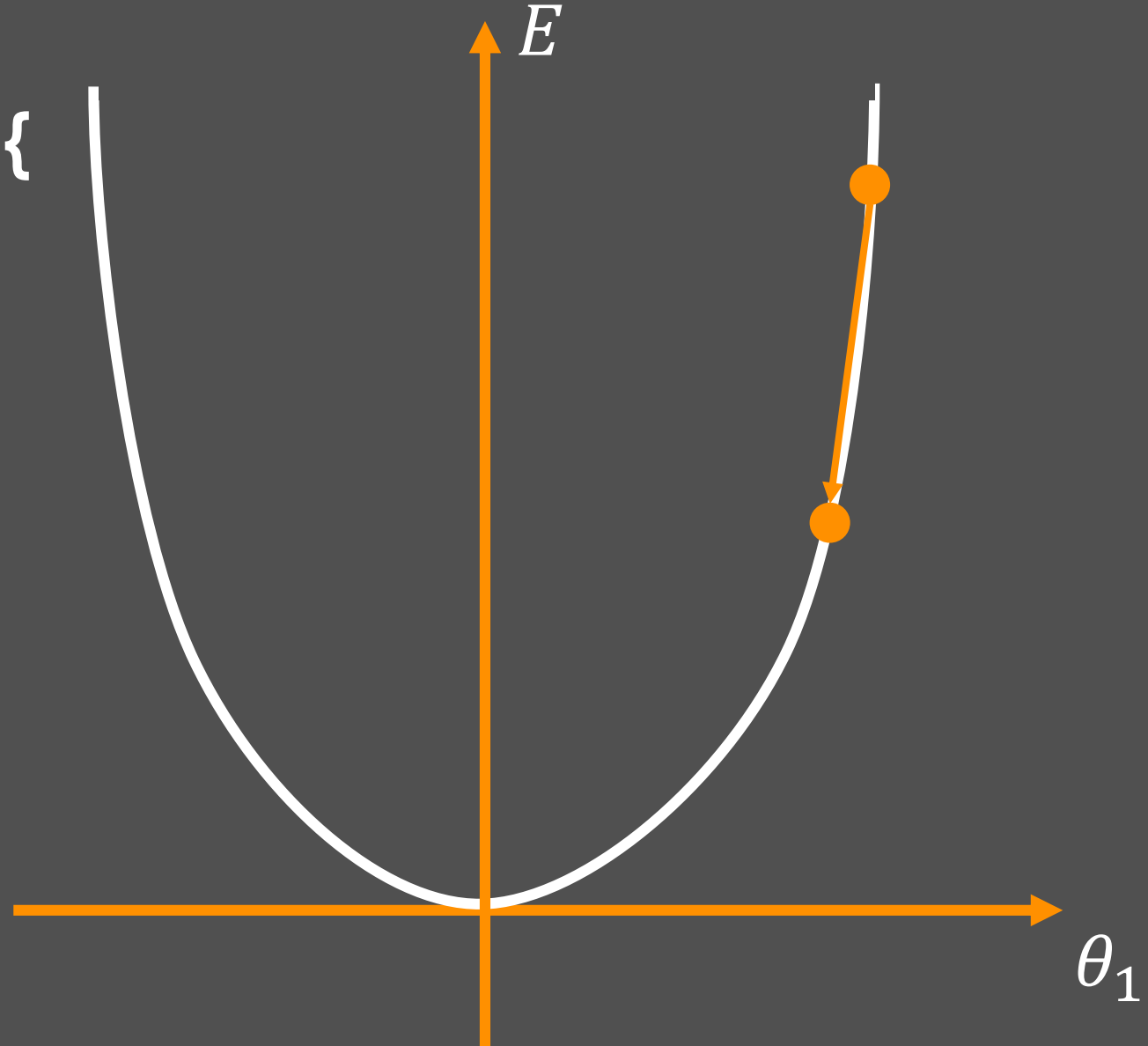
repeat until convergence{

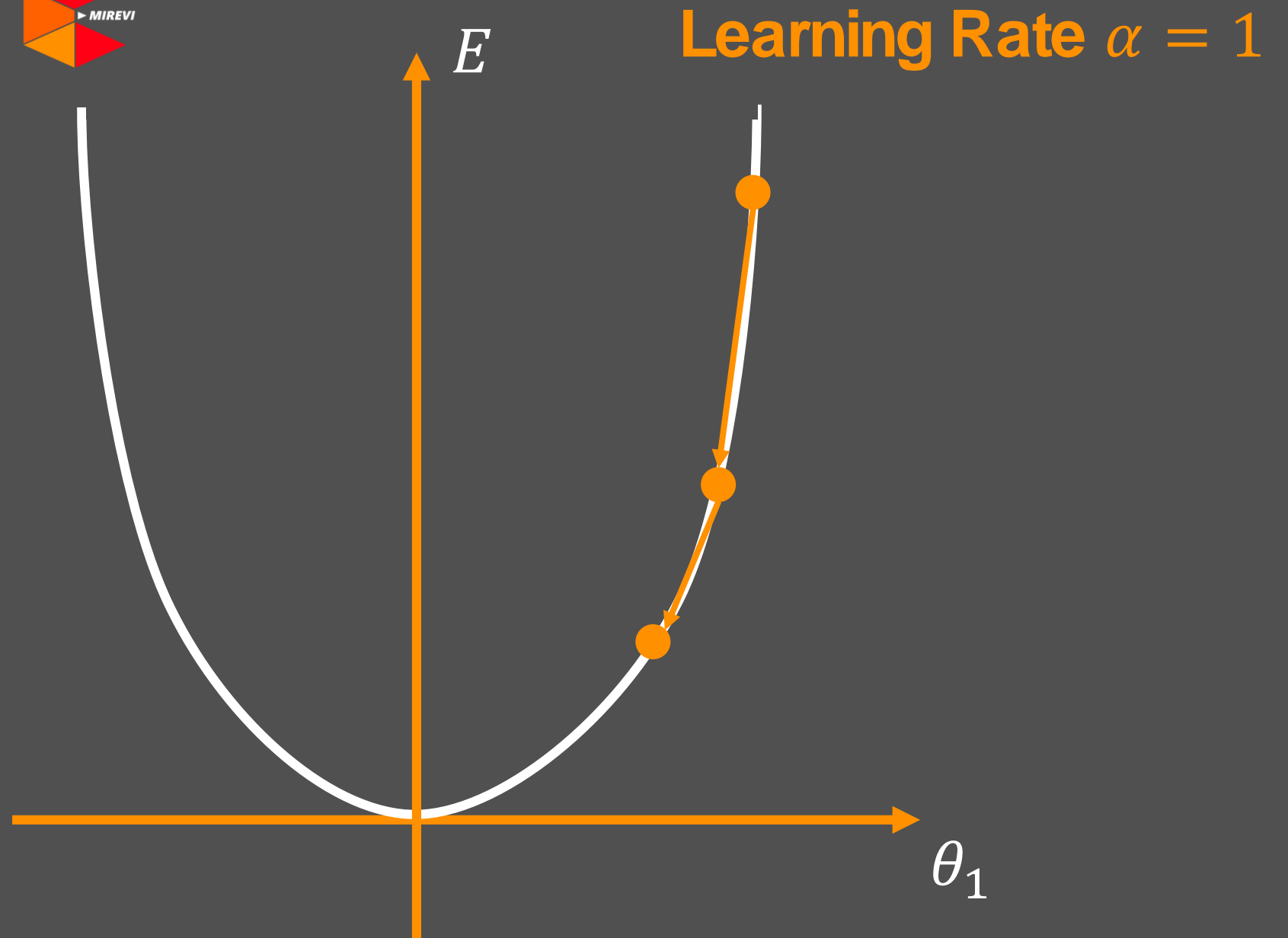
$$\theta_1 := \theta_1 - \alpha * \frac{\partial E(\theta_1)}{\partial \theta_1}$$

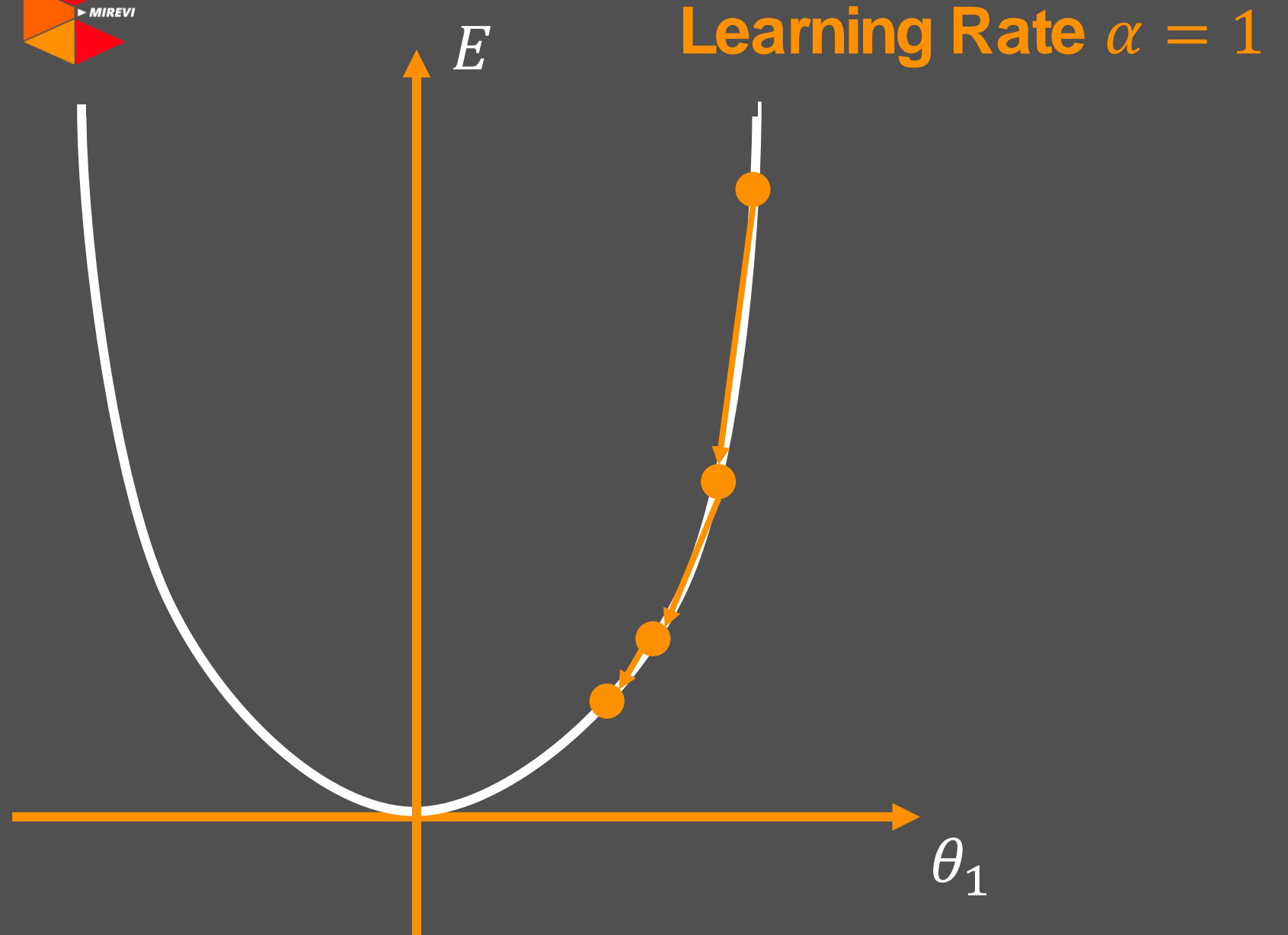
}

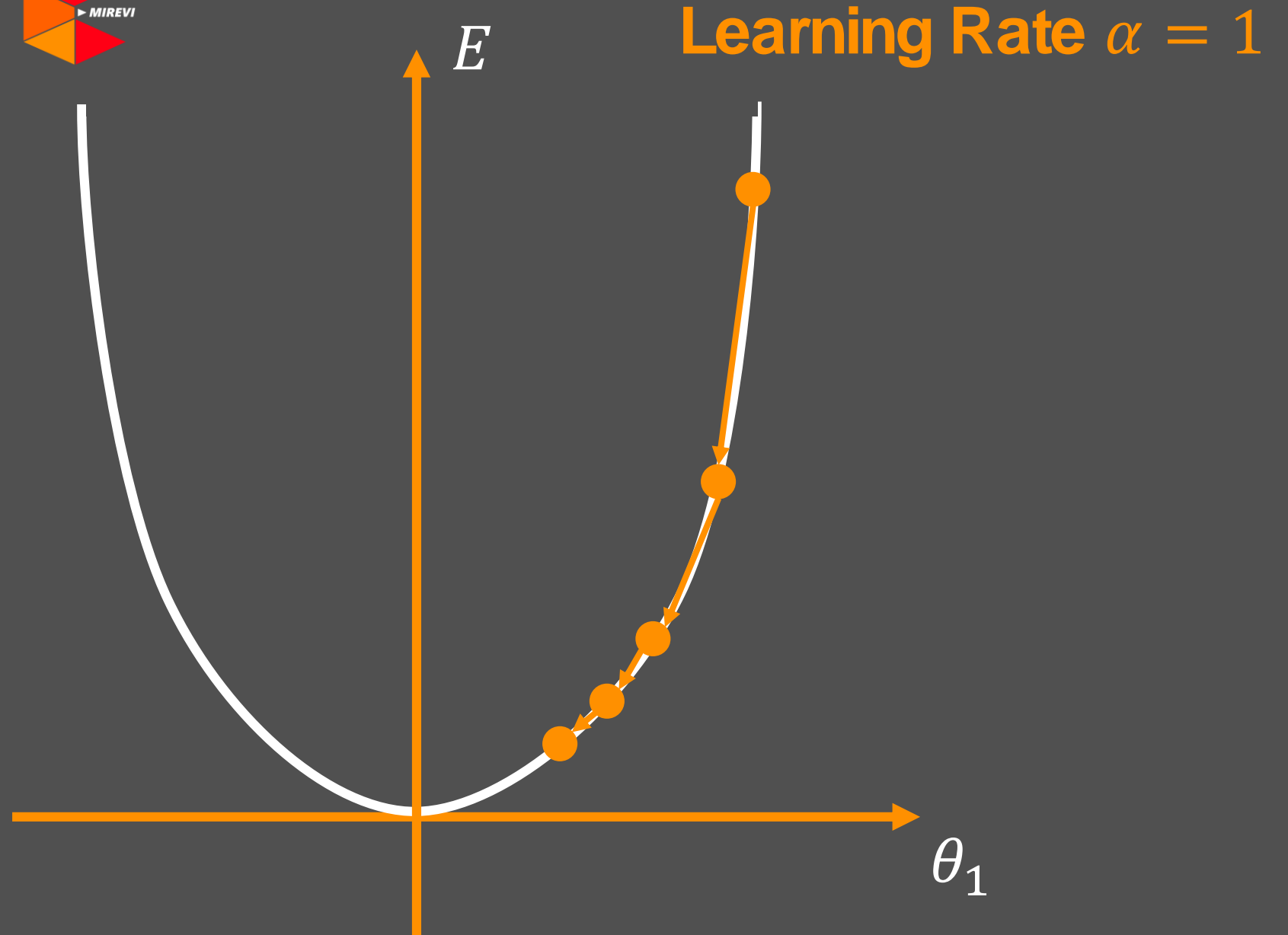
$\alpha$ : Learning Rate

$$\alpha = 1$$

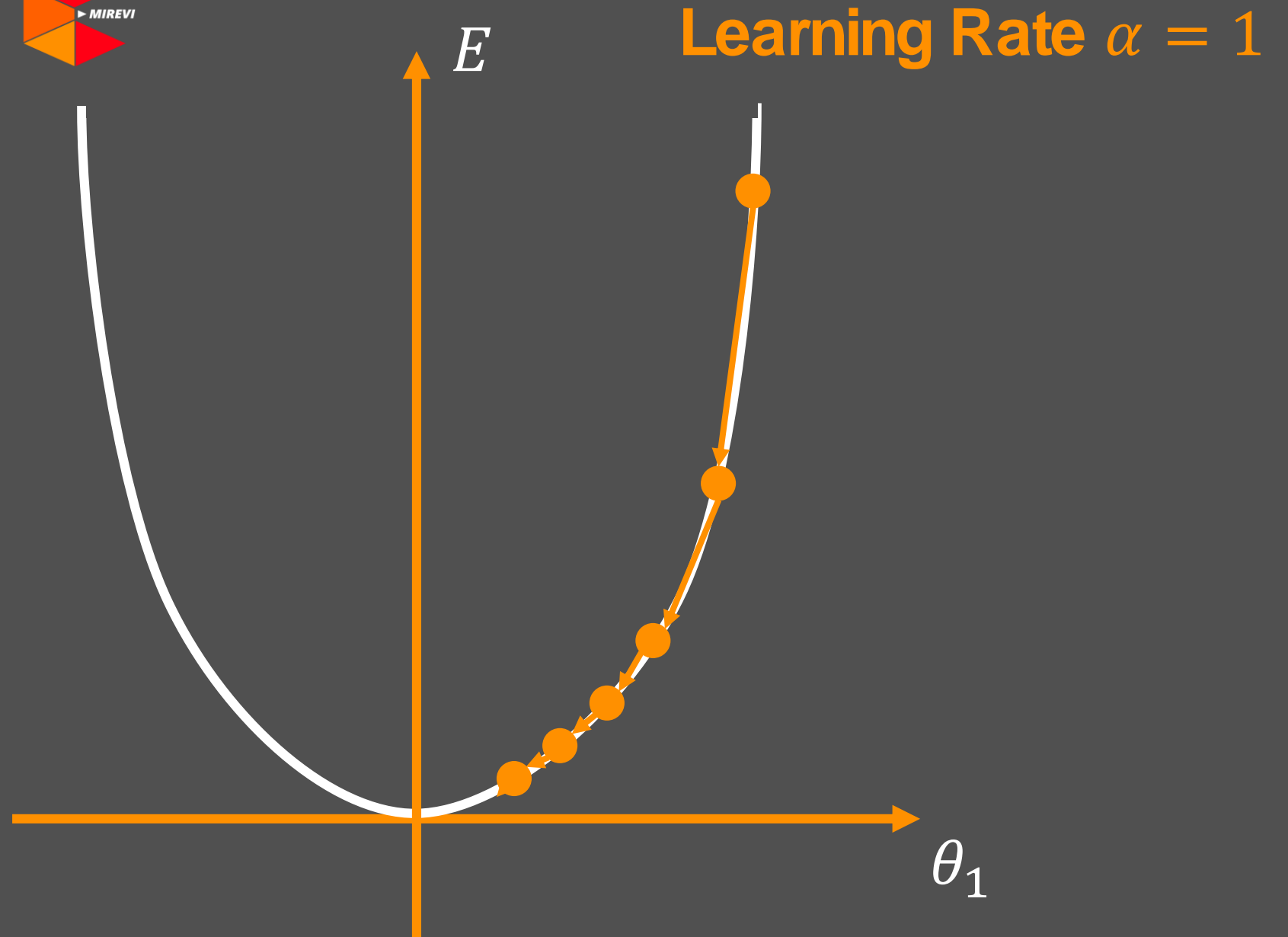






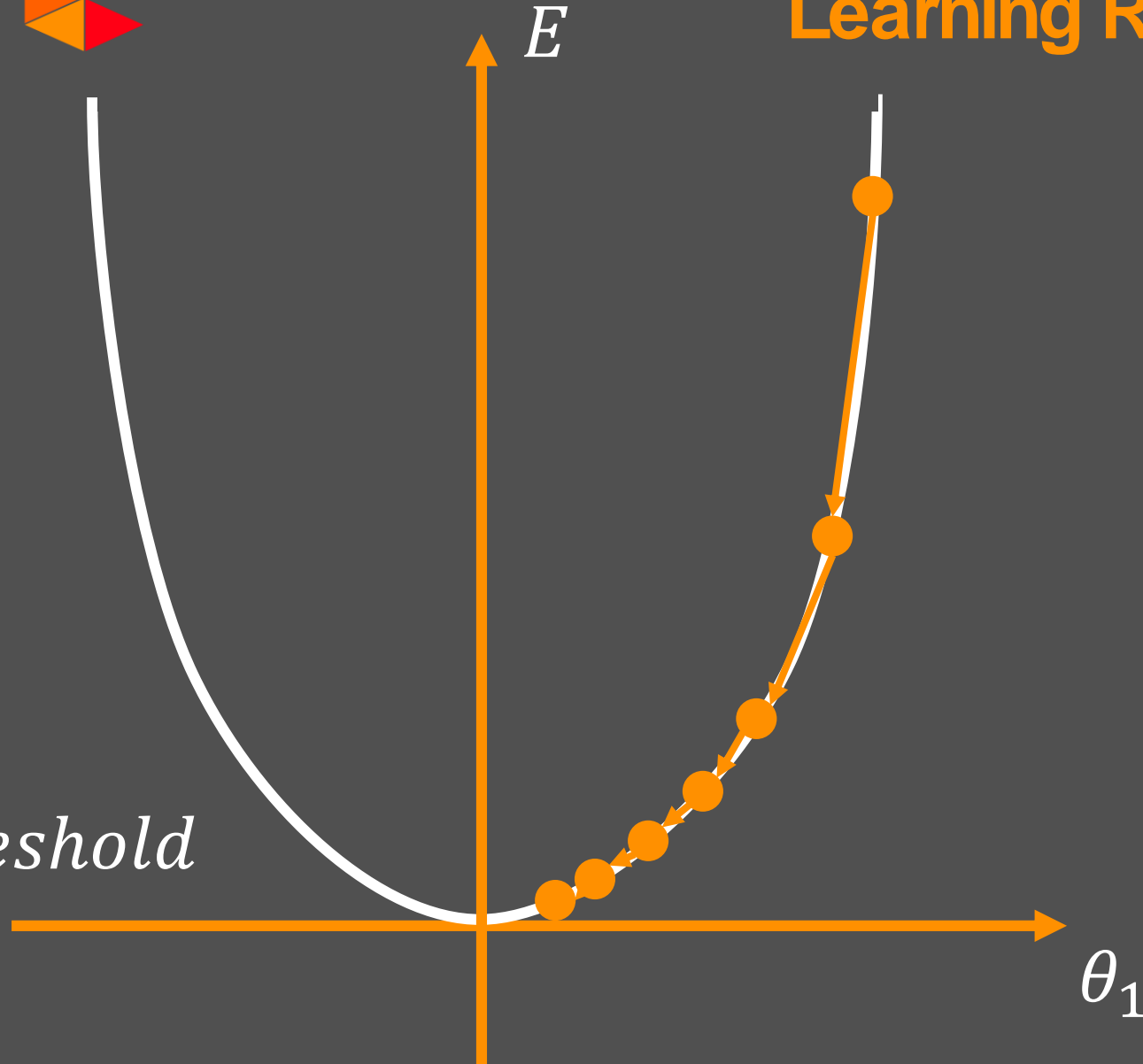




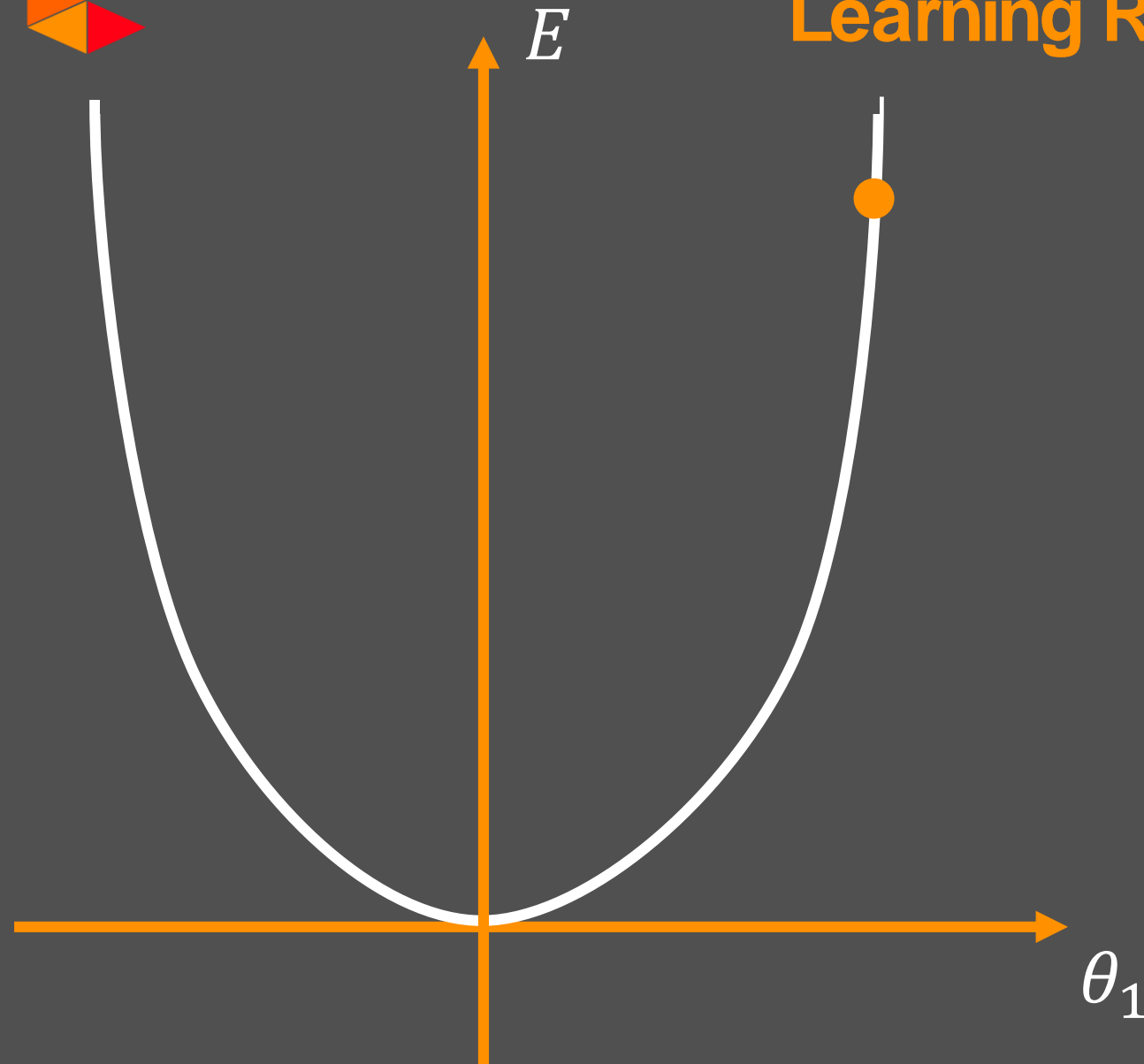


Learning Rate  $\alpha = 1$

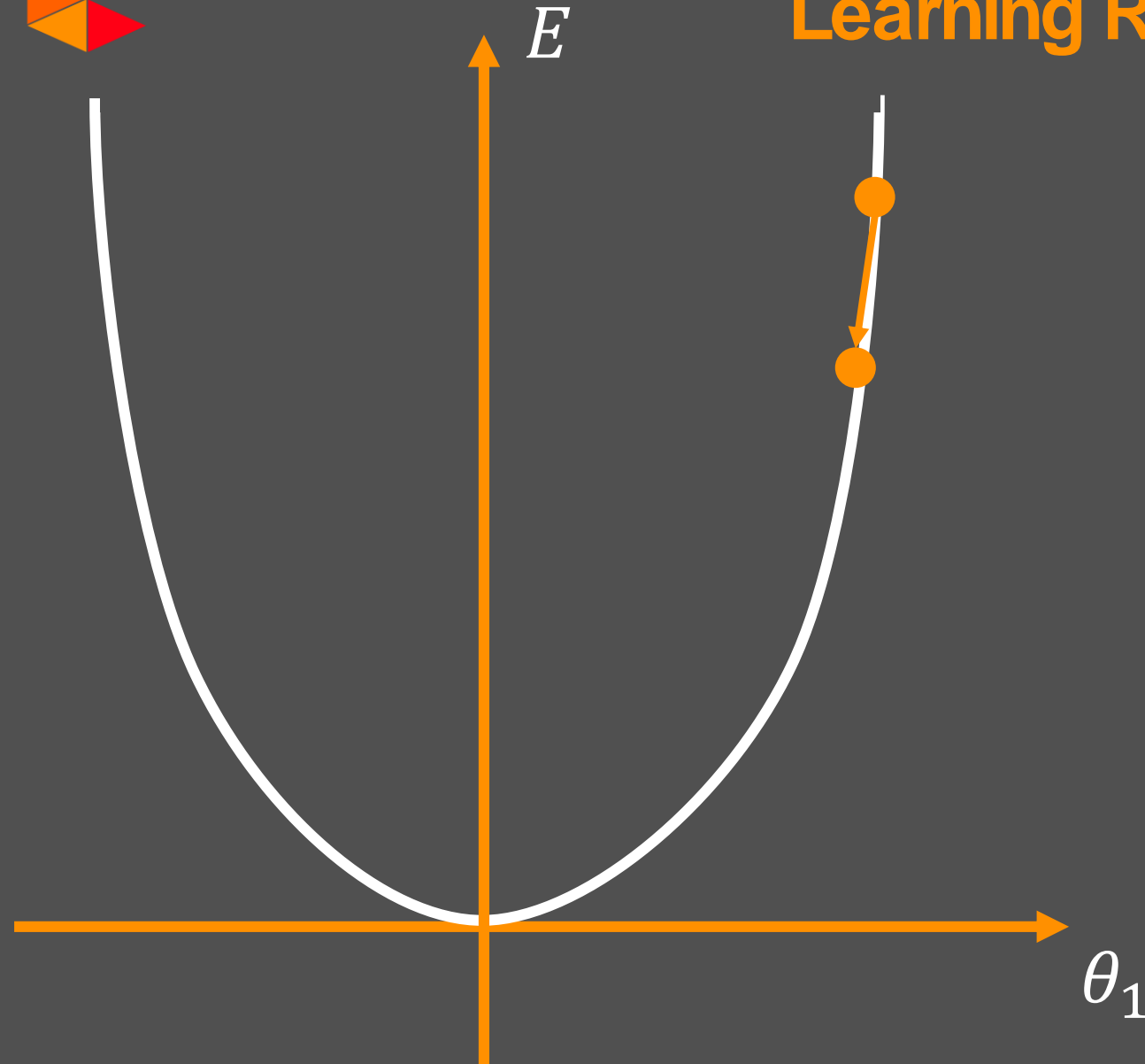
*e.g.:  $E < \text{threshold}$*

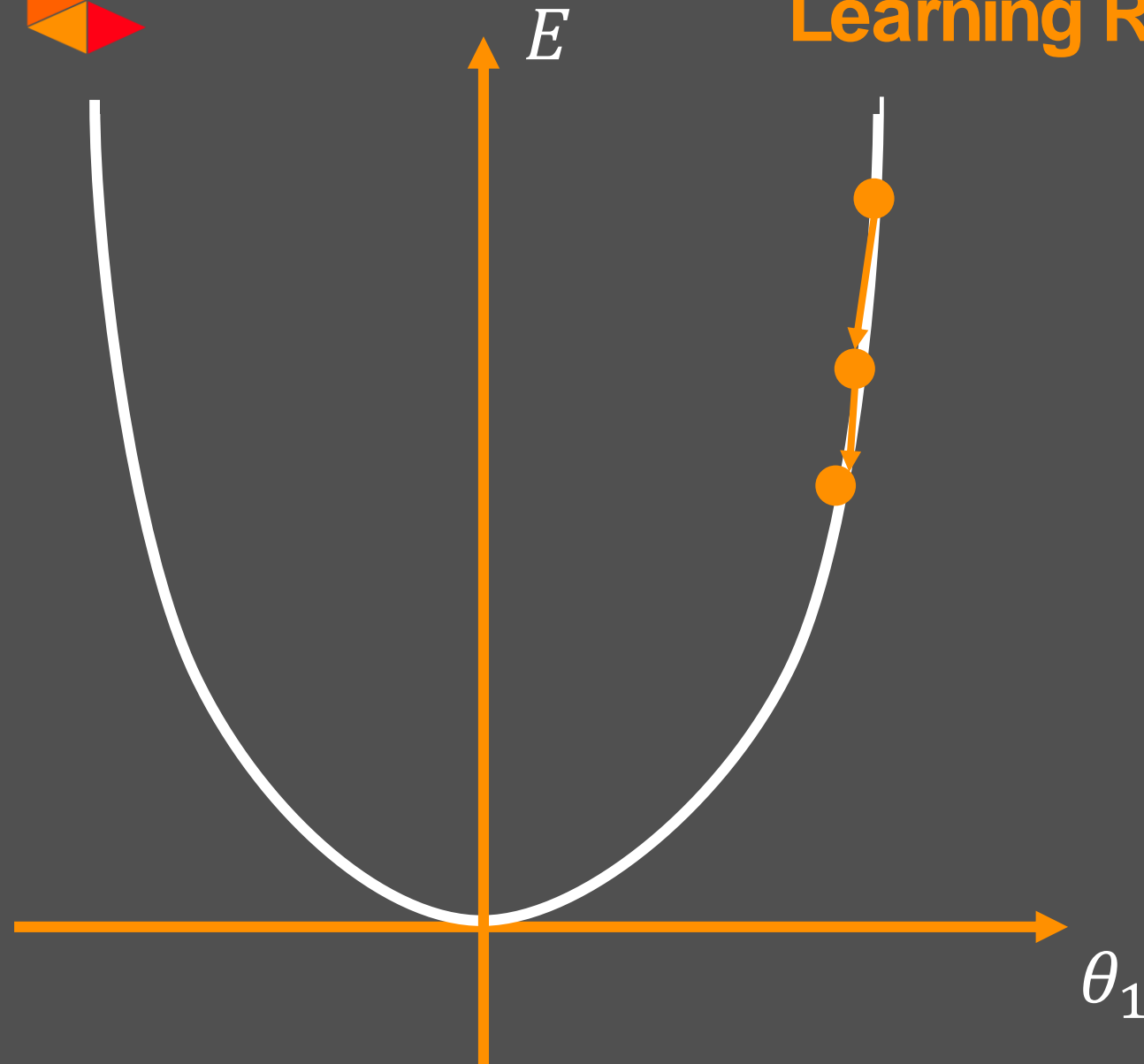


$$\alpha = 0.5$$

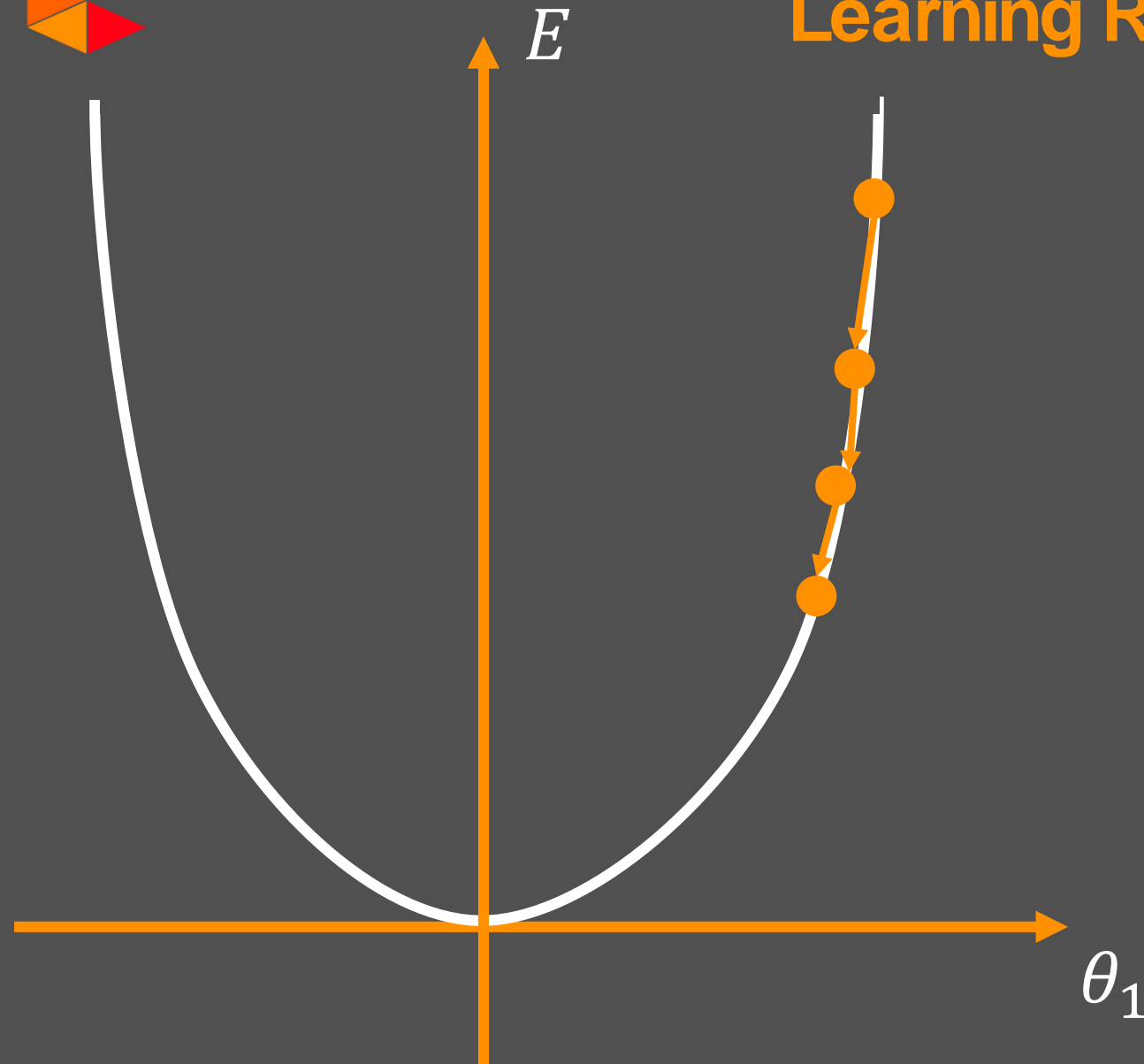


**Learning Rate  $\alpha = 0.5$**

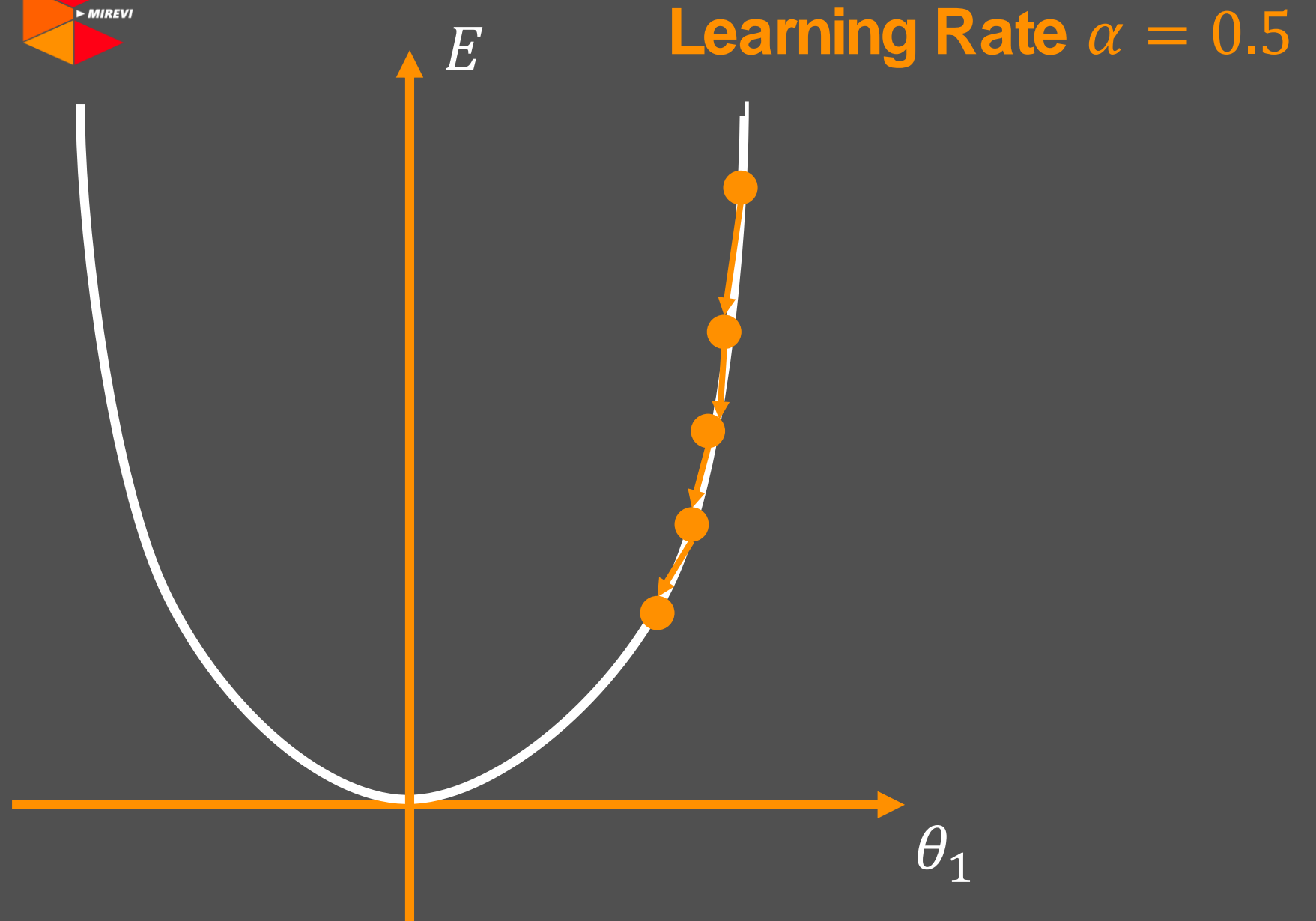


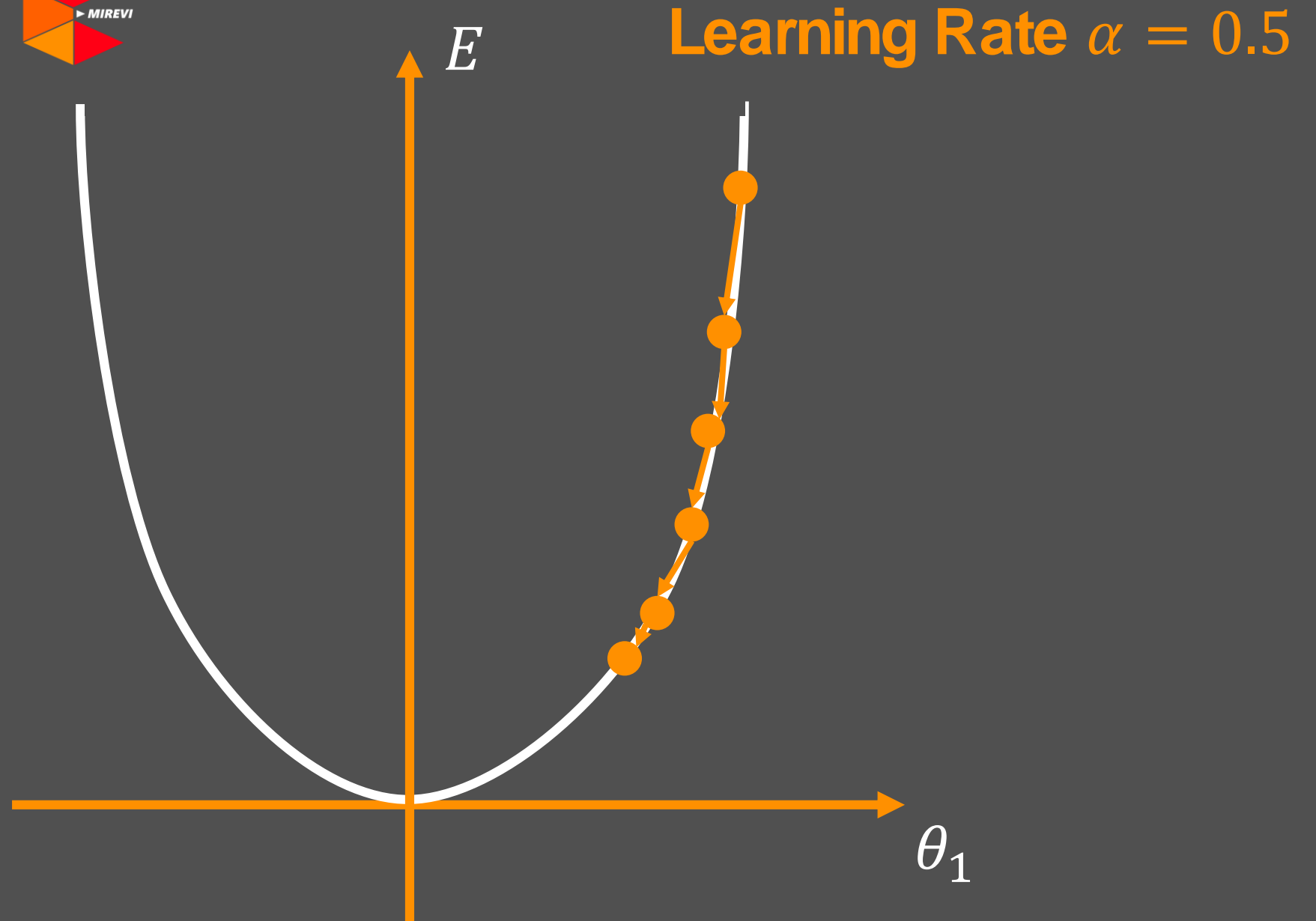


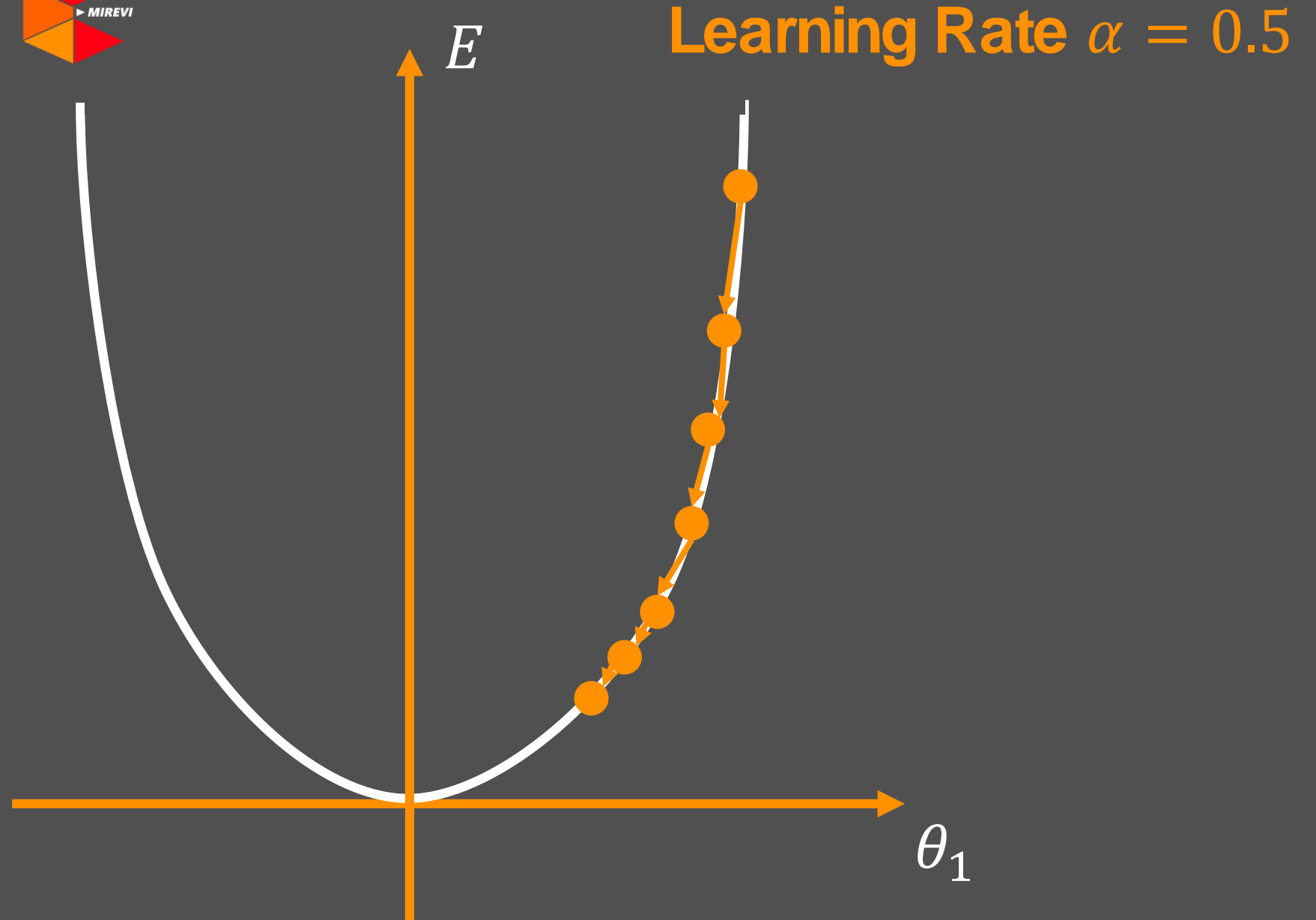


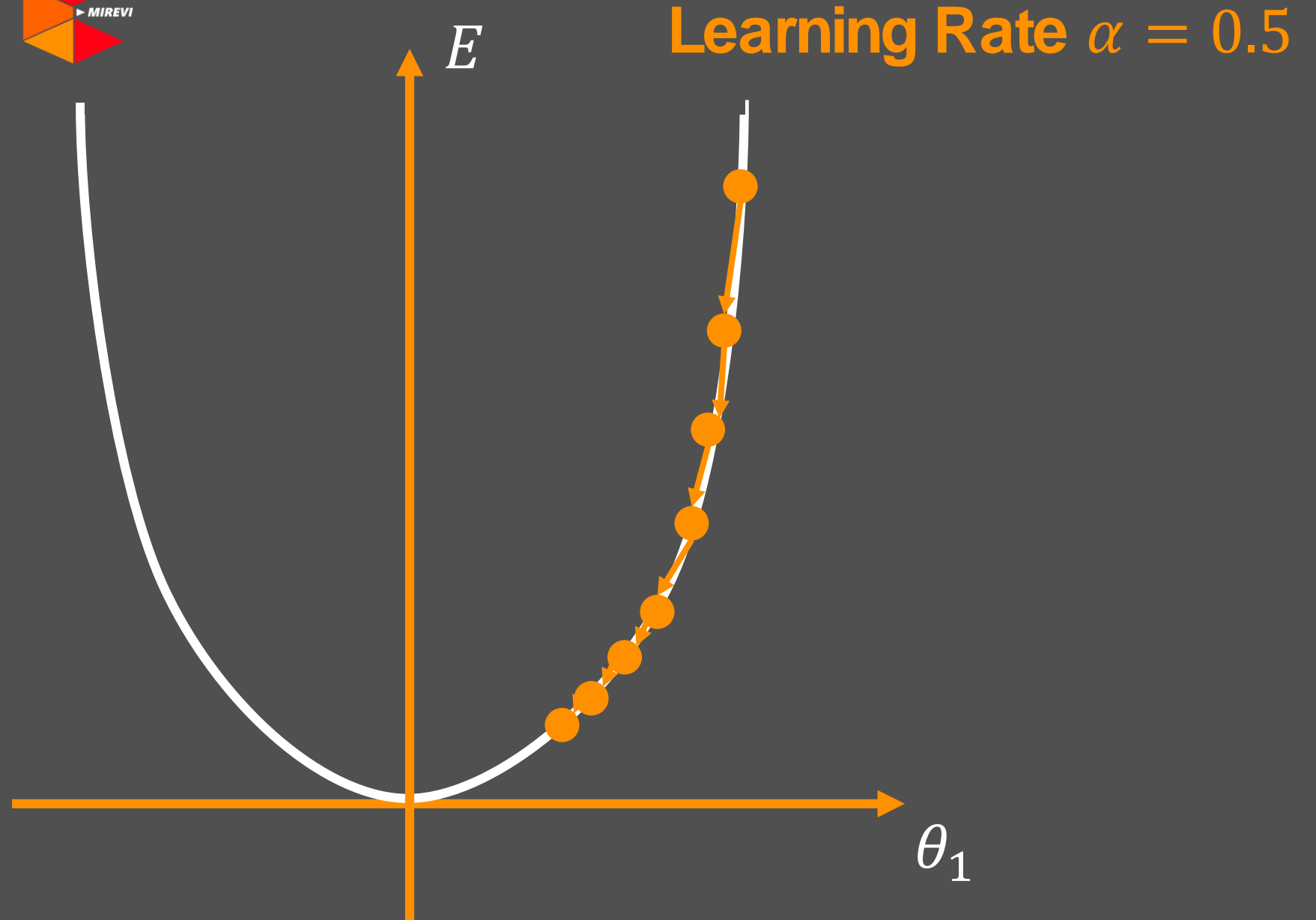


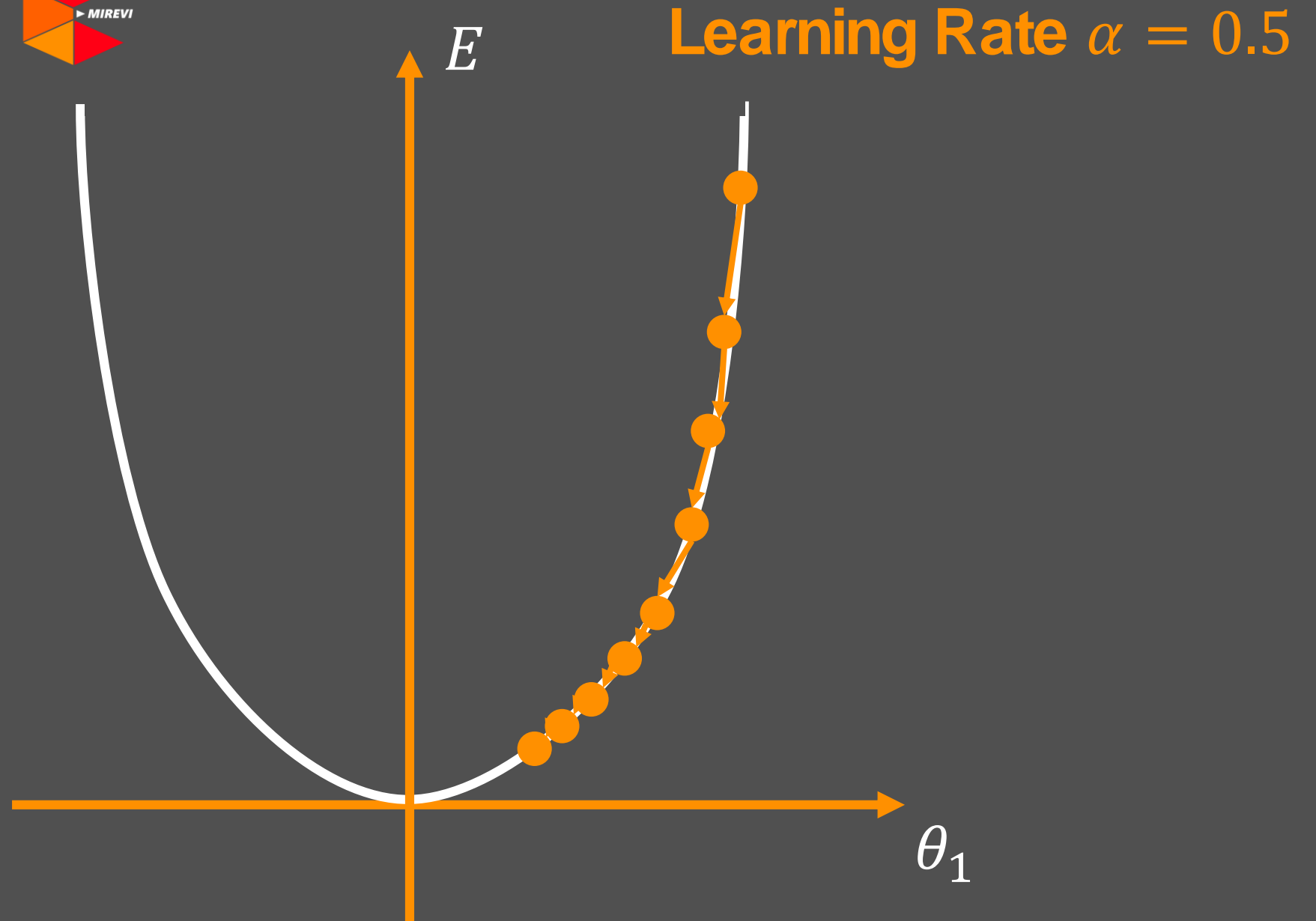
**Learning Rate  $\alpha = 0.5$**



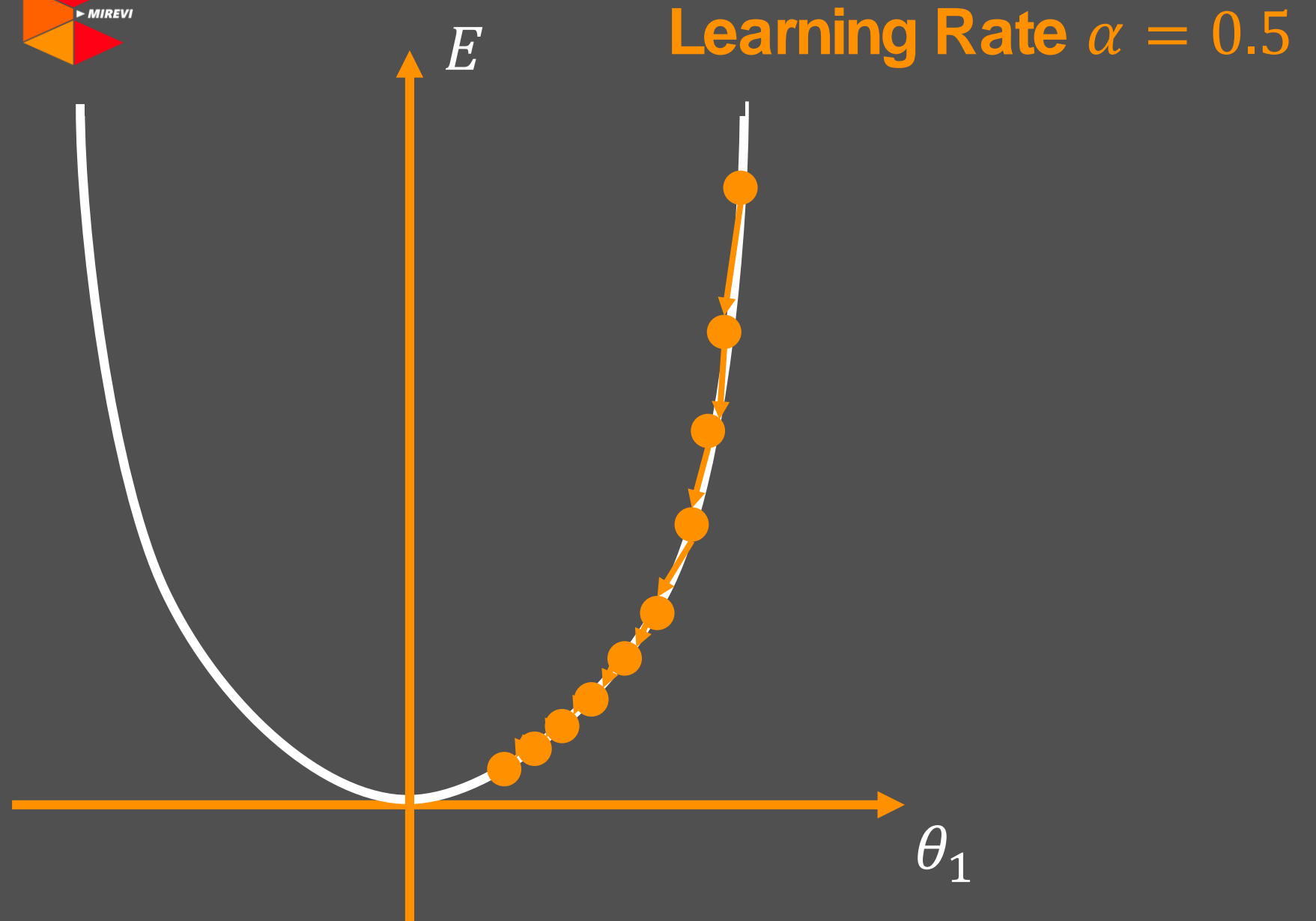




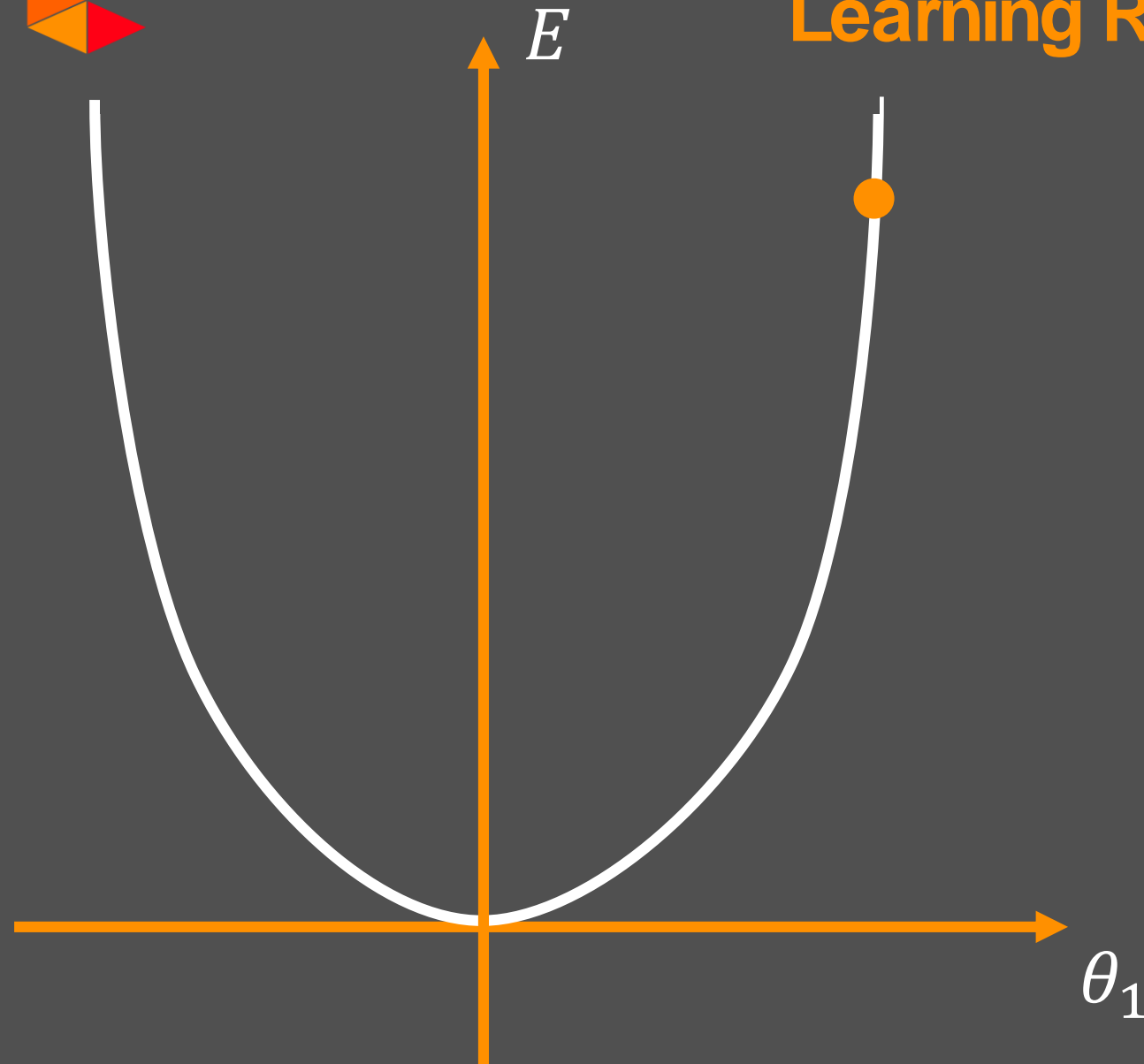




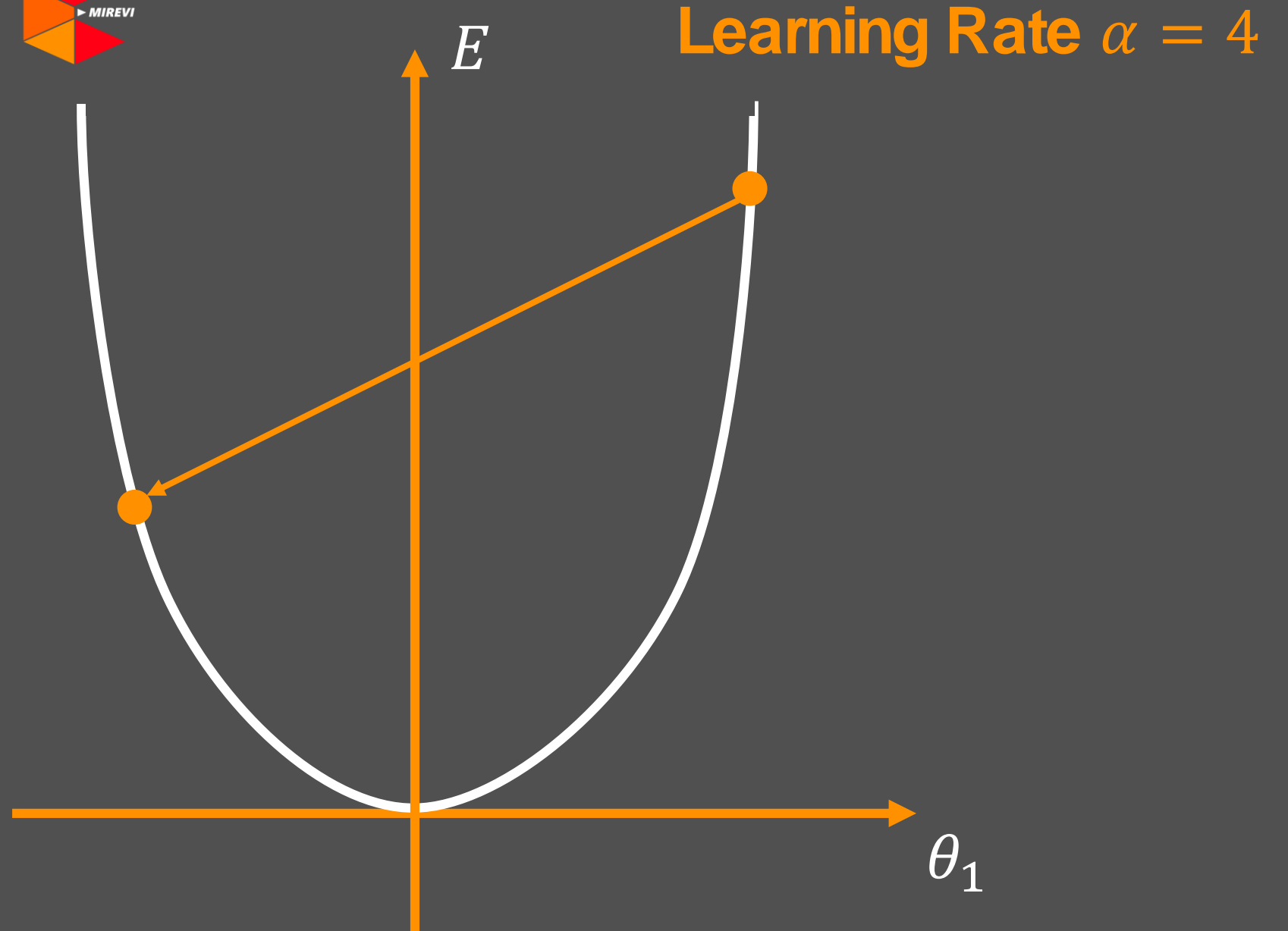




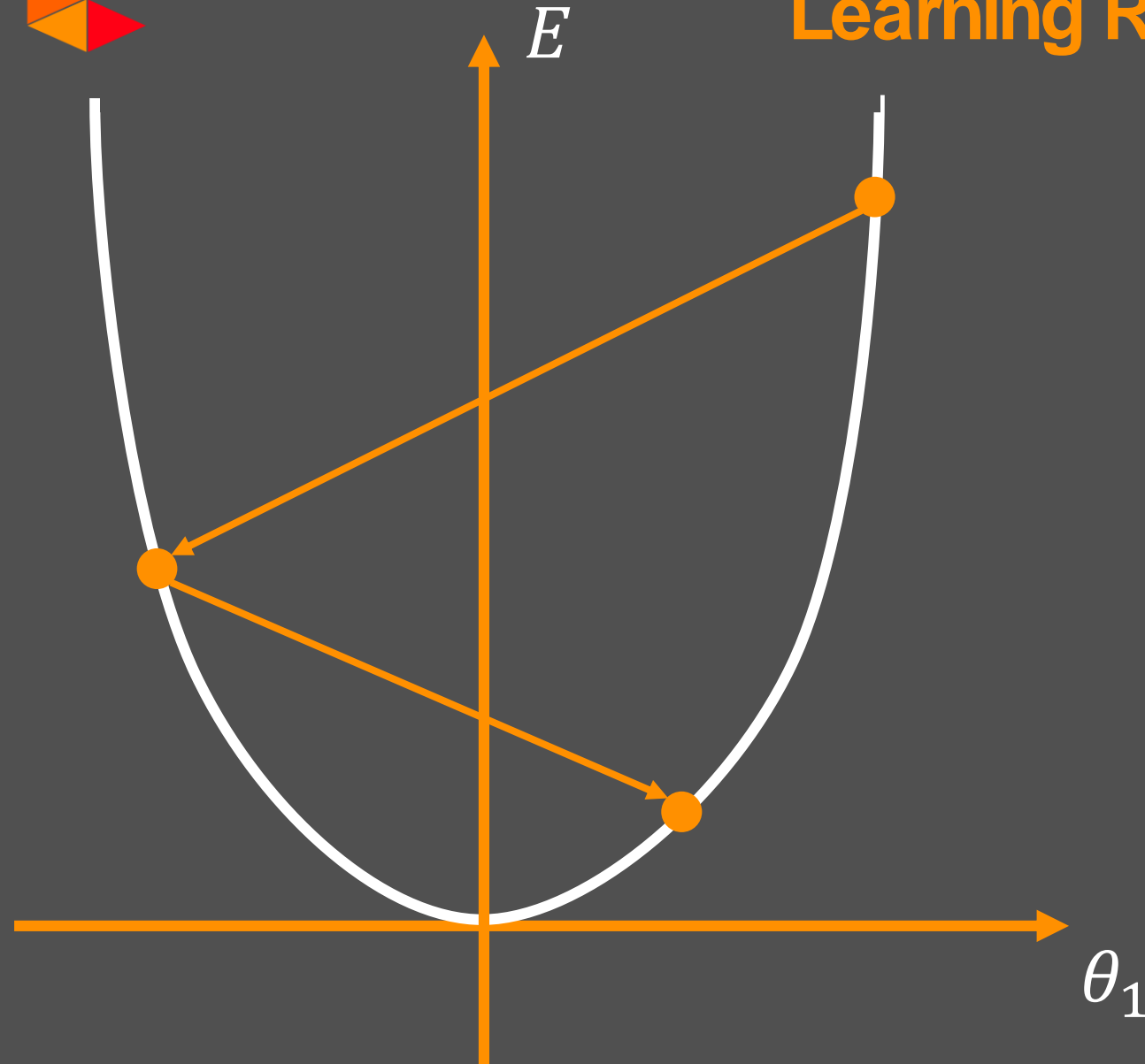
$$\alpha = 4$$



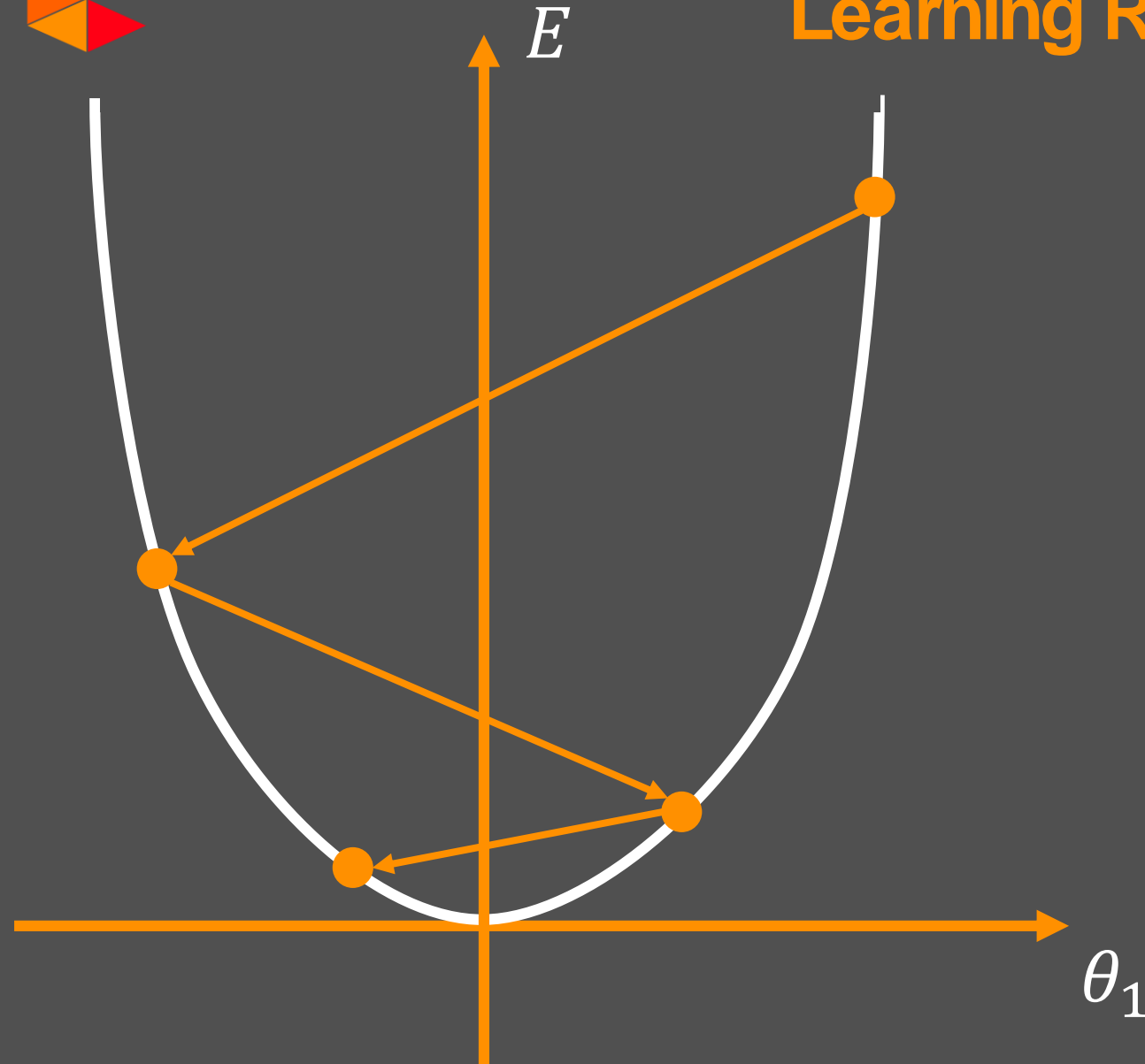
**Learning Rate  $\alpha = 4$**

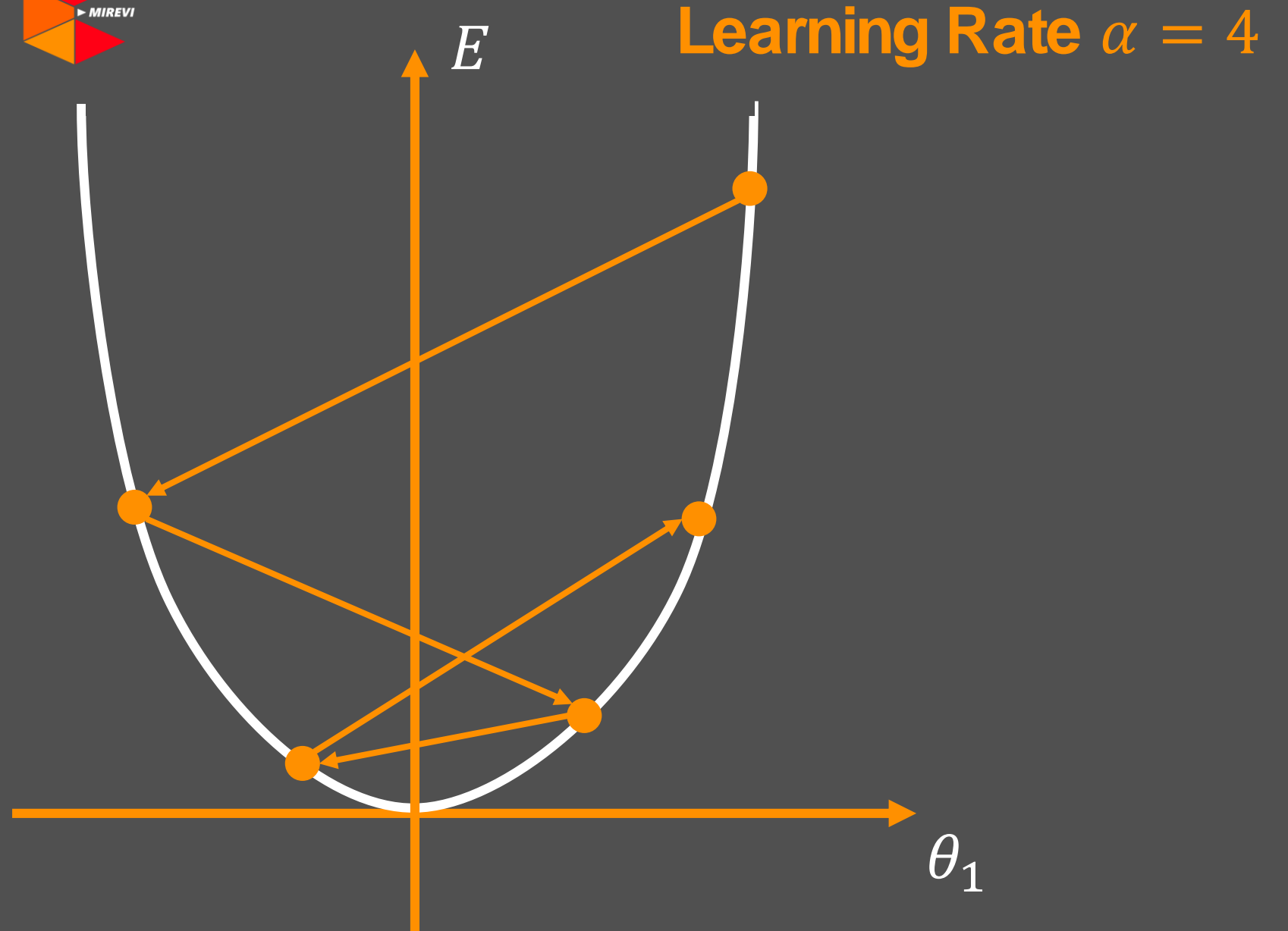


Learning Rate  $\alpha = 4$

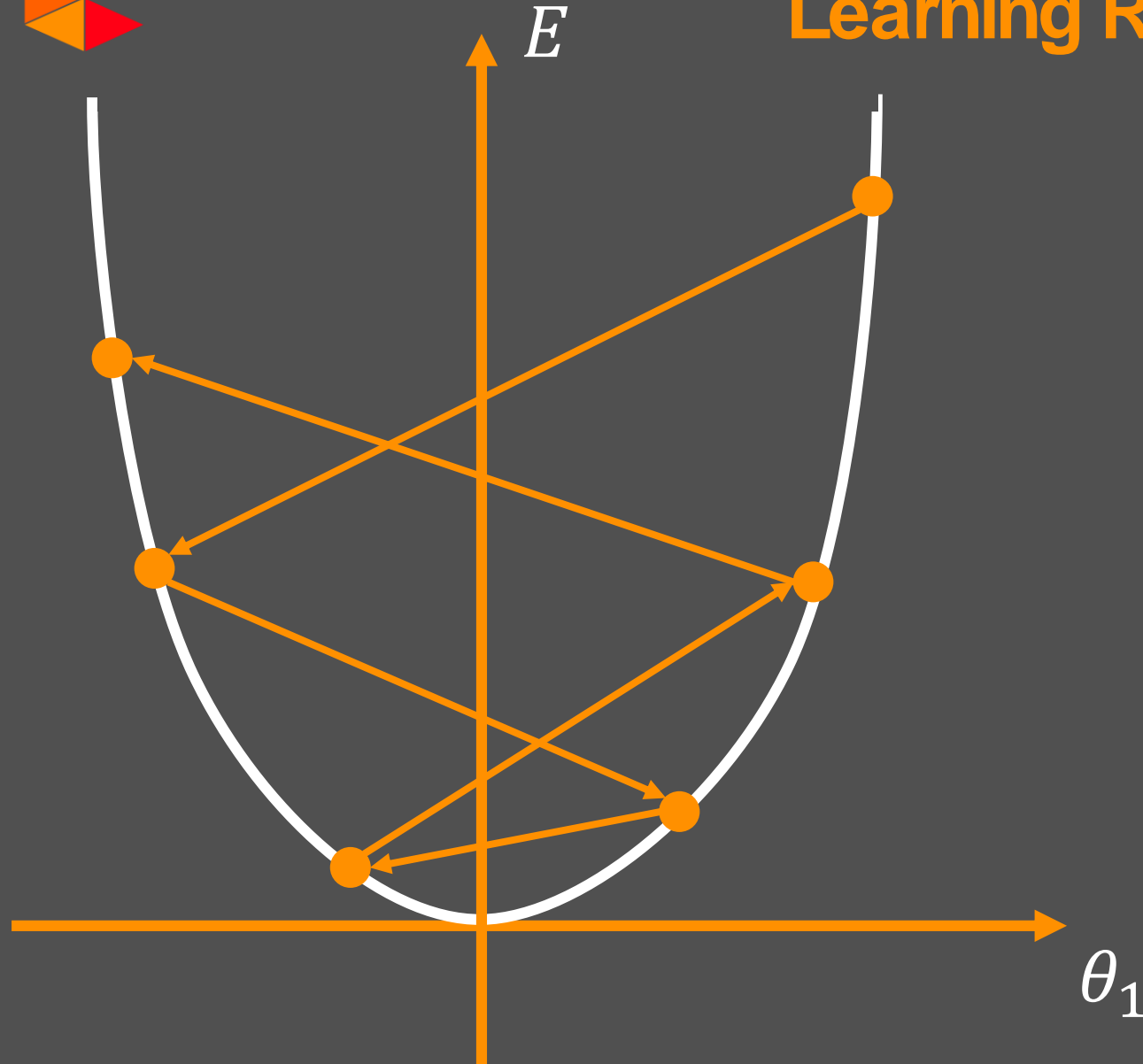


Learning Rate  $\alpha = 4$

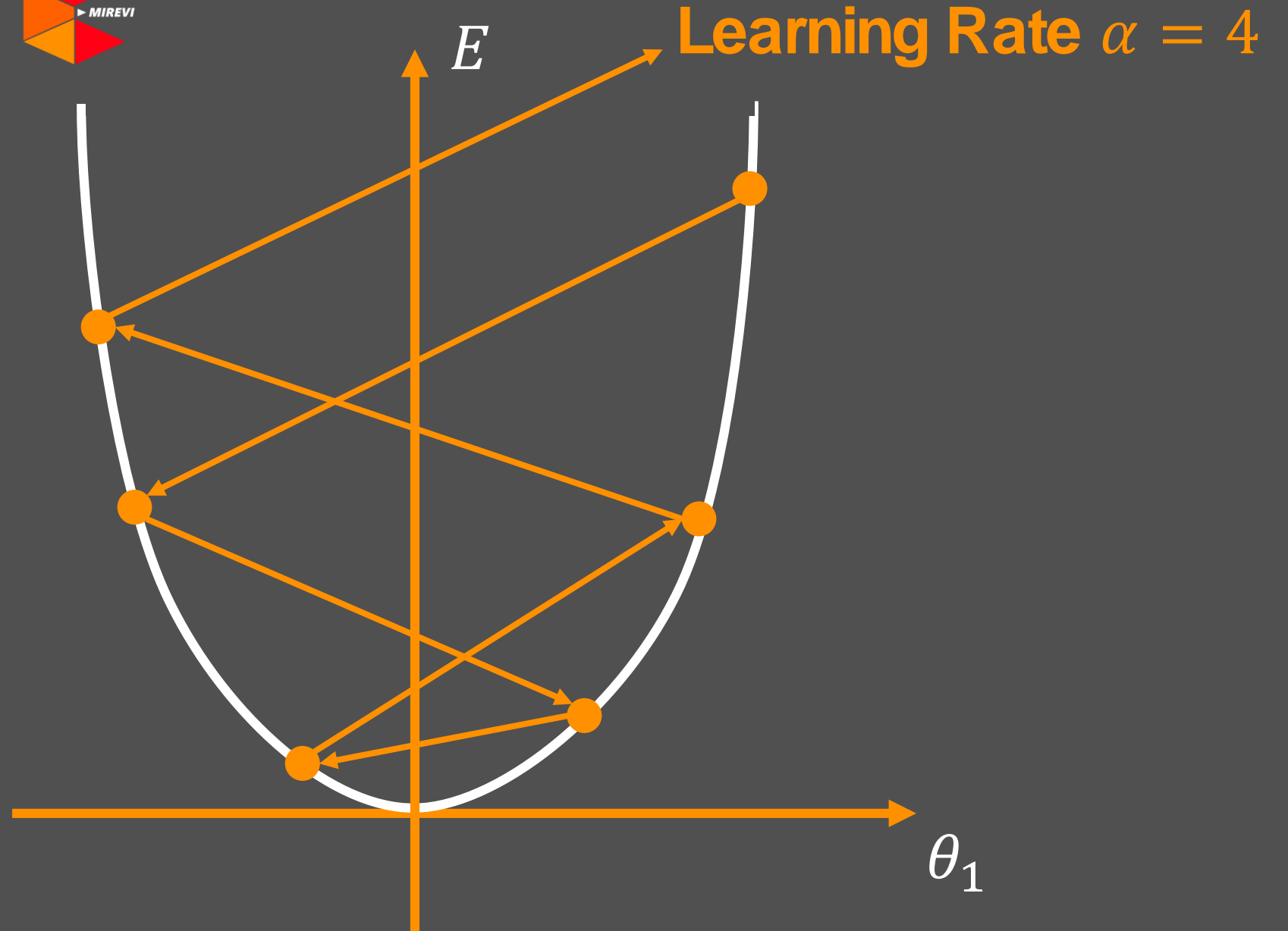




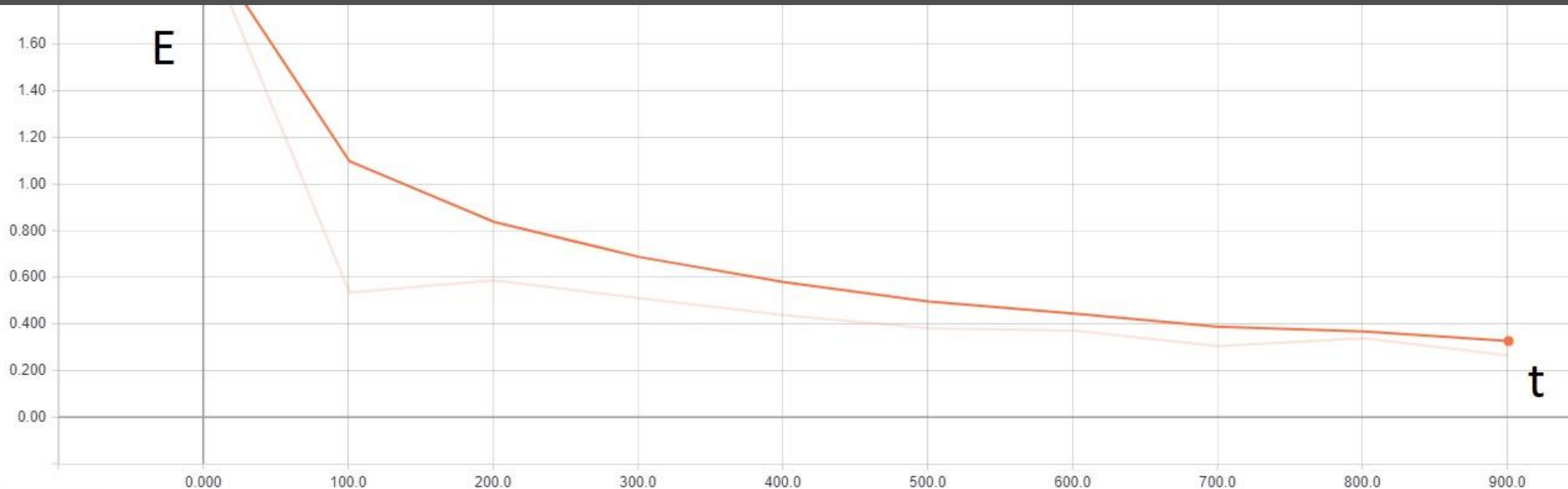
Learning Rate  $\alpha = 4$



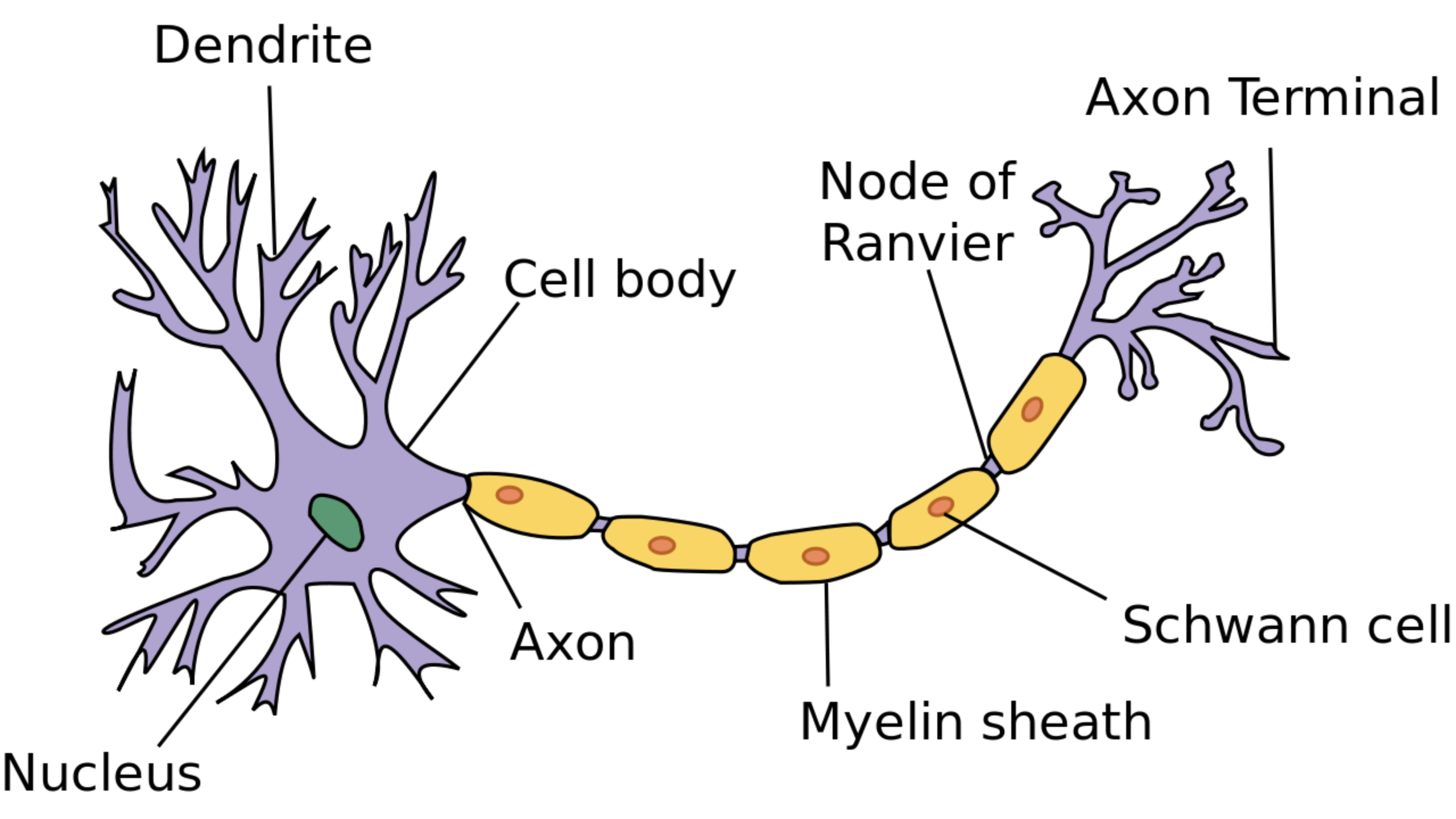


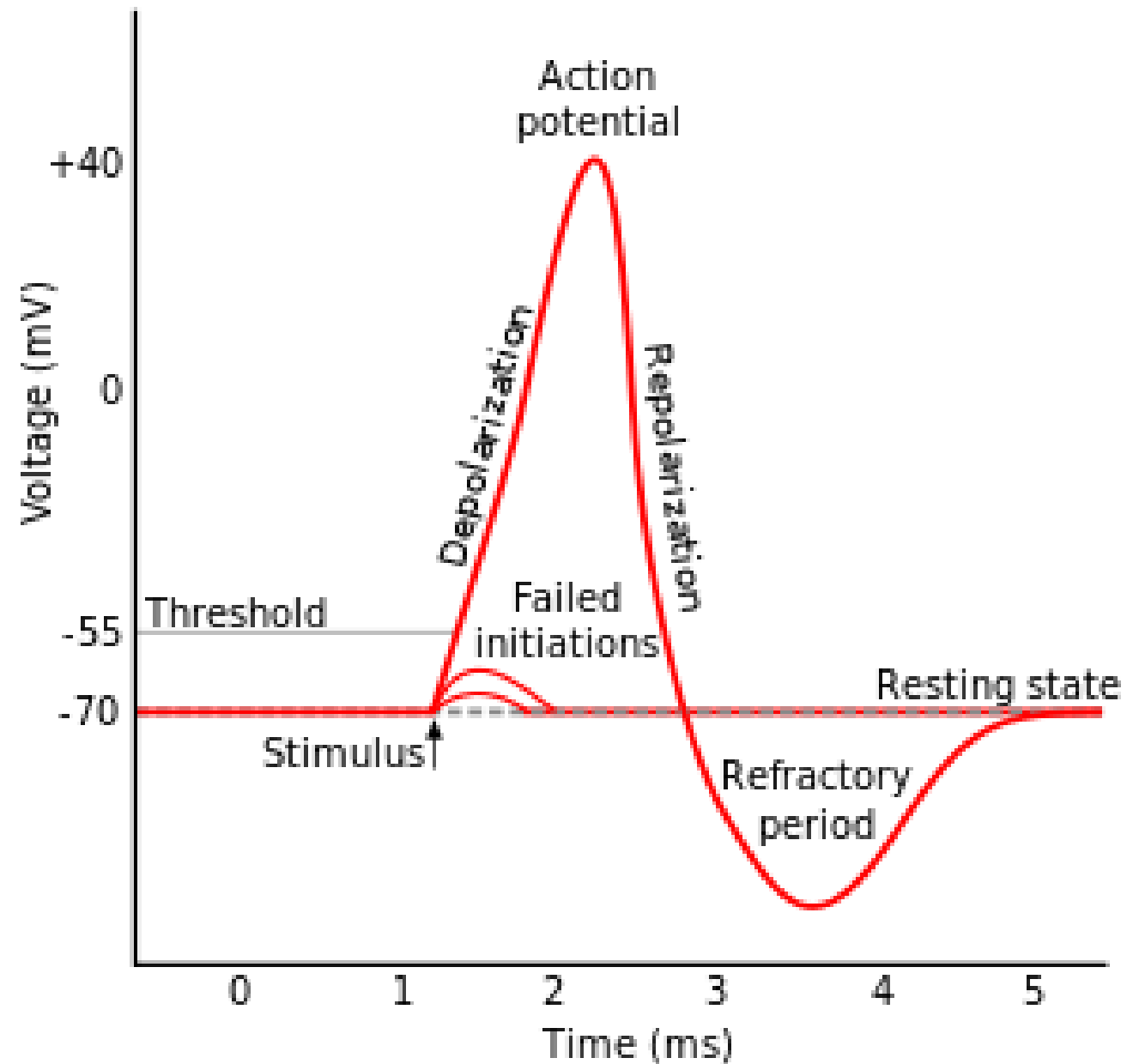


- Choose good learning rate  $\alpha$  by testing
- e.g.  $\alpha = i, i \in \{0.01, 0.03, 0.09, 0.1, 0.3, 0.9, 1, 3, 9\}$



# Towards Neural Nets





# Perceptron

- **Binary Classification**
- **Element of Neural Net**
- **Weighted Sum**
- **Activation Function**
- **Decision Boundary**

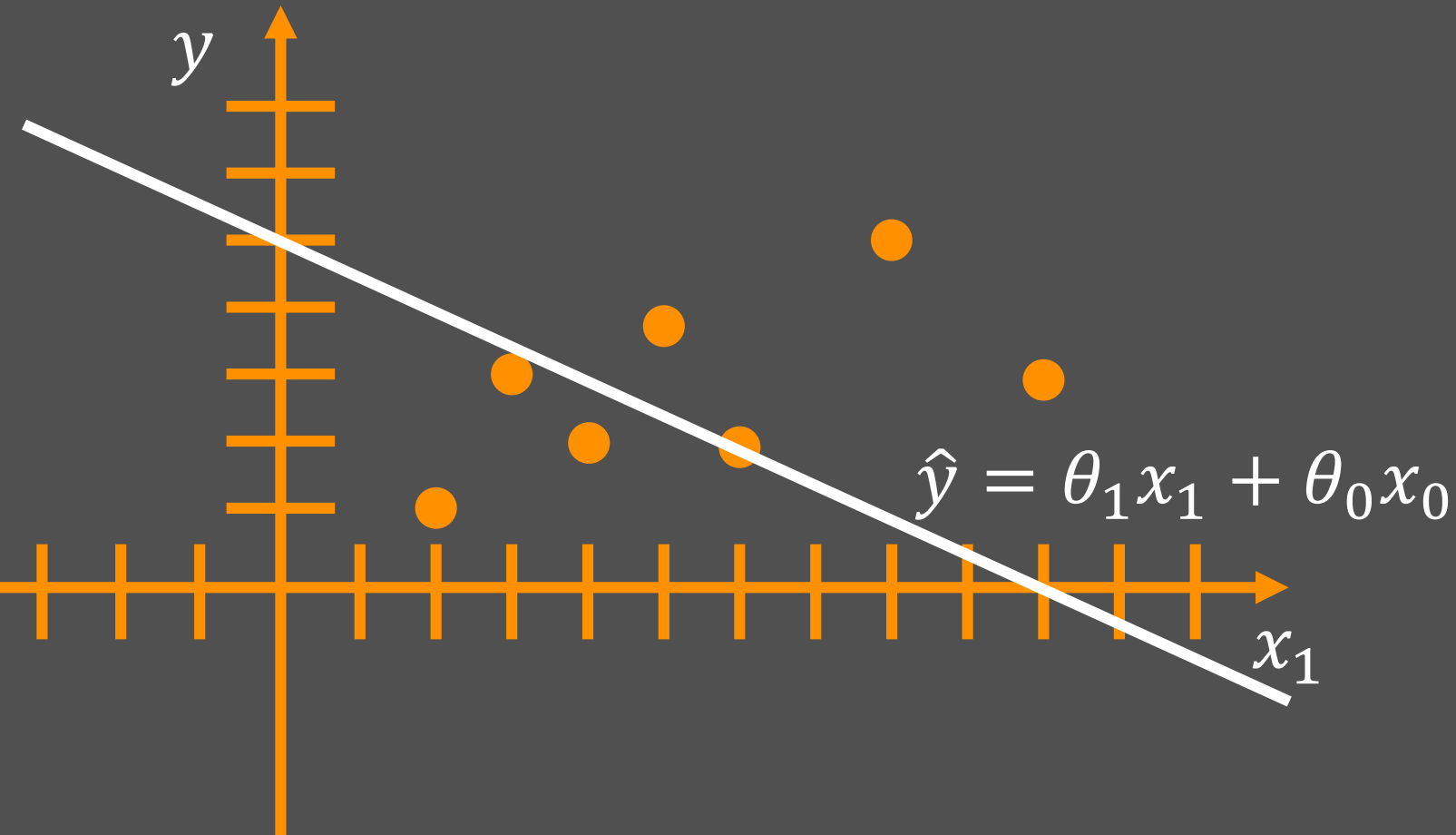
$$y=f(x)$$

# Elements of Perceptron

- **Activation Function**  $y = a(z)$
- **Weighted Sum**  $z = \theta x$

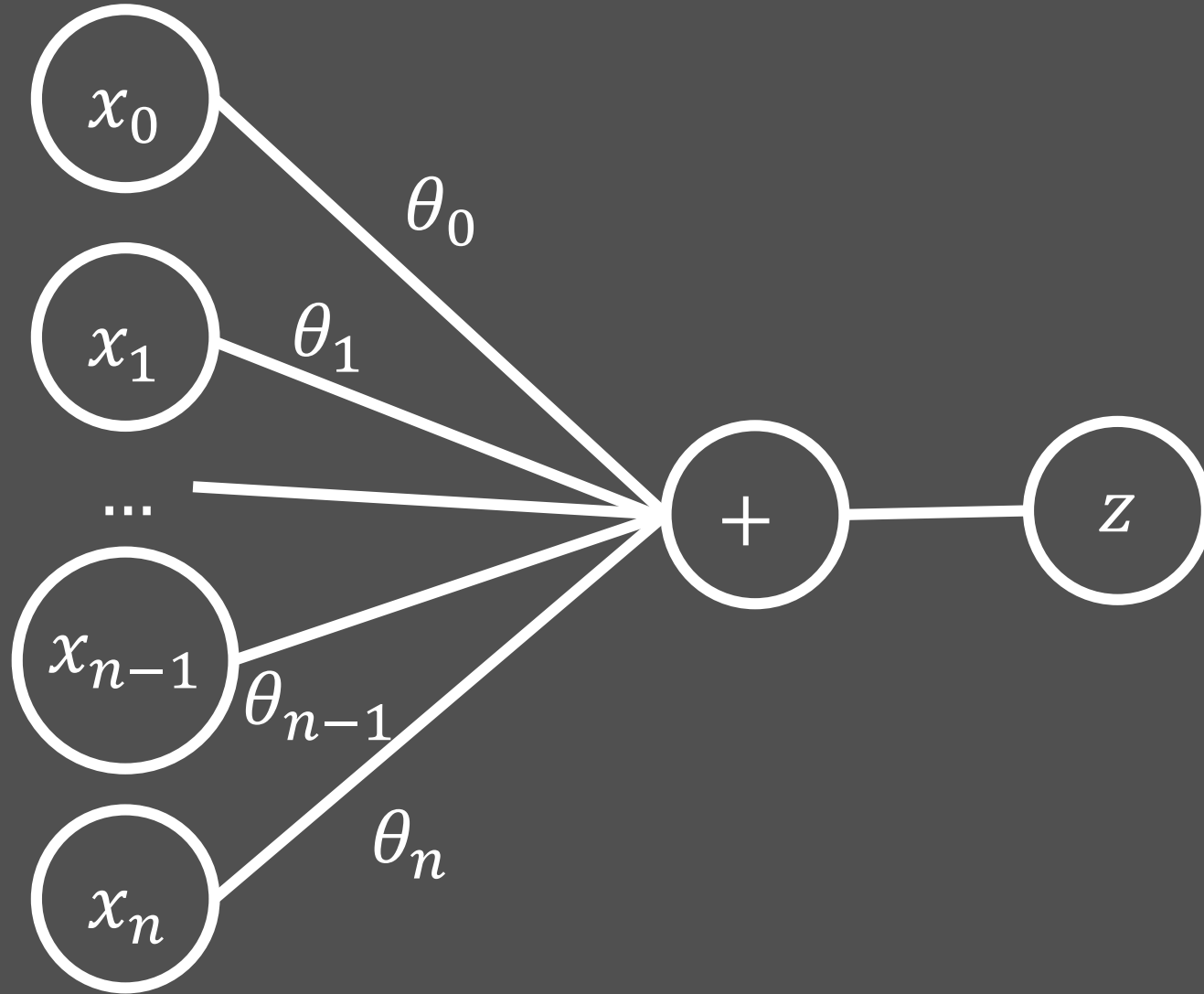


## Weighted Sum:



## Weighted Sum $z = \theta x$

- $z = \theta x = \theta_n * x_n + \theta_{n-1} * x_{n-1} + \dots + \theta_1 * x_1 + \theta_0 * x_0$
- $\theta$ : Weights
- $x$ : Data
- $x_0 = 1$

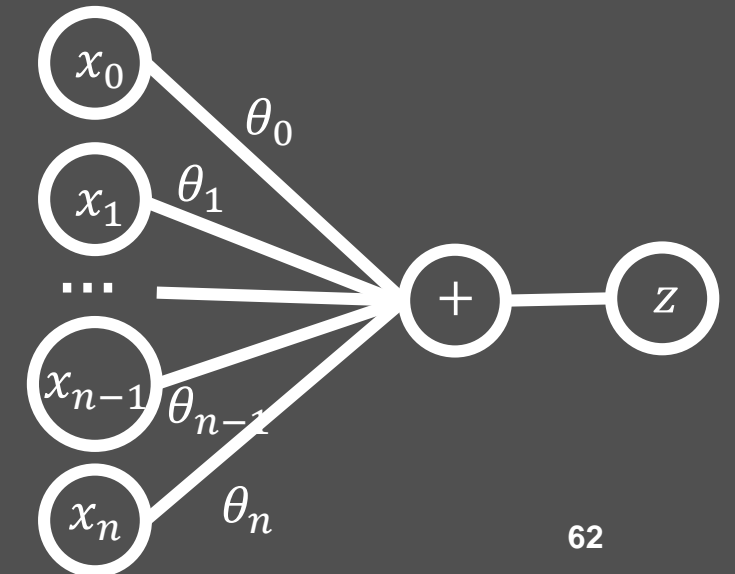
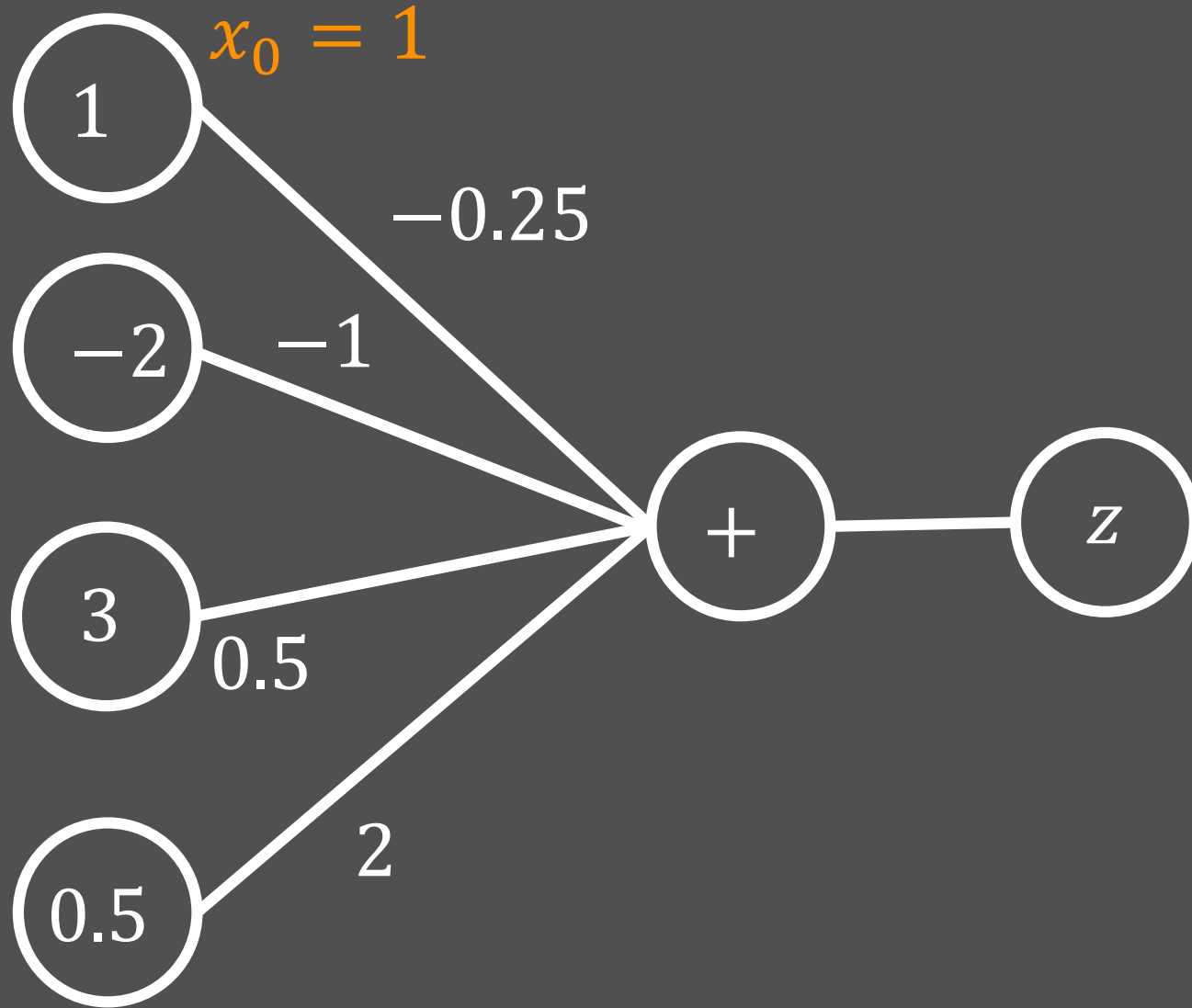


## Weighted Sum

- $z = \theta x$
- $\theta$ : **Weights**
- $x$ : **Data**
- $x_0 = 1$

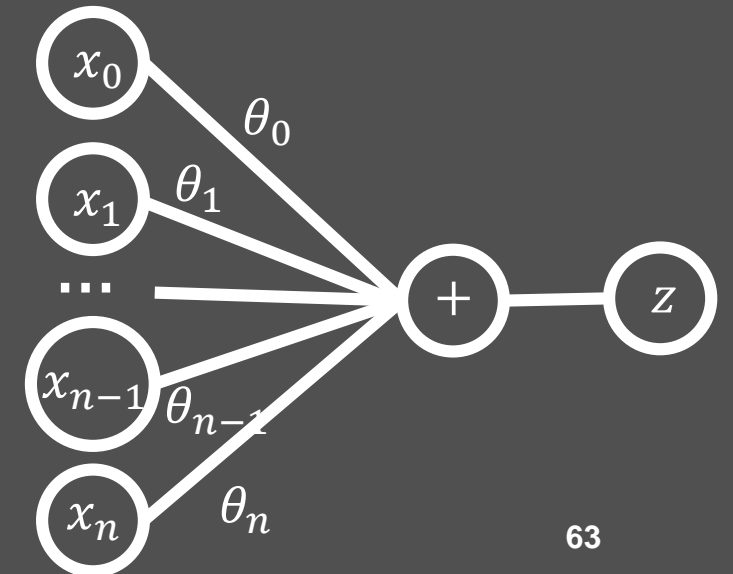
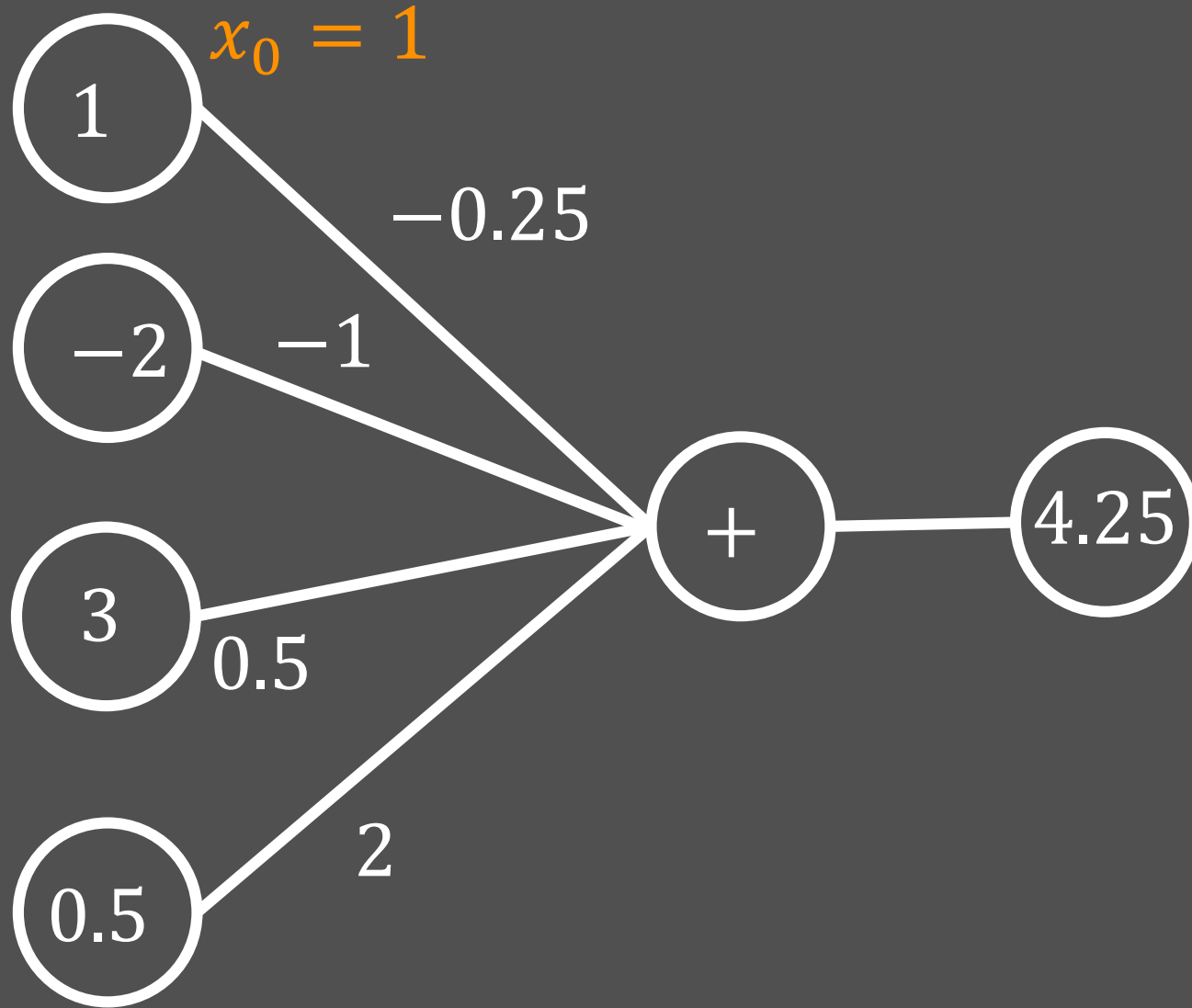
## Weighted Sum

- $z = \theta x$
- $\theta$ : **Weights**
- $x$ : **Data**
- $x_0 = 1$



## Weighted Sum

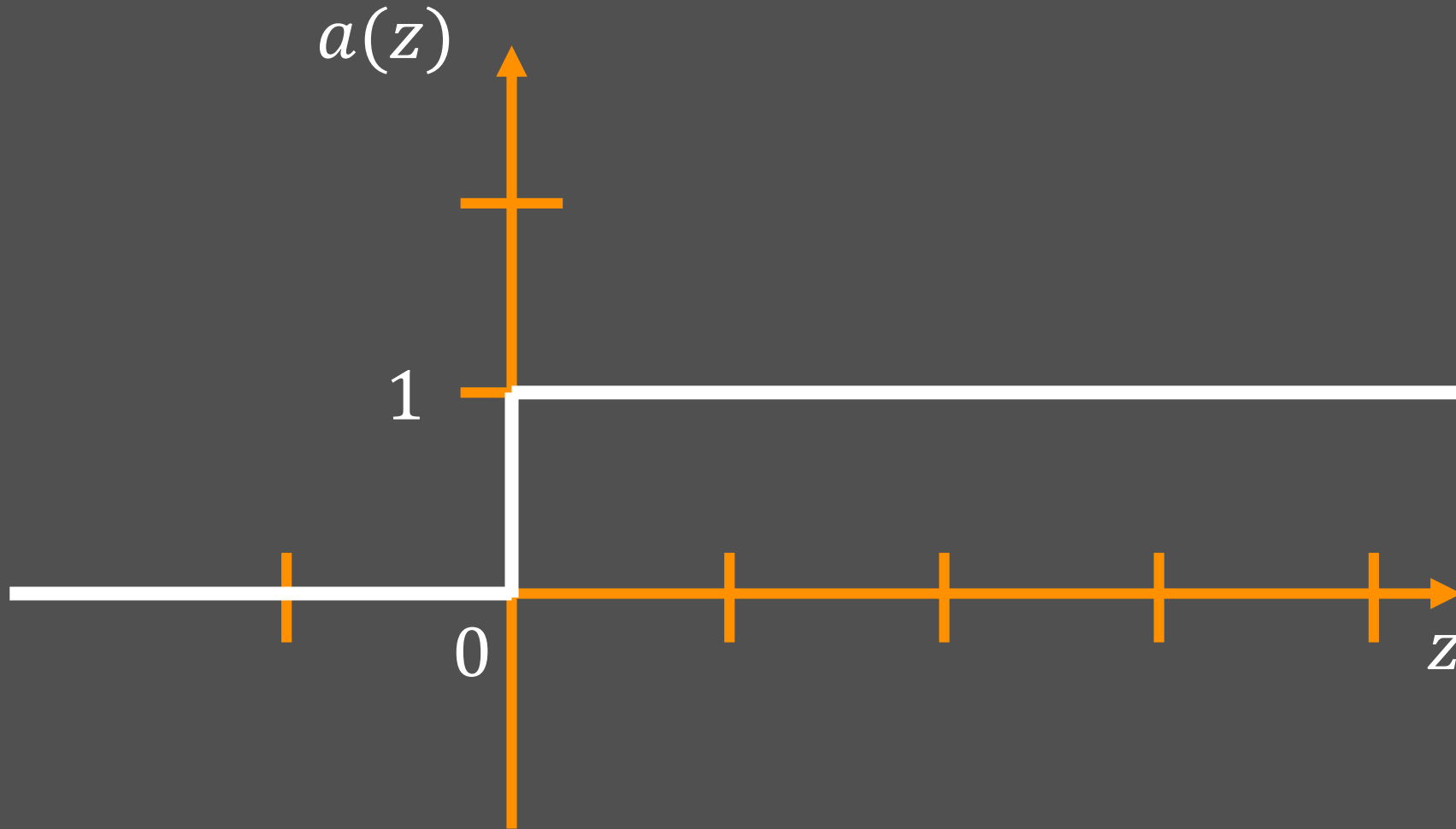
- $z = \theta x$
- $\theta$ : **Weights**
- $x$ : **Data**
- $x_0 = 1$



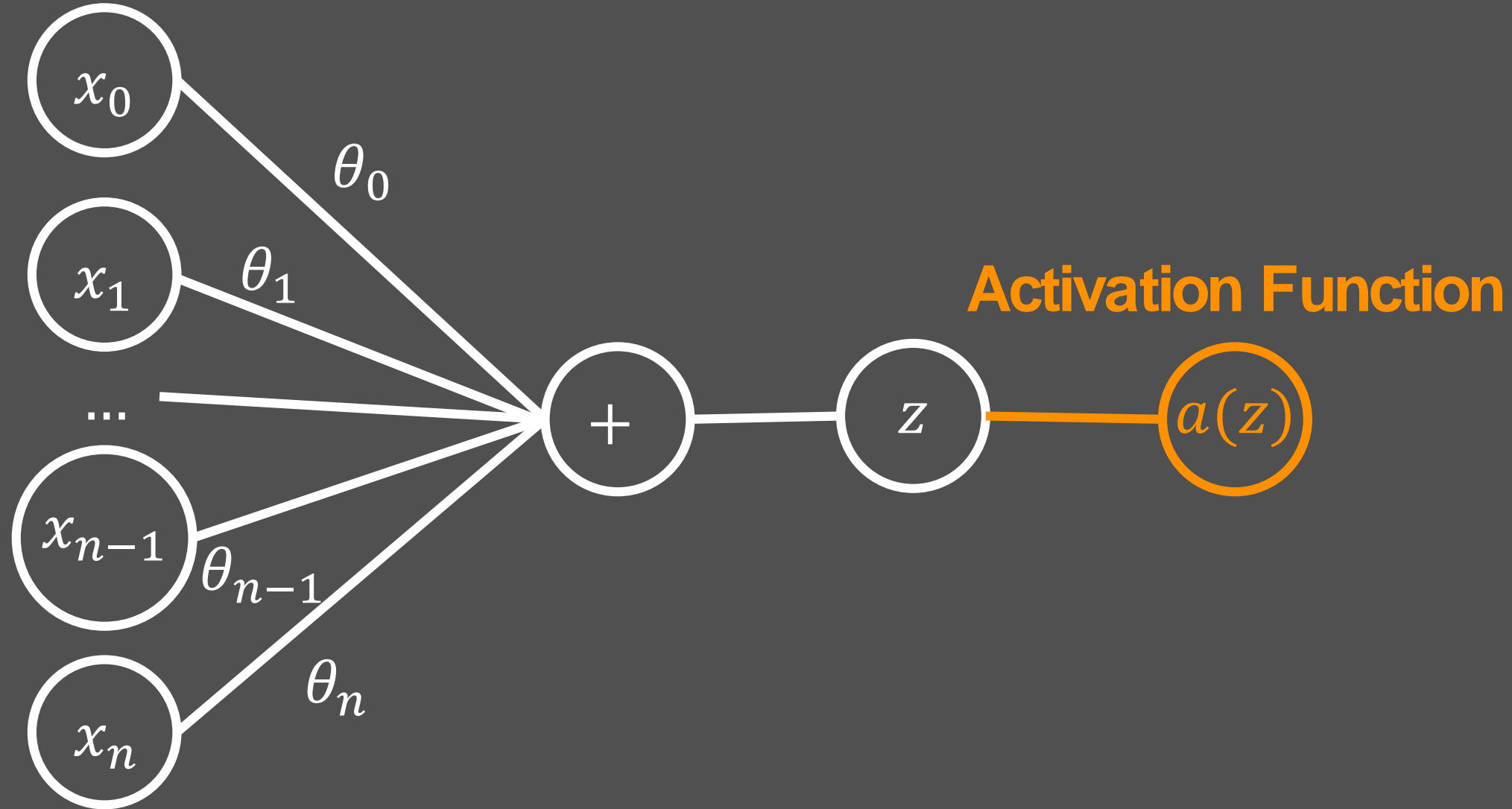
**Activation Function**  $y = a(z)$

**Binary Threshold Unit:**  $a(z) = \begin{cases} 0, & z < 0 \\ 1, & z \geq 0 \end{cases}$

# Perceptron

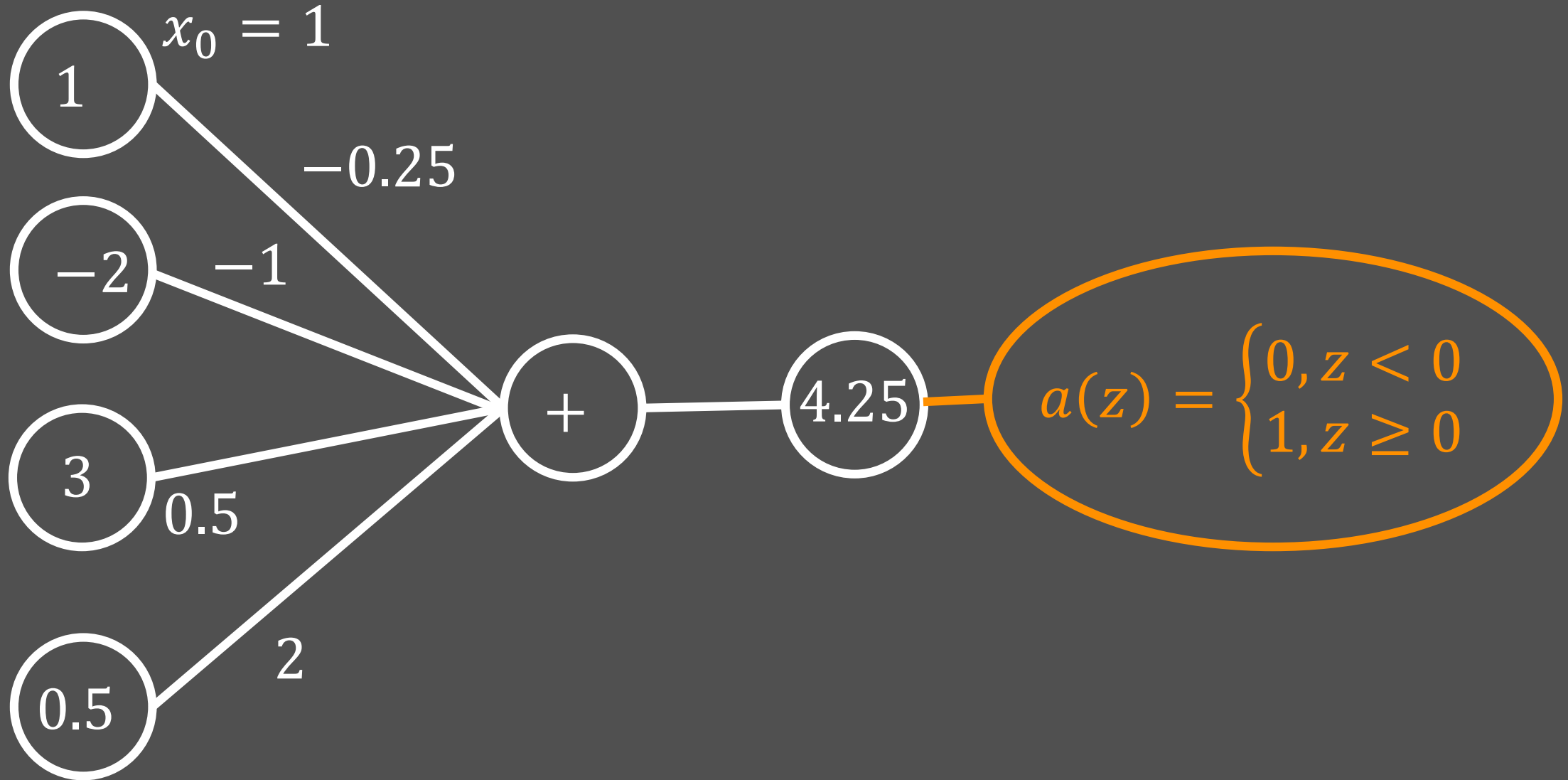


# Perceptron

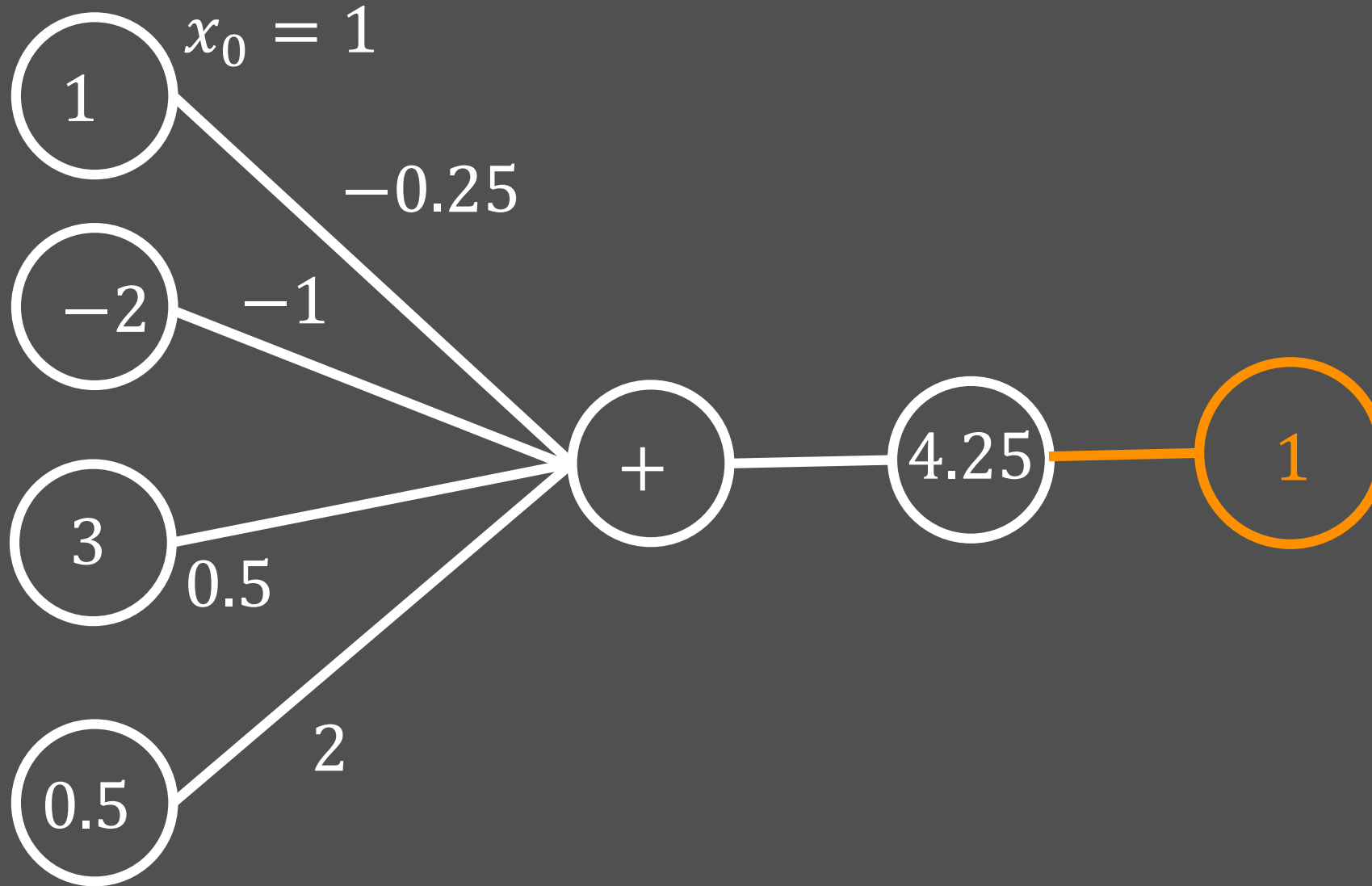




# Perceptron

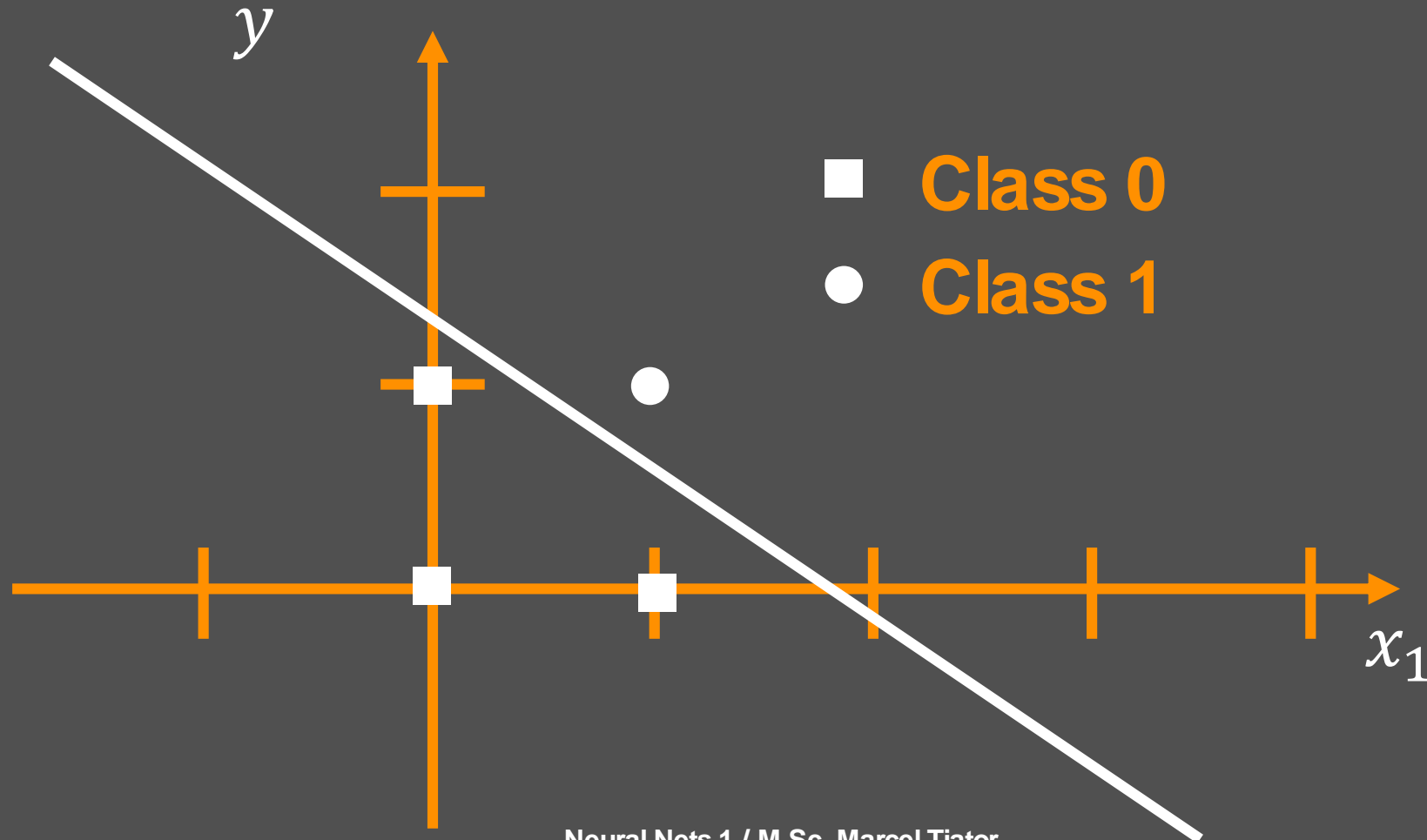


# Perceptron

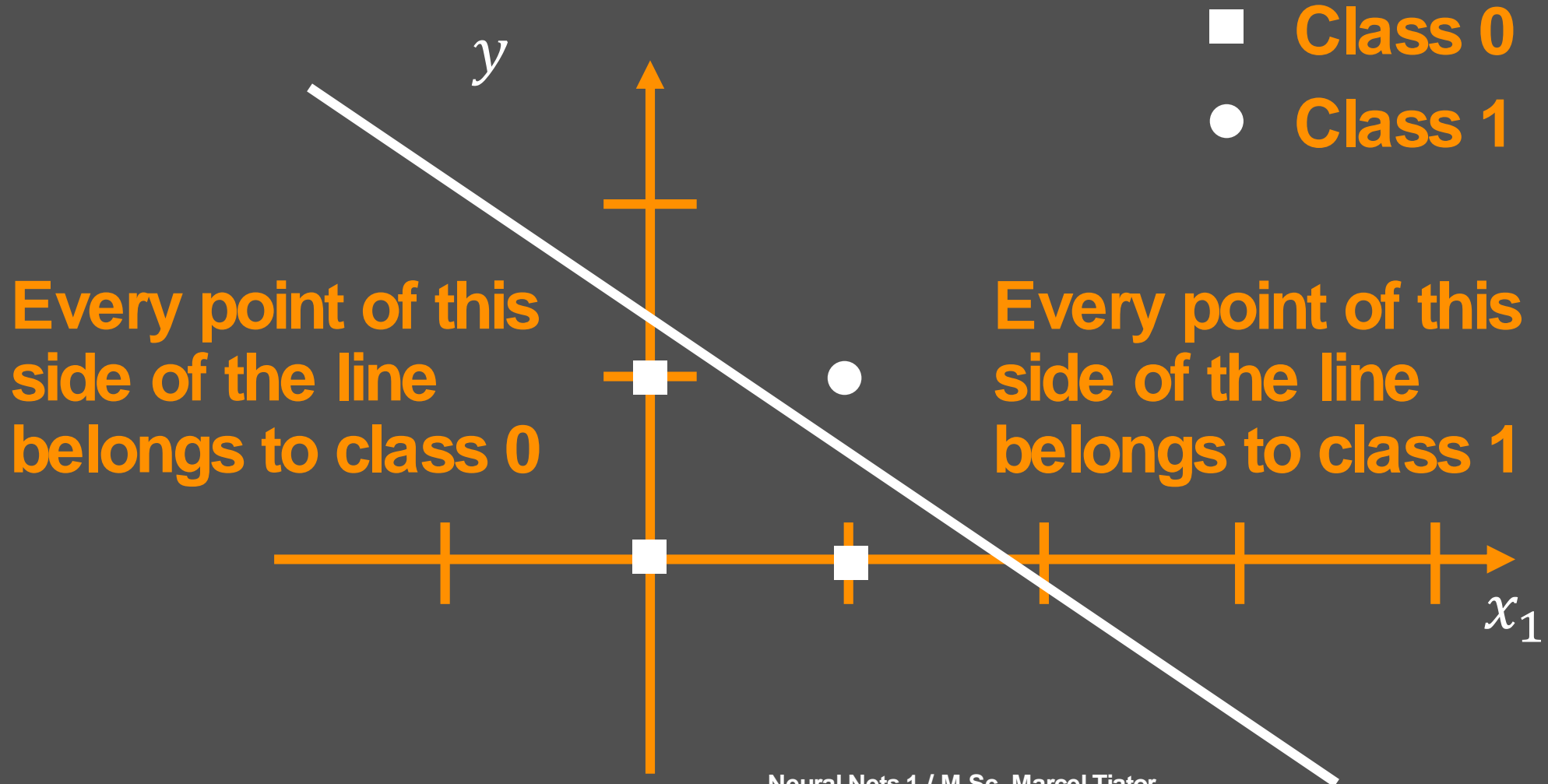


# Linear Classifier

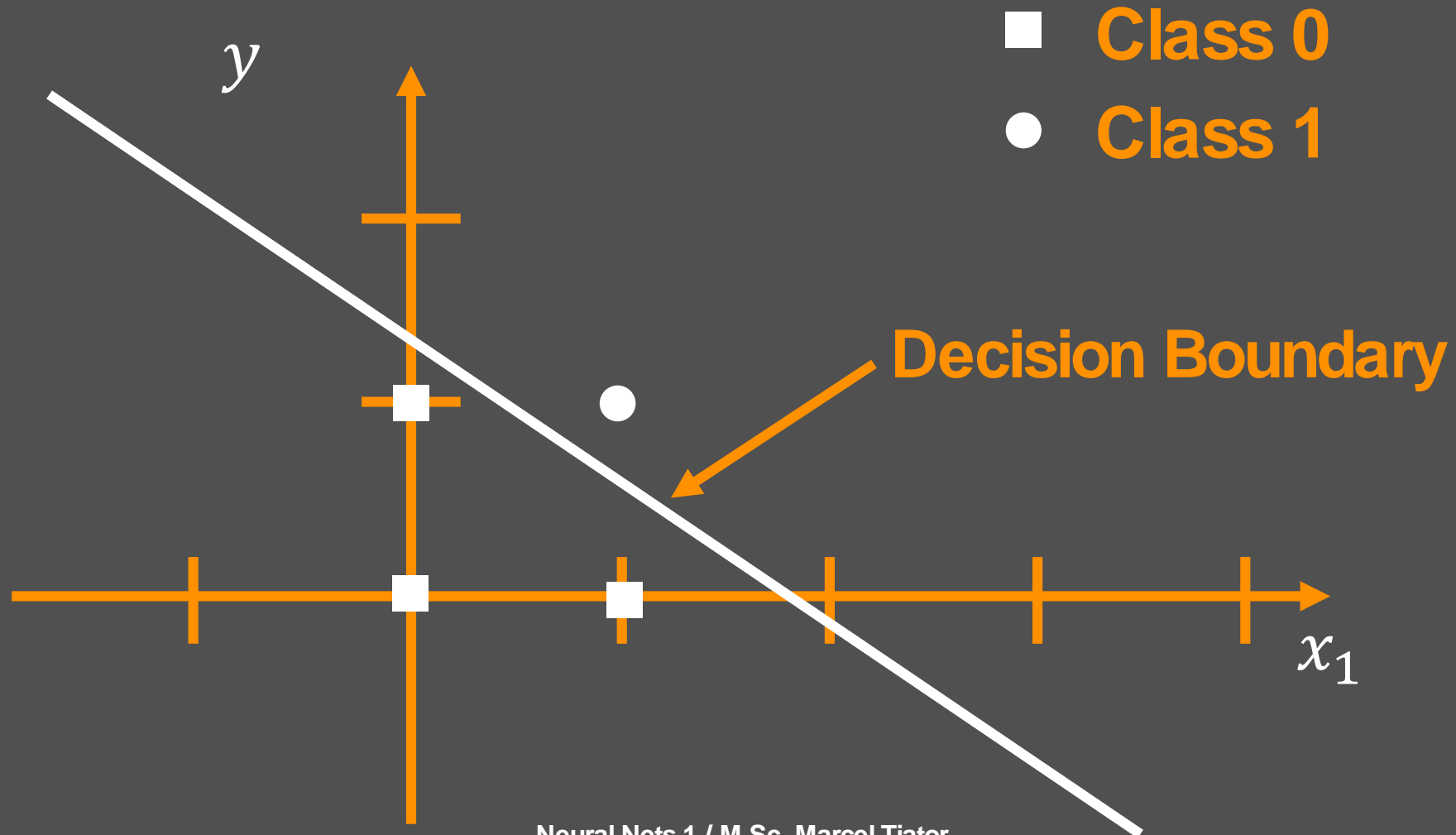
# Binary Classification



# Binary Classification



# Binary Classification



# Perceptron

Training:

Initialize weights randomly

do{

    error=false

$d=0$

    foreach  $x$  in trainingsset{

$\hat{y}=\text{Prediction}(x)$ ,  $\hat{y} \in \{0,1\}$

$d = y - \hat{y}$ ,  $y \in \{0,1\}$

        if( $d == 1$ )  $\theta = \theta + x$

        elif ( $d == -1$ )  $\theta = \theta - x$

        error =error |  $d \neq 0$

    }

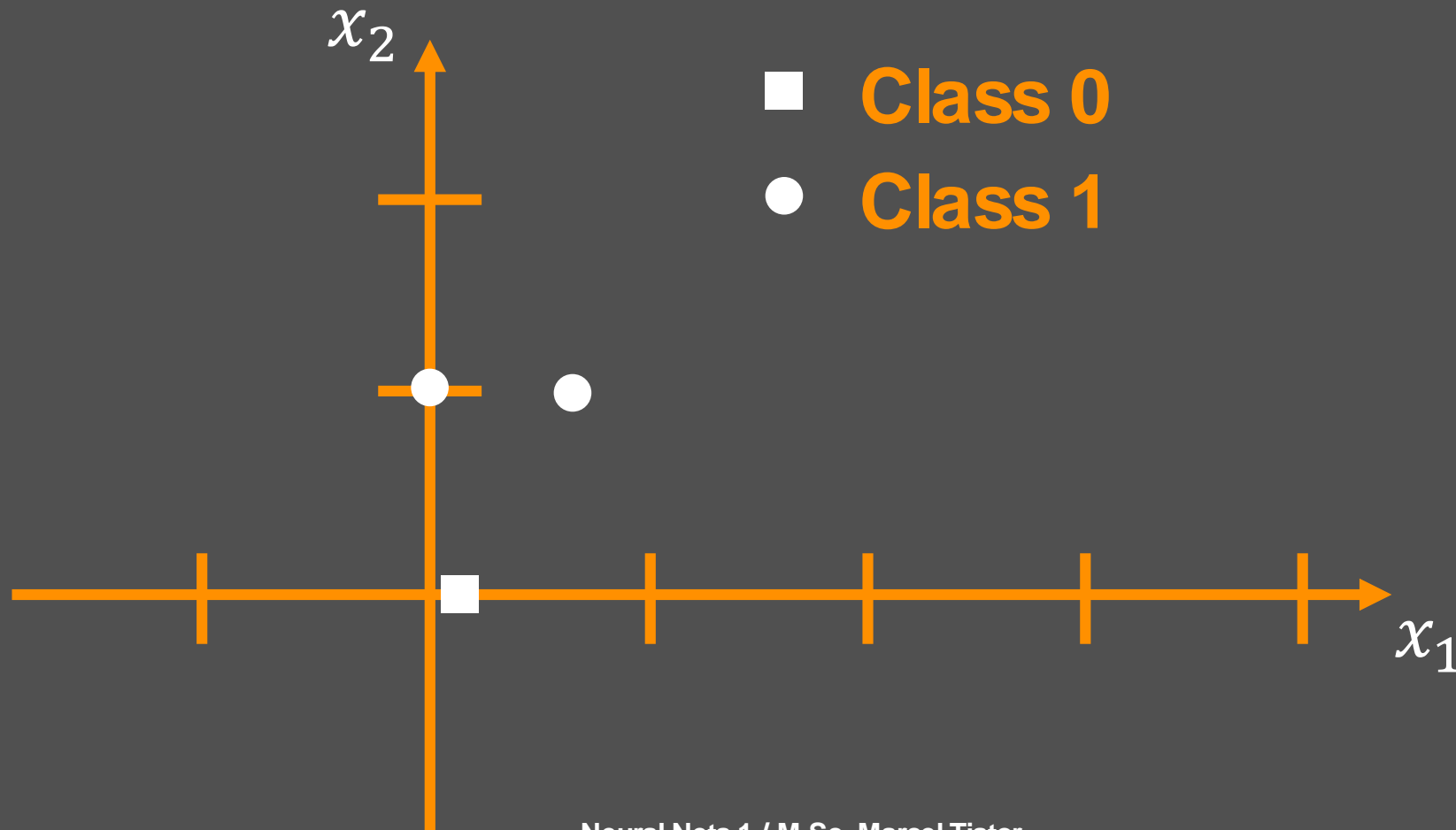
} while(error)

# Perceptron

| $x_1$ | $x_2$ | $y$ |
|-------|-------|-----|
| 0.1   | 0     | 0   |
| 0.6   | 1     | 1   |
| 0     | 1     | 1   |



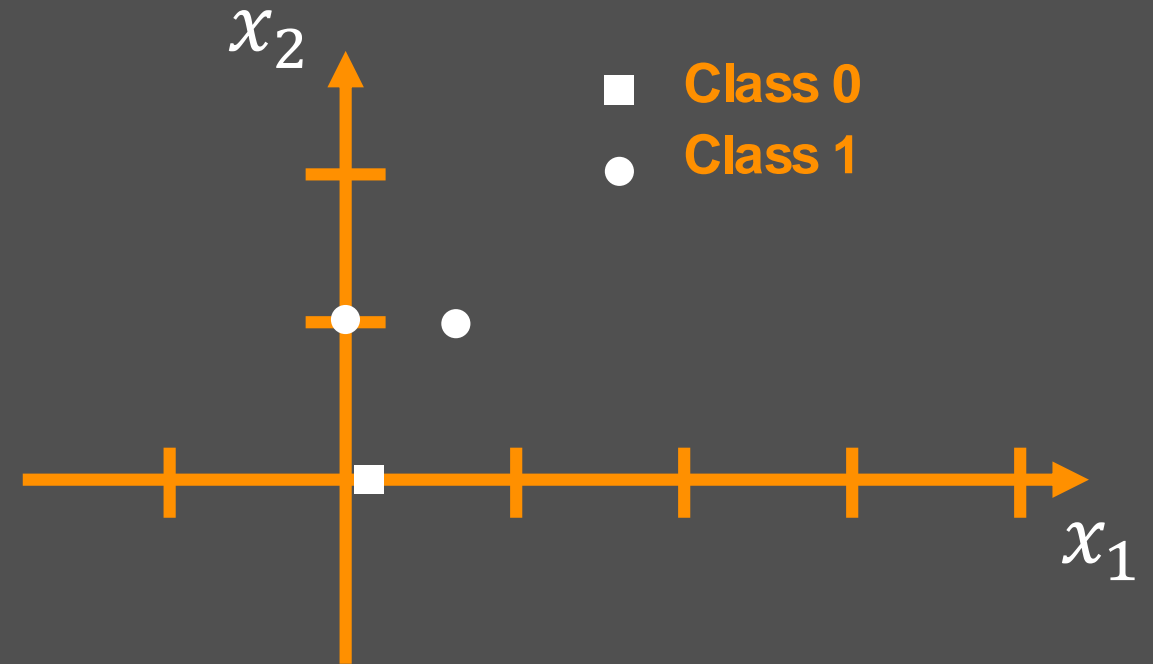
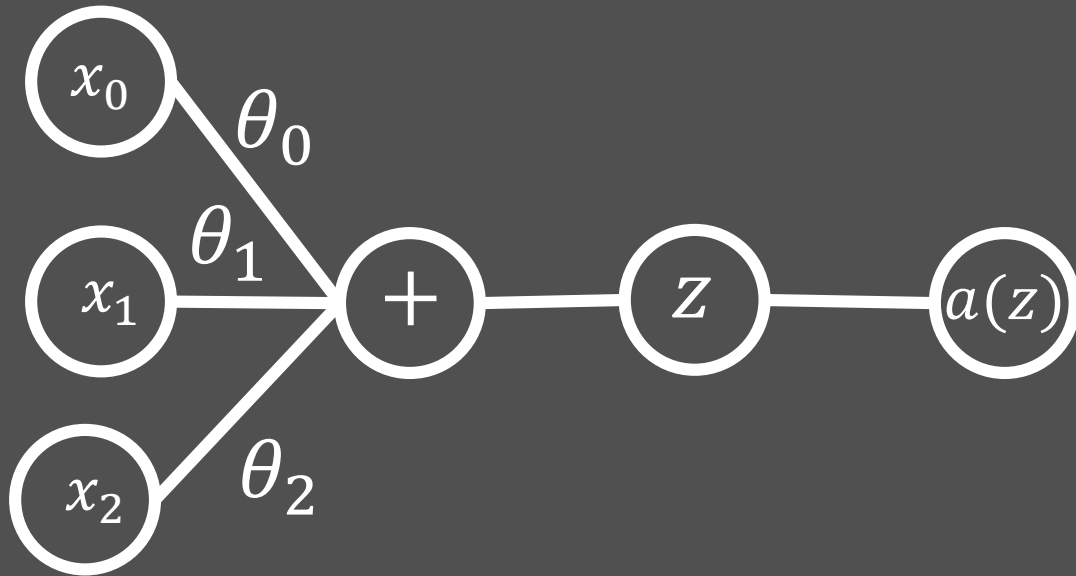
## Visualization of data



## Append $x_0$

| $x_0$ | $x_1$ | $x_2$ | $y$ |
|-------|-------|-------|-----|
| 1     | 0.1   | 0     | 0   |
| 1     | 0.6   | 1     | 1   |
| 1     | 0     | 1     | 1   |

## Perceptron-Model



# Perceptron

**Initialize weights randomly**

**foreach**  $x$  **in** **trainingsset**{

$\hat{y} = \text{Prediction}(x)$ ,  $\hat{y} \in \{0, 1\}$

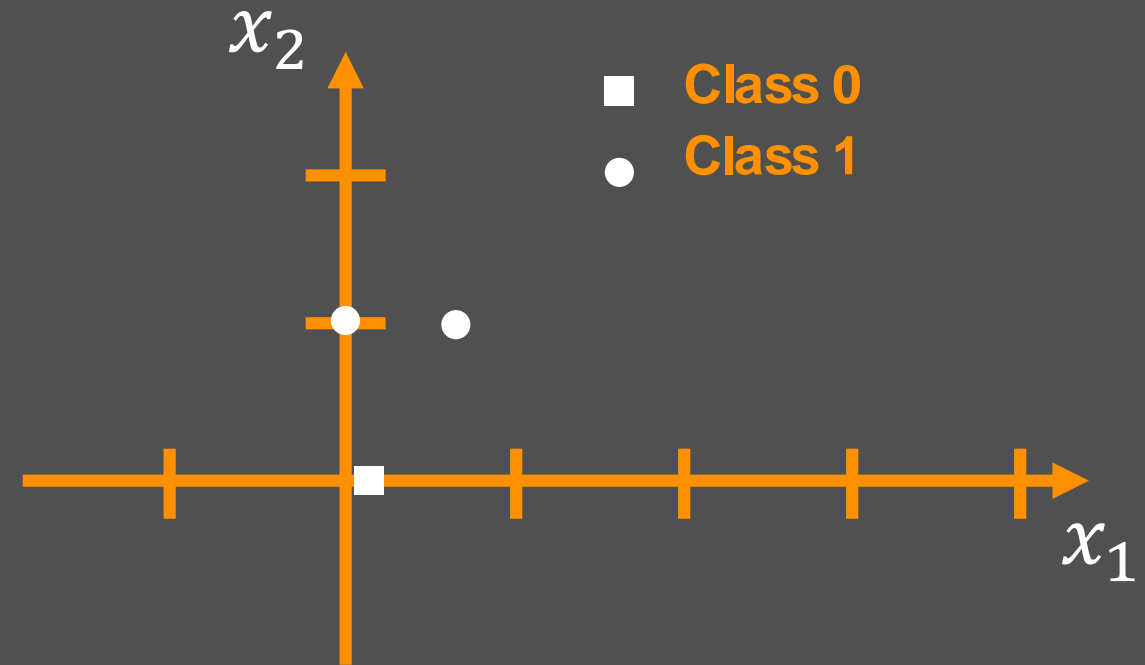
$d = y - \hat{y}$ ,  $y \in \{0, 1\}$

**if** ( $d == 1$ )  $\theta = \theta + x$

**elif** ( $d == -1$ )  $\theta = \theta - x$

}

$$\theta_0 = -0.5, \theta_1 = 1, \theta_2 = -1$$



# Perceptron

Initialize weights randomly  
foreach  $x$  in trainingsset{

$\hat{y} = \text{Prediction}(x), \hat{y} \in \{0, 1\}$

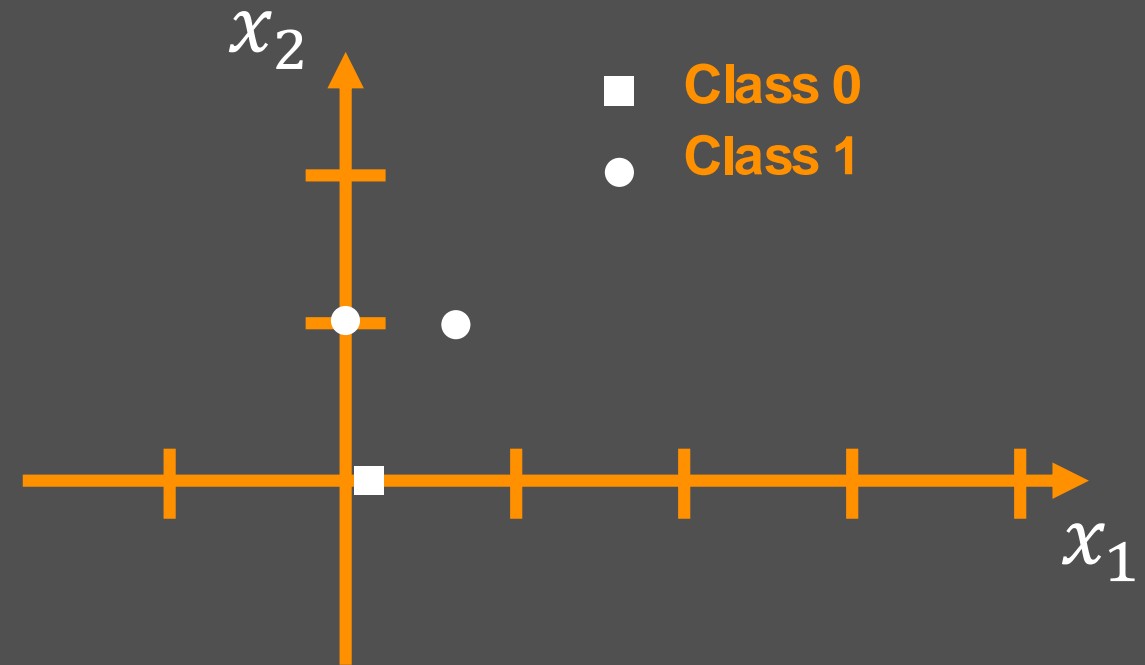
$d = y - \hat{y}, y \in \{0, 1\}$

if ( $d == 1$ )  $\theta = \theta + x$

elif ( $d == -1$ )  $\theta = \theta - x$

}

$$\theta_0 = -0.5, \theta_1 = 1, \theta_2 = -1$$

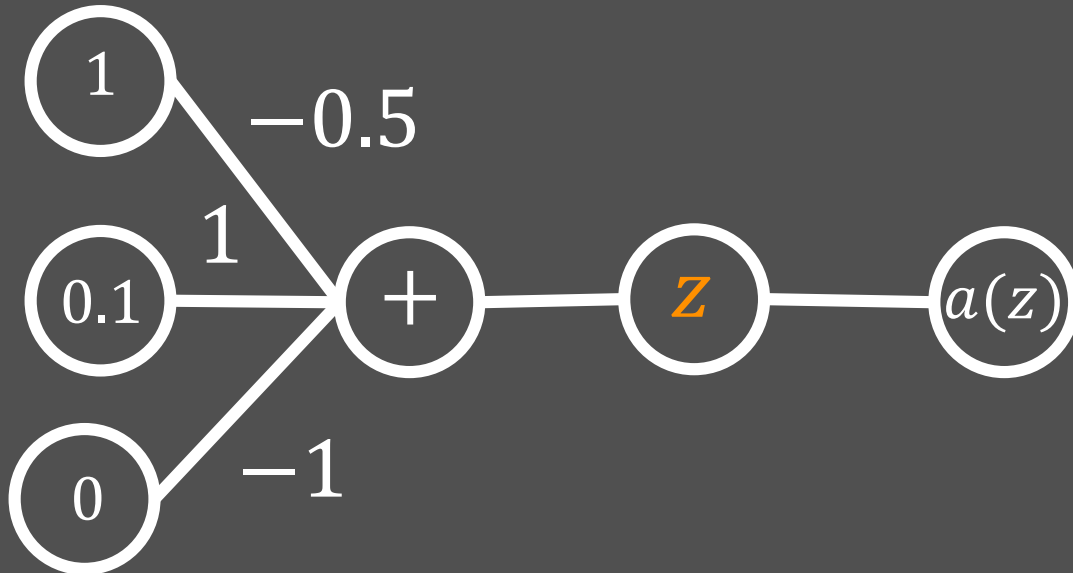


# Perceptron

## Perceptron-Model

$$z = \theta_2 * x_2 + \theta_1 * x_1 + \theta_0 * x_0$$

$$\theta_0 = -0.5, \theta_1 = 1, \theta_2 = -1$$



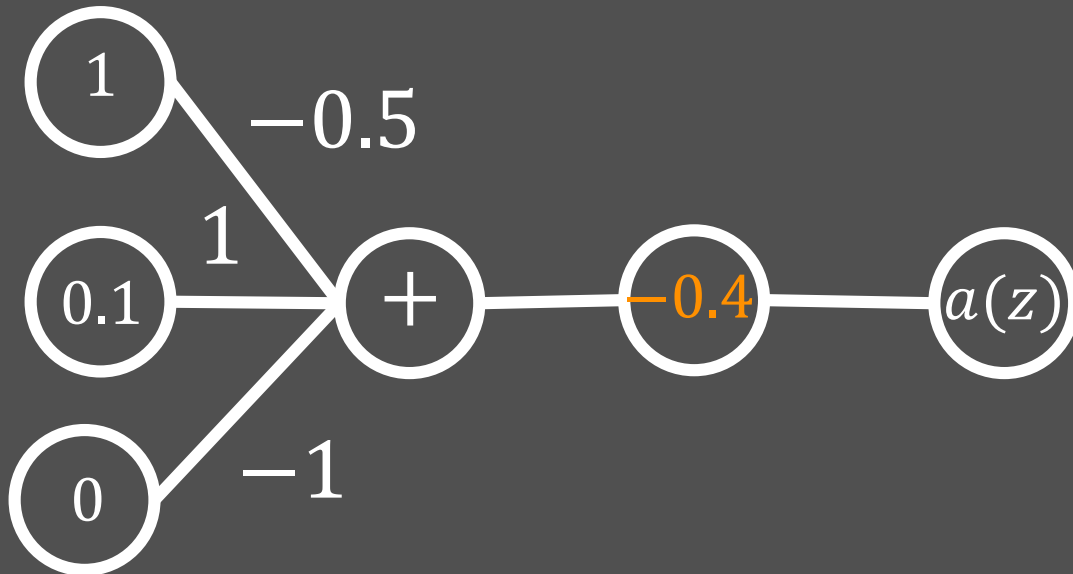
| $x_0$    | $x_1$      | $x_2$    | $y$      |
|----------|------------|----------|----------|
| <b>1</b> | <b>0.1</b> | <b>0</b> | <b>0</b> |
| <b>1</b> | <b>0.6</b> | <b>1</b> | <b>1</b> |
| <b>1</b> | <b>0</b>   | <b>1</b> | <b>1</b> |

# Perceptron

## Perceptron-Model

$$z = \theta_2 * x_2 + \theta_1 * x_1 + \theta_0 * x_0$$

$$\theta_0 = -0.5, \theta_1 = 1, \theta_2 = -1$$

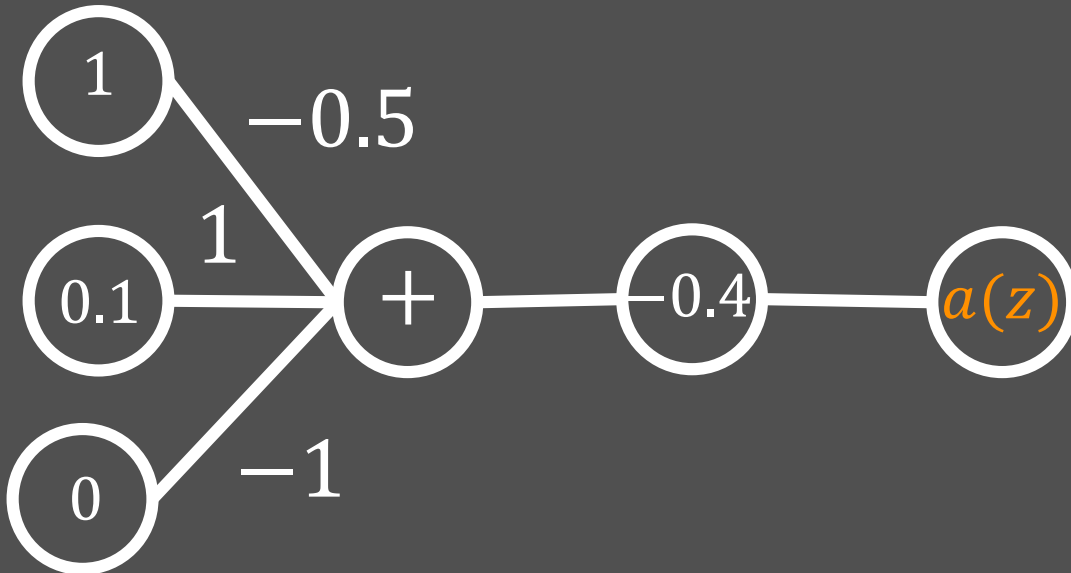


| $x_0$    | $x_1$      | $x_2$    | $y$      |
|----------|------------|----------|----------|
| <b>1</b> | <b>0.1</b> | <b>0</b> | <b>0</b> |
| <b>1</b> | <b>0.6</b> | <b>1</b> | <b>1</b> |
| <b>1</b> | <b>0</b>   | <b>1</b> | <b>1</b> |

# Perceptron

## Perceptron-Model

$$a(z) = \begin{cases} 0, & z < 0 \\ 1, & z \geq 0 \end{cases}$$



$$\theta_0 = -0.5, \theta_1 = 1, \theta_2 = -1$$

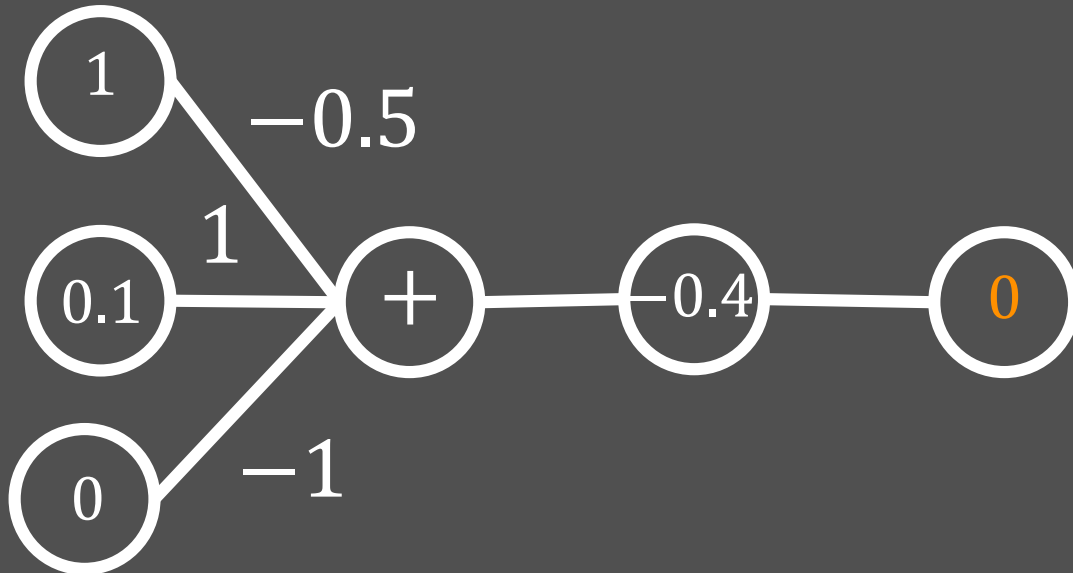
| $x_0$ | $x_1$ | $x_2$ | $y$ |
|-------|-------|-------|-----|
| 1     | 0.1   | 0     | 0   |
| 1     | 0.6   | 1     | 1   |
| 1     | 0     | 1     | 1   |



# Perceptron

## Perceptron-Model

$$\hat{y} = a(z) = \begin{cases} 0, & z < 0 \\ 1, & z \geq 0 \end{cases}$$



$$\theta_0 = -0.5, \theta_1 = 1, \theta_2 = -1$$

| $x_0$ | $x_1$ | $x_2$ | $y$ |
|-------|-------|-------|-----|
| 1     | 0.1   | 0     | 0   |
| 1     | 0.6   | 1     | 1   |
| 1     | 0     | 1     | 1   |

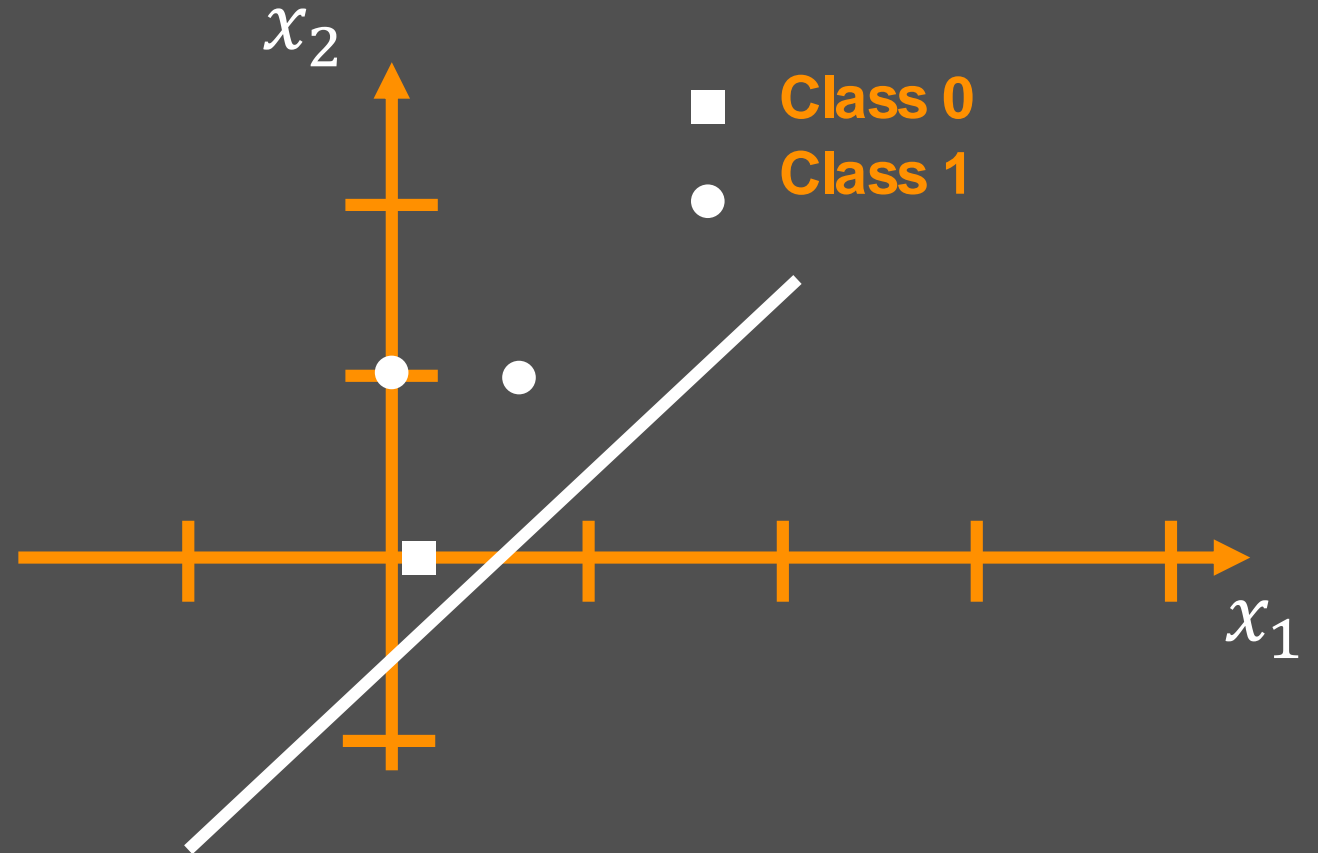
$$x_2\theta_2 + x_1\theta_1 + x_0\theta_0 = 0$$

$$x_2 = \frac{-(x_1\theta_1 + x_0\theta_0)}{\theta_2}$$

$$x_2 = \frac{-(1x_1 - 0.5)}{-1}$$

$$x_2 = x_1 - 0.5$$

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$d = y - \hat{y}$ ,  $y \in \{0, 1\}$

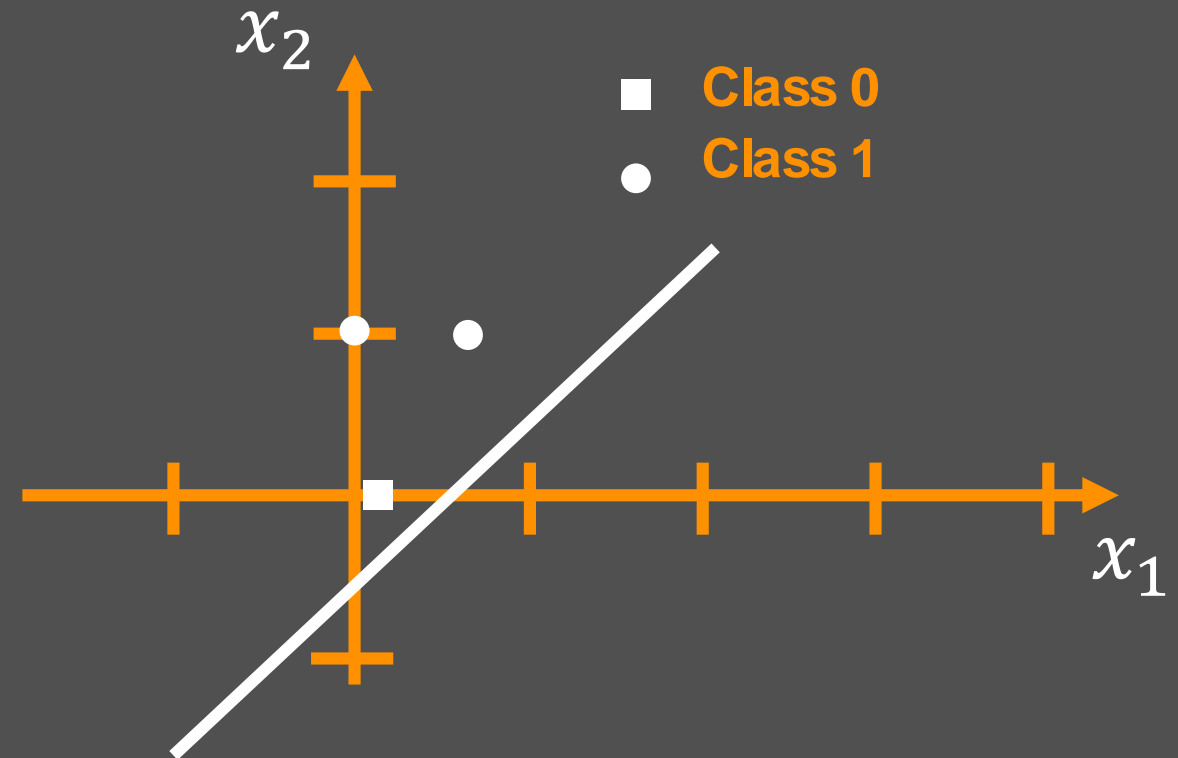
if ( $d == 1$ )  $\theta = \theta + x$

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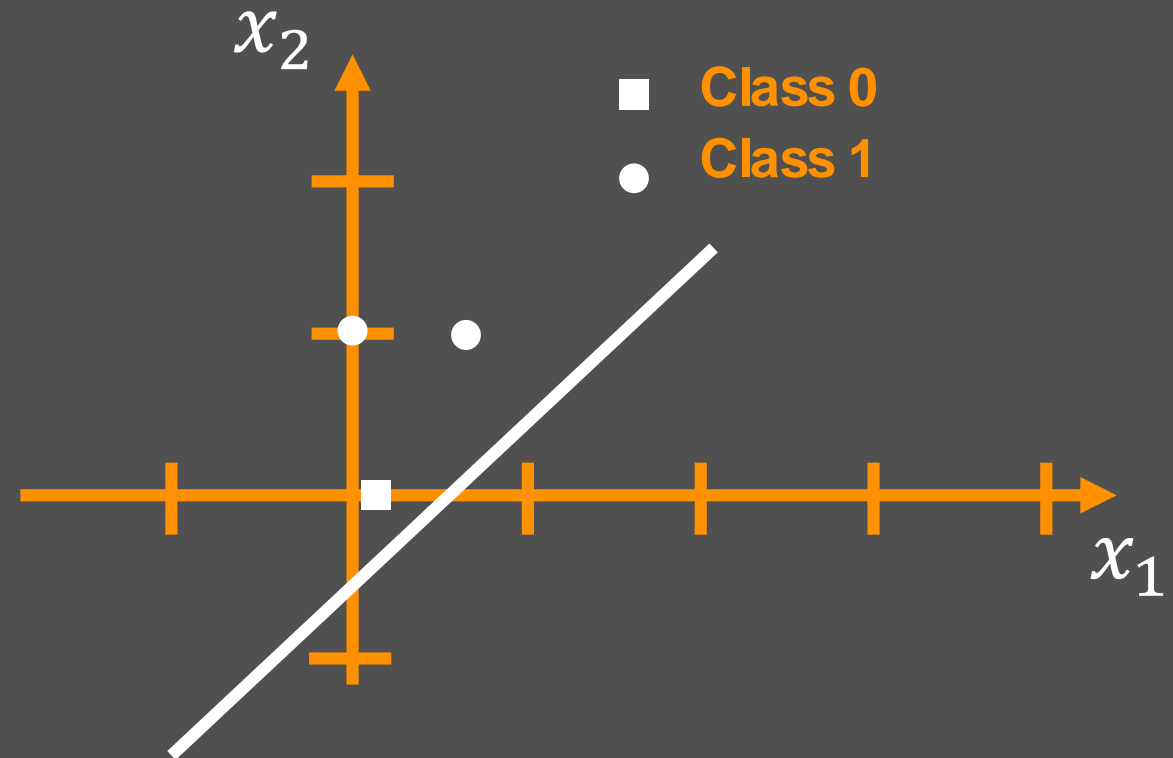
$\hat{y} = 0$



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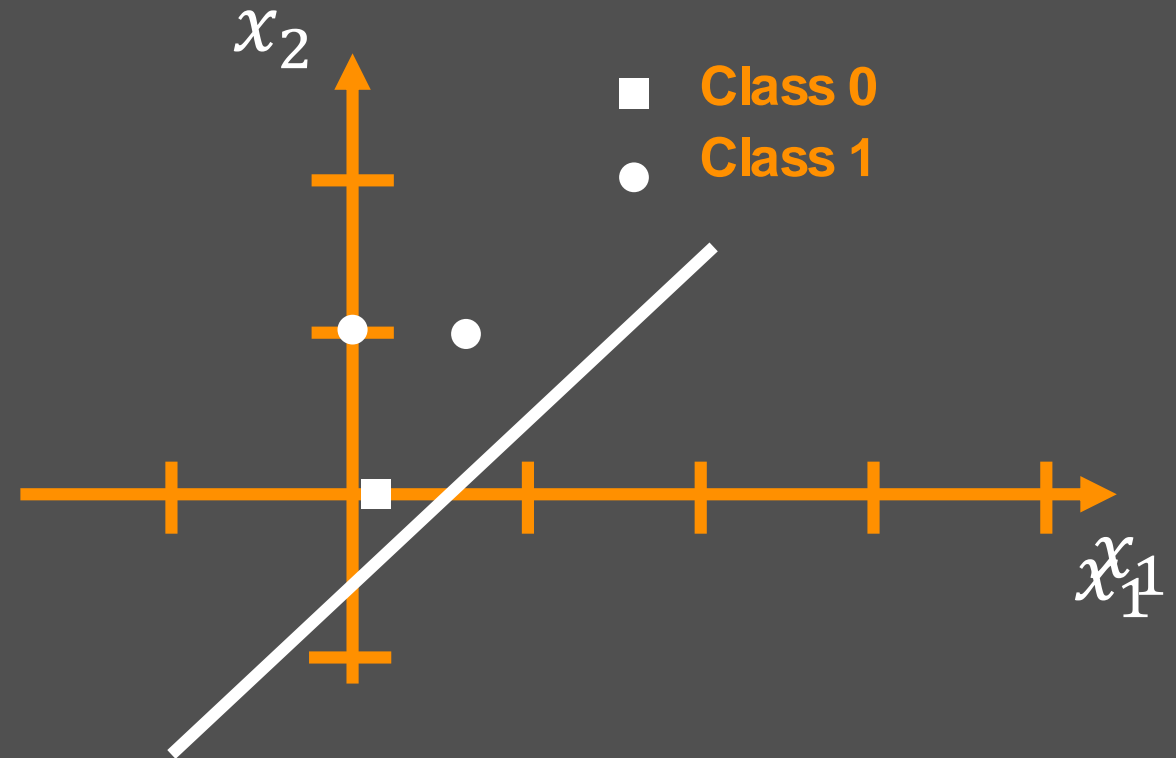
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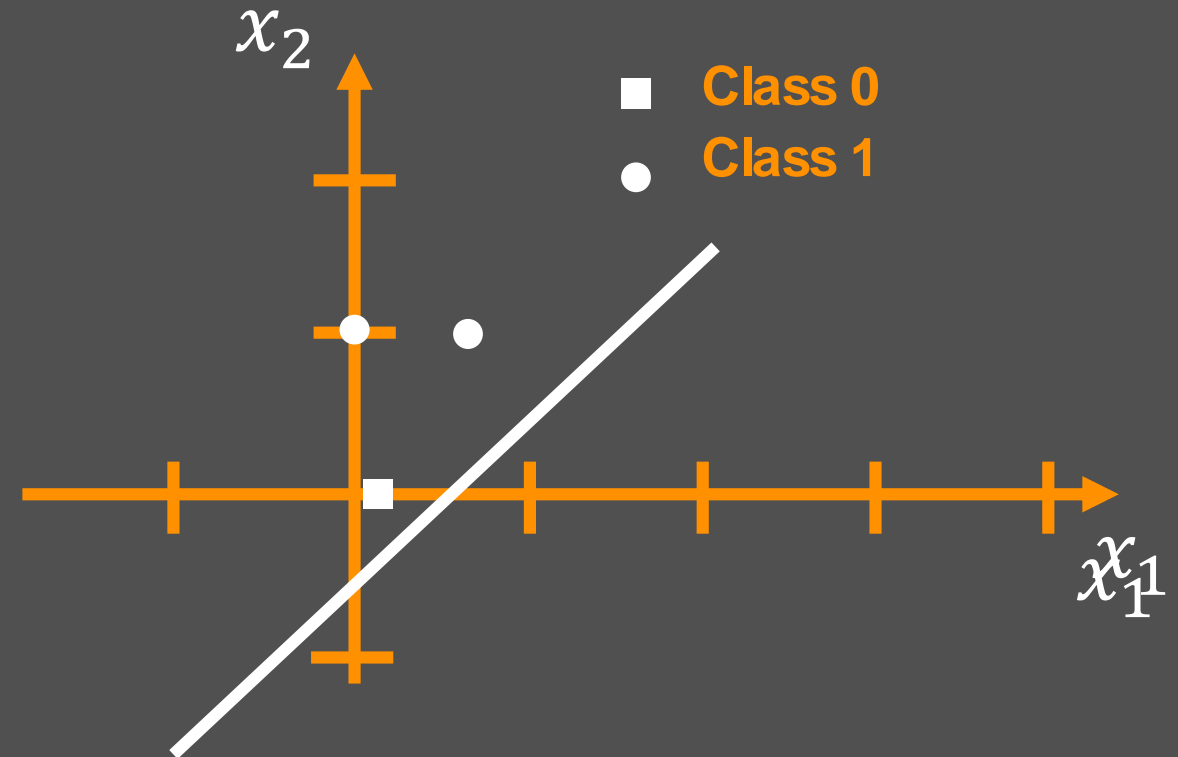
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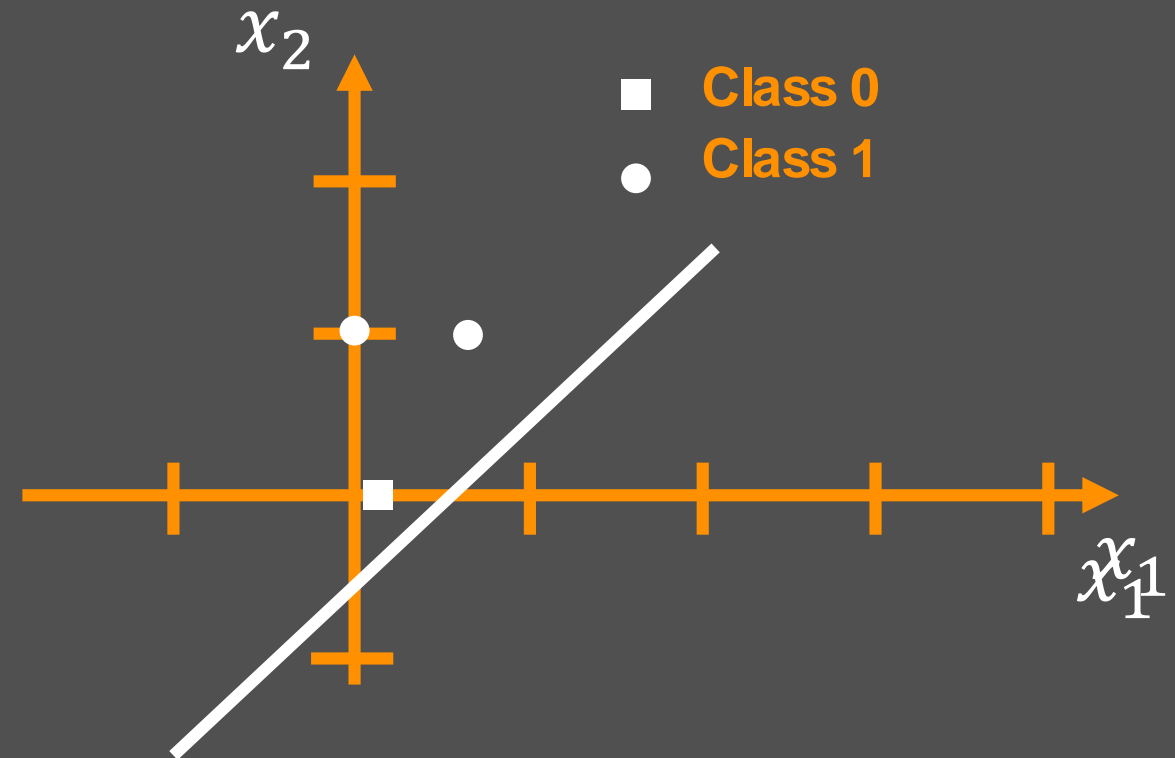
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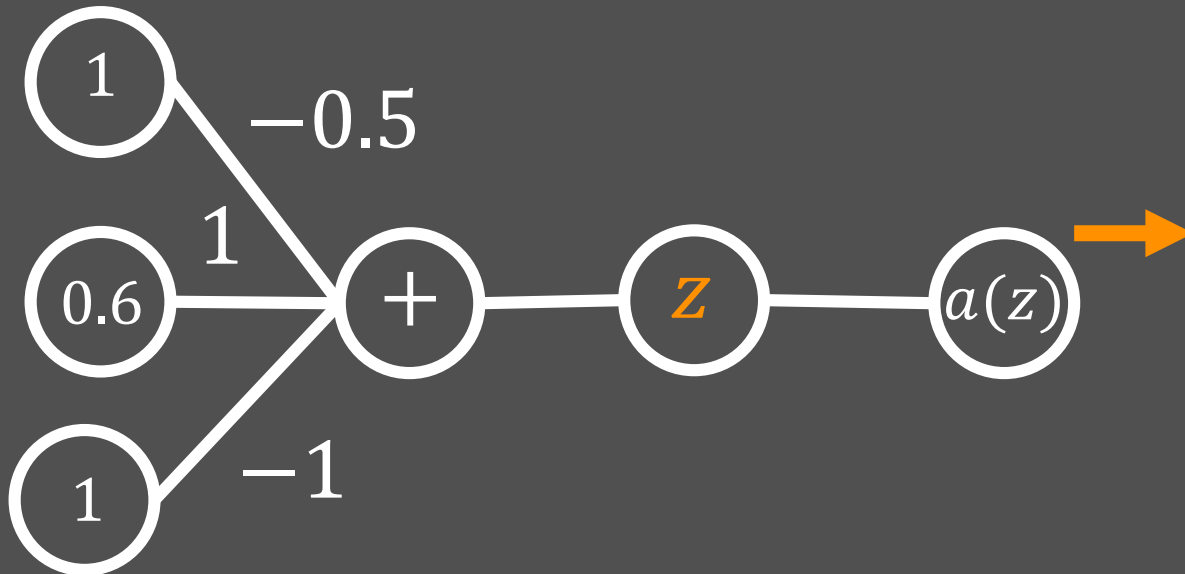


# Perceptron

## Perceptron-Model

$$z = \theta_2 * x_2 + \theta_1 * x_1 + \theta_0 * x_0$$

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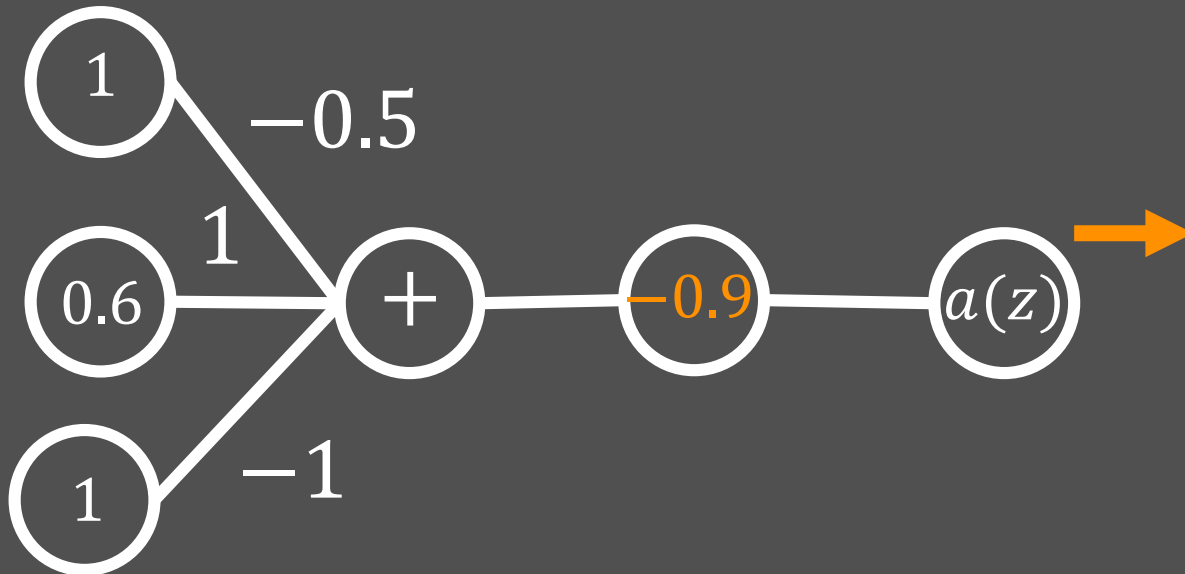
| $x_0$    | $x_1$      | $x_2$    | $y$      |
|----------|------------|----------|----------|
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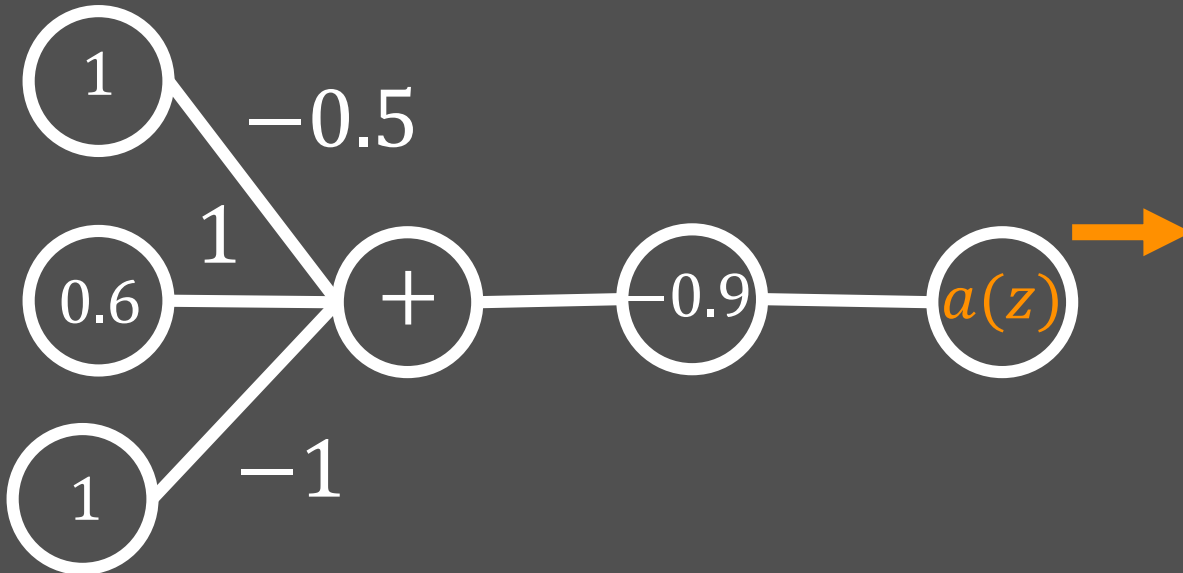


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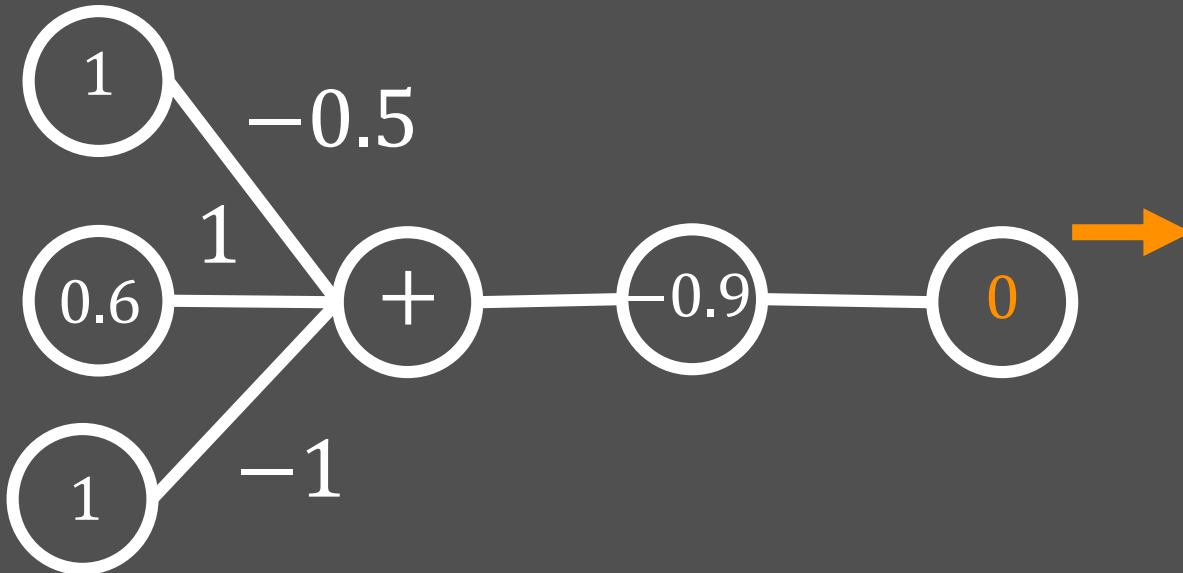
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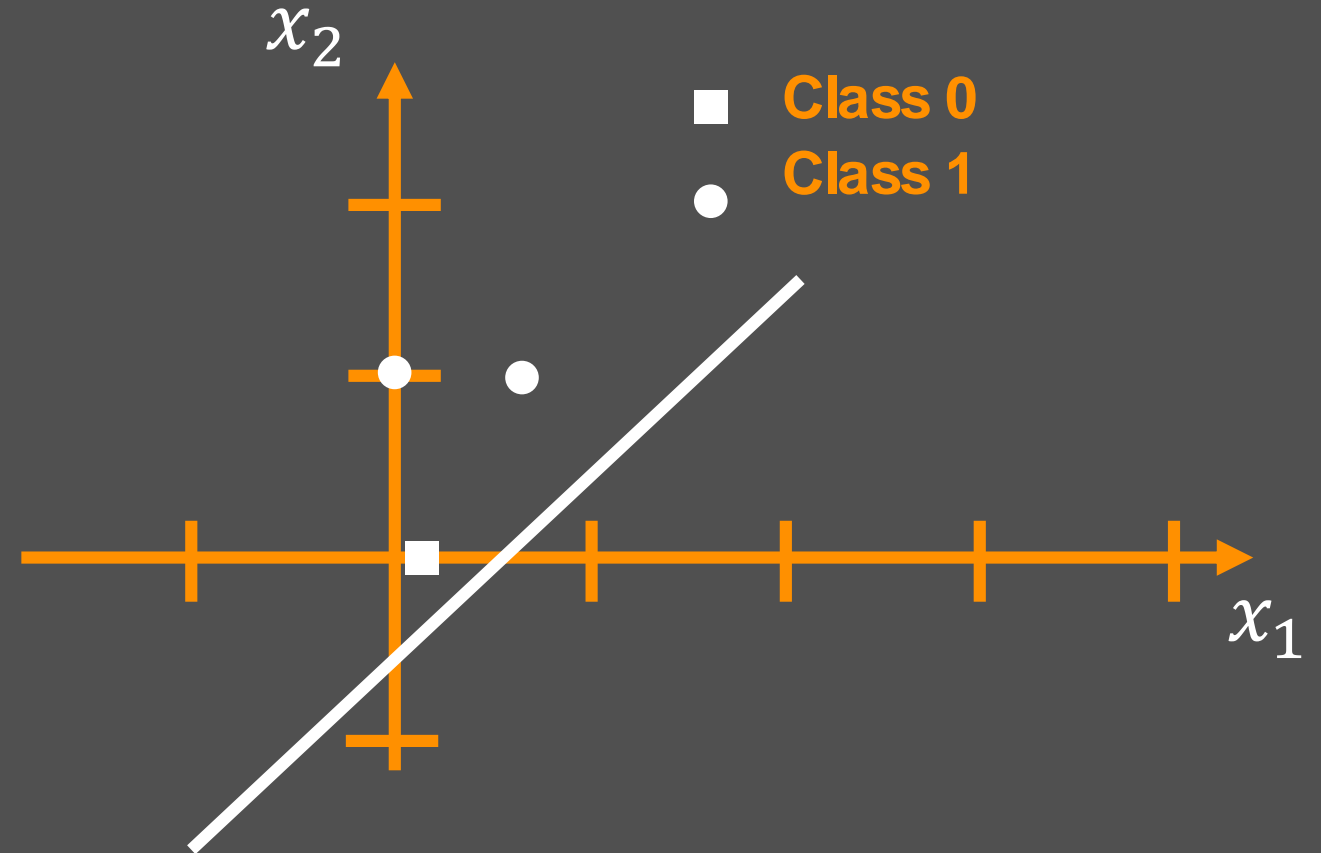
$$x_2\theta_2 + x_1\theta_1 + x_0\theta_0 = 0$$

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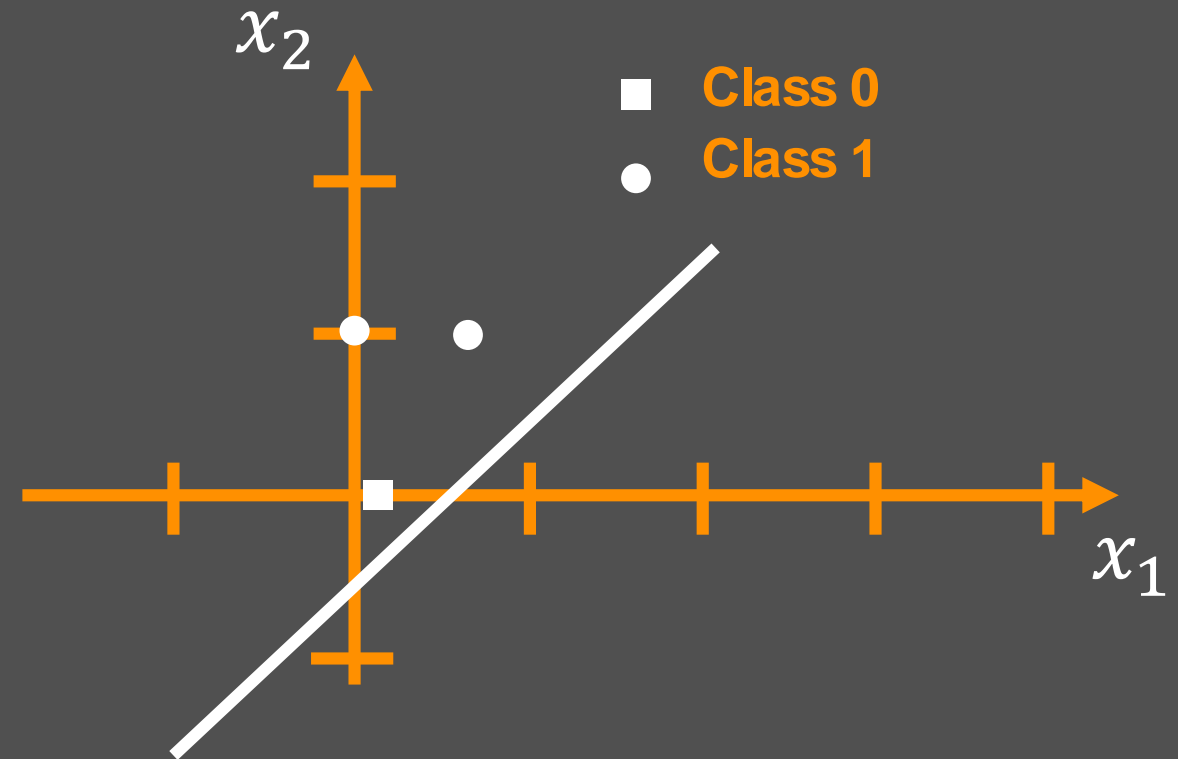
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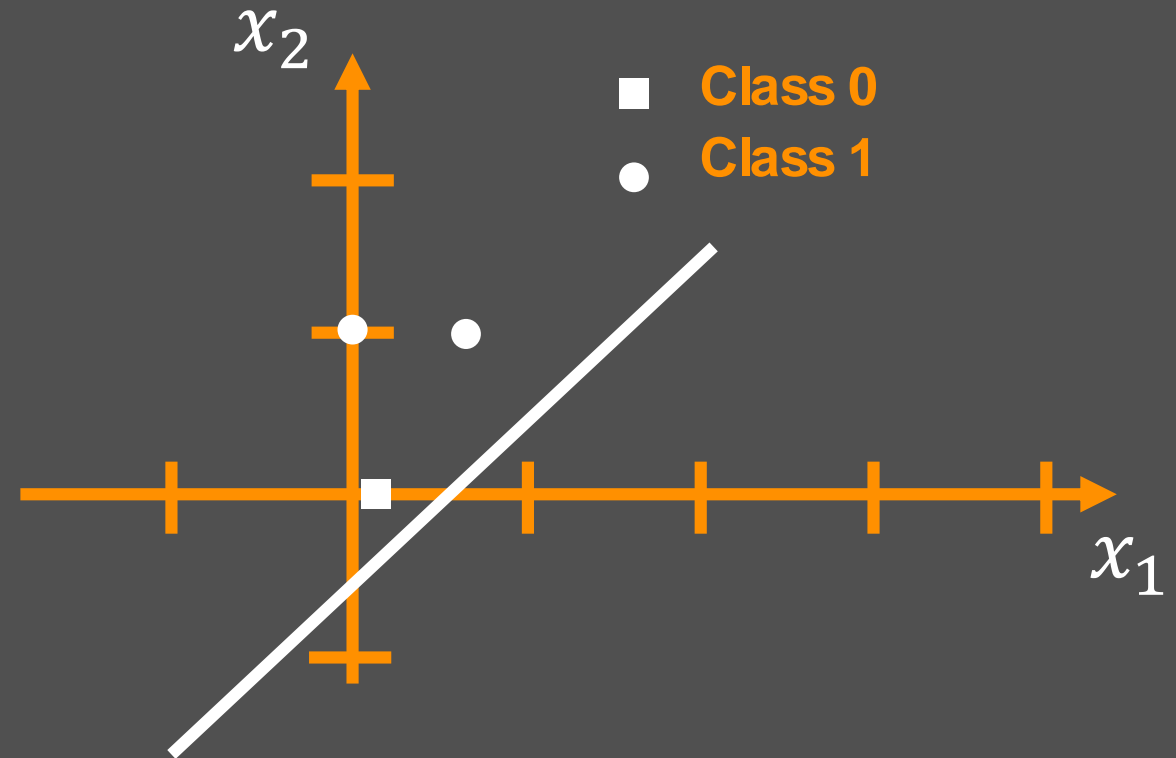
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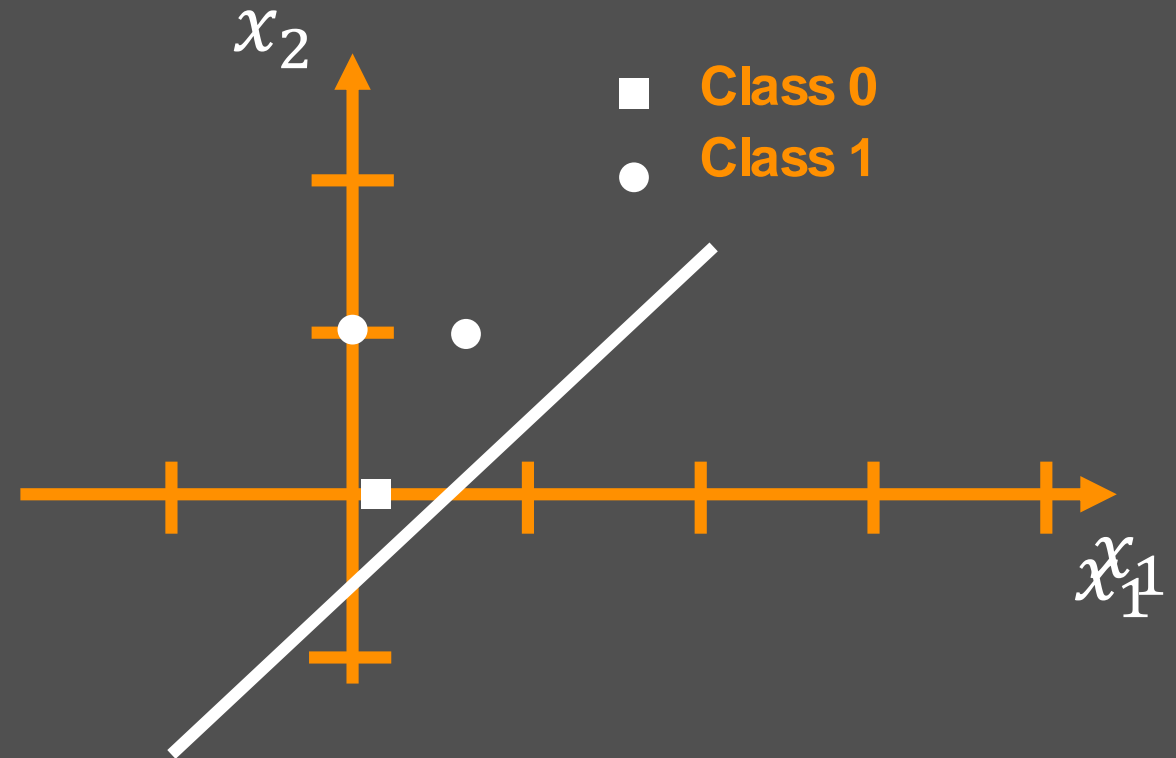
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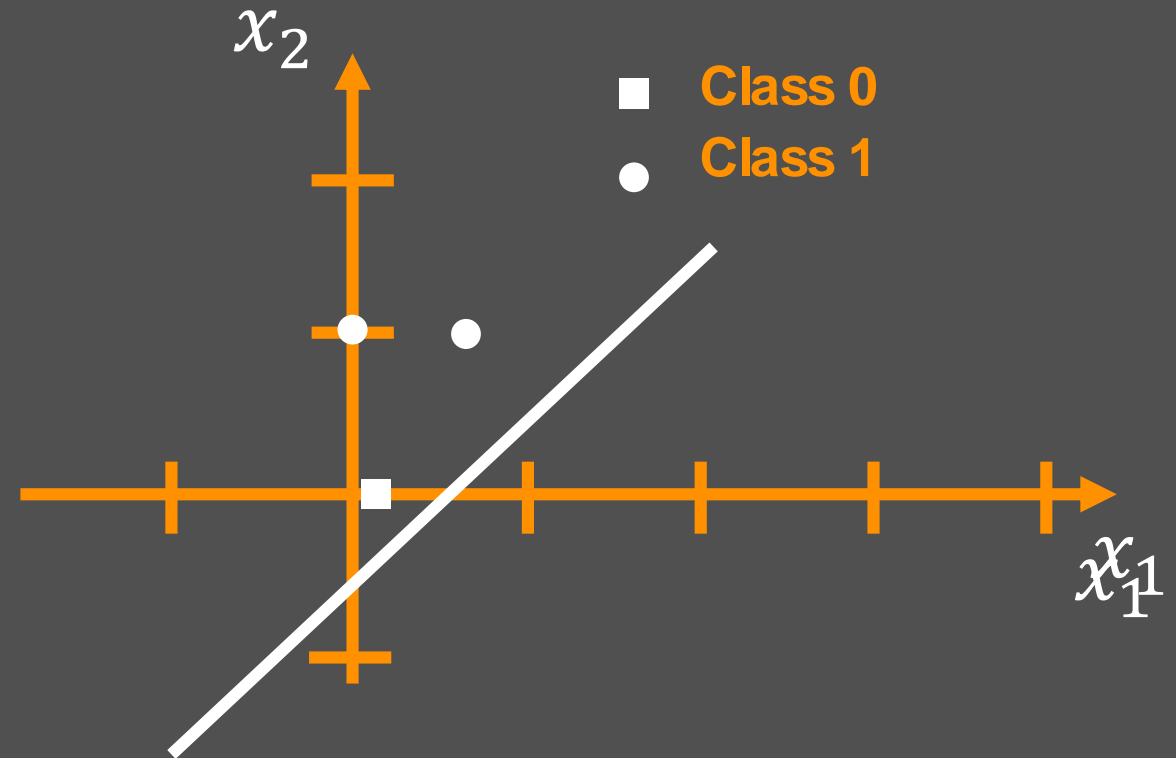
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$$\theta_0 = -0.5, \theta_1 = 1, \theta_2 = -1, \\ \hat{y} = 0, d = 1$$



if( $d==1$ )  $\theta = \theta + x$

$$\theta = \begin{pmatrix} -0.5 \\ 1 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0.6 \\ 1 \end{pmatrix}$$

$$\theta = \begin{pmatrix} 0.5 \\ 1.6 \\ 0 \end{pmatrix}$$

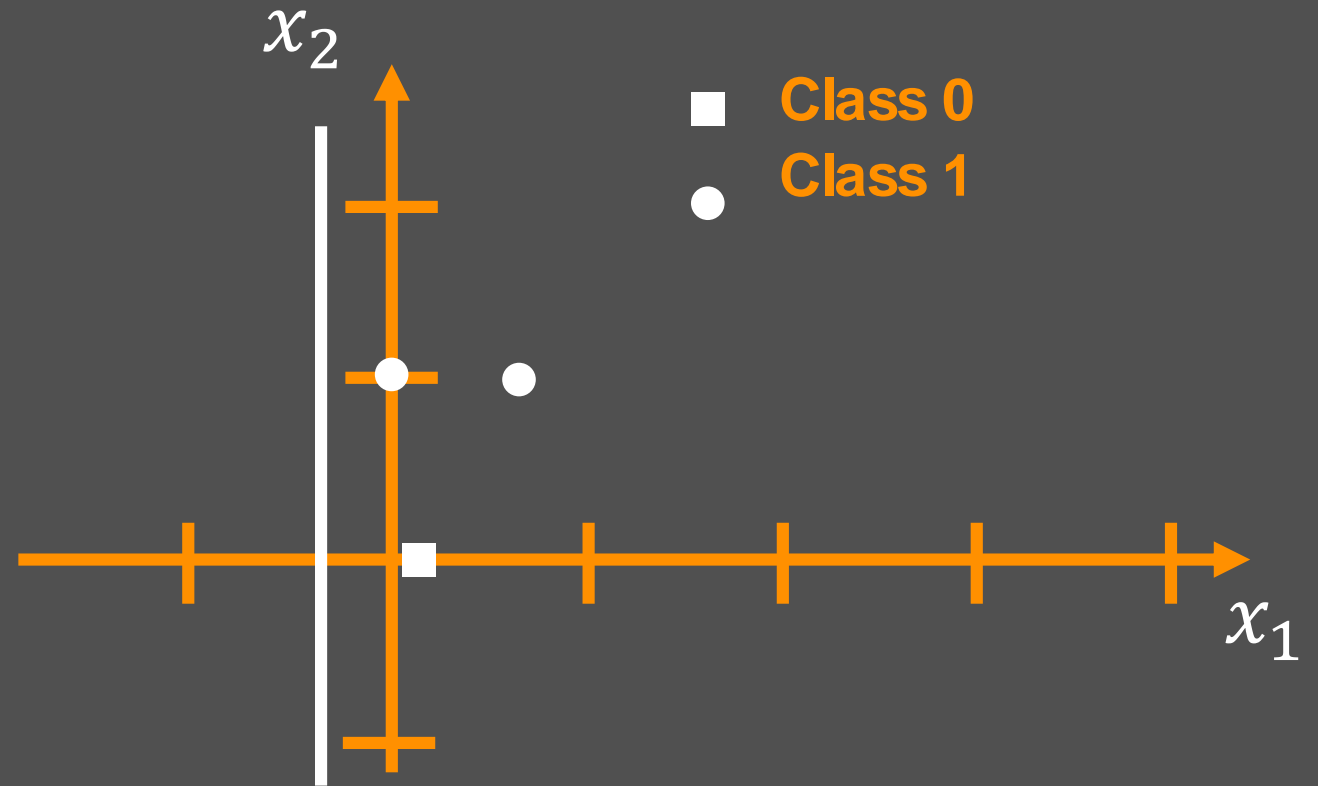
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$$\begin{aligned}x_2\theta_2 + x_1\theta_1 + x_0\theta_0 &= 0 \\1.6x_1 + 0.5 &= 0 \\x_1 &= -\frac{0.5}{1.6} = -0.3125\end{aligned}$$

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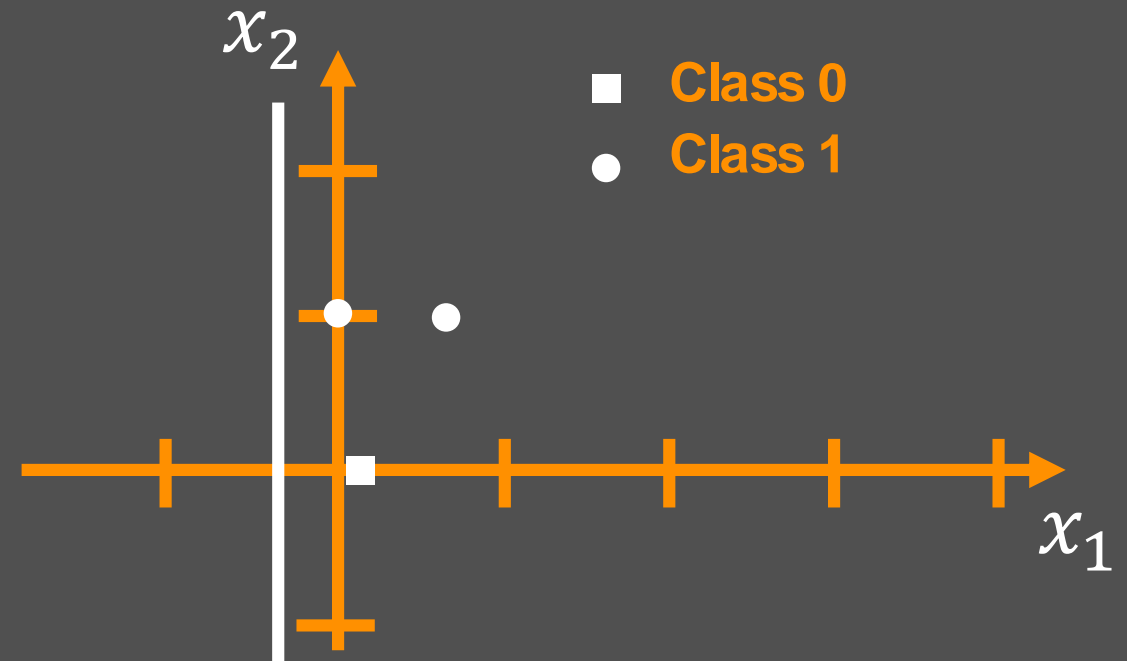
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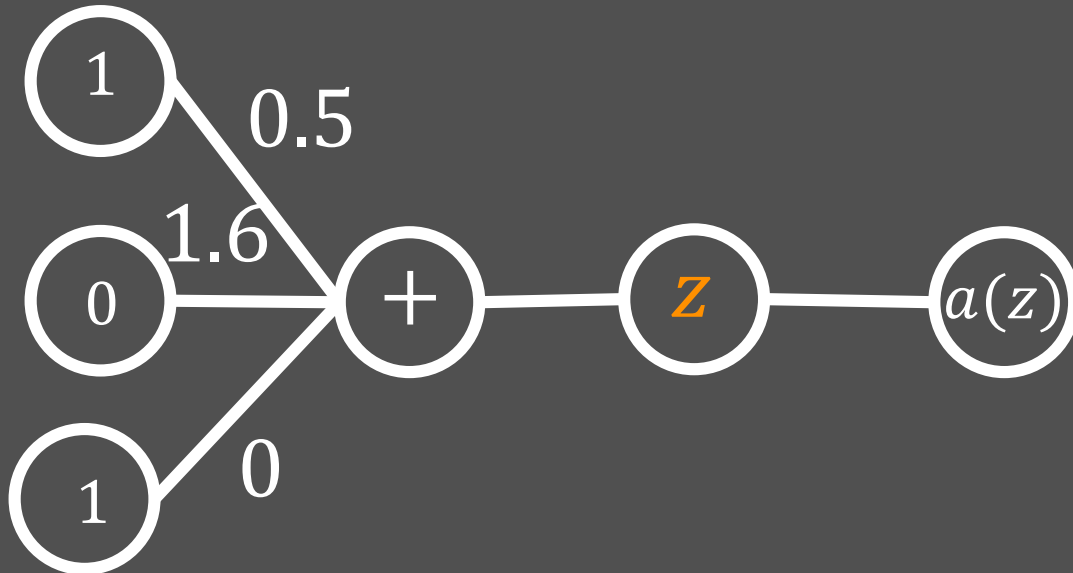


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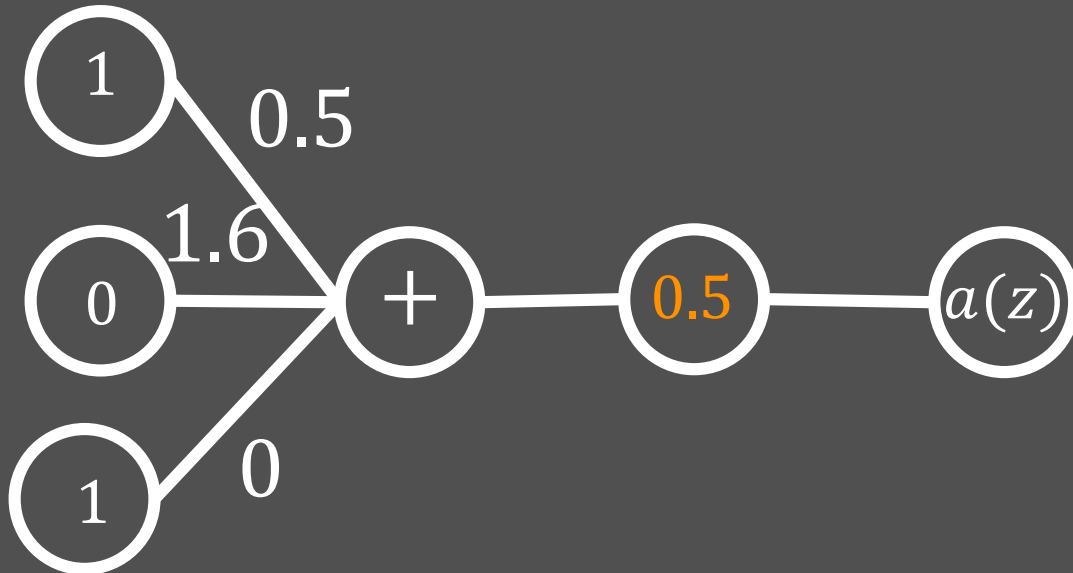
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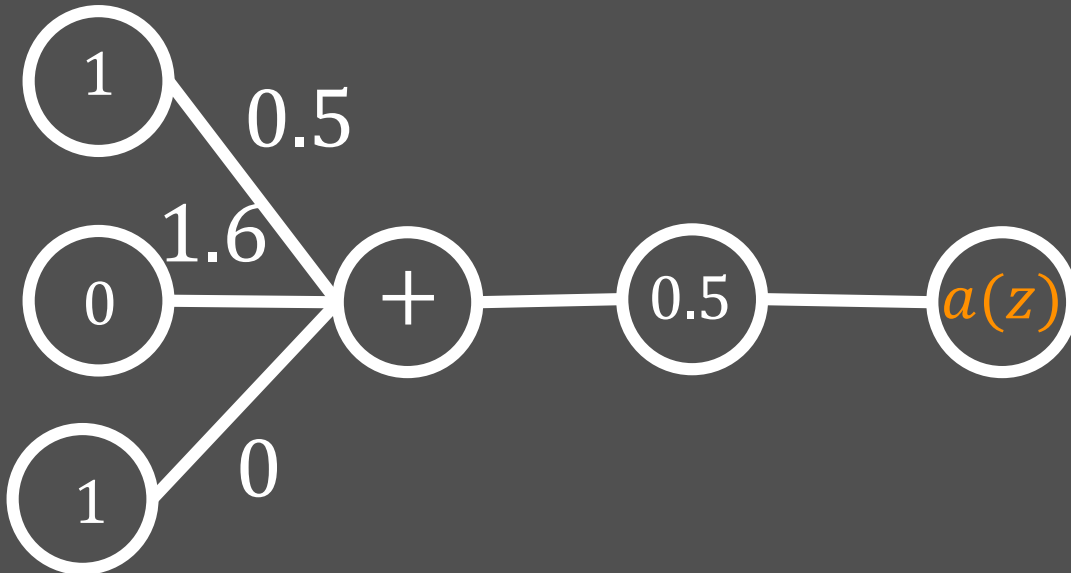


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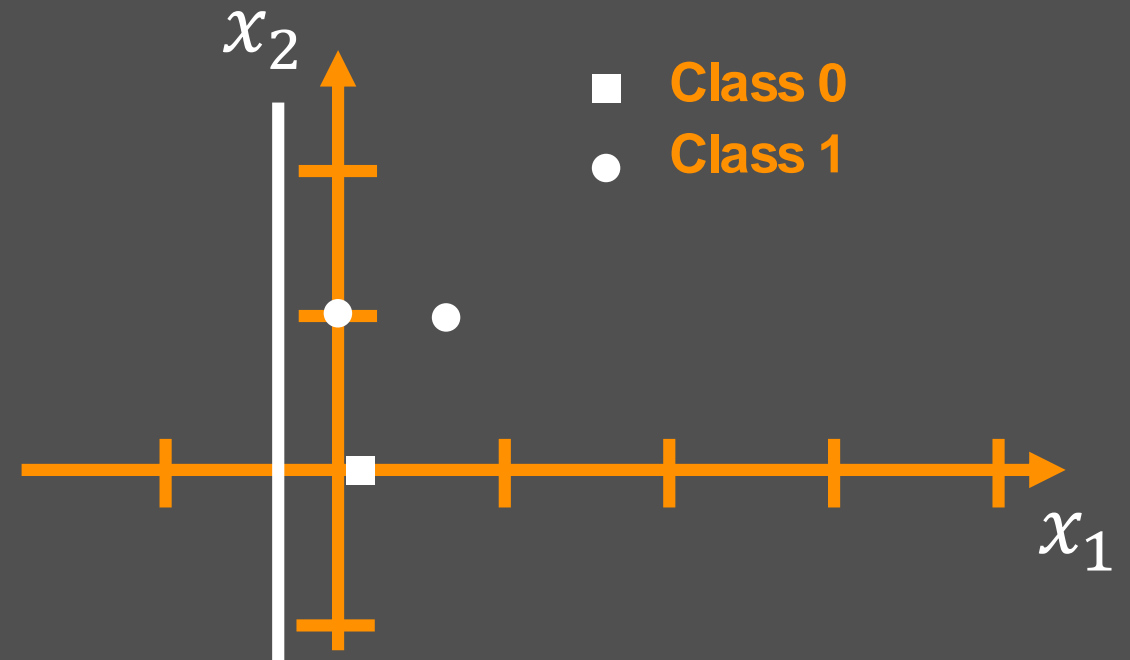
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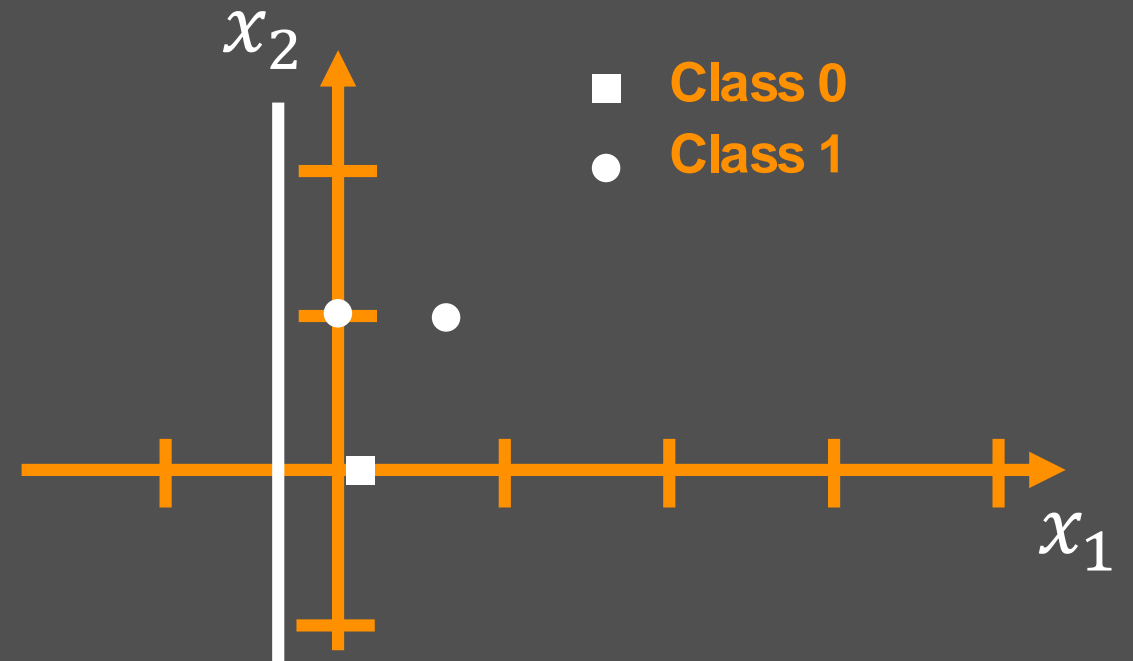
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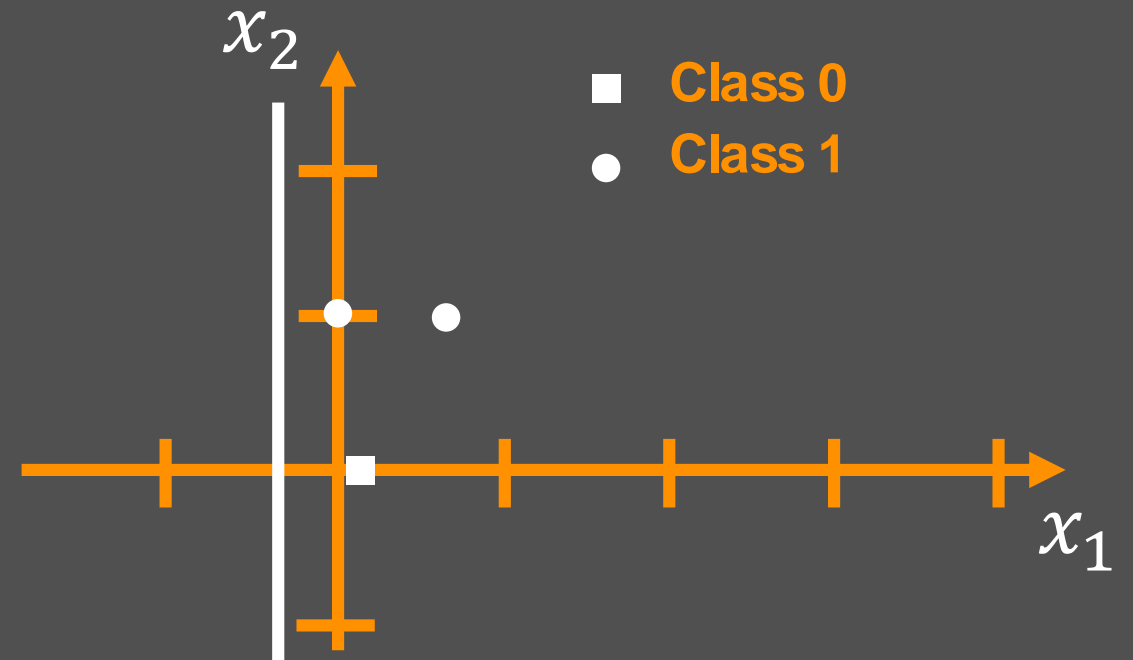
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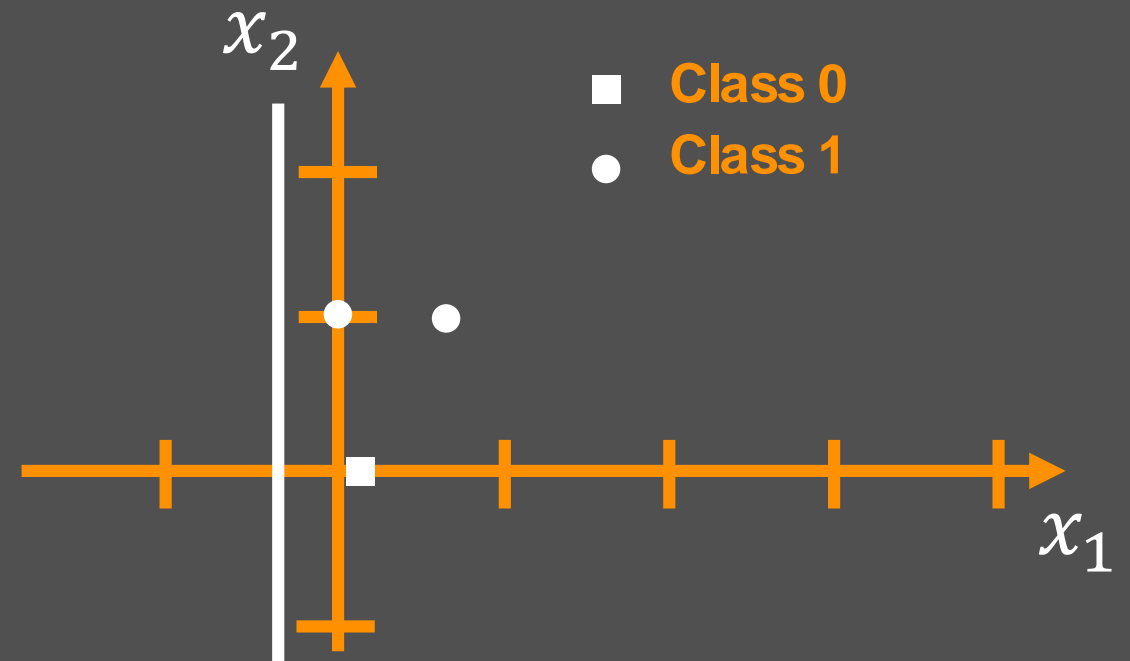
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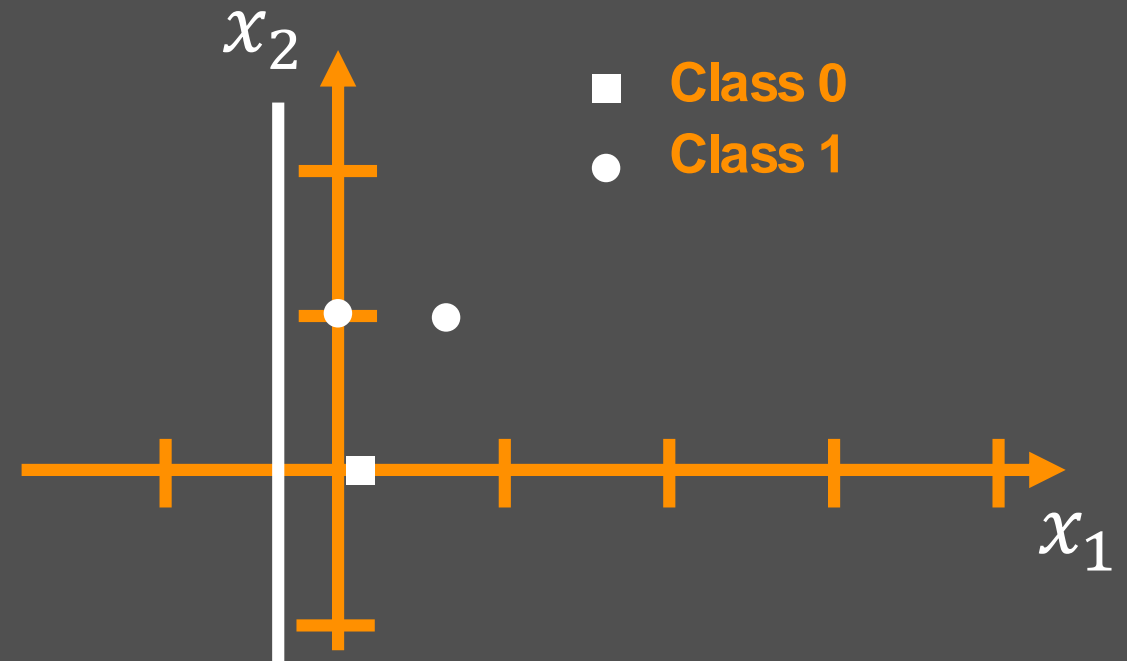
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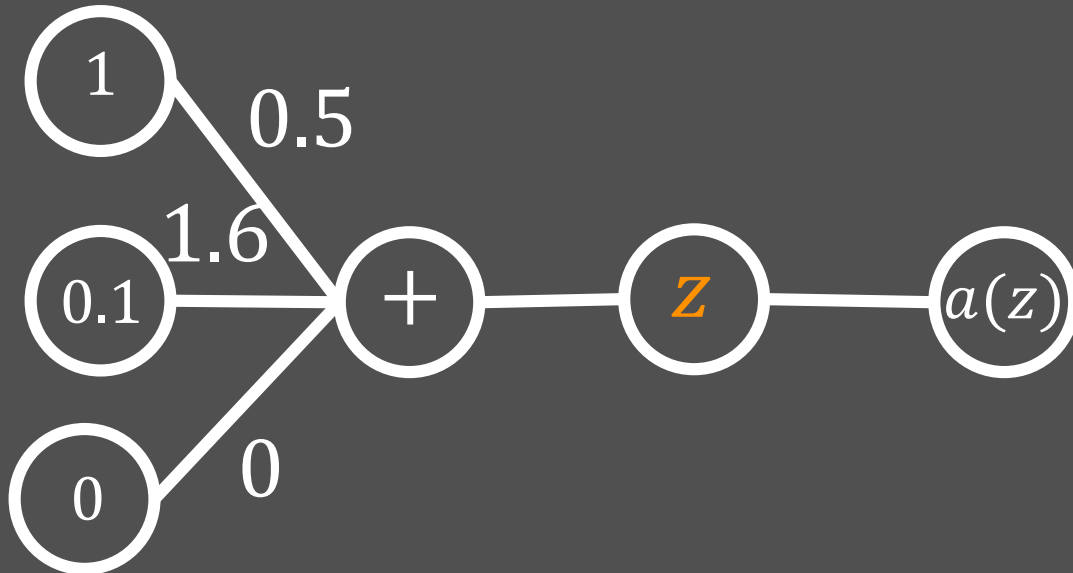


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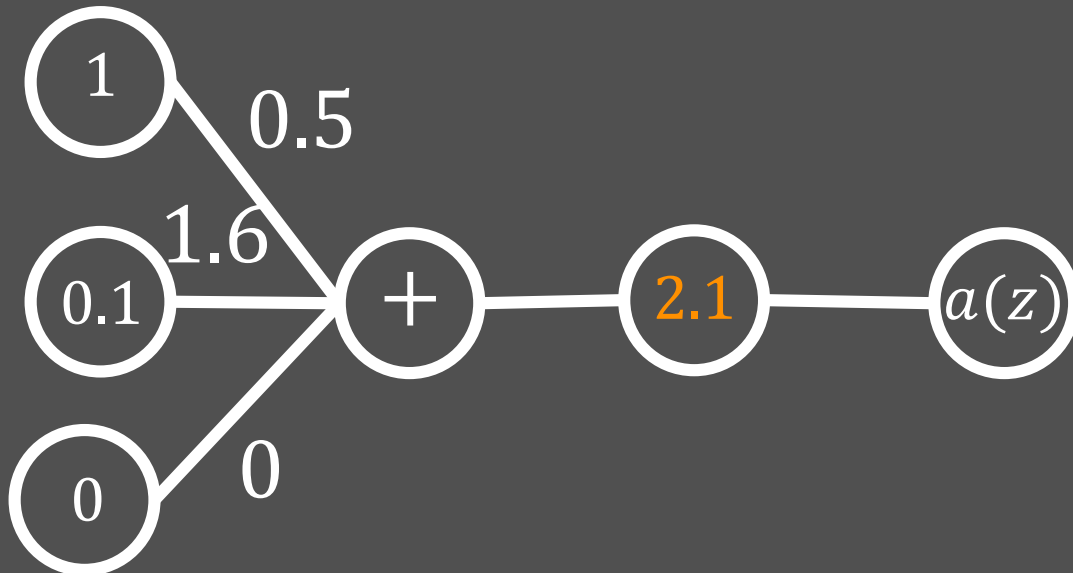


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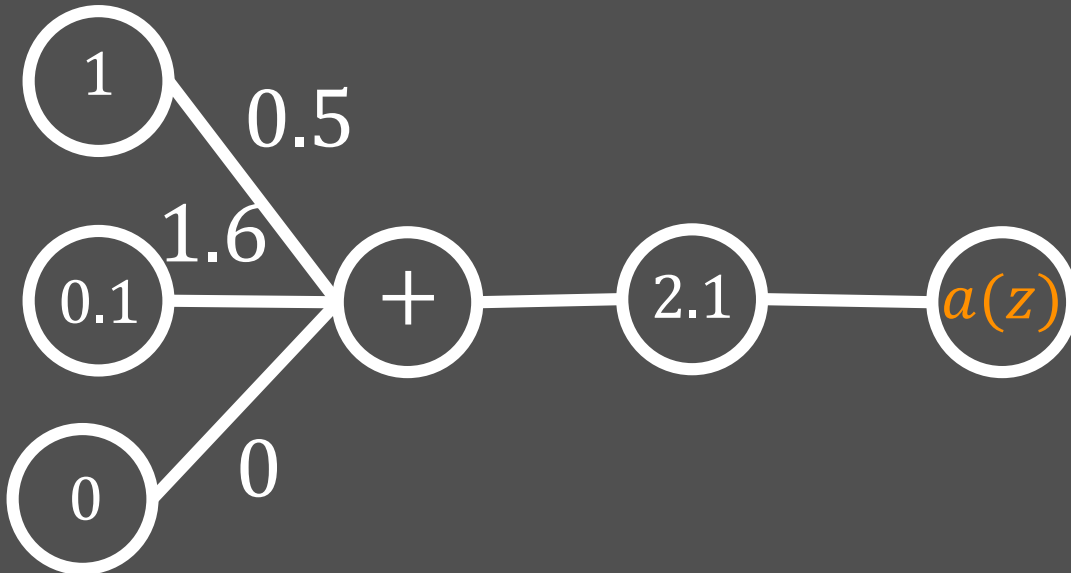
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## Perceptron-Model

$$\theta_0 = 0.5, \theta_1 = 1.6, \theta_2 = 0$$

$$a(z) = \begin{cases} 0, & z < 0 \\ 1, & z \geq 0 \end{cases}$$

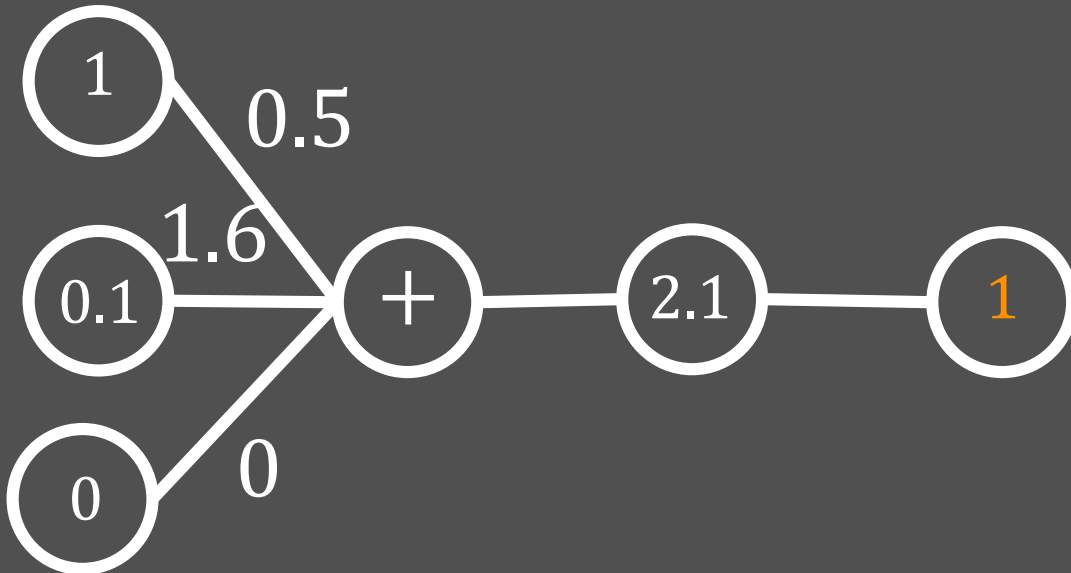


| $x_0$    | $x_1$      | $x_2$    | $y$      |
|----------|------------|----------|----------|
| <b>1</b> | <b>0.1</b> | <b>0</b> | <b>0</b> |
| <b>1</b> | <b>0.6</b> | <b>1</b> | <b>1</b> |
| <b>1</b> | <b>0</b>   | <b>1</b> | <b>1</b> |

# Perceptron

## Perceptron-Model

$$\hat{y} = a(z) = \begin{cases} 0, & z < 0 \\ 1, & z \geq 0 \end{cases}$$



$$\theta_0 = 0.5, \theta_1 = 1.6, \theta_2 = 0$$

| $x_0$ | $x_1$ | $x_2$ | $y$ |
|-------|-------|-------|-----|
| 1     | 0.1   | 0     | 0   |
| 1     | 0.6   | 1     | 1   |
| 1     | 0     | 1     | 1   |

# Perceptron

Initialize weights randomly  
foreach  $x$  in trainingsset{

$\hat{y} = \text{Prediction}(x), \hat{y} \in \{0, 1\}$

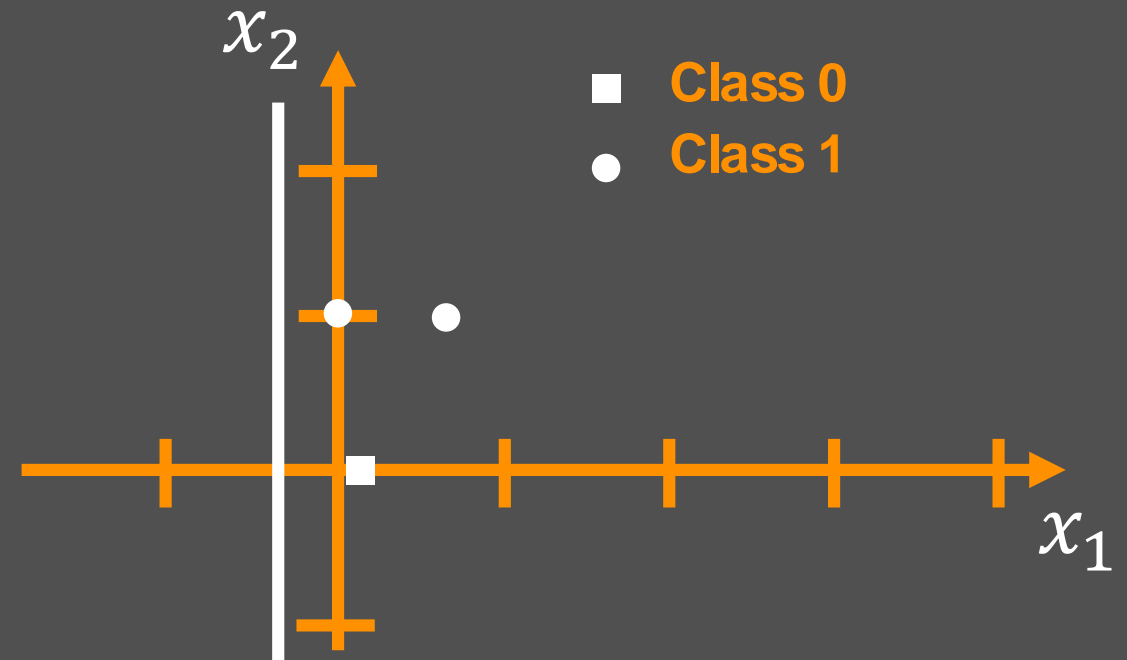
$d = y - \hat{y}, y \in \{0, 1\}$

if ( $d == 1$ )  $\theta = \theta + x$

elif ( $d == -1$ )  $\theta = \theta - x$

}

$\theta_0 = 0.5, \theta_1 = 1.6, \theta_2 = 0,$   
 $\hat{y} = 1$

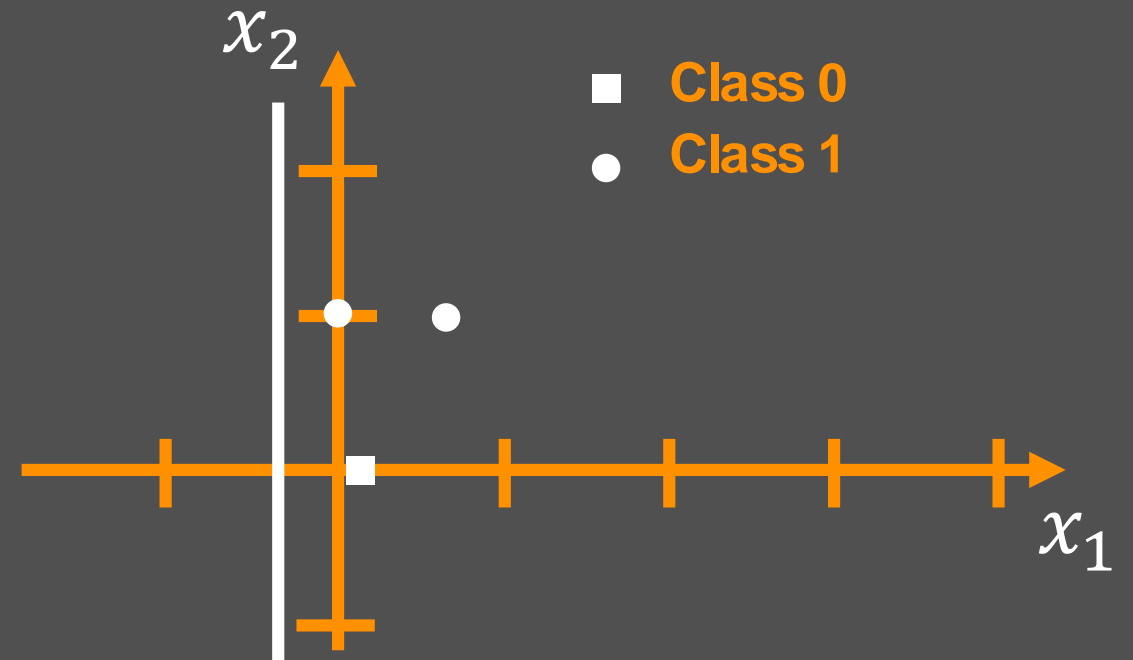


# Perceptron

```

Initialize weights randomly
foreach  $x$  in trainingsset{
     $\hat{y} = \text{Prediction}(x)$ ,  $\hat{y} \in \{0, 1\}$ 
     $d = y - \hat{y}$ ,  $y \in \{0, 1\}$ 
    if ( $d == 1$ )  $\theta = \theta + x$ 
    elif ( $d == -1$ )  $\theta = \theta - x$ 
}
    
```

$$\theta_0 = 0.5, \theta_1 = 1.6, \theta_2 = 0, \\ \hat{y} = 1$$



# Perceptron

Initialize weights randomly

foreach  $x$  in trainingsset{

$\hat{y} = \text{Prediction}(x)$ ,  $\hat{y} \in \{0, 1\}$

$d = y - \hat{y}$ ,  $y \in \{0, 1\}$

if ( $d == 1$ )  $\theta = \theta + x$

elif ( $d == -1$ )  $\theta = \theta - x$

}

$$\theta_0 = 0.5, \theta_1 = 1.6, \theta_2 = 0, \\ \hat{y} = 1, d = -1$$



| $x_0$ | $x_1$ | $x_2$ | $y$ |
|-------|-------|-------|-----|
| 1     | 0.1   | 0     | 0   |
| 1     | 0.6   | 1     | 1   |
| 1     | 0     | 1     | 1   |

# Perceptron

Initialize weights randomly

foreach  $x$  in trainingsset{

$\hat{y} = \text{Prediction}(x)$ ,  $\hat{y} \in \{0, 1\}$

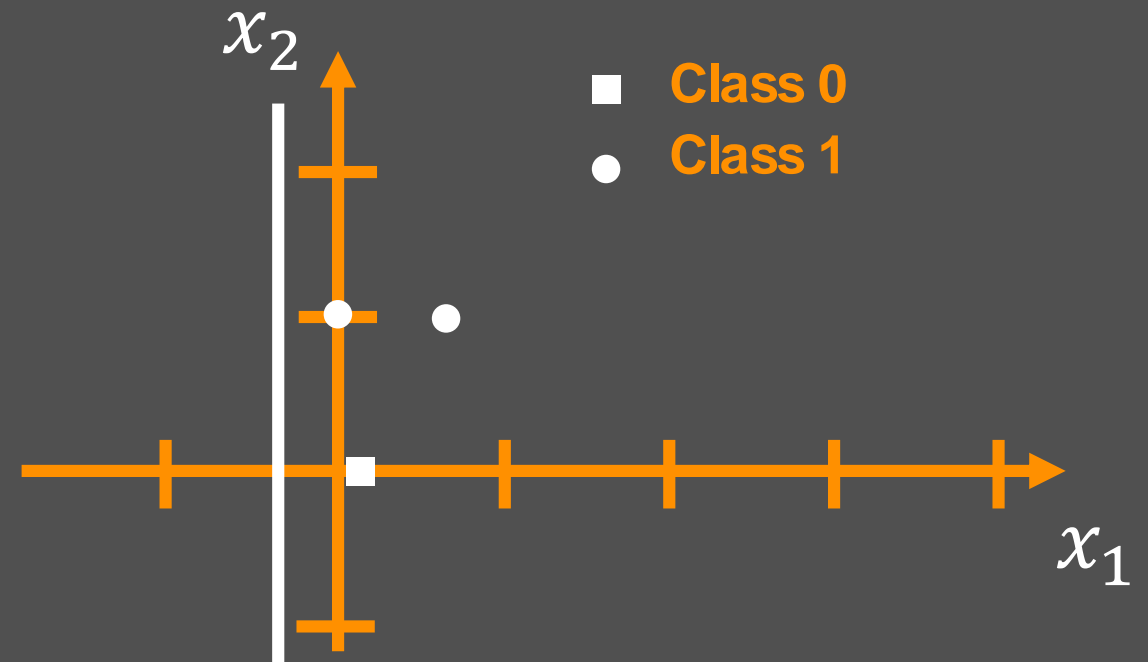
$d = y - \hat{y}$ ,  $y \in \{0, 1\}$

if ( $d == 1$ )  $\theta = \theta + x$

elif ( $d == -1$ )  $\theta = \theta - x$

}

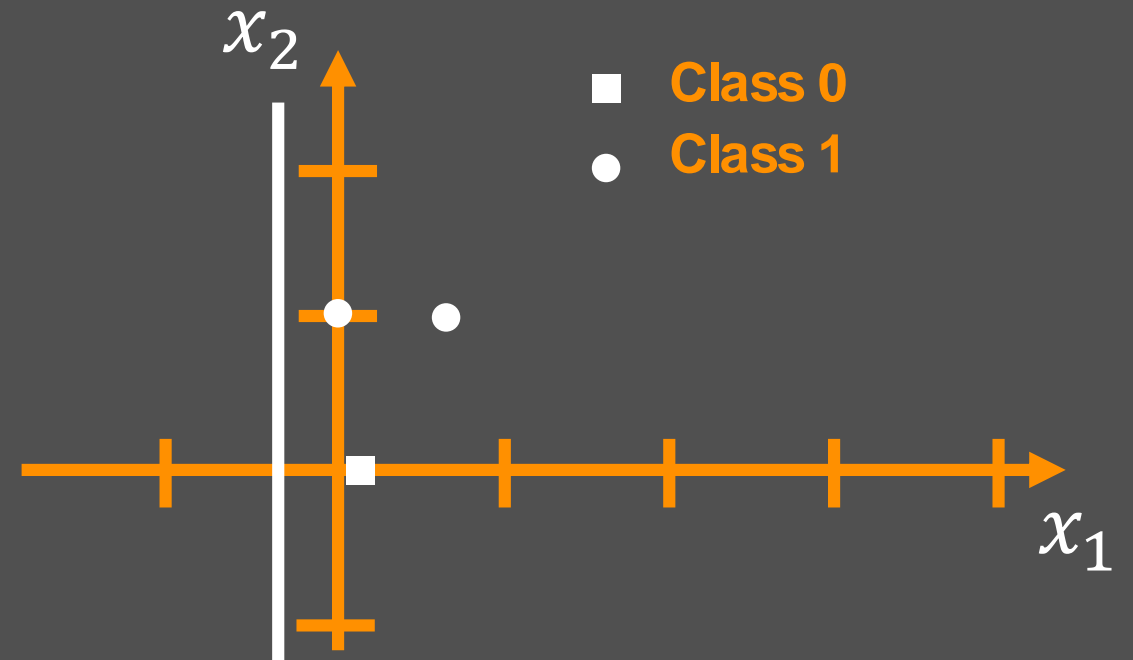
$$\theta_0 = 0.5, \theta_1 = 1.6, \theta_2 = 0, \\ \hat{y} = 1, d = -1$$



# Perceptron

```
Initialize weights randomly
foreach  $x$  in trainingsset{
     $\hat{y} = \text{Prediction}(x)$ ,  $\hat{y} \in \{0, 1\}$ 
     $d = y - \hat{y}$ ,  $y \in \{0, 1\}$ 
    if ( $d == 1$ )  $\theta = \theta + x$ 
    elif ( $d == -1$ )  $\theta = \theta - x$ 
}
```

$$\theta_0 = 0.5, \theta_1 = 1.6, \theta_2 = 0,$$
$$\hat{y} = 1, d = -1$$





# Perceptron

Initialize weights randomly

foreach  $x$  in trainingsset{

$\hat{y} = \text{Prediction}(x)$ ,  $\hat{y} \in \{0, 1\}$

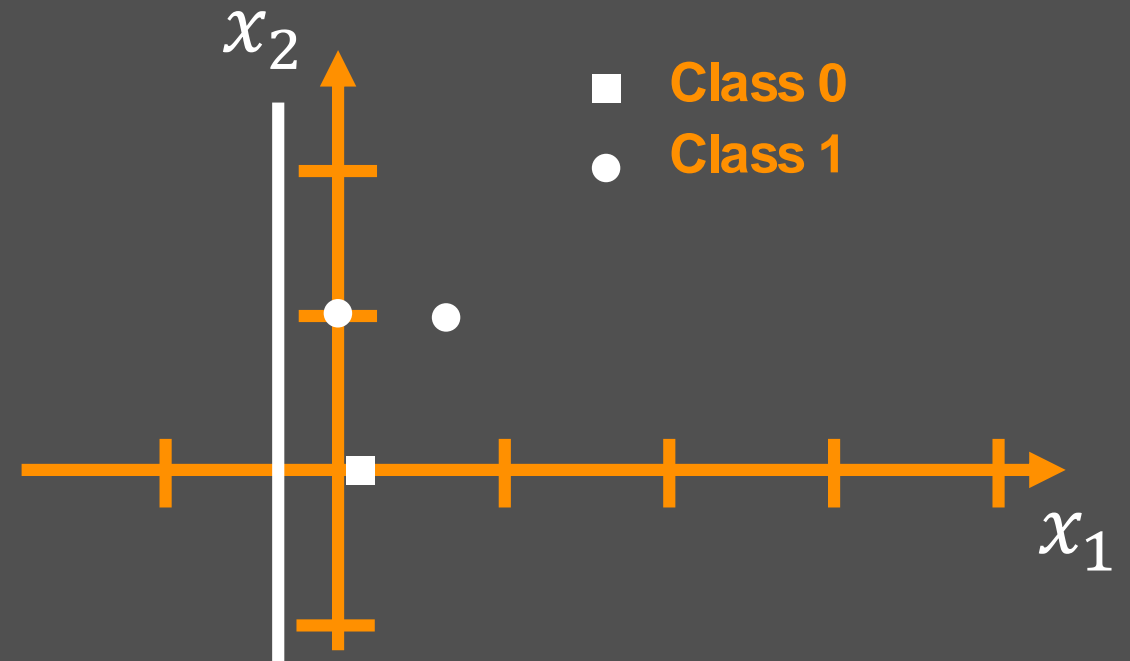
$d = y - \hat{y}$ ,  $y \in \{0, 1\}$

if ( $d == 1$ )  $\theta = \theta + x$

elif ( $d == -1$ )  $\theta = \theta - x$

}

$$\theta_0 = 0.5, \theta_1 = 1.6, \theta_2 = 0, \\ \hat{y} = 1, d = -1$$



if( $d == -1$ )  $\theta = \theta - x$

$$\theta = \begin{pmatrix} 0.5 \\ 1.6 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0.1 \\ 0 \end{pmatrix}$$

$$\theta = \begin{pmatrix} -0.5 \\ 1.5 \\ 0 \end{pmatrix}$$

$$\theta_0 = 0.5, \theta_1 = 1.6, \theta_2 = 0, \\ \hat{y} = 0, d = -1$$



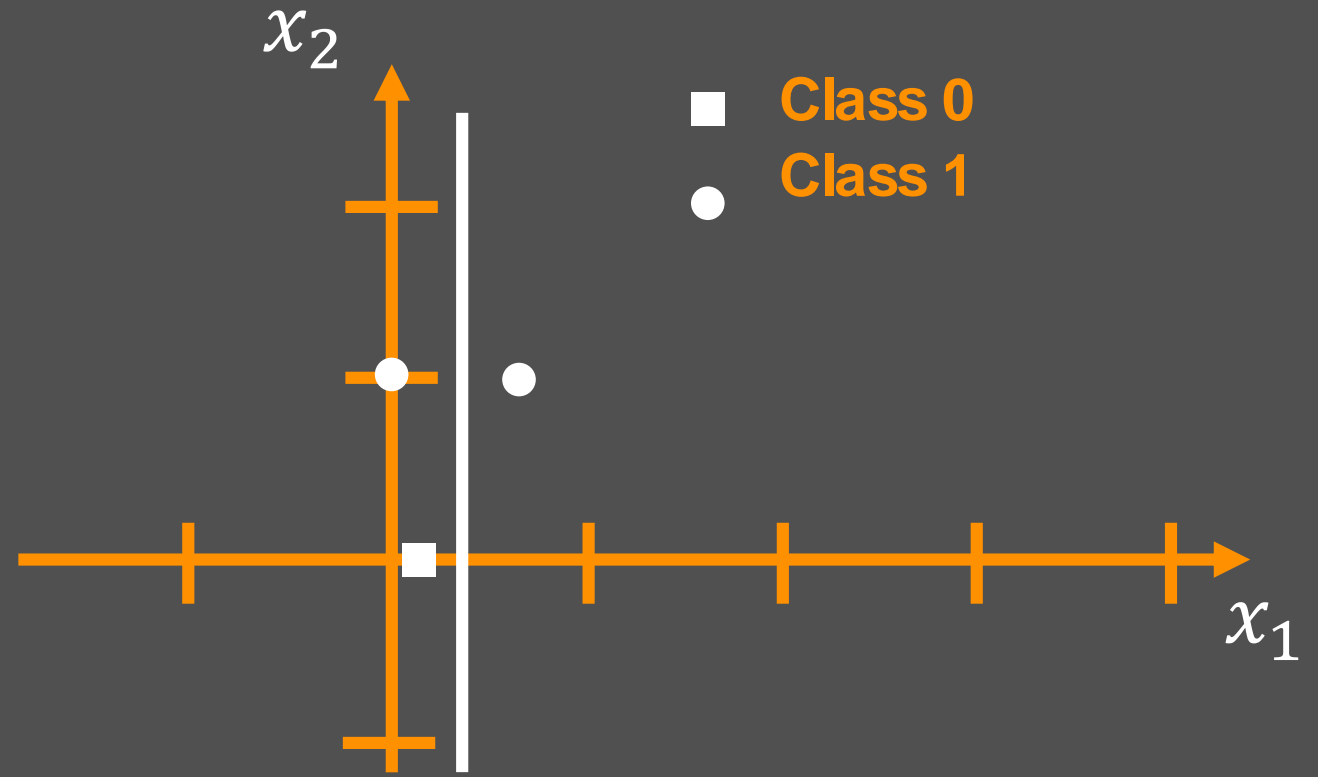
| $x_0$    | $x_1$      | $x_2$    | $y$      |
|----------|------------|----------|----------|
| <b>1</b> | <b>0.1</b> | <b>0</b> | <b>0</b> |
| <b>1</b> | <b>0.6</b> | <b>1</b> | <b>1</b> |
| <b>1</b> | <b>0</b>   | <b>1</b> | <b>1</b> |

$$x_2\theta_2 + x_1\theta_1 + x_0\theta_0 = 0$$

$$1.5x_1 - 0.5 = 0$$

$$x_1 = \frac{0.5}{1.5} = \frac{1}{3}$$

$$\theta_0 = -0.5, \theta_1 = 1.5, \theta_2 = 0$$



# Two iterations later...

# Perceptron

Initialize weights randomly  
foreach  $x$  in trainingsset{

$\hat{y} = \text{Prediction}(x)$ ,  $\hat{y} \in \{0, 1\}$

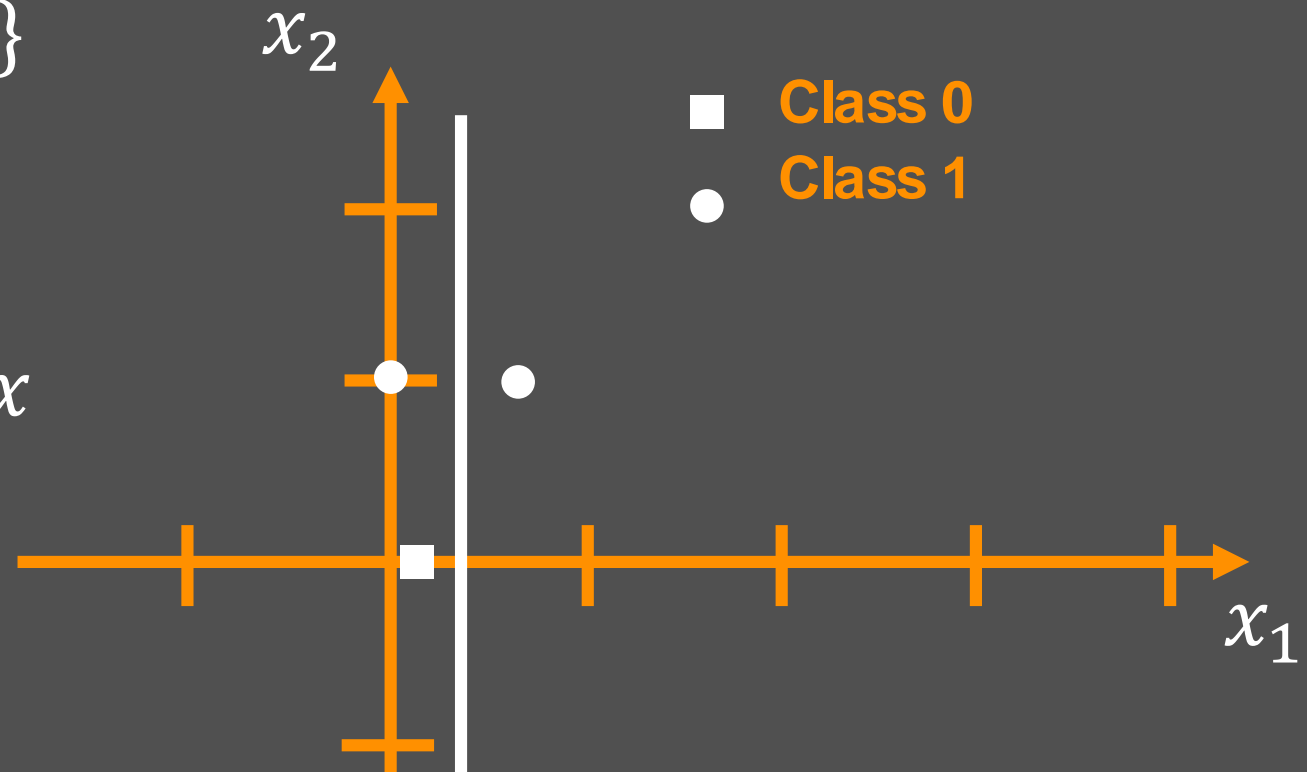
$d = y - \hat{y}$ ,  $y \in \{0, 1\}$

if ( $d == 1$ )  $\theta = \theta + x$

elif ( $d == -1$ )  $\theta = \theta - x$

}

$$\theta_0 = -0.5, \theta_1 = 1.5, \theta_2 = 0, \\ \hat{y} = 0, d = 1$$



if( $d==1$ )  $\theta = \theta + x$

$$\theta = \begin{pmatrix} 0.5 \\ 1.5 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\theta = \begin{pmatrix} 1.5 \\ 1.5 \\ 1 \end{pmatrix}$$

$$\theta_0 = -0.5, \theta_1 = 1.5, \theta_2 = 0, \\ \hat{y} = 0, d = 1$$



| $x_0$ | $x_1$ | $x_2$ | $y$ |
|-------|-------|-------|-----|
| 1     | 0.1   | 0     | 0   |
| 1     | 0.6   | 1     | 1   |
| 1     | 0     | 1     | 1   |

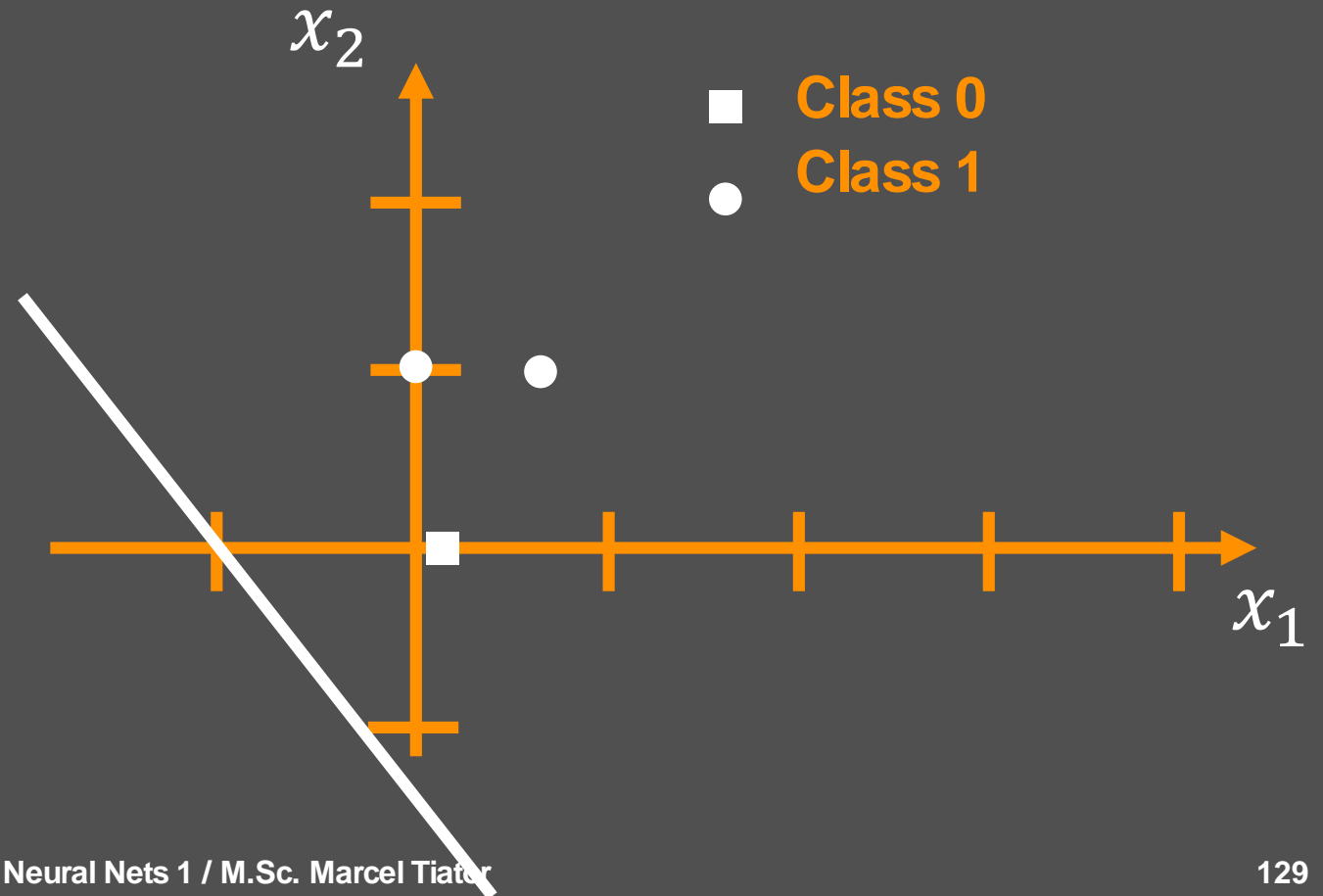
$$x_2\theta_2 + x_1\theta_1 + x_0\theta_0 = 0$$

$$x_2 = \frac{-(x_1\theta_1 + x_0\theta_0)}{\theta_2}$$

$$x_2 = \frac{-(1.5x_1 + 1.5)}{1}$$

$$x_2 = -1.5(x_1 + 1)$$

$$\theta_0 = 1.5, \theta_1 = 1.5, \theta_2 = 1$$



# One iteration later...



# Perceptron

Initialize weights randomly

foreach  $x$  in trainingsset{

$\hat{y} = \text{Prediction}(x)$ ,  $\hat{y} \in \{0, 1\}$

$d = y - \hat{y}$ ,  $y \in \{0, 1\}$

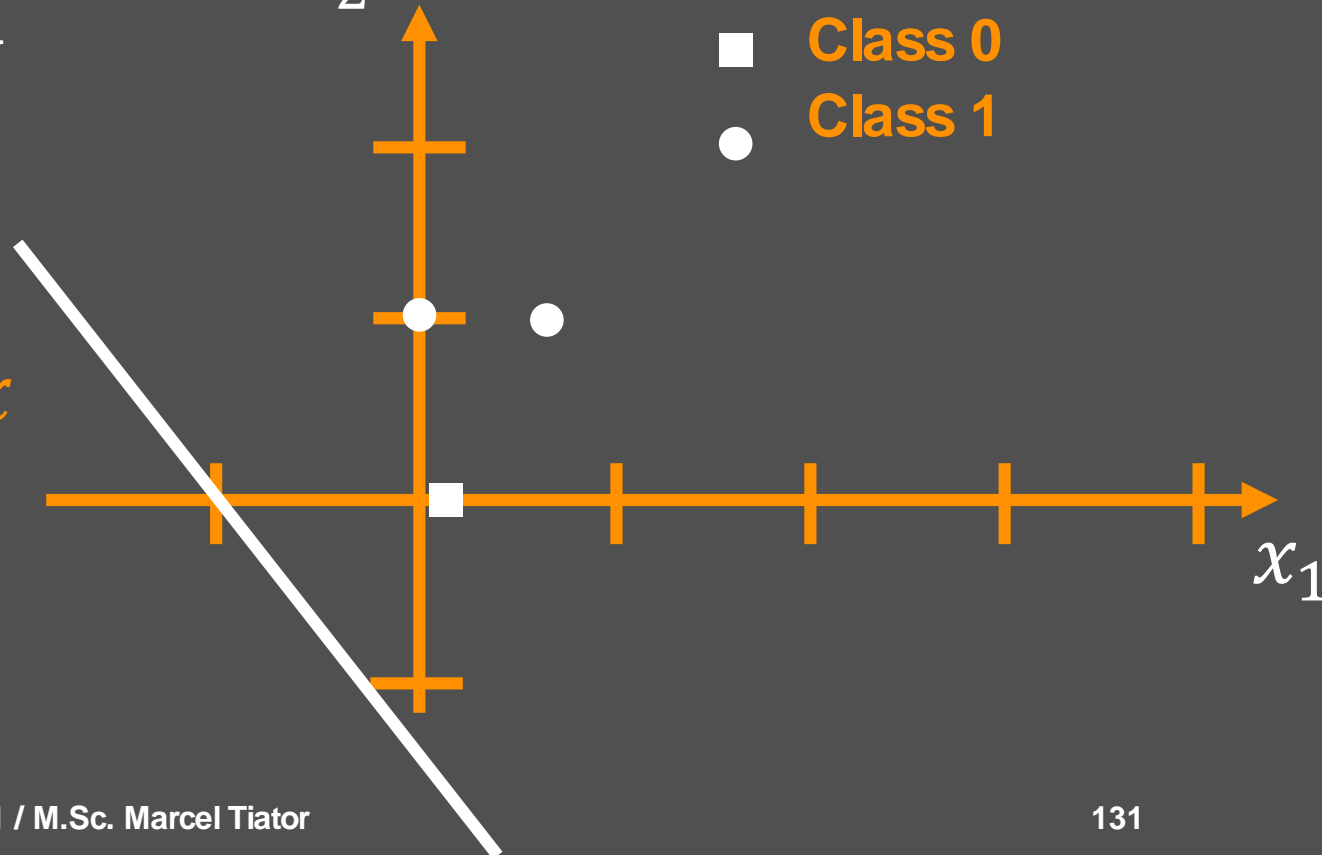
if ( $d == 1$ )  $\theta = \theta + x$

elif ( $d == -1$ )  $\theta = \theta - x$

}

$$\theta_0 = 1.5, \theta_1 = 1.5, \theta_2 = 1,$$

$$\hat{y} = 1, d = -1$$



if( $d == -1$ )  $\theta = \theta - x$

$$\theta = \begin{pmatrix} 1.5 \\ 1.5 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 0.1 \\ 0 \end{pmatrix}$$

$$\theta = \begin{pmatrix} 0.5 \\ 1.4 \\ 1 \end{pmatrix}$$

$$\theta_0 = 1.5, \theta_1 = 1.5, \theta_2 = 1, \\ \hat{y} = 1, d = -1$$



| $x_0$    | $x_1$      | $x_2$    | $y$      |
|----------|------------|----------|----------|
| <b>1</b> | <b>0.1</b> | <b>0</b> | <b>0</b> |
| <b>1</b> | <b>0.6</b> | <b>1</b> | <b>1</b> |
| <b>1</b> | <b>0</b>   | <b>1</b> | <b>1</b> |

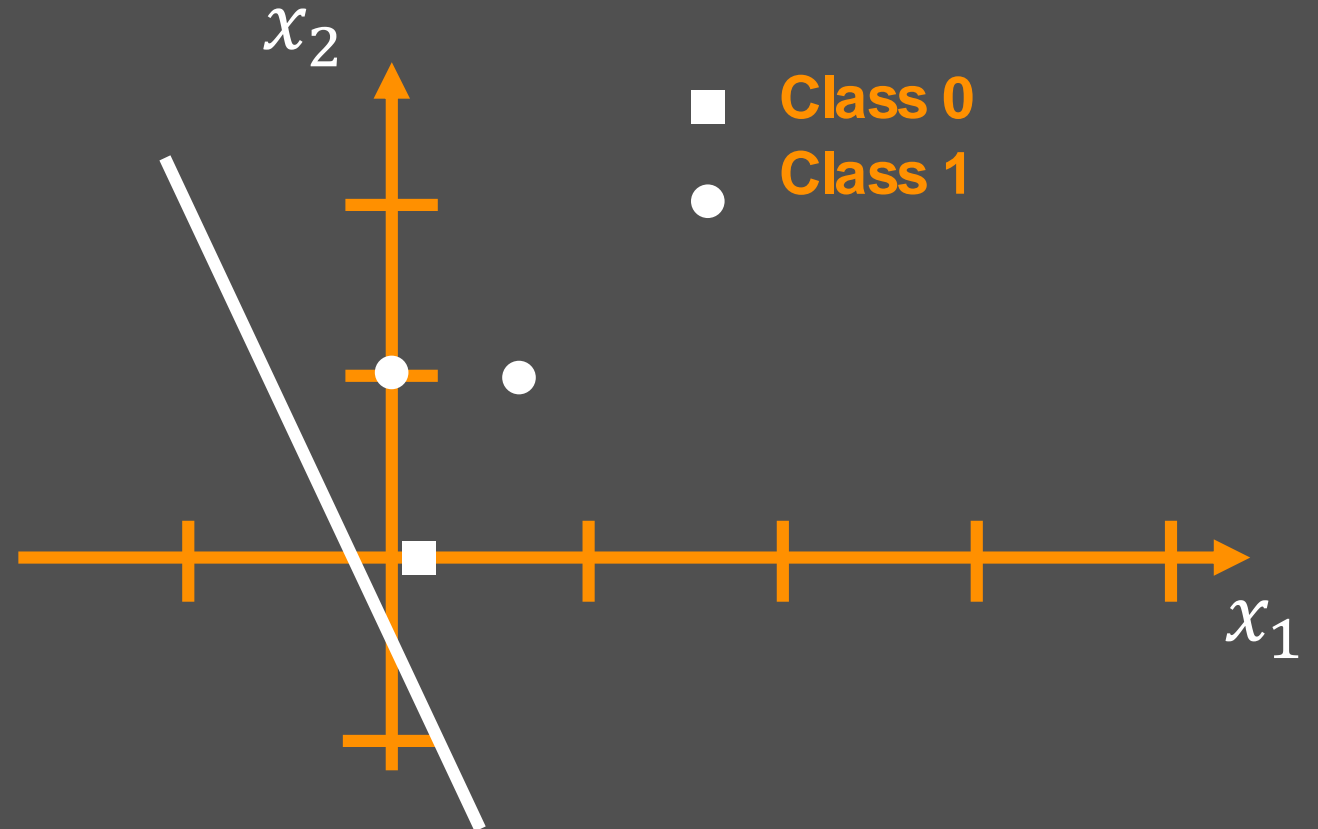
$$x_2\theta_2 + x_1\theta_1 + x_0\theta_0 = 0$$

$$x_2 = \frac{-(x_1\theta_1 + x_0\theta_0)}{\theta_2}$$

$$x_2 = \frac{-(1.4x_1 + 0.5)}{1}$$

$$x_2 = -1.4(x_1 + 0.357)$$

$$\theta_0 = 0.5, \theta_1 = 1.4, \theta_2 = 1$$



# Two iterations later...

# Perceptron

Initialize weights randomly

foreach  $x$  in trainingsset{

$\hat{y} = \text{Prediction}(x)$ ,  $\hat{y} \in \{0, 1\}$

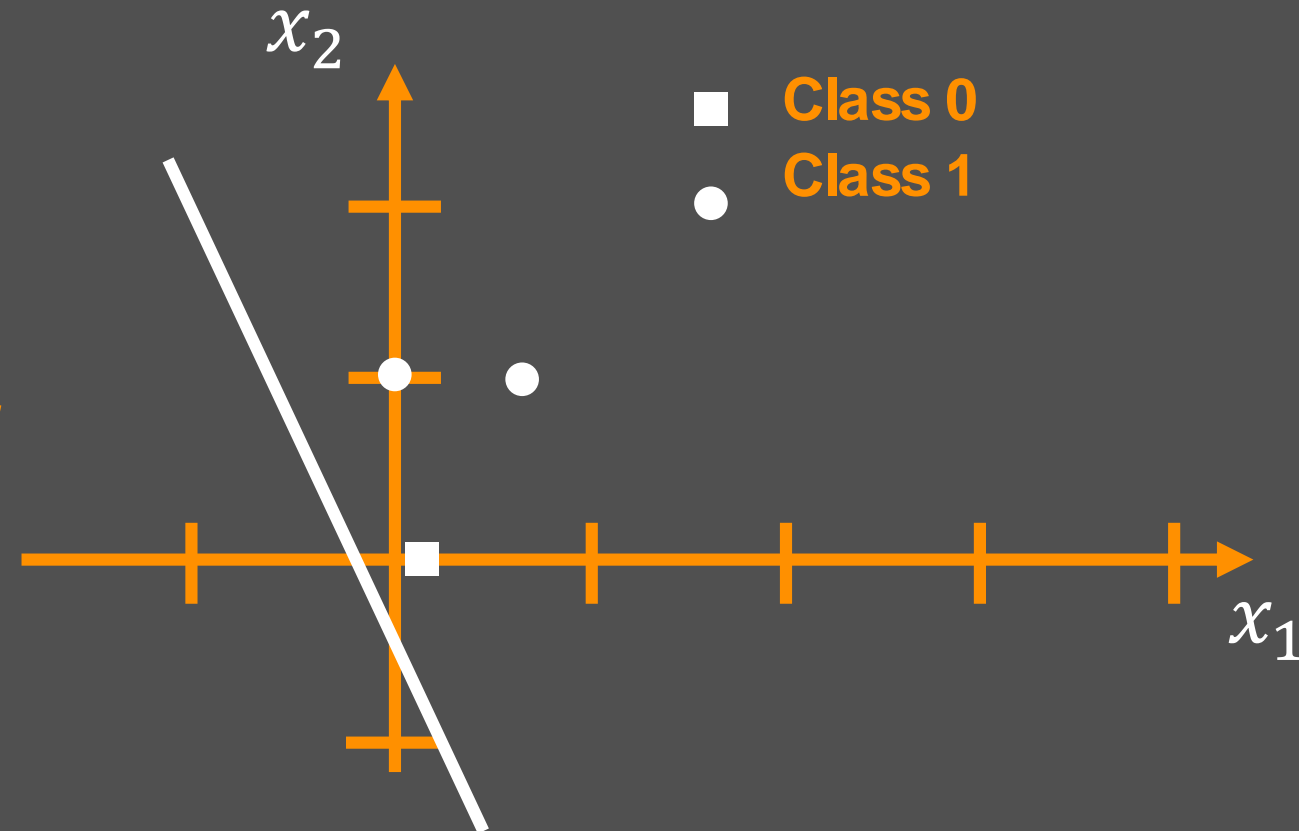
$d = y - \hat{y}$ ,  $y \in \{0, 1\}$

if ( $d == 1$ )  $\theta = \theta + x$

elif ( $d == -1$ )  $\theta = \theta - x$

}

$\theta_0 = 0.5, \theta_1 = 1.4, \theta_2 = 1$ ,  
 $\hat{y} = 1, d = -1$



if( $d == -1$ )  $\theta = \theta - x$

$$\theta = \begin{pmatrix} 0.5 \\ 1.4 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 0.1 \\ 0 \end{pmatrix}$$

$$\theta = \begin{pmatrix} -0.5 \\ 1.3 \\ 1 \end{pmatrix}$$

$$\theta_0 = 0.5, \theta_1 = 1.4, \theta_2 = 1, \\ \hat{y} = 1, d = -1$$



| $x_0$    | $x_1$      | $x_2$    | $y$      |
|----------|------------|----------|----------|
| <b>1</b> | <b>0.1</b> | <b>0</b> | <b>0</b> |
| <b>1</b> | <b>0.6</b> | <b>1</b> | <b>1</b> |
| <b>1</b> | <b>0</b>   | <b>1</b> | <b>1</b> |

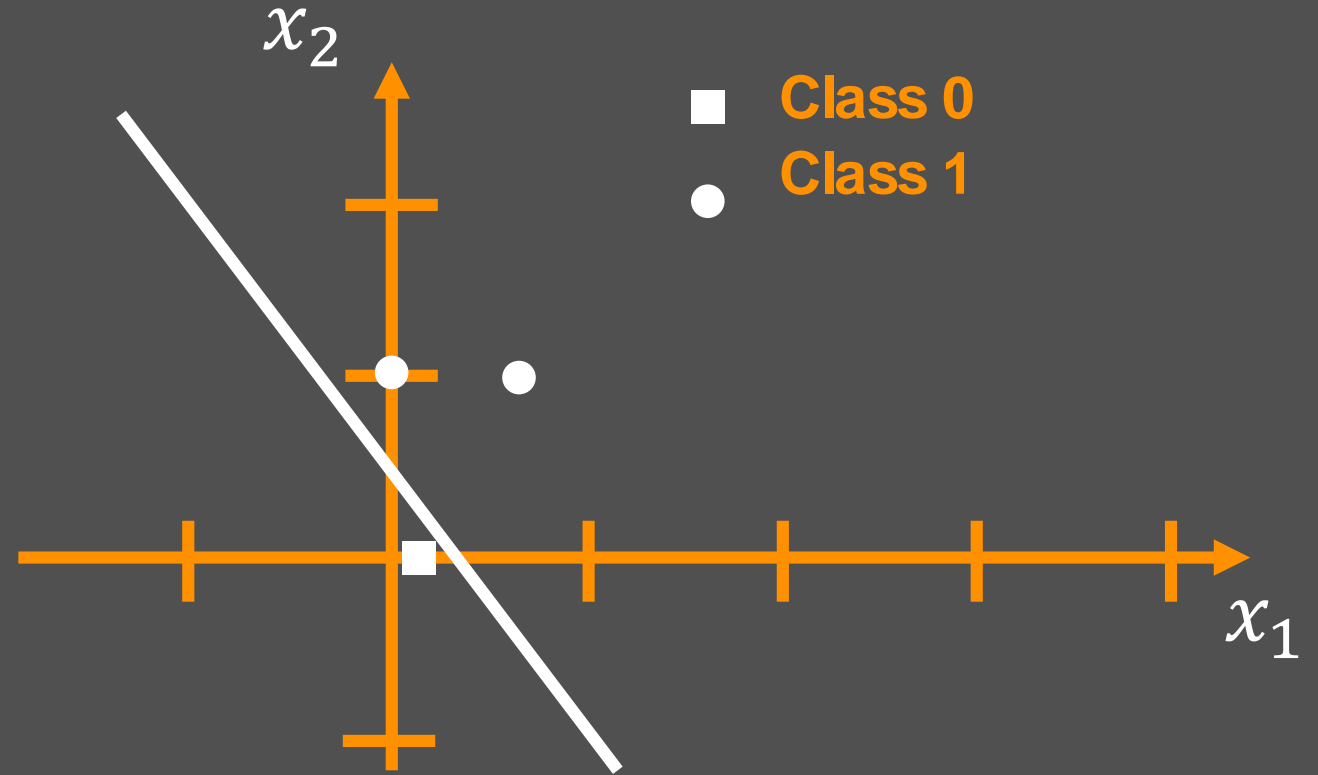
$$x_2\theta_2 + x_1\theta_1 + x_0\theta_0 = 0$$

$$x_2 = \frac{-(x_1\theta_1 + x_0\theta_0)}{\theta_2}$$

$$x_2 = \frac{-(1.3x_1 - 0.5)}{1}$$

$$x_2 = -1.3(x_1 - 0.385)$$

$$\theta_0 = -0.5, \theta_1 = 1.3, \theta_2 = 1$$

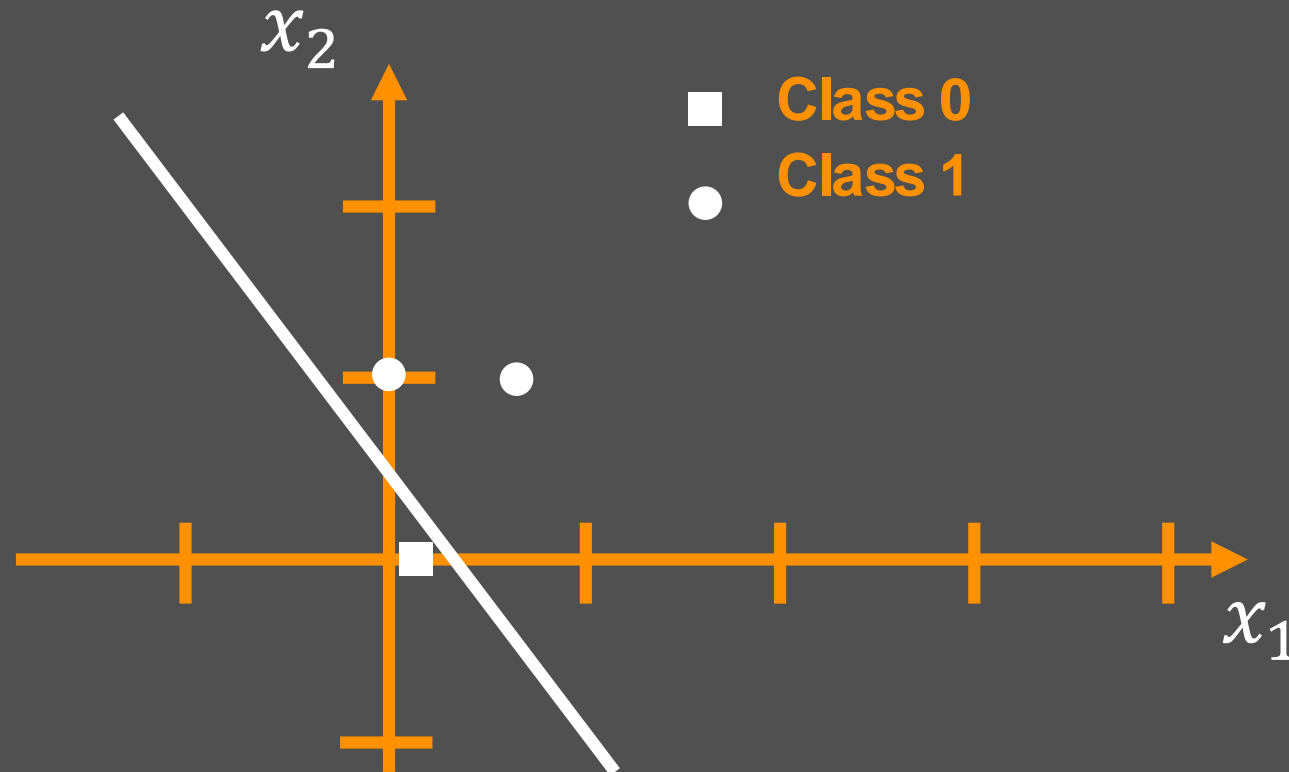


# Training Finished!

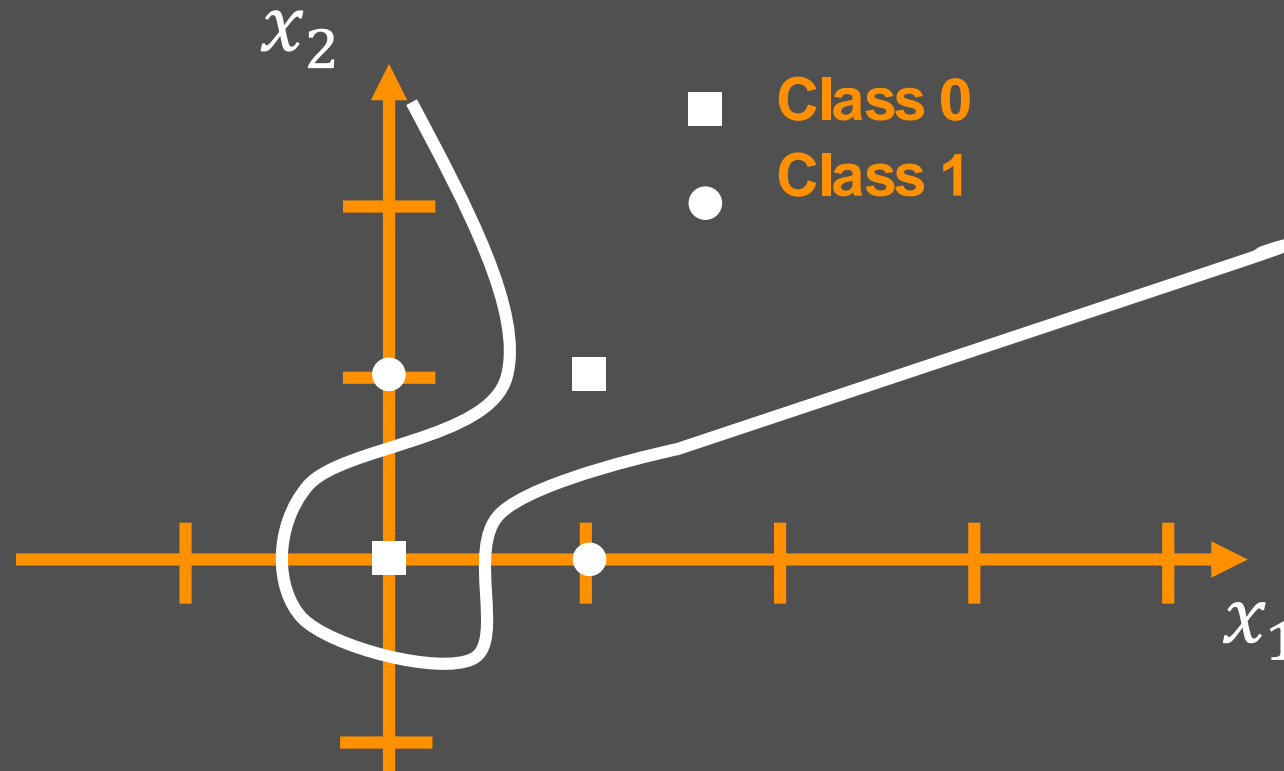


# Binary Classification with Perceptron

- Need linearly separable dataset



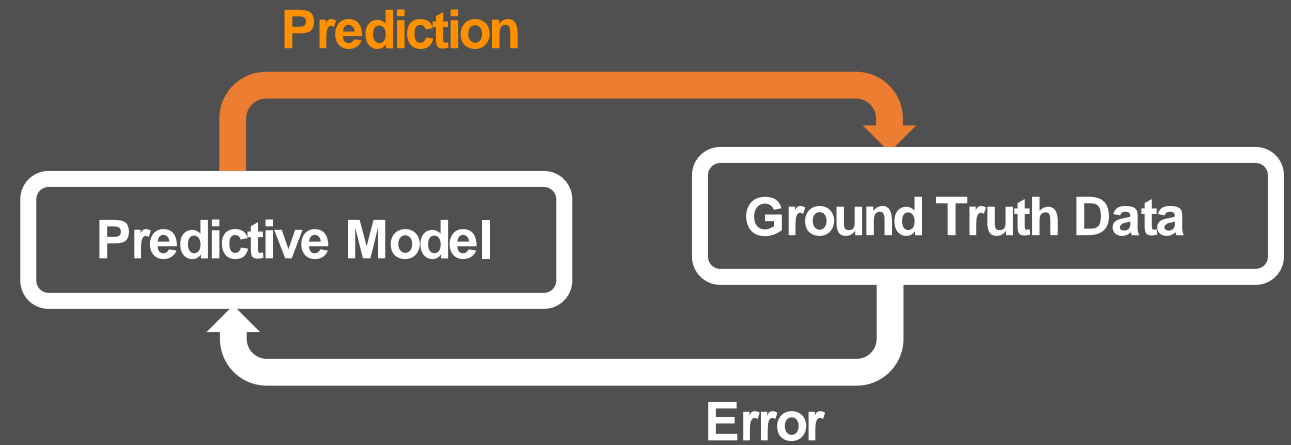
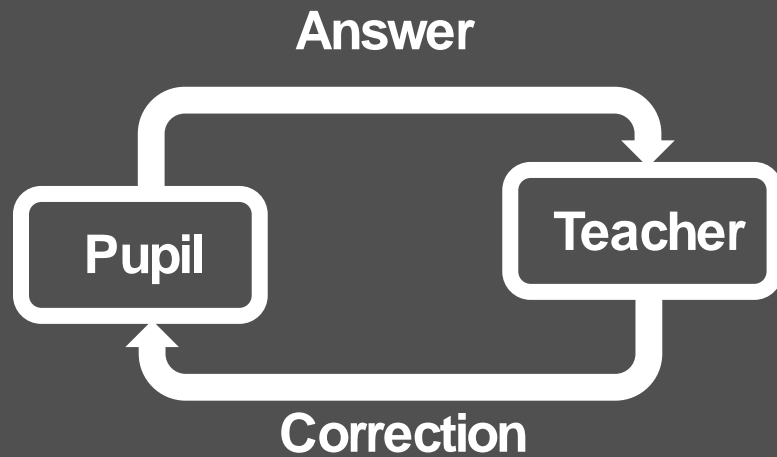
# Assume a more complex problem



# Multilayer Perceptron (MLP)

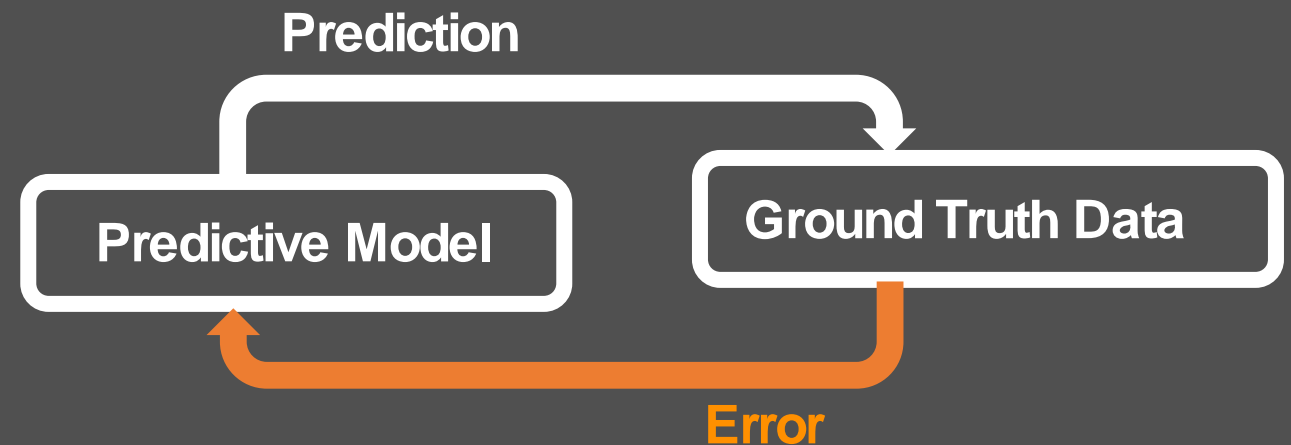
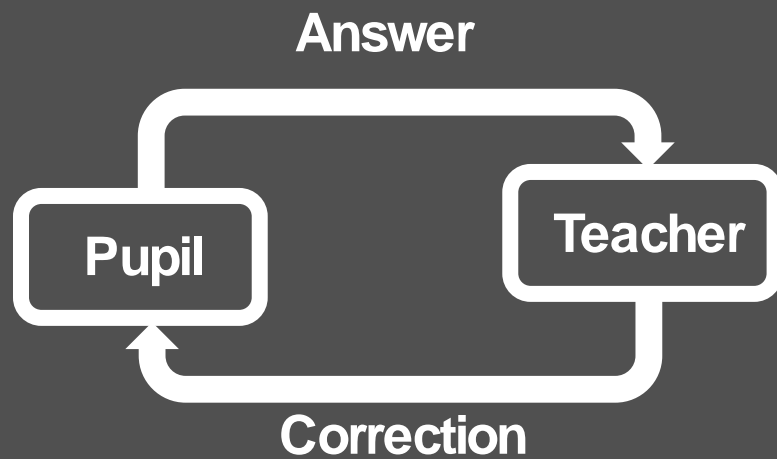
## Two Processes:

- **Forward Propagation (Prediction)**
- **Backward Propagation (Weight Tuning)**



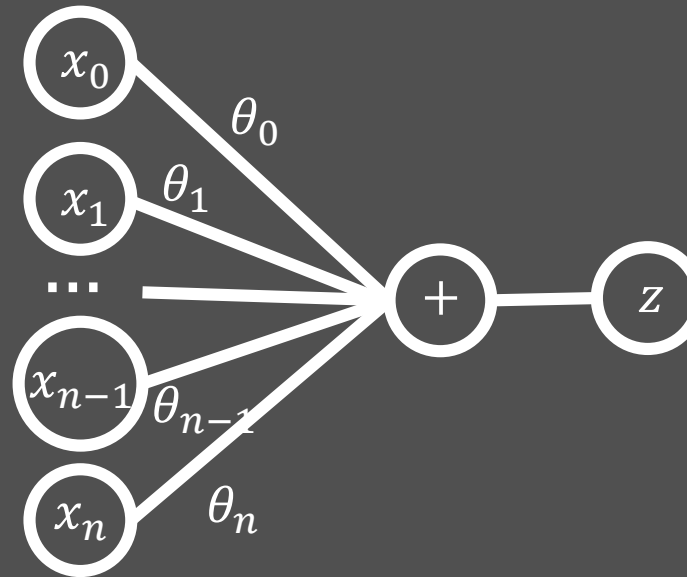
## Two Processes:

- Forward Propagation (Prediction)
- **Backward Propagation (Weight Tuning)**

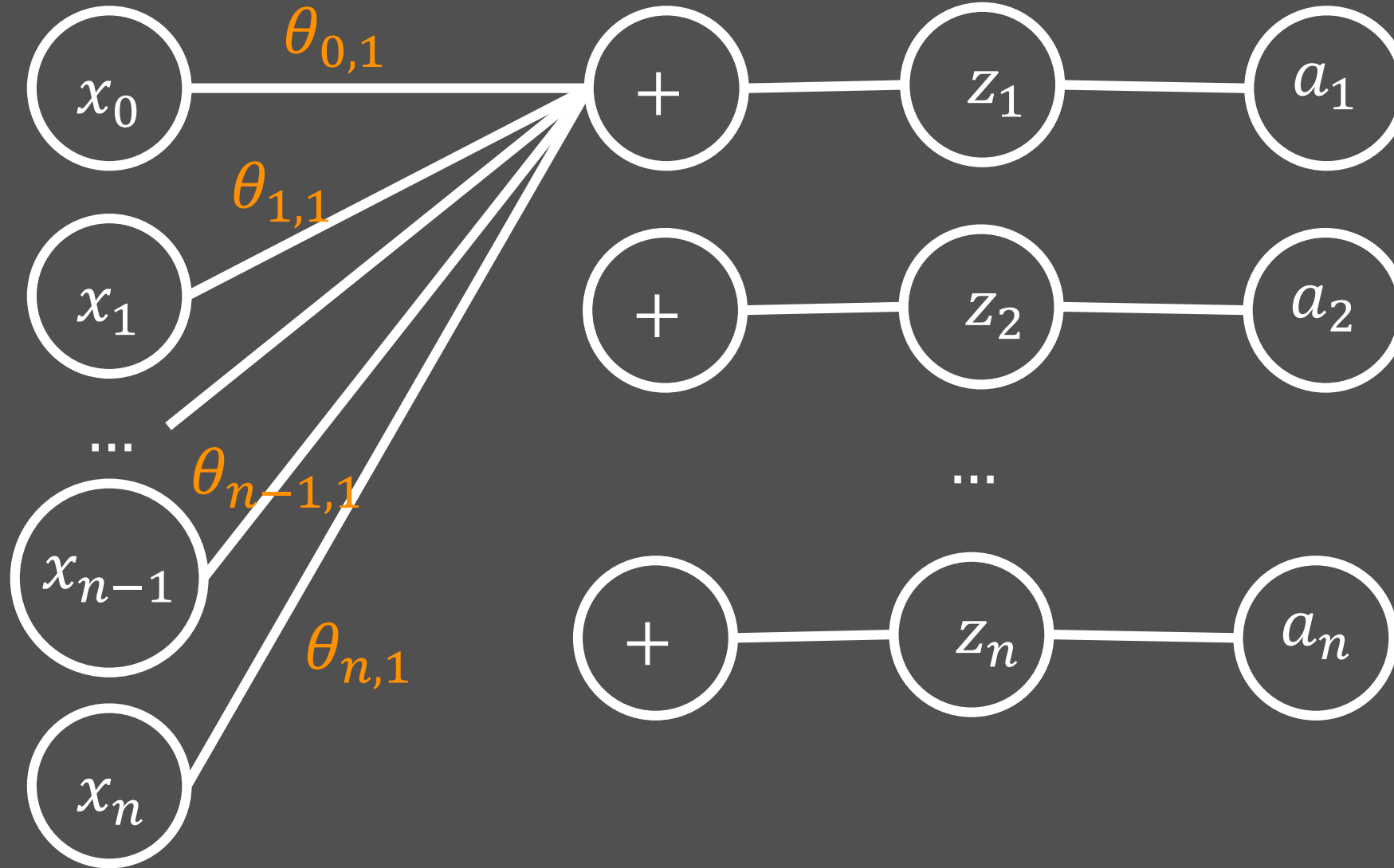


# Forward Propagation

- Compute multiple weighted sums  $z_i$
- Activate them to get  $a_i(z_i)$
- Remember from Perceptron:  $z = \theta x$

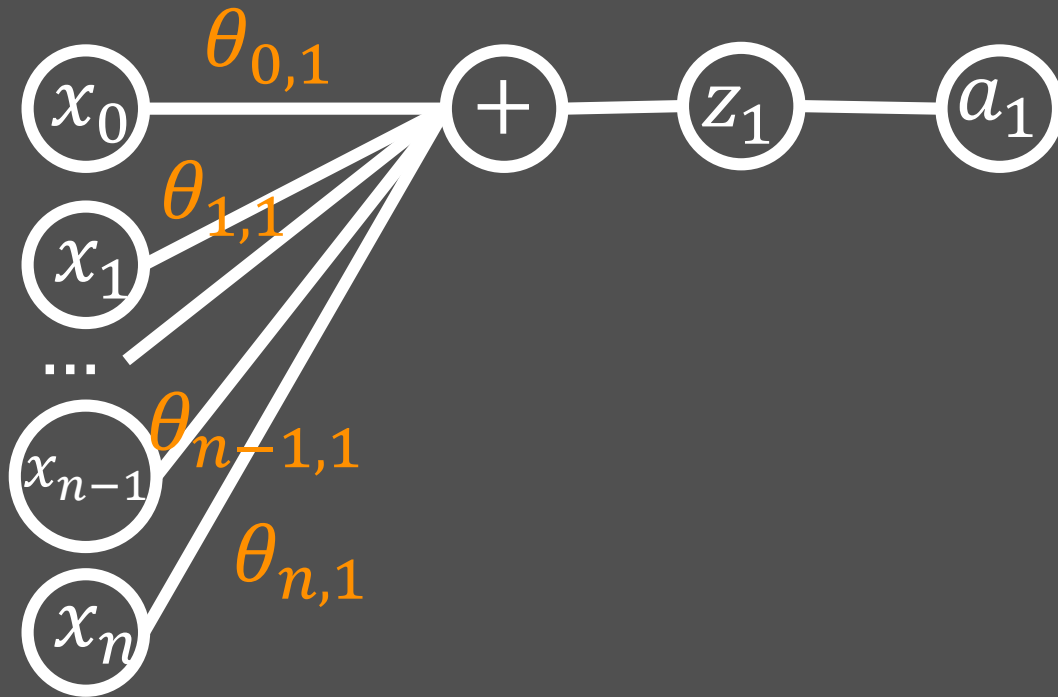


# Forward Propagation



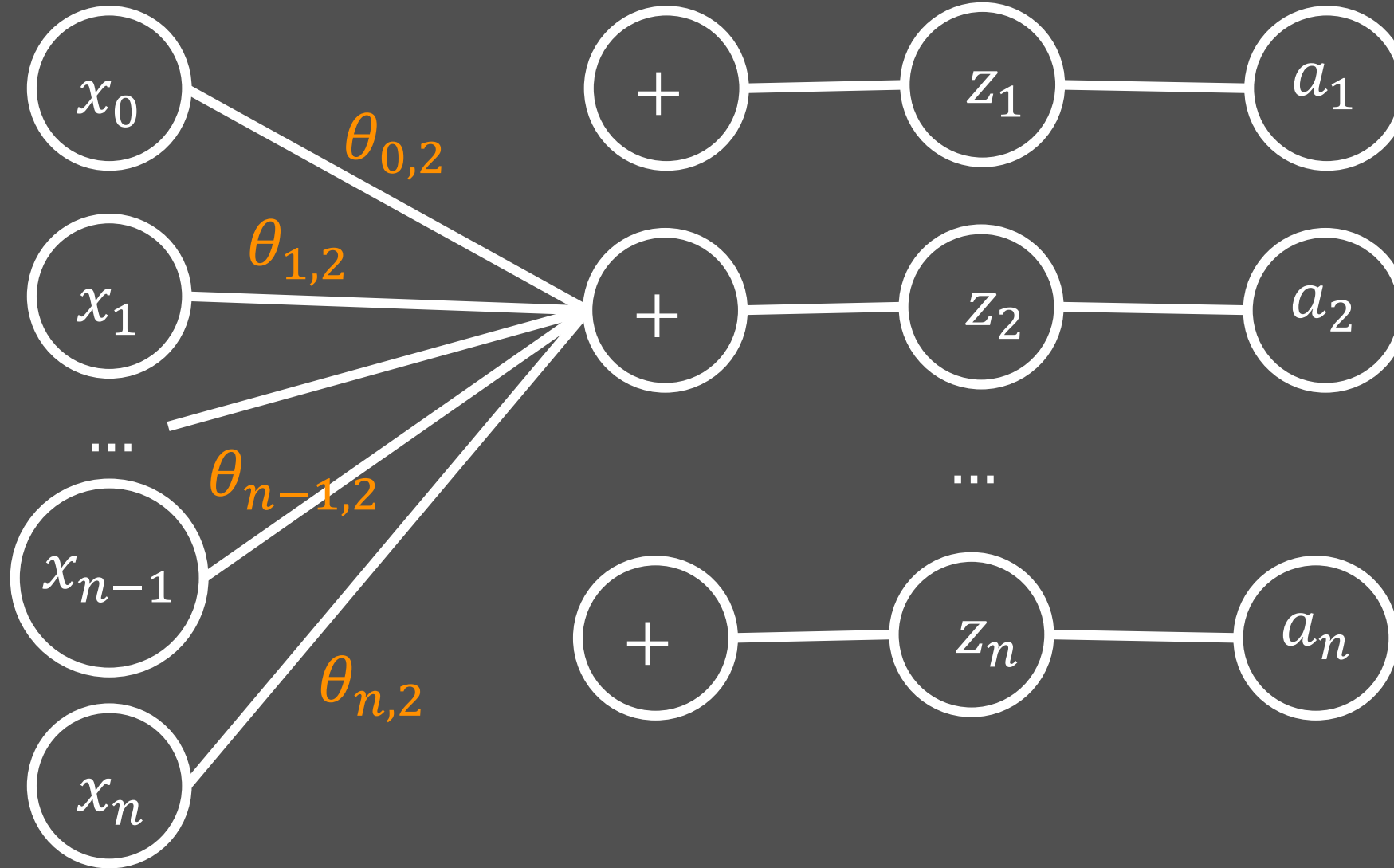
$$z_{to} = \theta_{from,to} x_{from}$$

**e.g.**  $z_1 = \theta_{0,1} x_0$

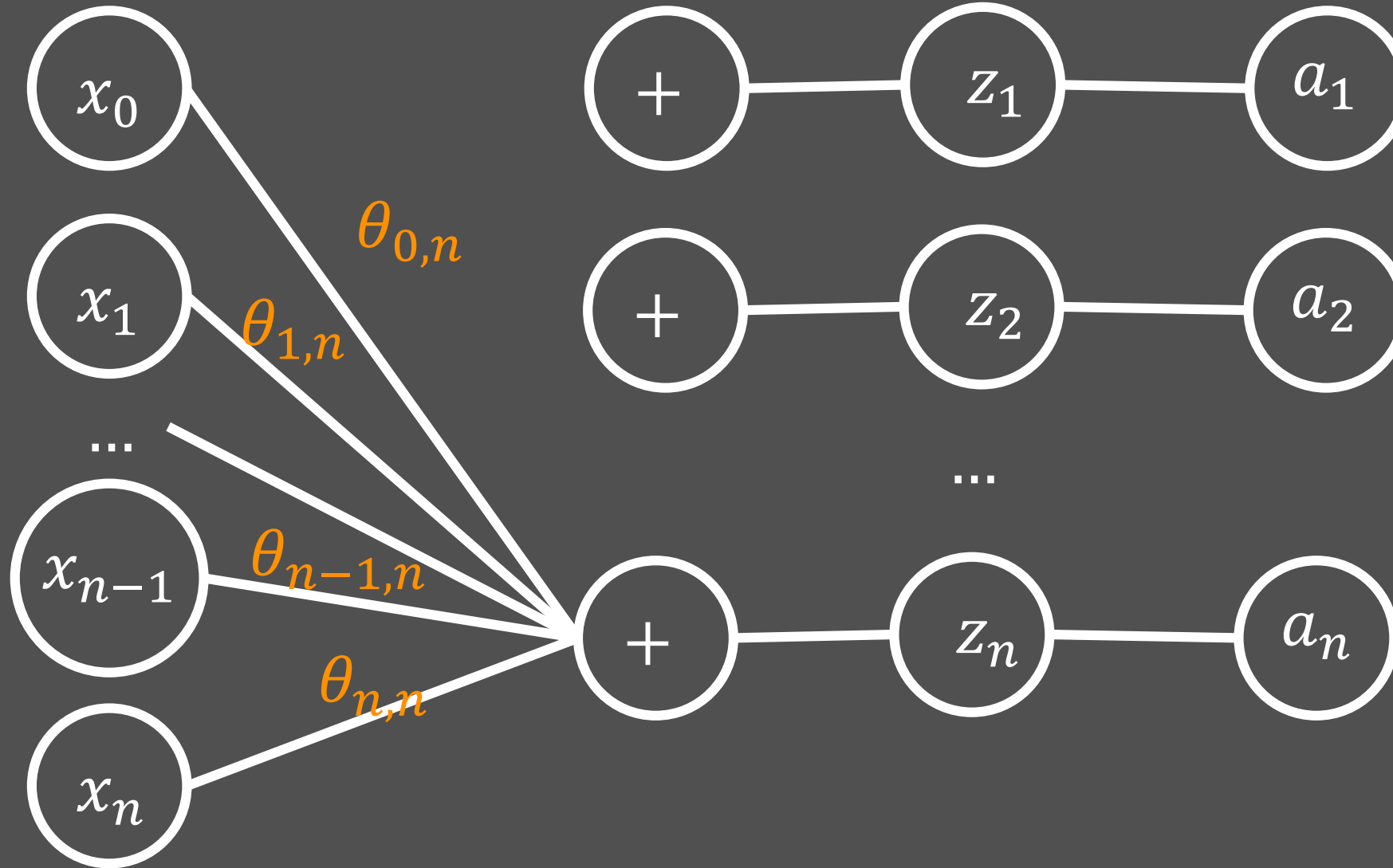




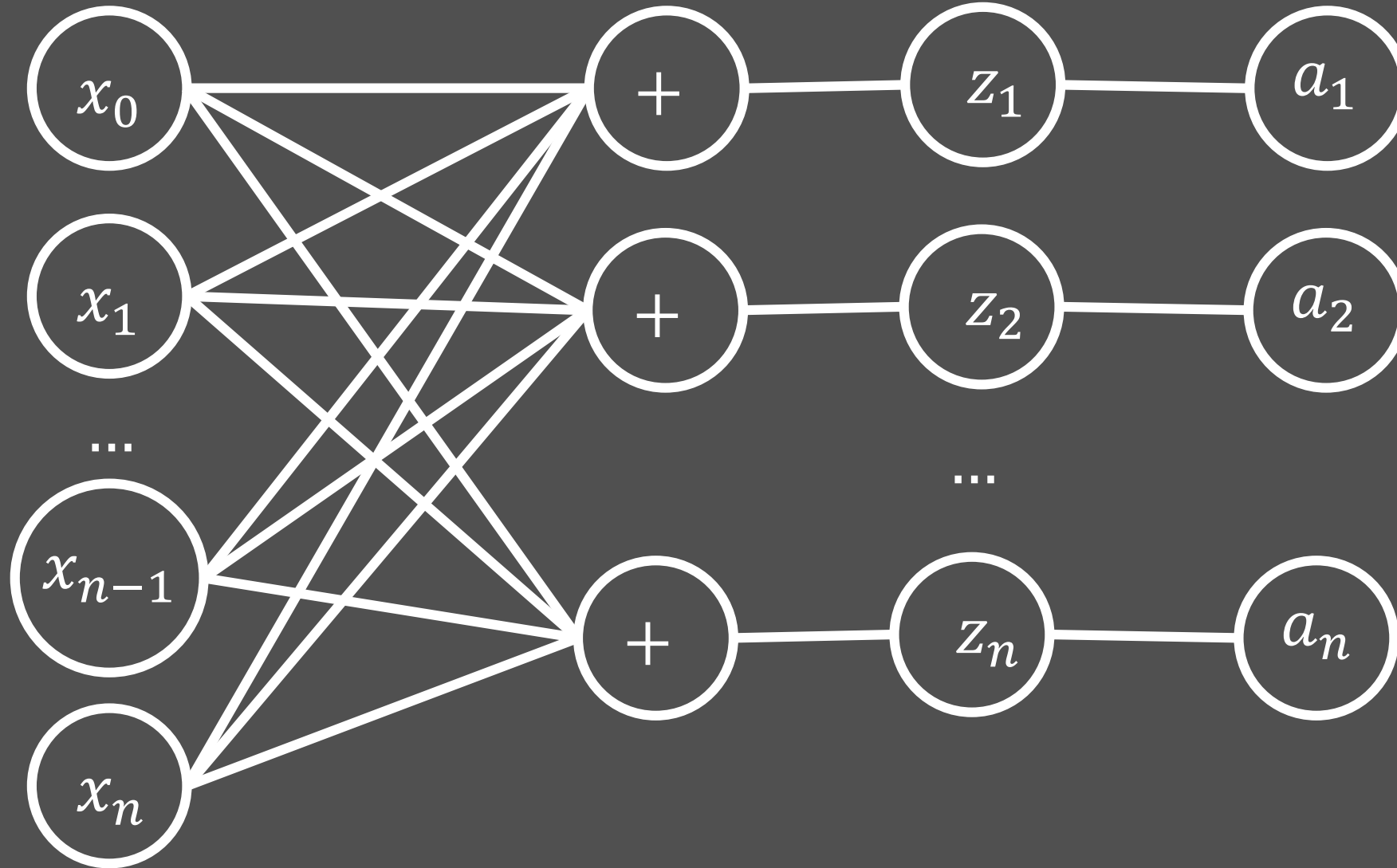
# Forward Propagation



# Forward Propagation

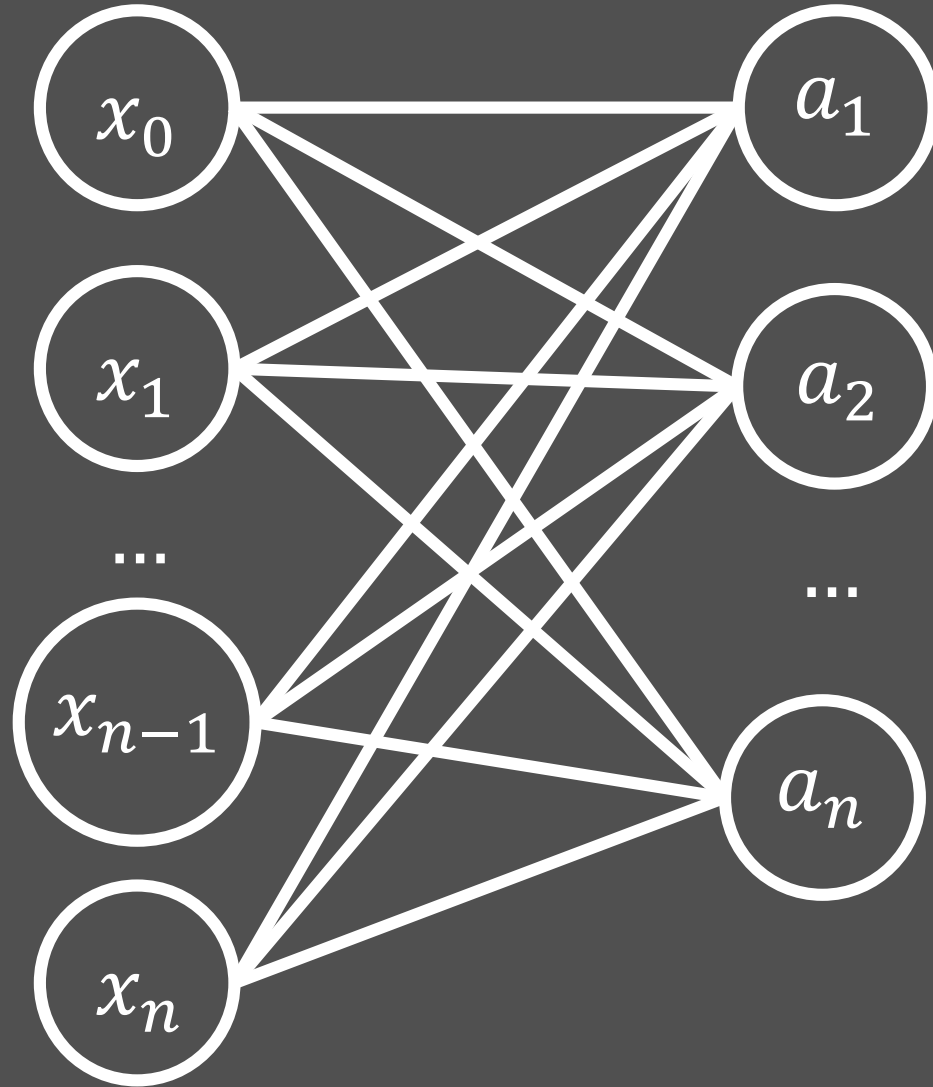


# Forward Propagation

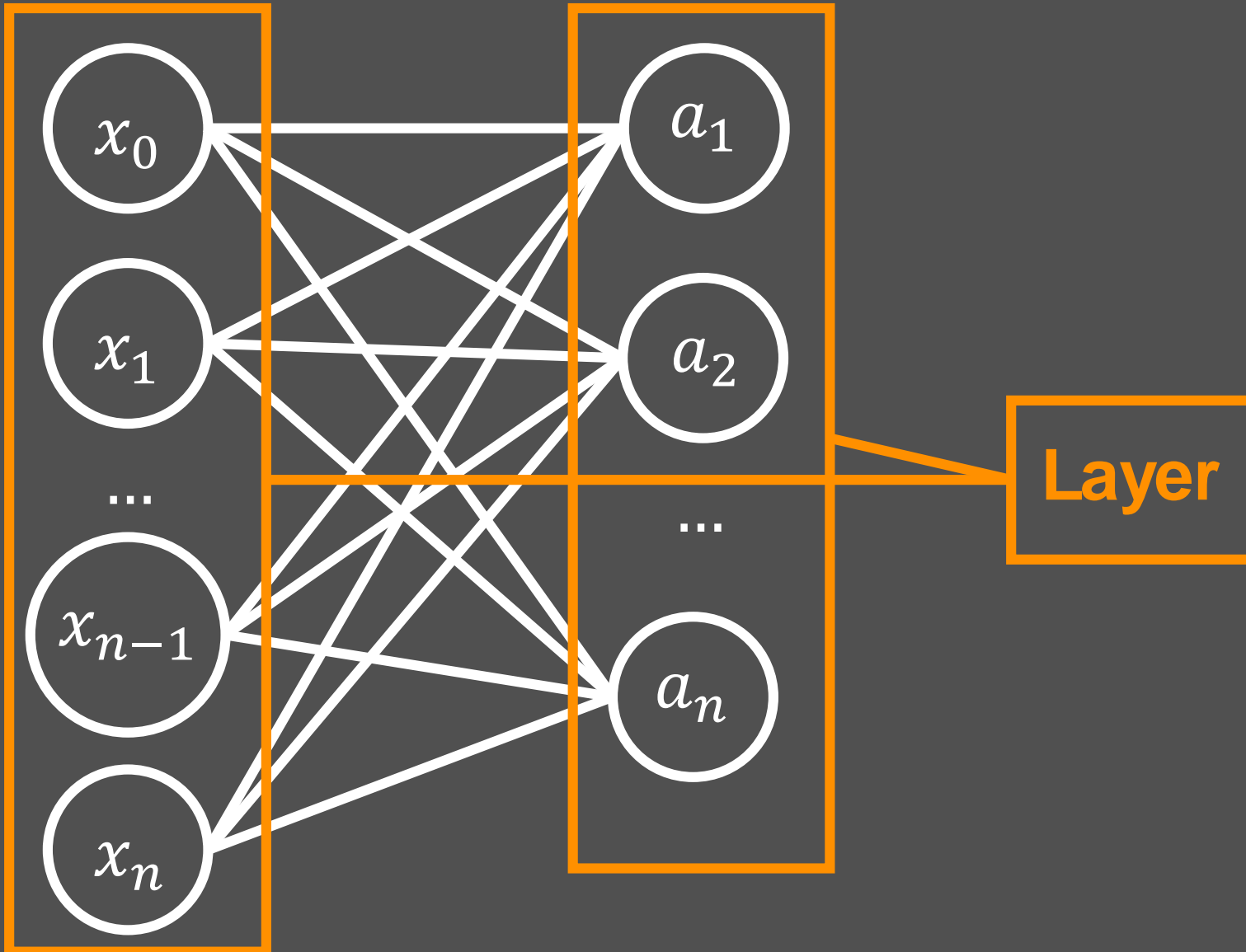


# In Short (without $z$ )

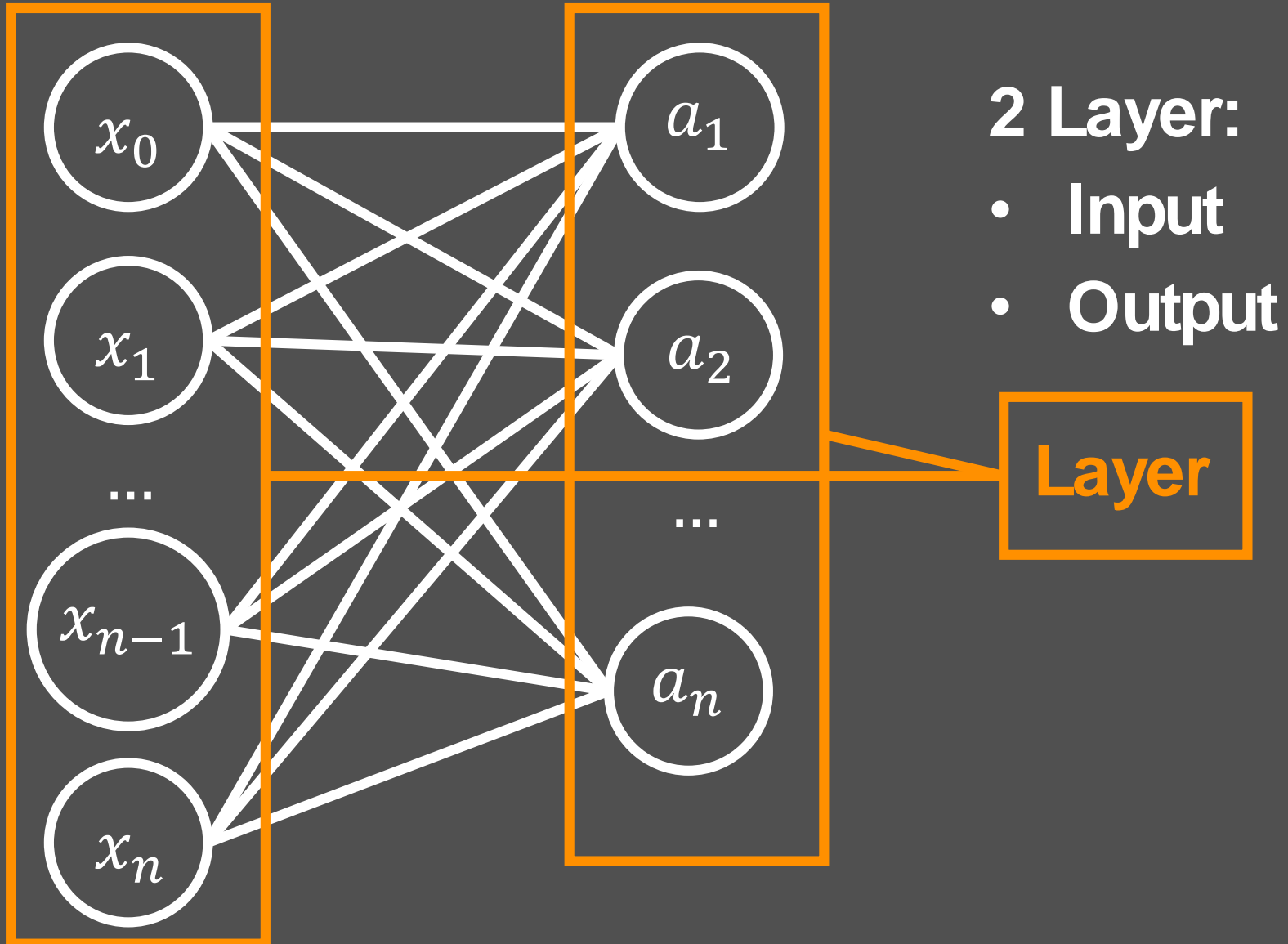
# Forward Propagation



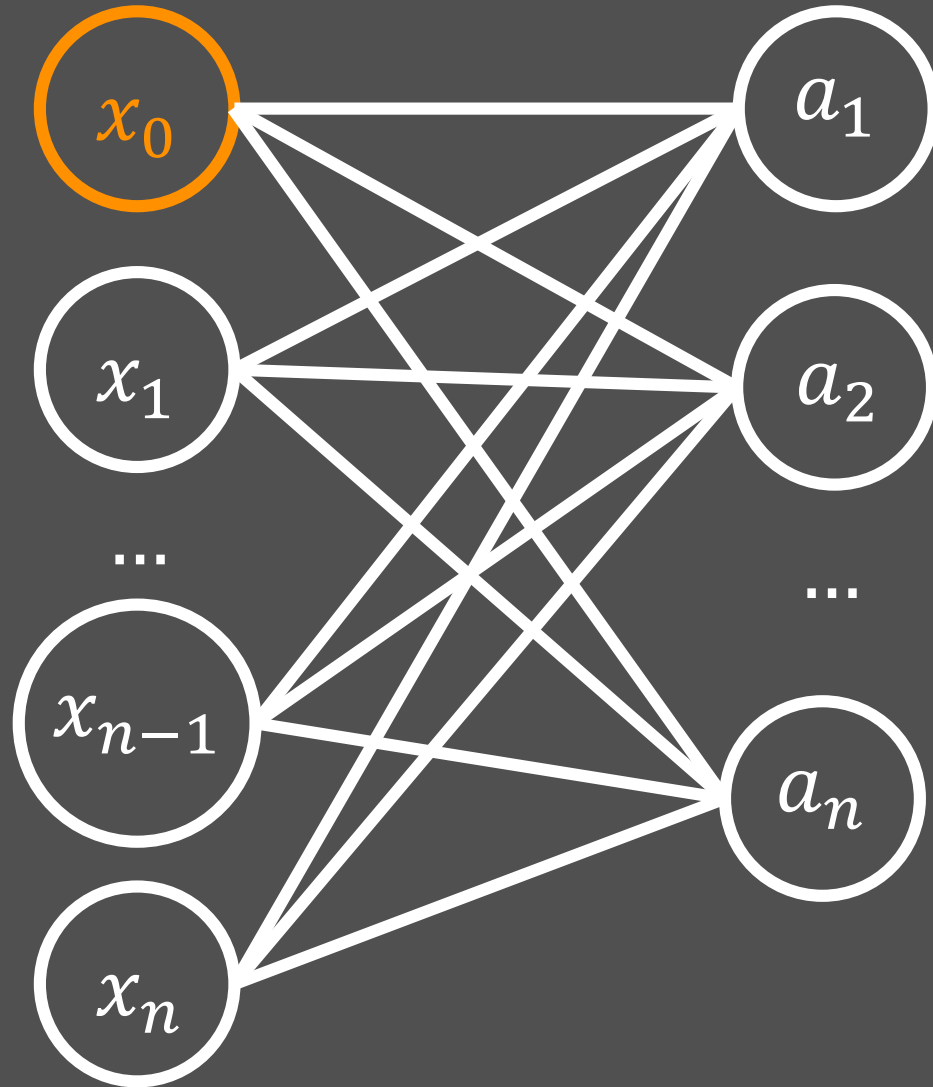
# Forward Propagation



# Forward Propagation



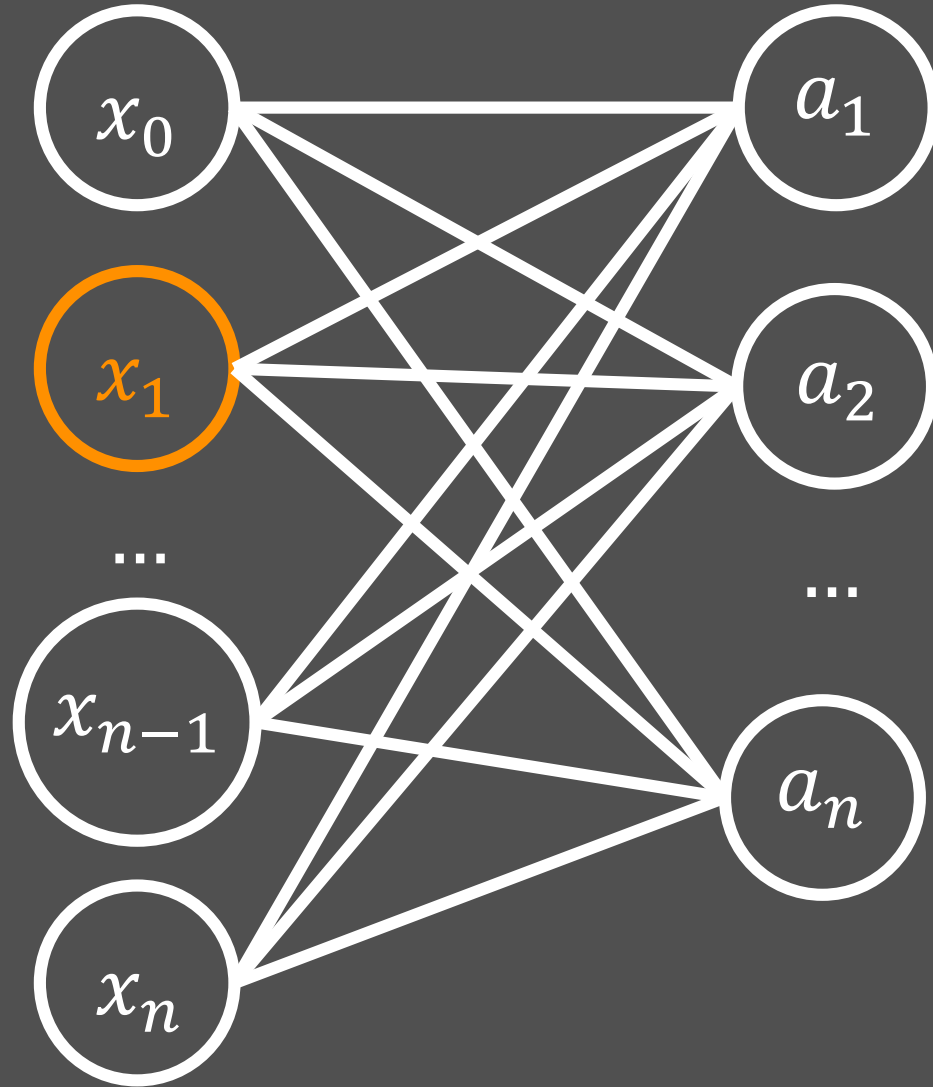
# Forward Propagation



**Input Neuron**

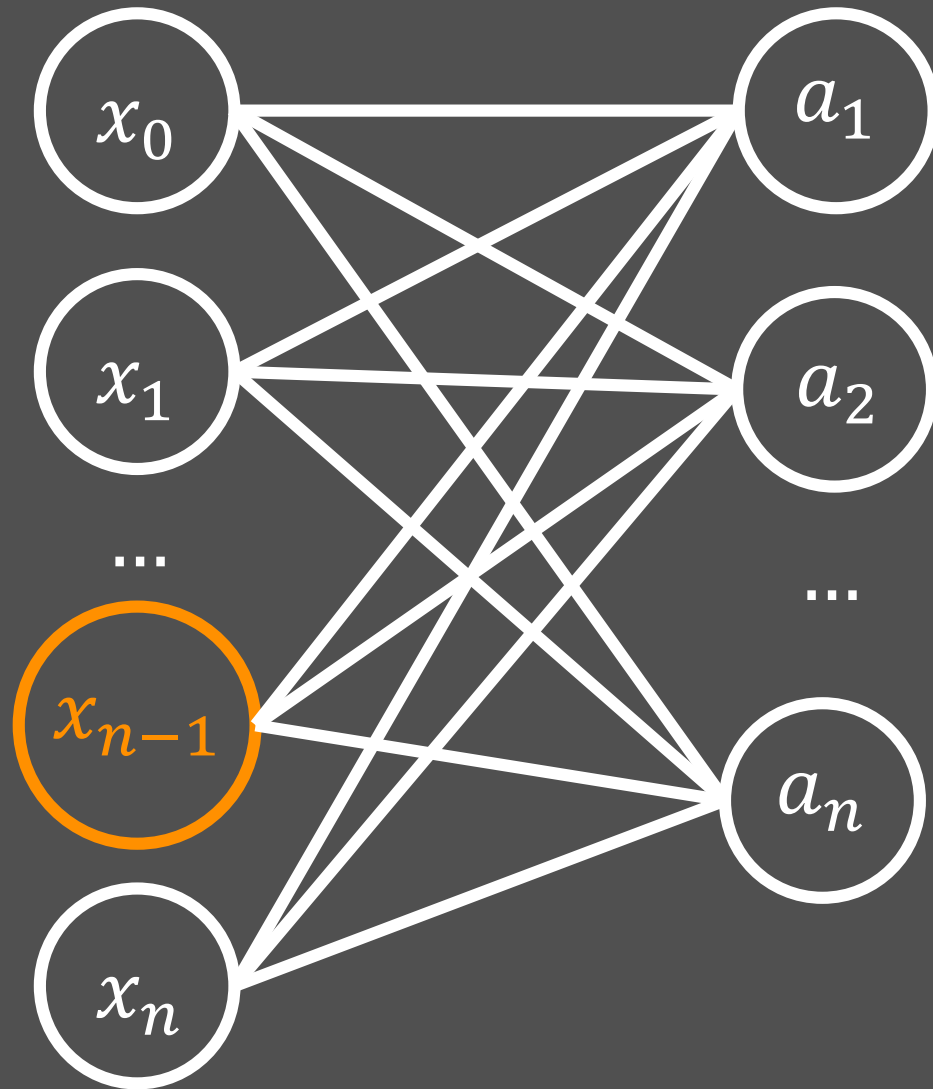


# Forward Propagation



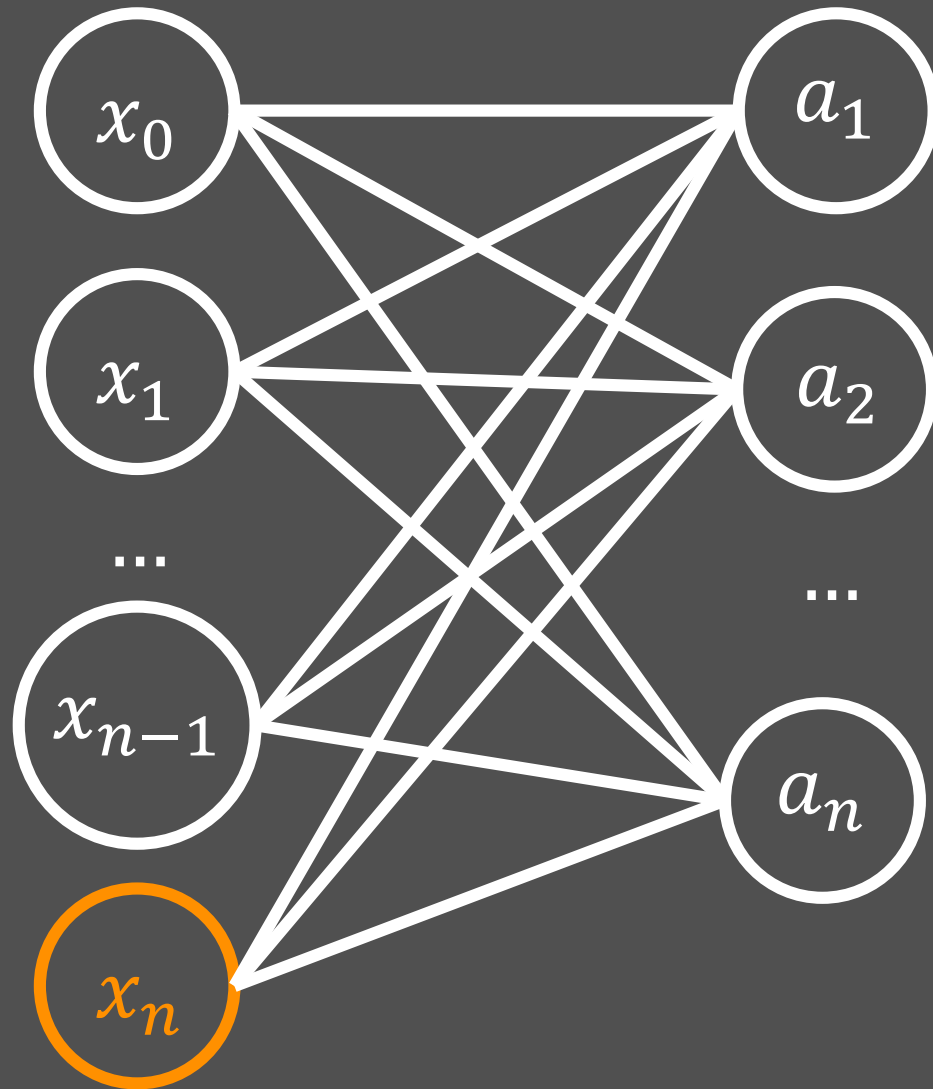
Input Neuron

# Forward Propagation



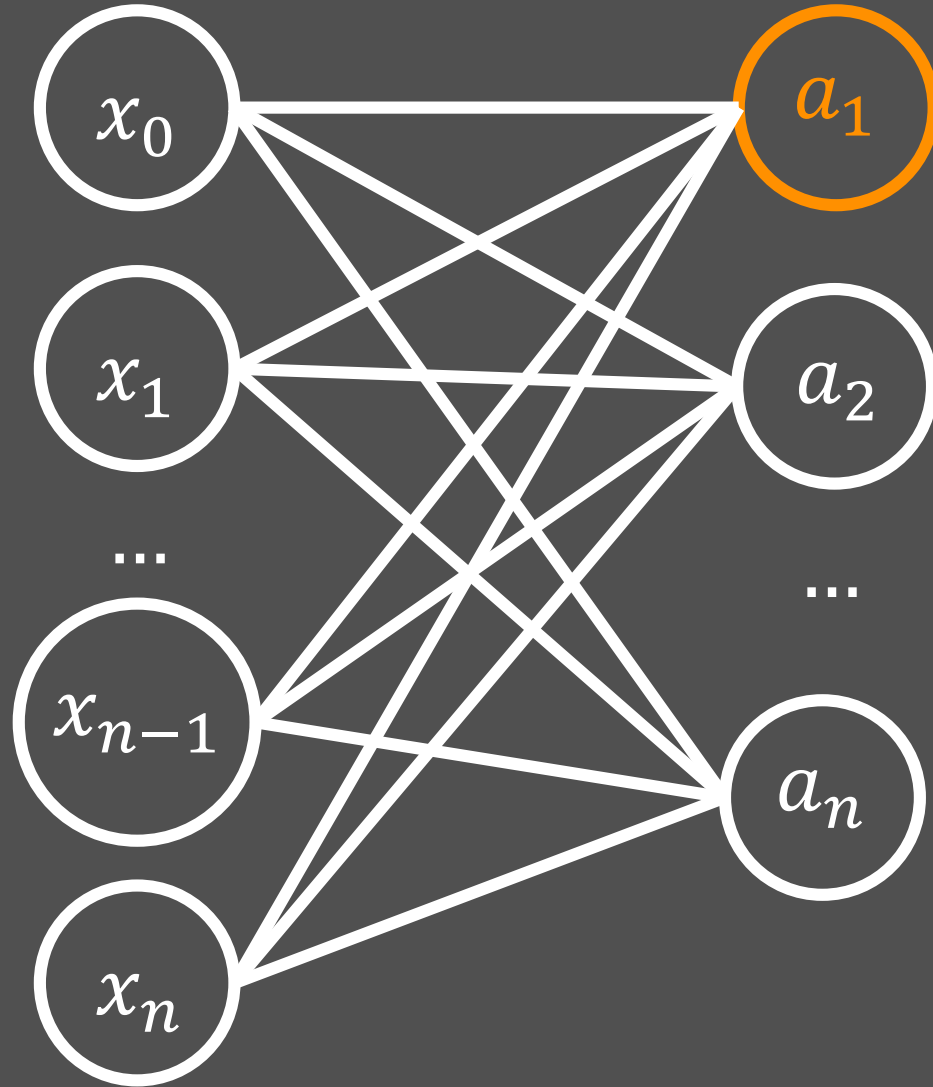
Input Neuron

# Forward Propagation



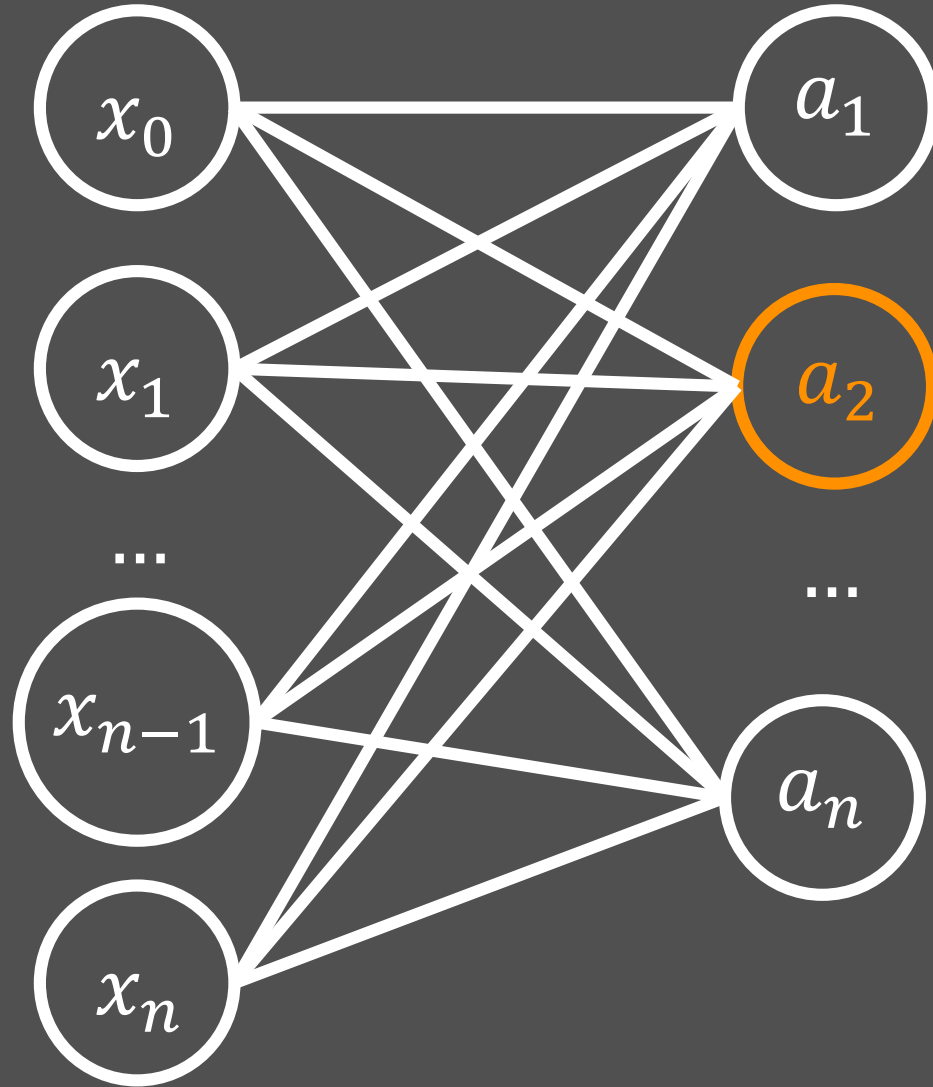
Input Neuron

# Forward Propagation



Output Neuron

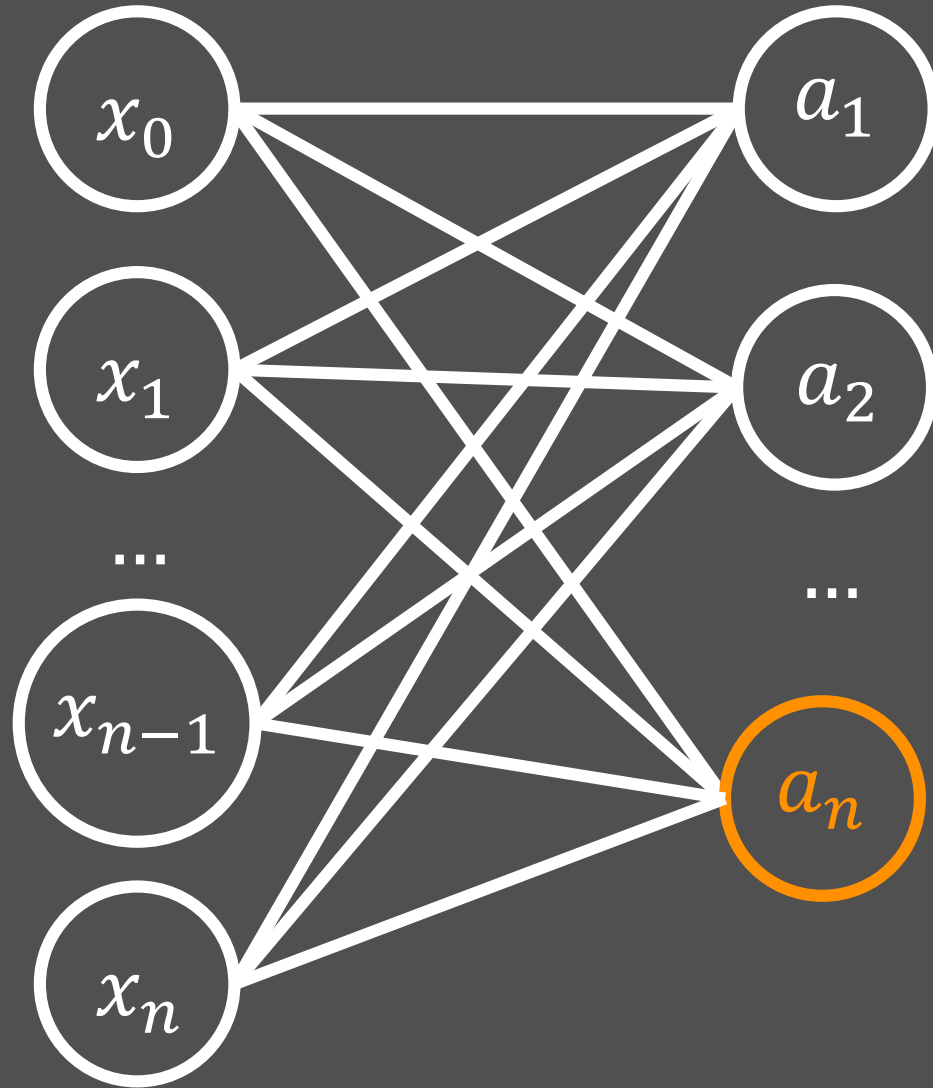
# Forward Propagation



**Output Neuron**

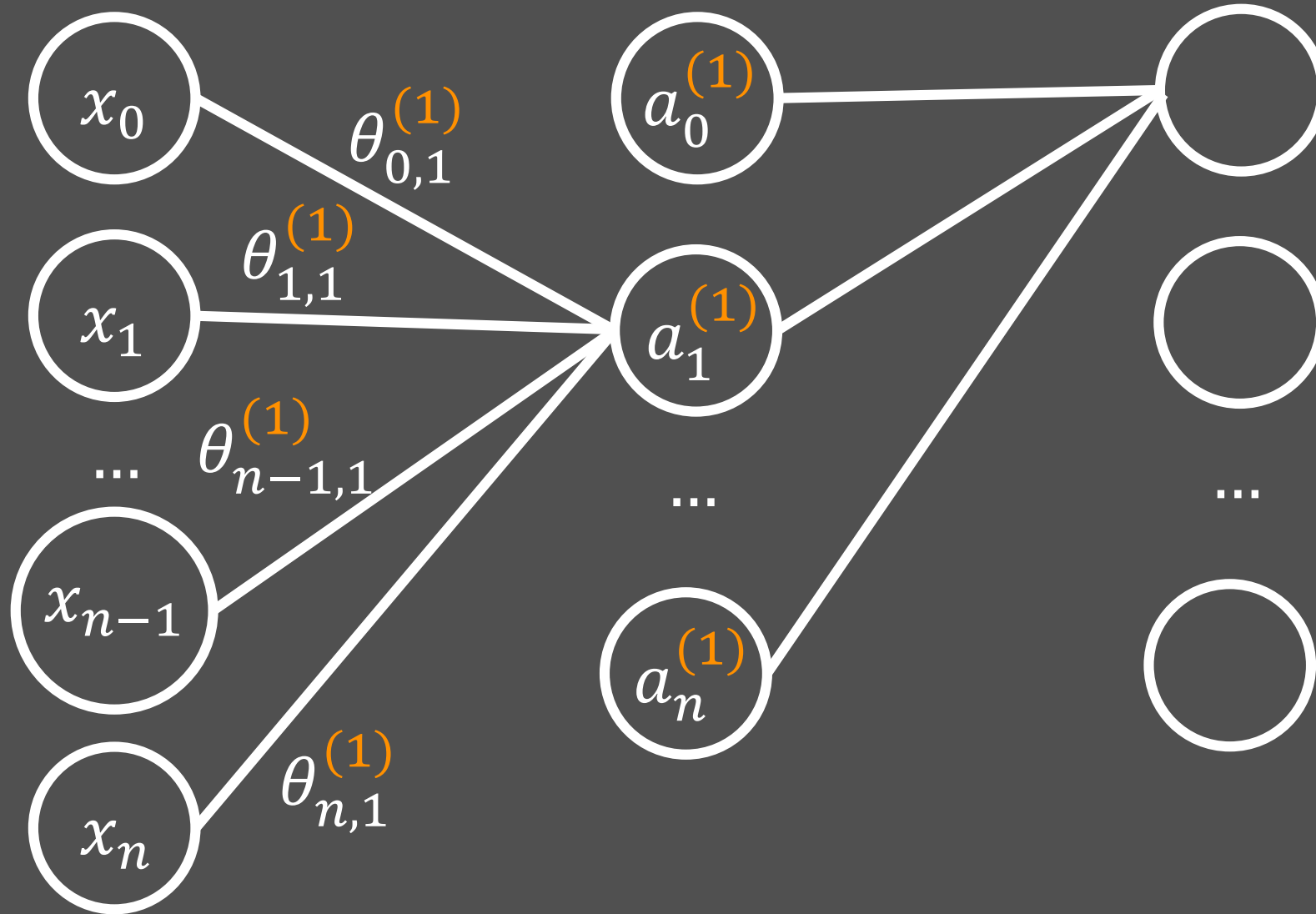
# Forward Propagation

## Output Neuron



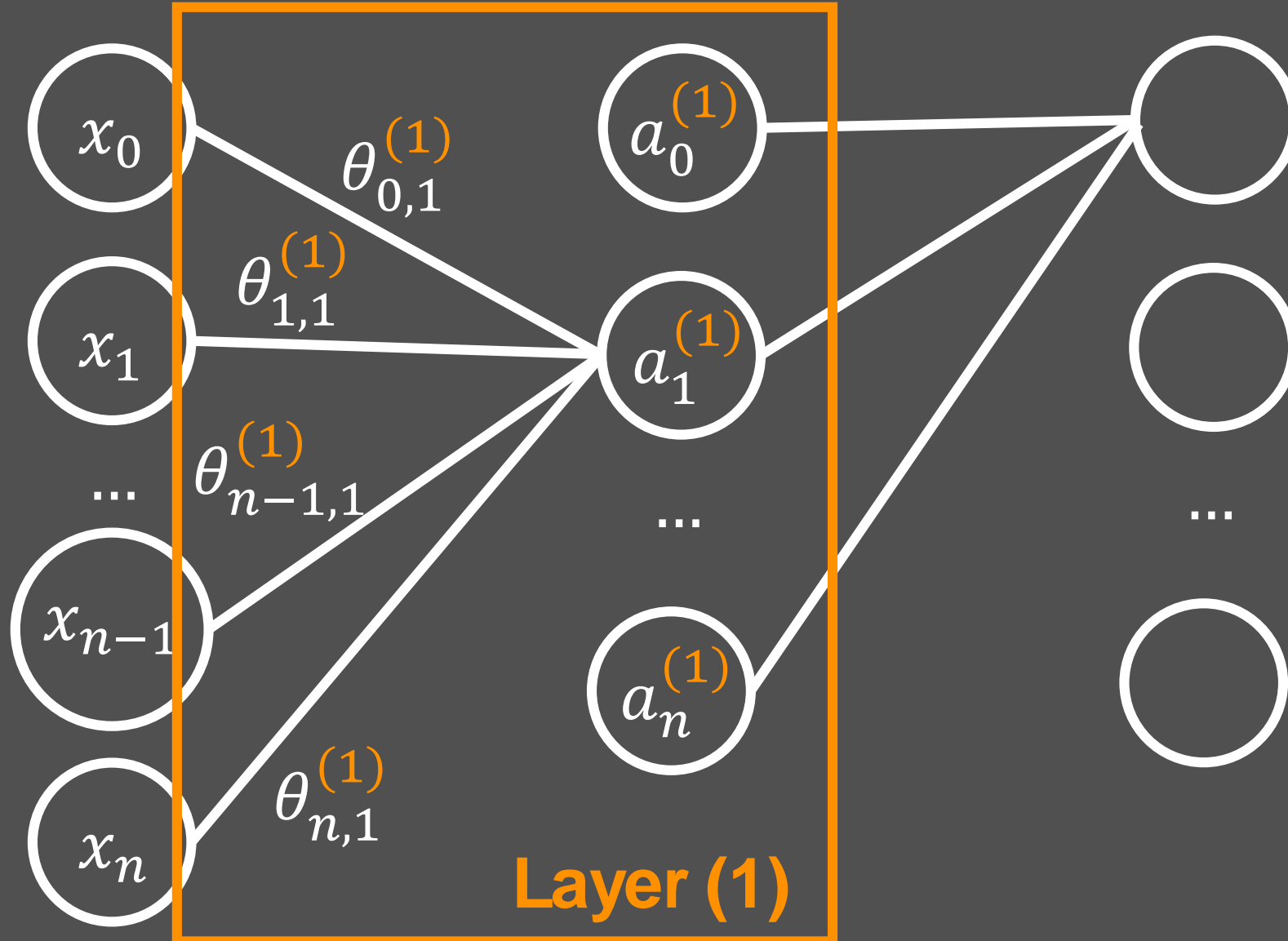
# Add More Layer!

# Forward Propagation

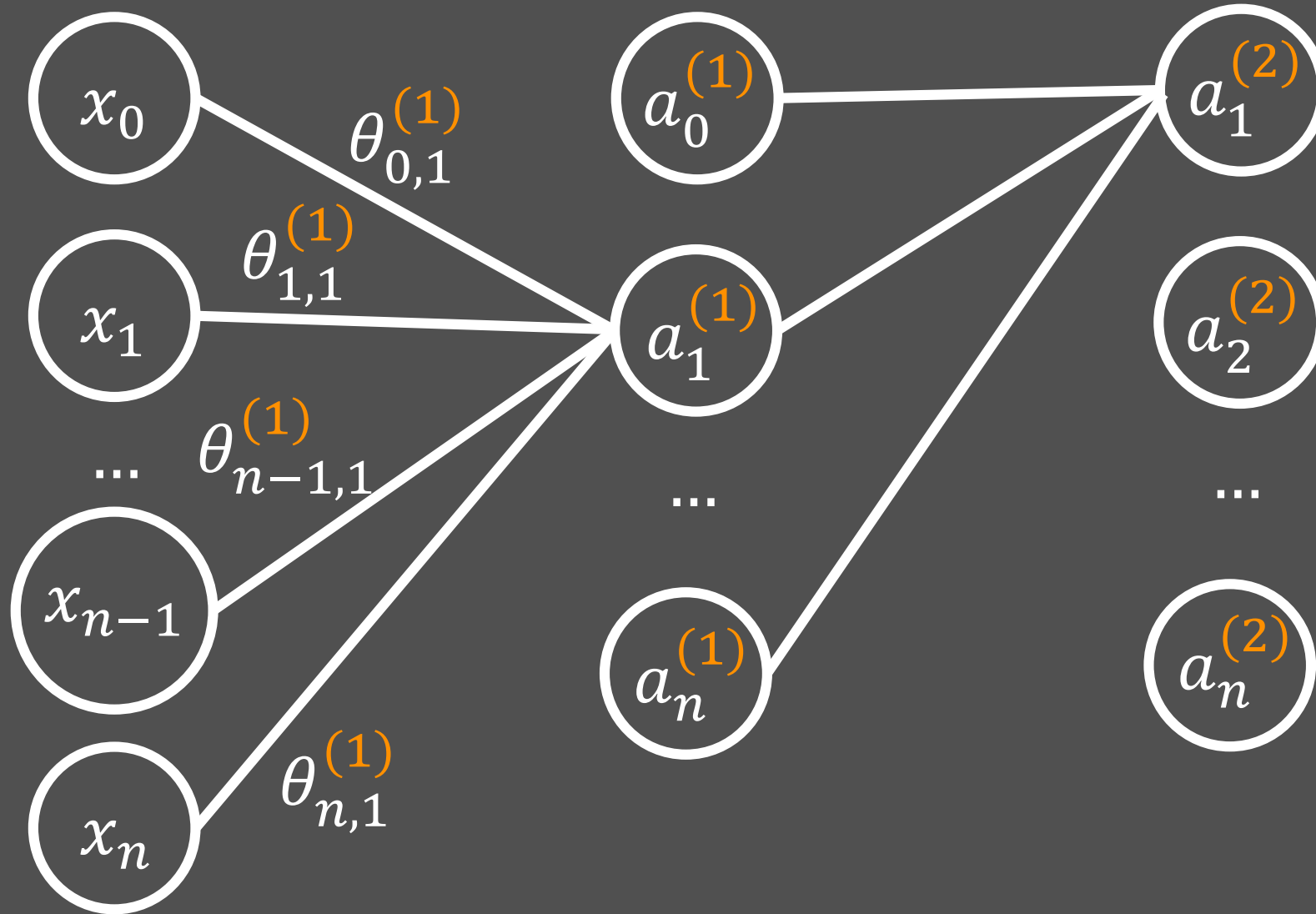




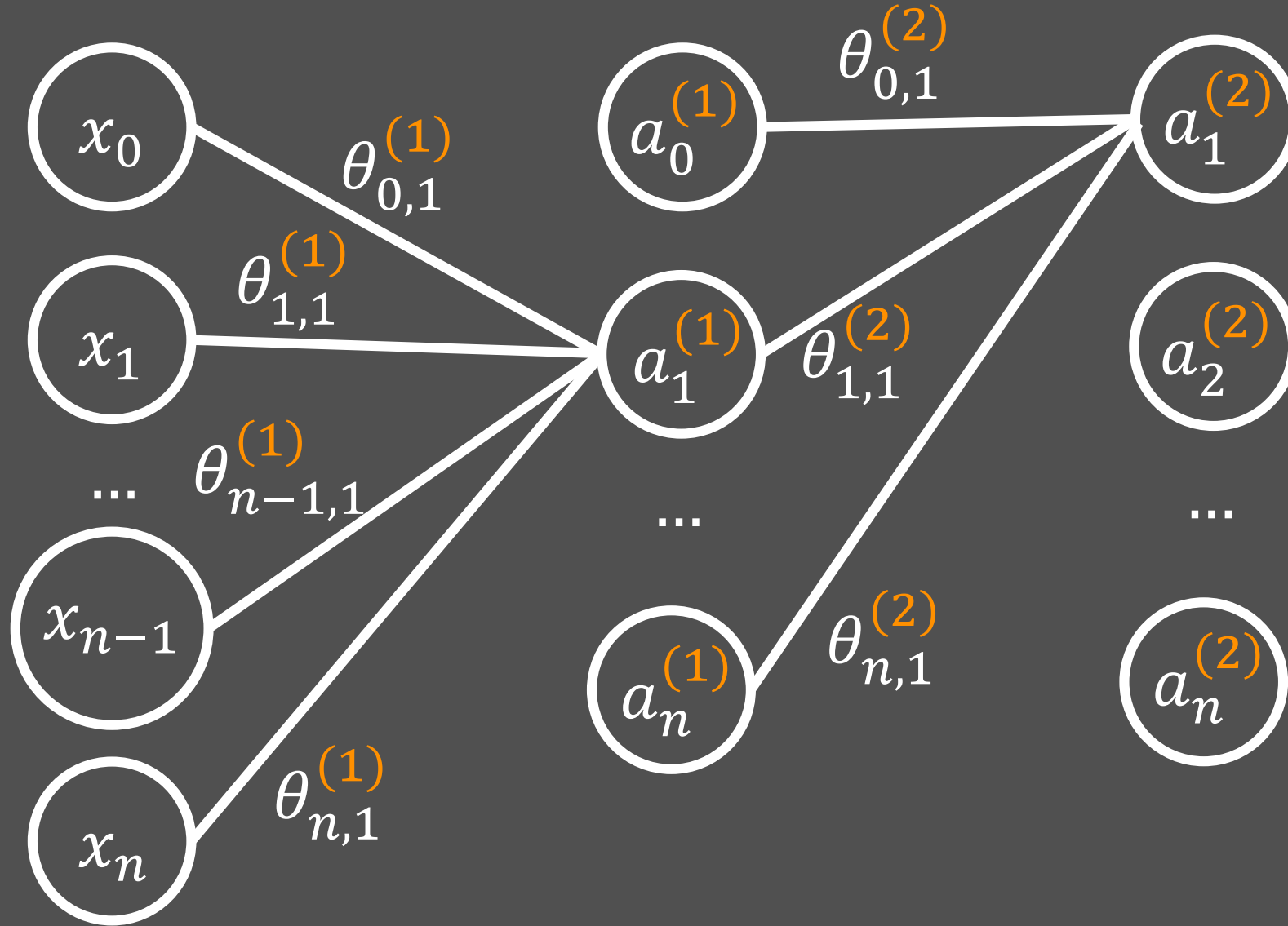
# Forward Propagation



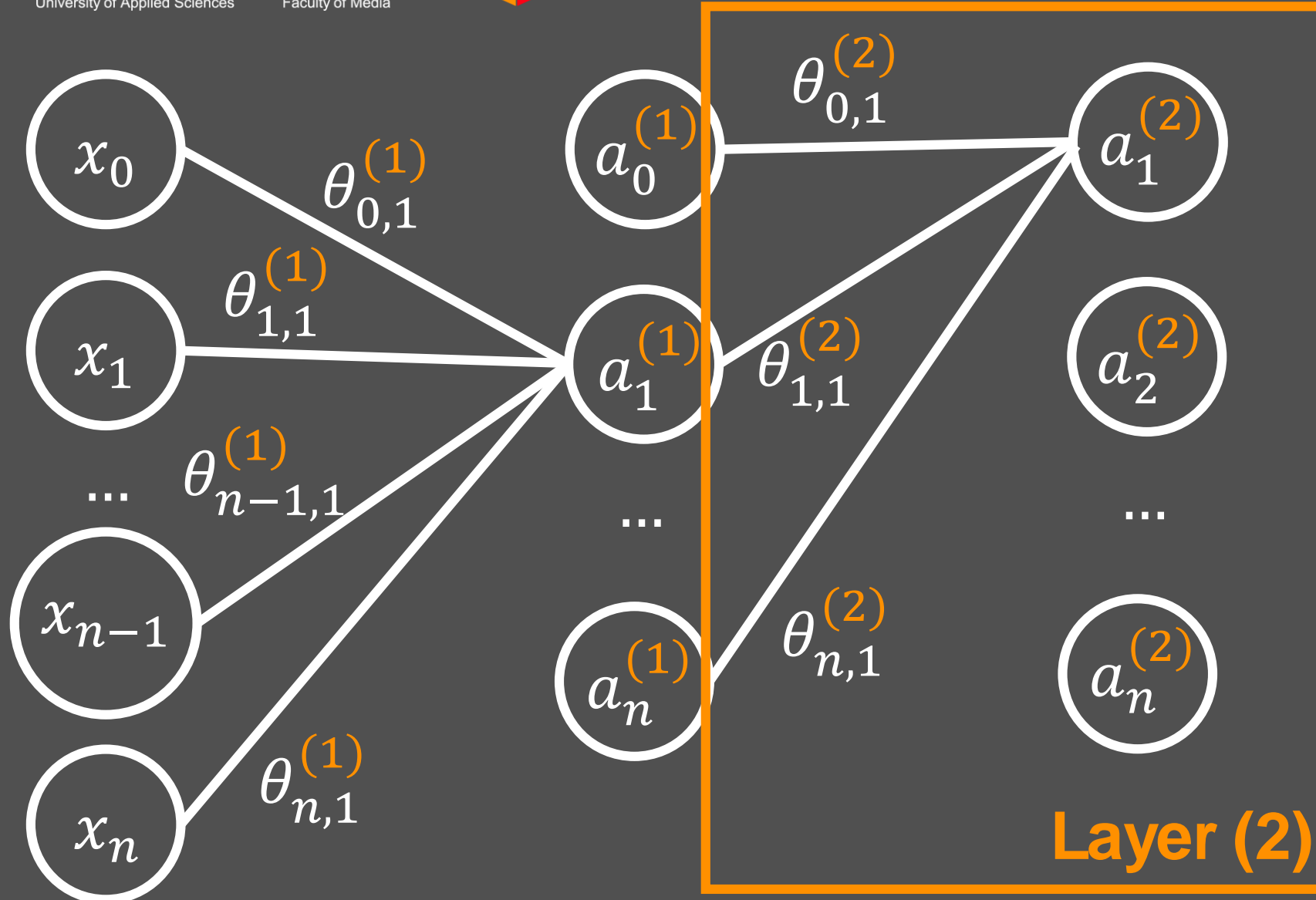
# Forward Propagation



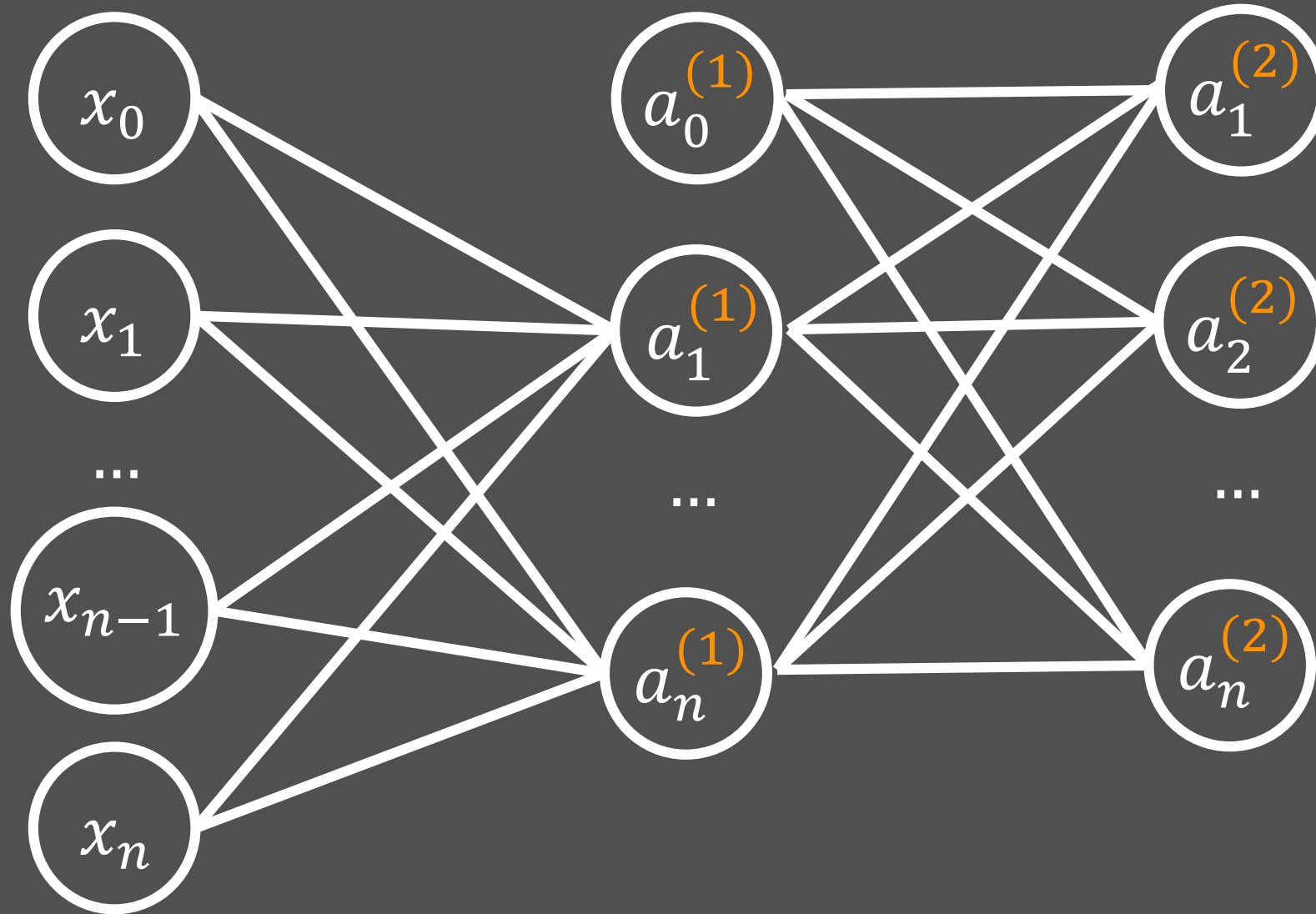
# Forward Propagation



# Forward Propagation

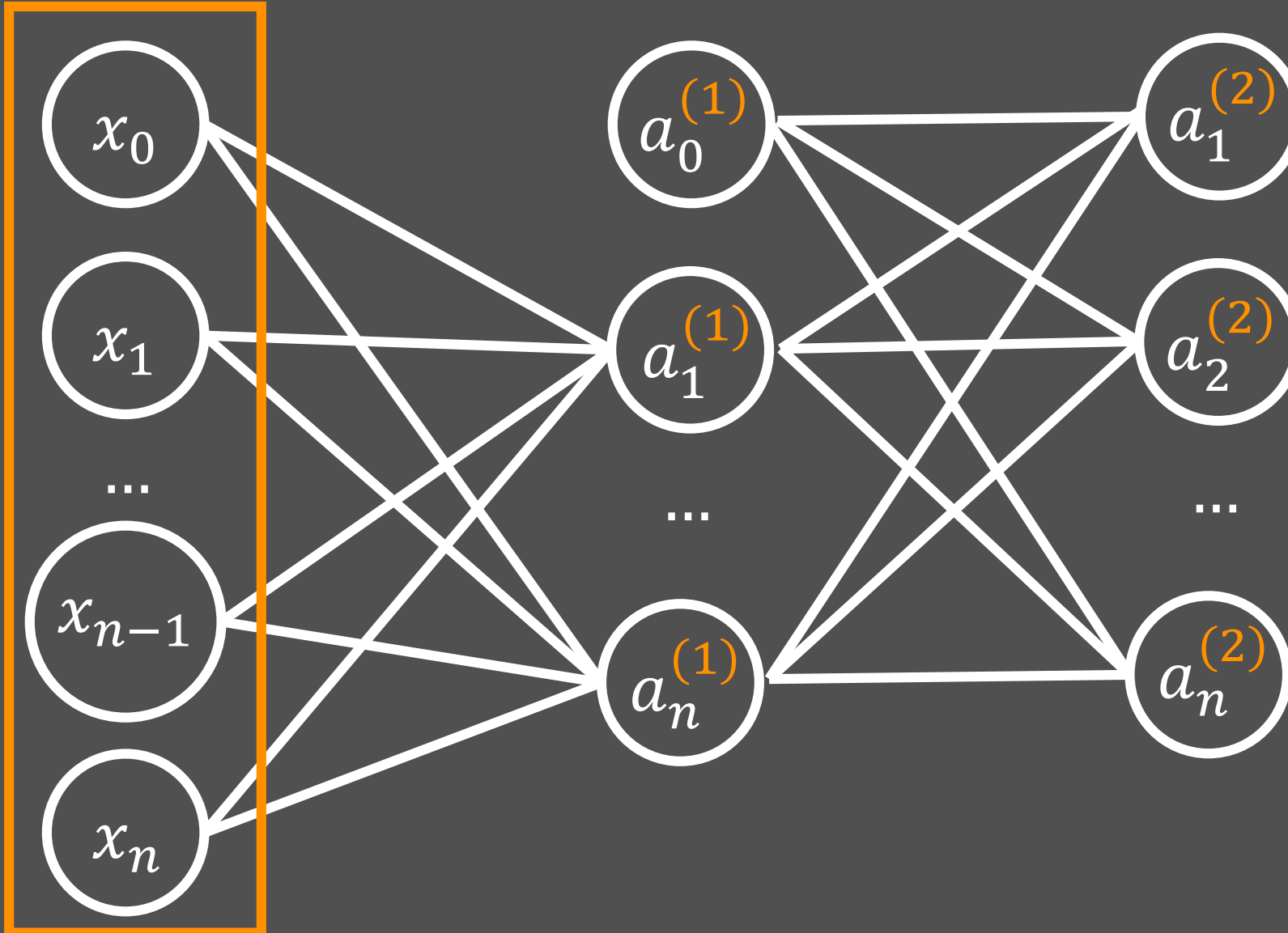


# Forward Propagation



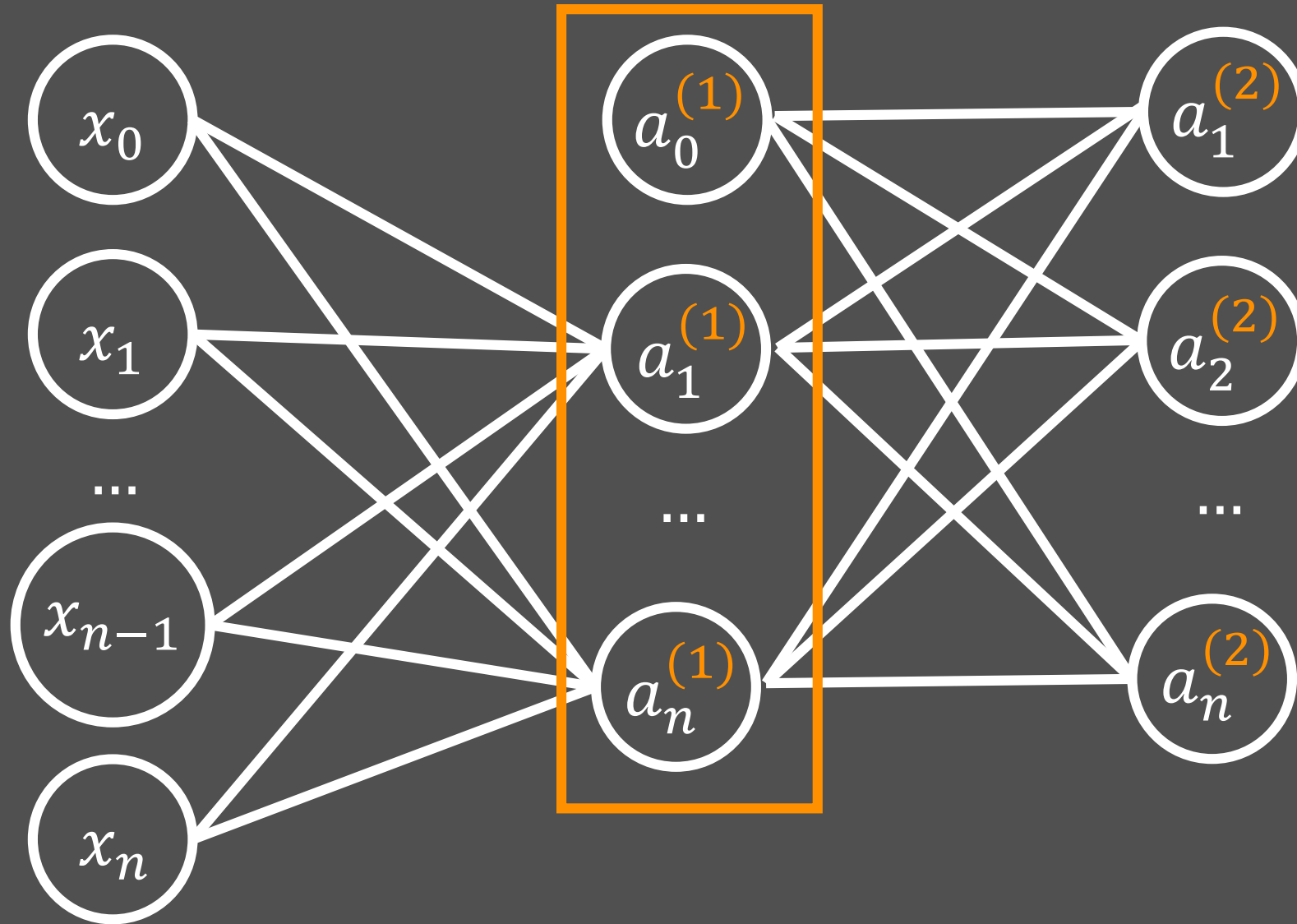
# Forward Propagation

## Input Layer



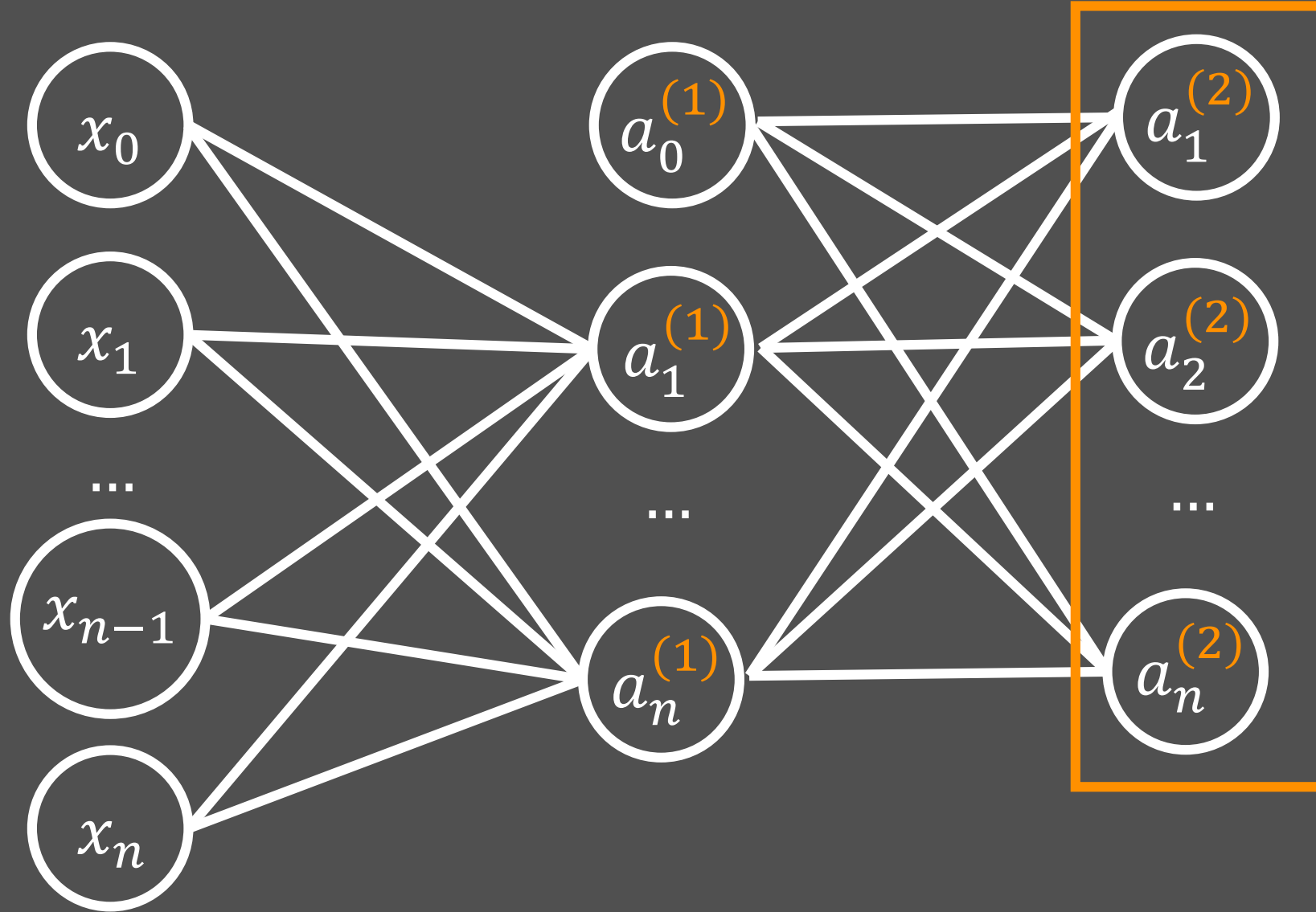
# Forward Propagation

## Hidden Layer



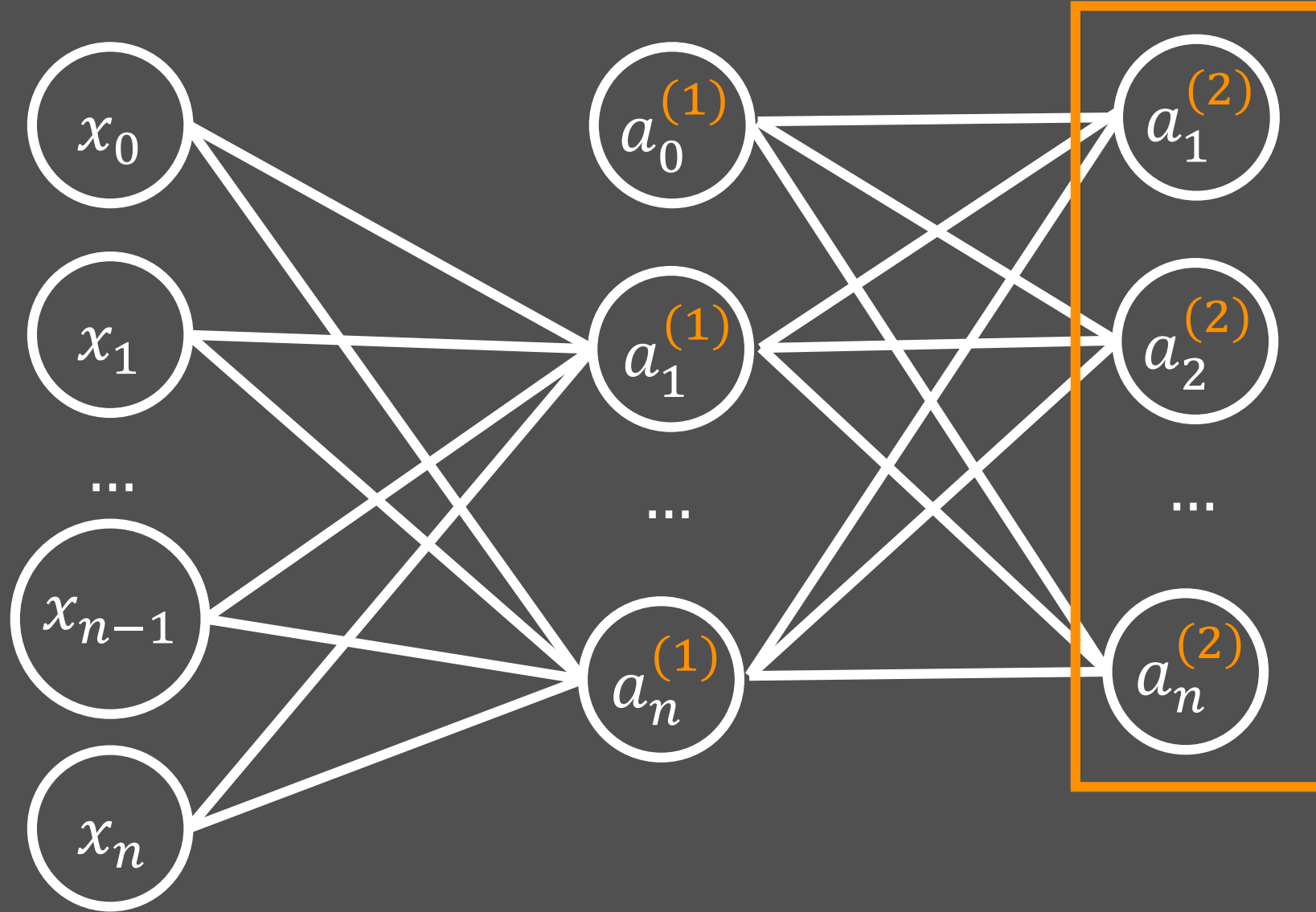
# Forward Propagation

## Output Layer





# Forward Propagation



## Output Layer:

- **Multiclass Classification**
- **Each neuron could be interpreted as class probability**

# Forward Propagation

- Object Recognition
- Assume 4 classes
- L-Layer (L is the last layer)

$$a_1^{(L)}$$

Car

$$a_2^{(L)}$$

Airplane

$$a_3^{(L)}$$

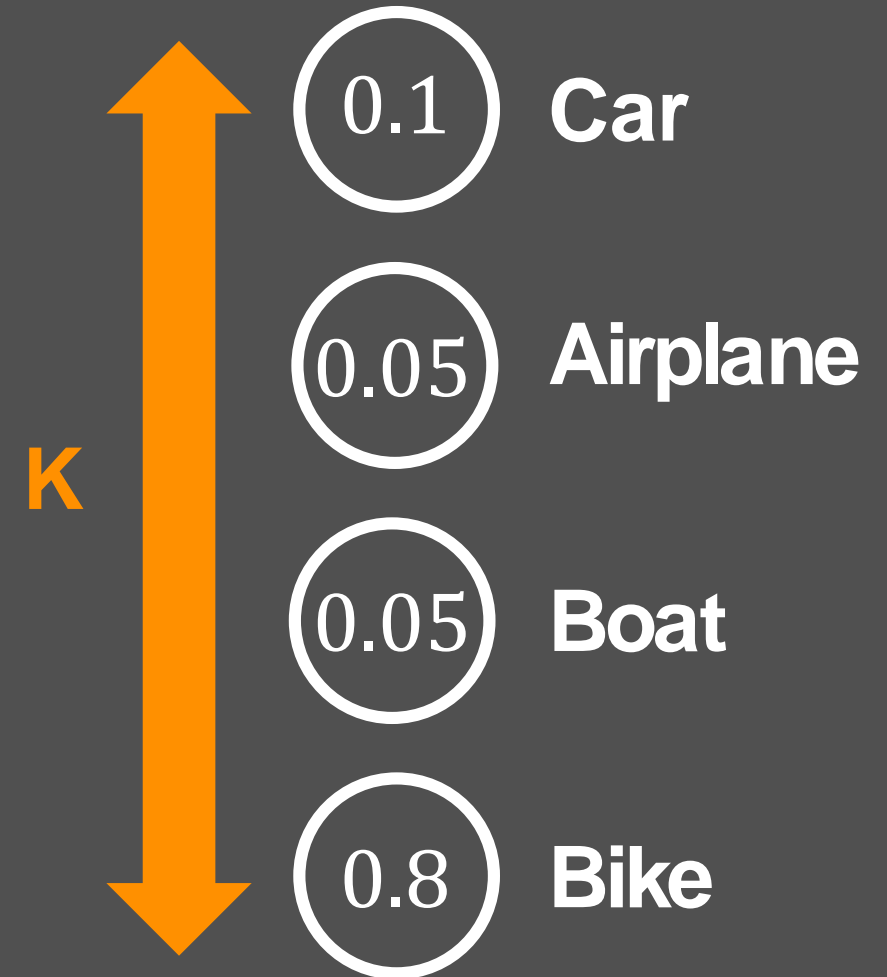
Boat

$$a_4^{(L)}$$

Bike

## Forward Propagation

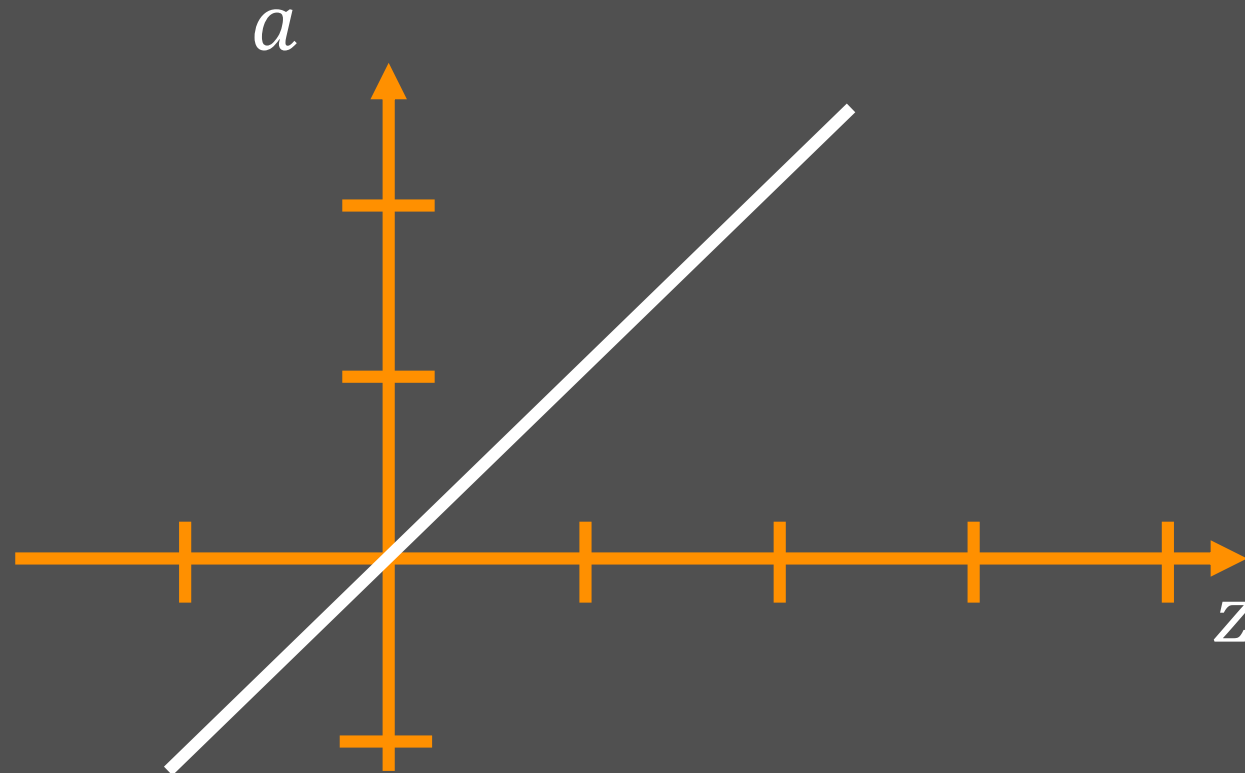
- Object Recognition
- Assume 4 classes
- L-Layer (L is the last layer)
- Softmax:  $a = \frac{e^{z_j}}{\sum_{k=1}^K e^{z_k}}$
- Use Softmax for the last Layer



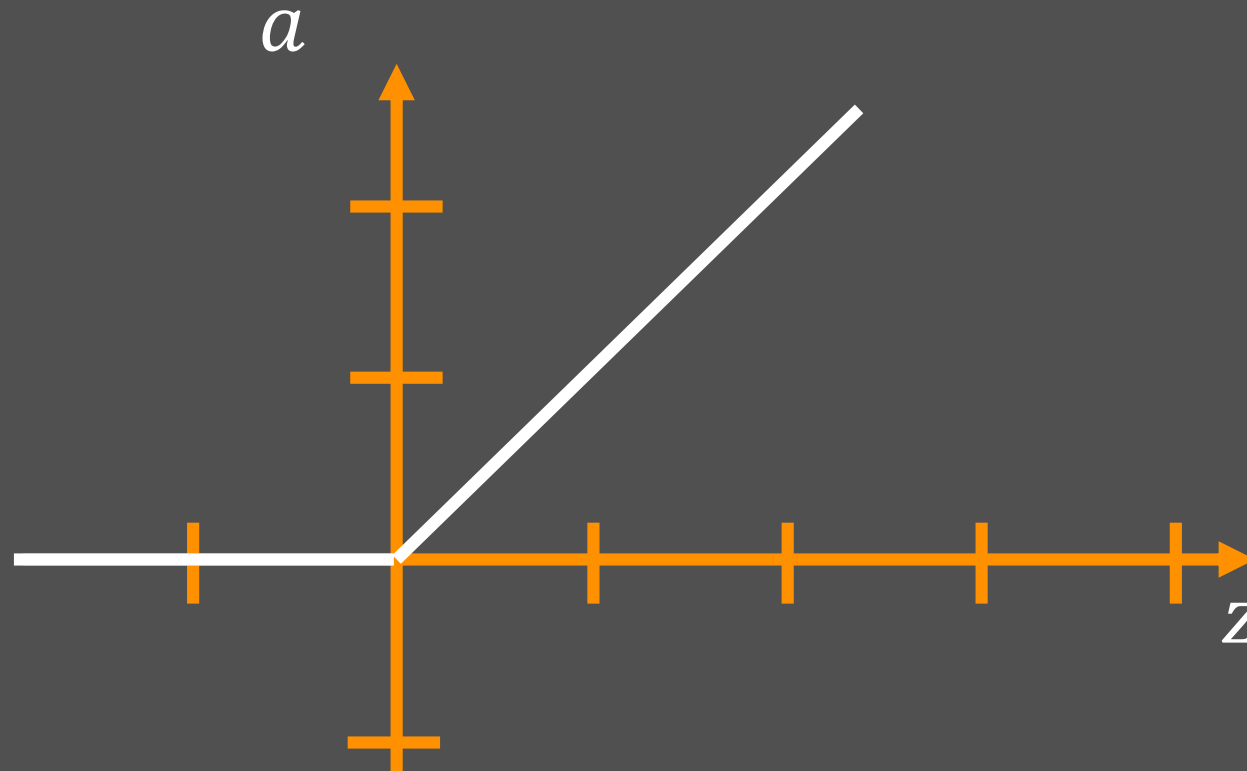
## Hidden Layer Activation Functions $a(z)$ :

- **Linear Neuron:**  $a(z) = z$
- **Binary Threshold Unit:**  $a(z) = \begin{cases} 0, & z < 0 \\ 1, & z \geq 0 \end{cases}$
- **Rectified Linear (ReLU):**  $a(z) = \begin{cases} 0, & z < 0 \\ z, & z \geq 0 \end{cases}$
- **Sigmoid:**  $a(z) = \frac{1}{1+e^{-z}}$
- **Tanh:**  $a(z) = \tanh(z)$

**Linear Neuron:**  $a(z) = z$

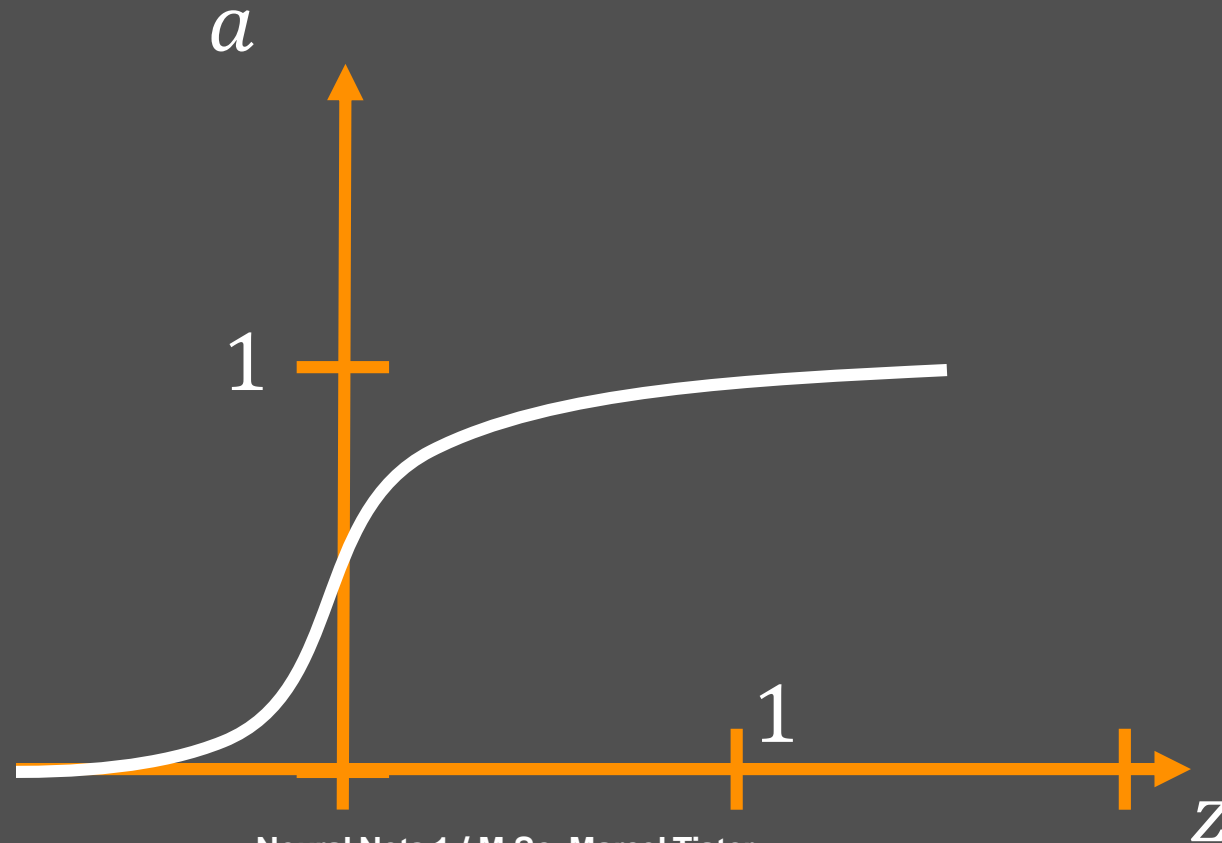


**Rectified Linear (ReLu):**  $a(z) = \begin{cases} 0, & z < 0 \\ z, & z \geq 0 \end{cases}$

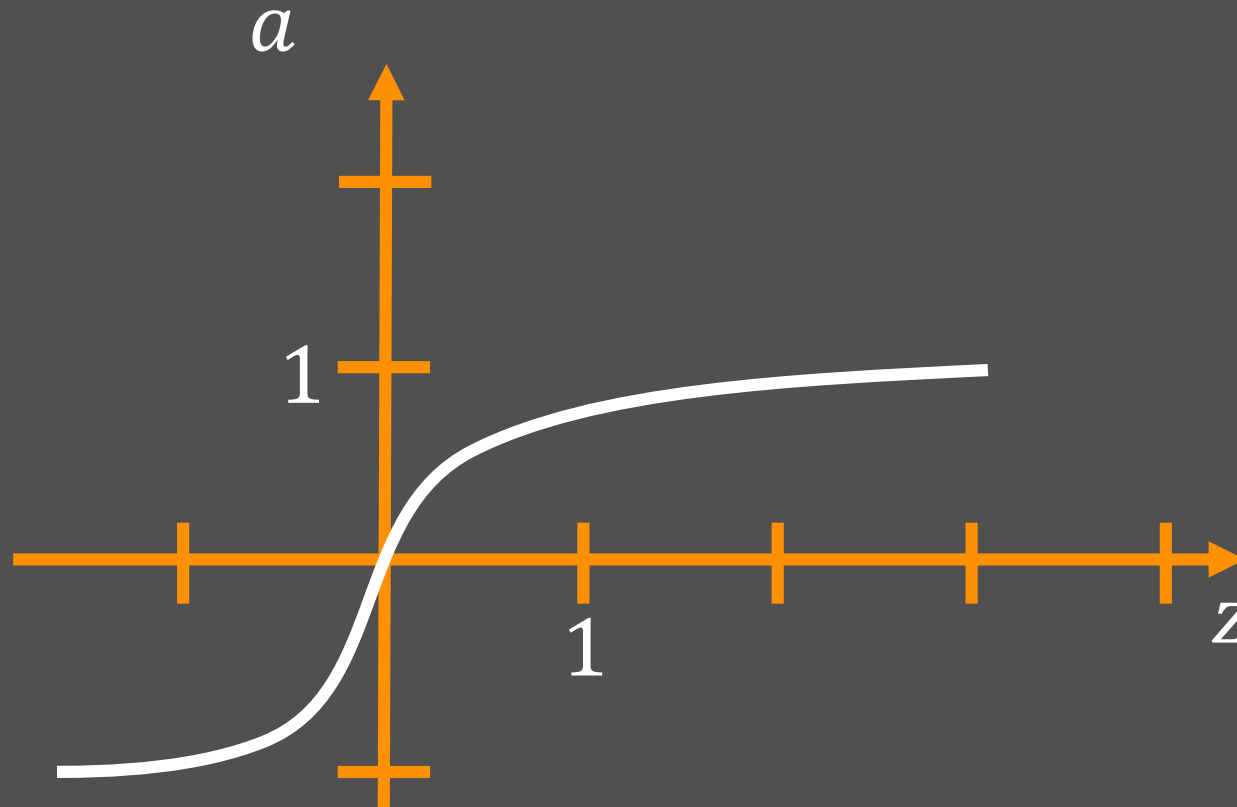


# Forward Propagation

- **Sigmoid:**  $a(z) = \frac{1}{1+e^{-z}}$



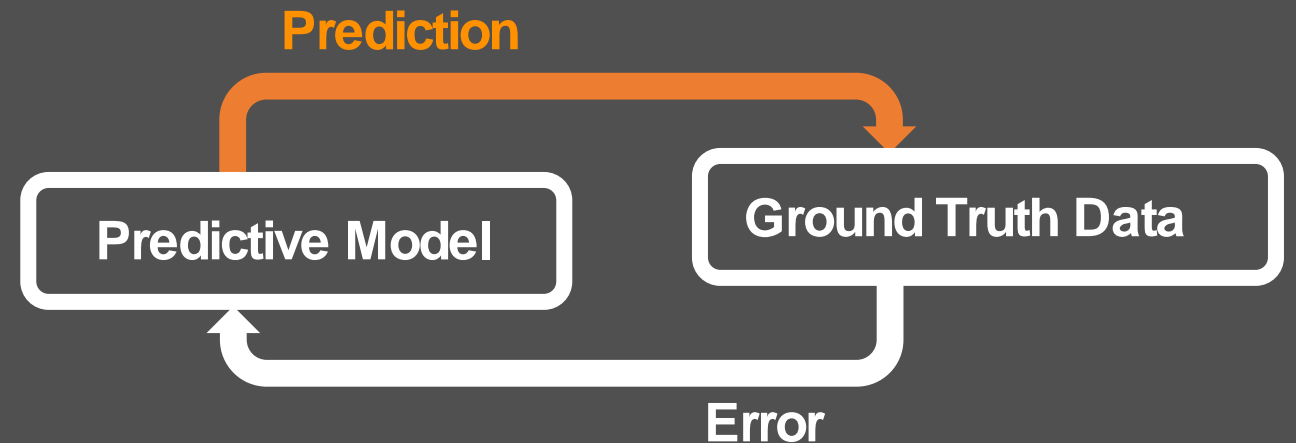
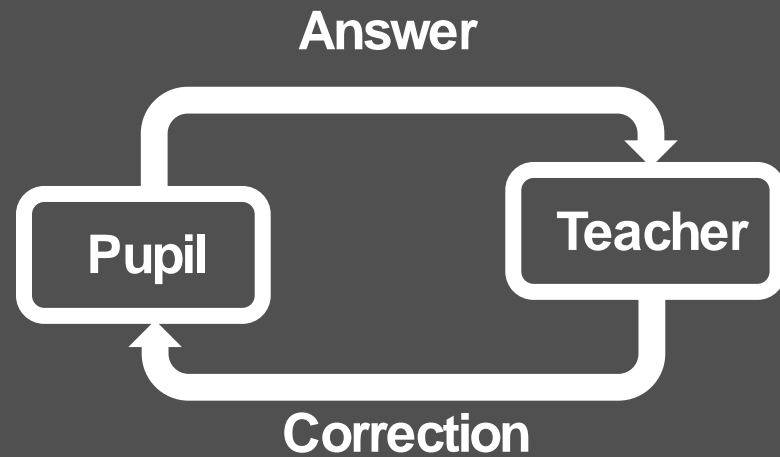
- **Tanh:**  $a(z) = \tanh(z)$





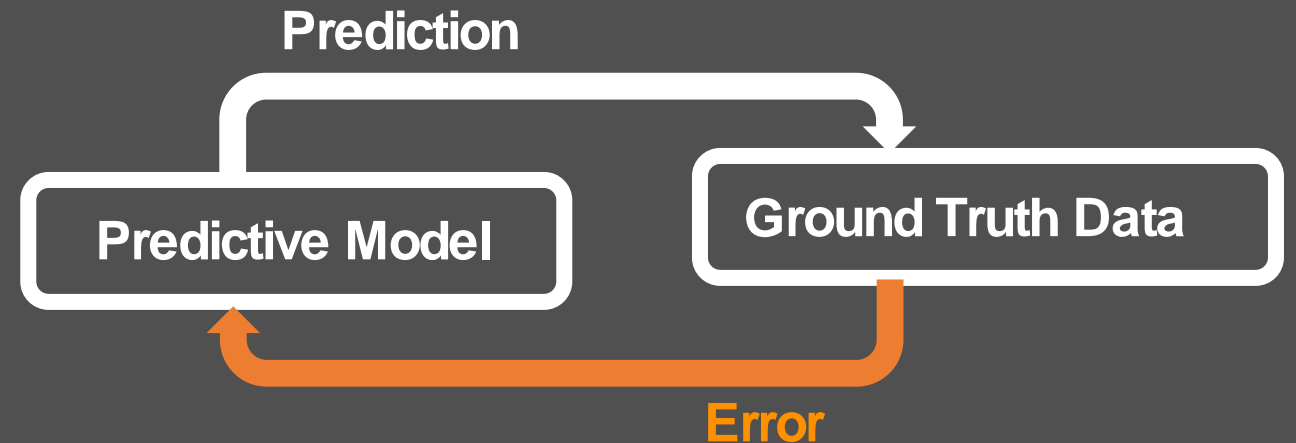
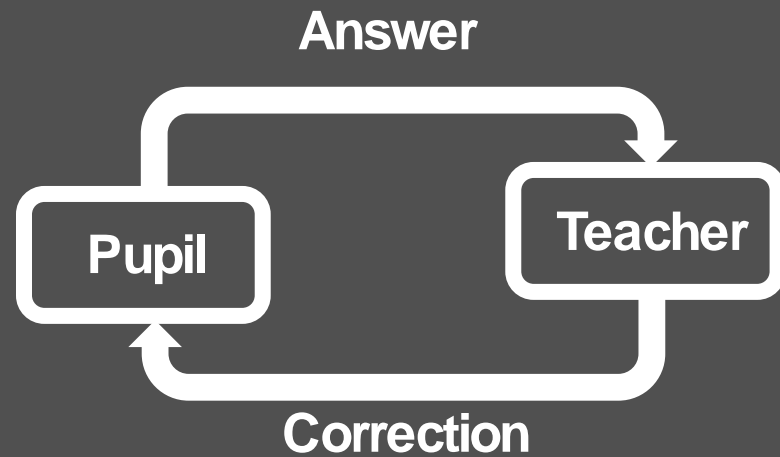
## Two Processes:

- **Forward Propagation (Prediction)**
- **Backward Propagation (Weight Tuning)**



## Two Processes:

- Forward Propagation (Prediction)
- **Backward Propagation (Weight Tuning)**

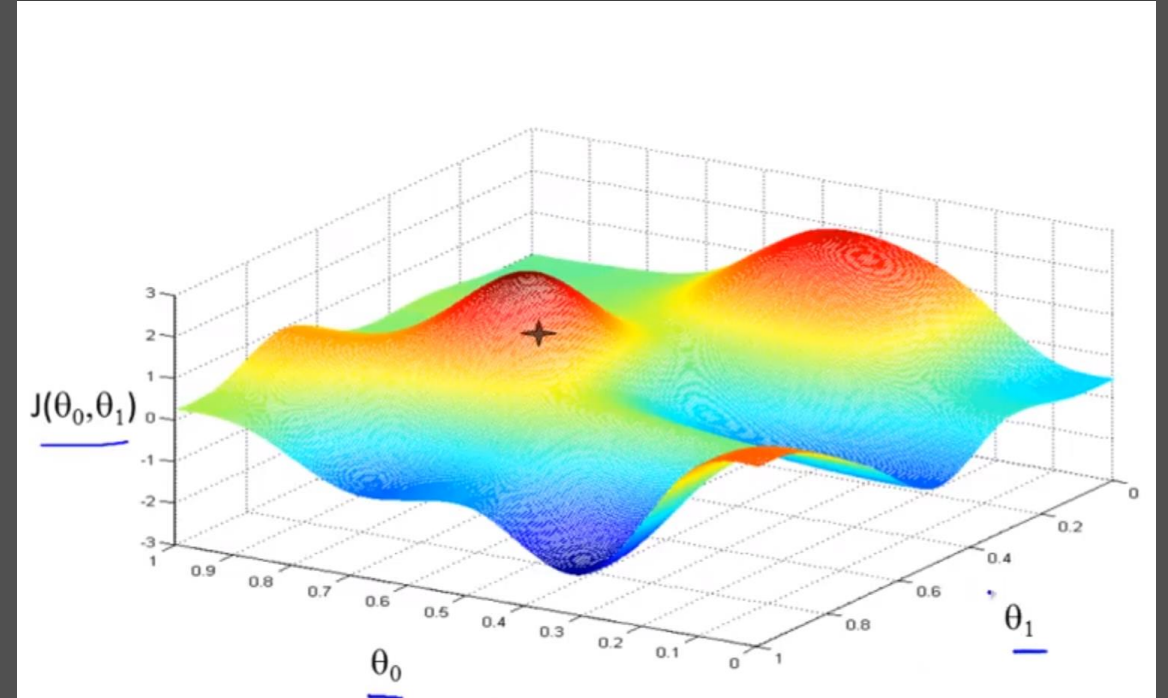
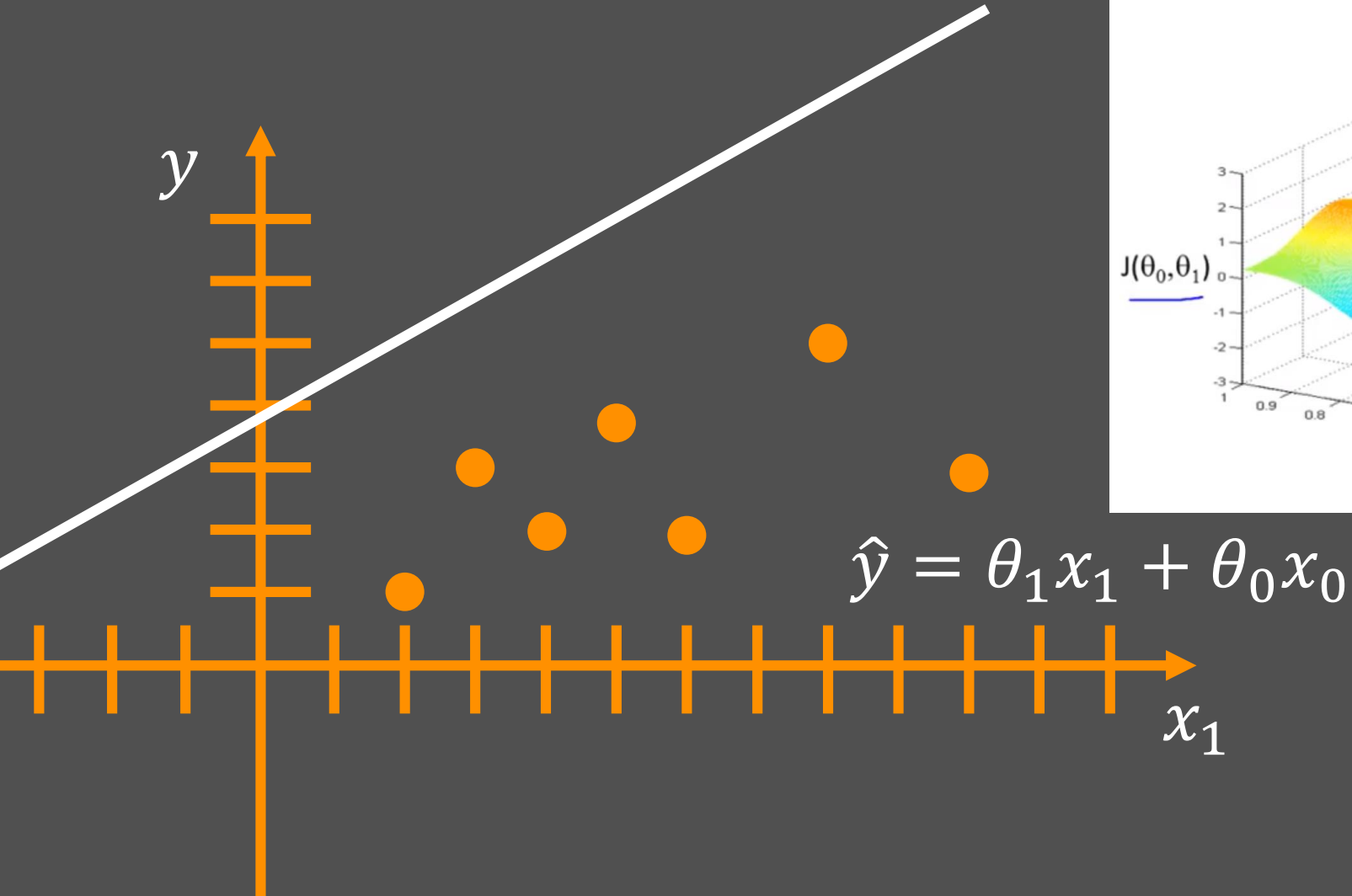


## Training of MLP:

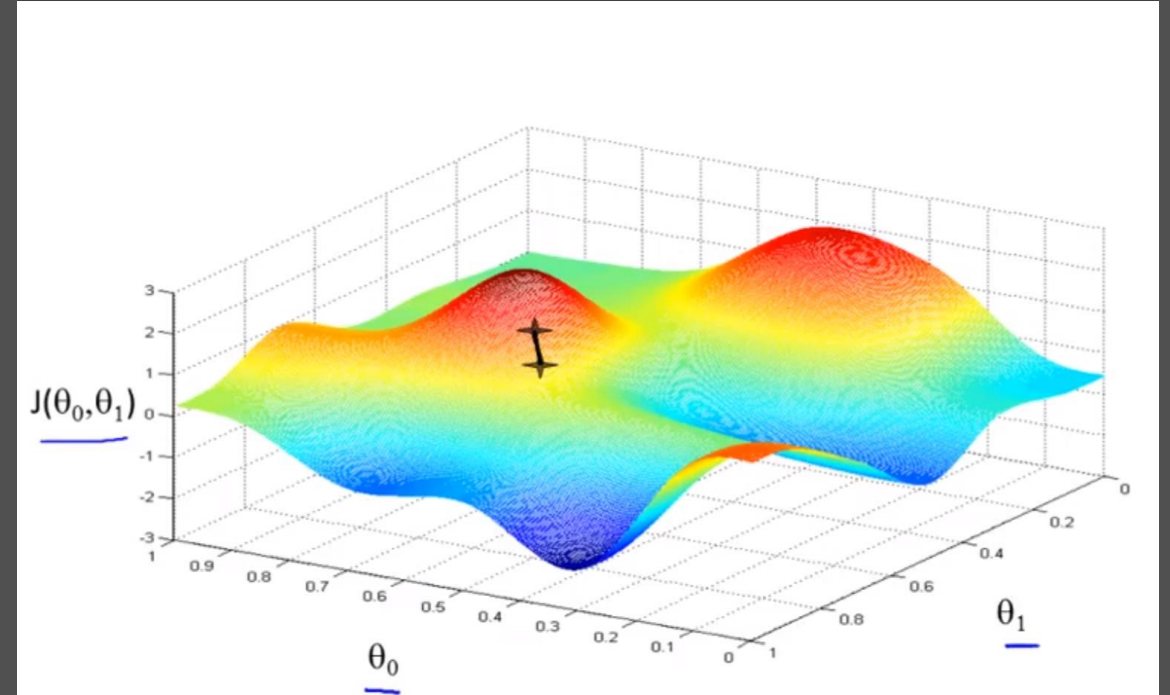
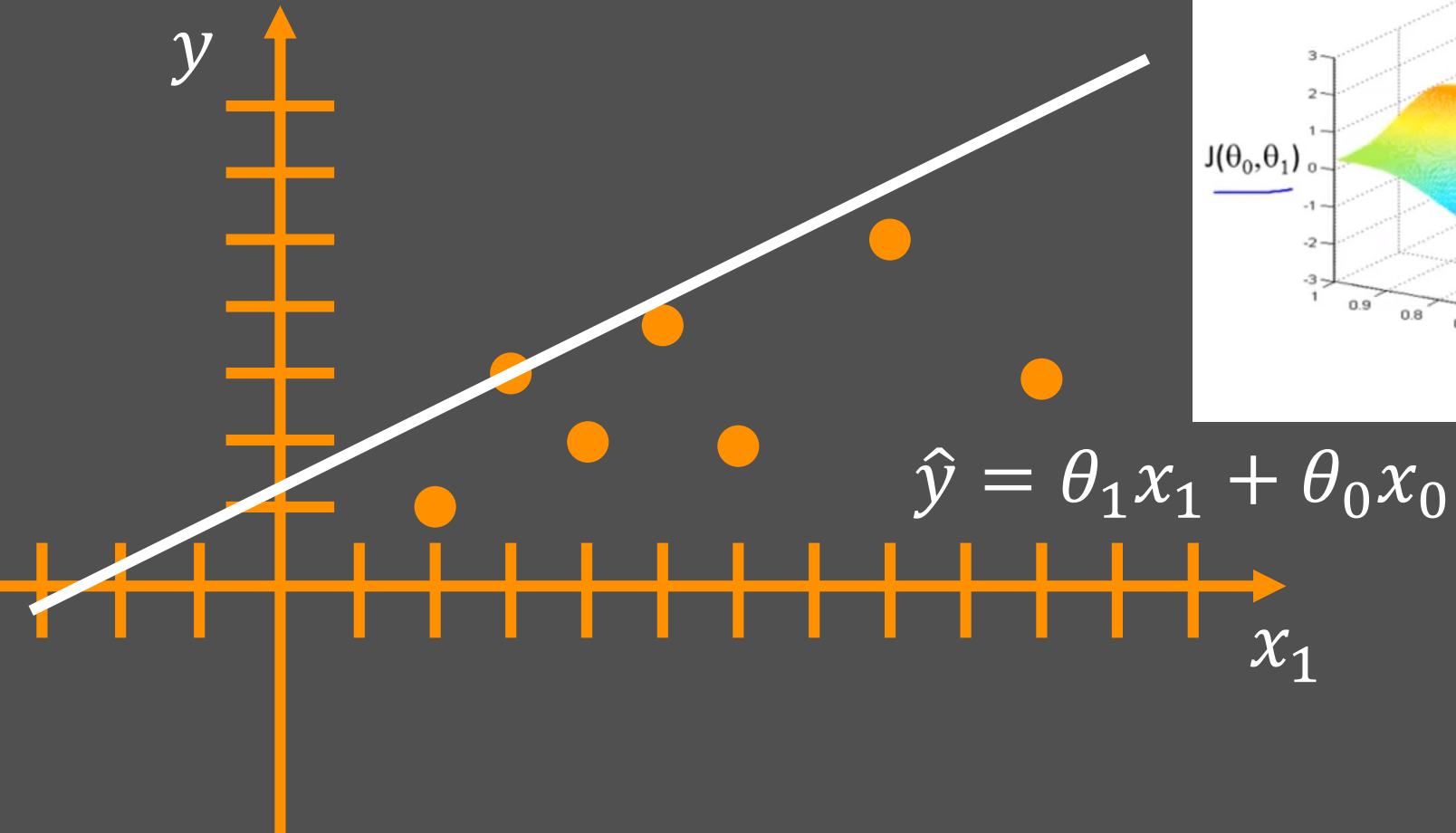
- Use Gradient-Based Algorithm: **Backpropagation**
- Similar to Gradient Descent
- Weight-Update is more complex

- Assume, we get some Output  $\hat{y}$  from MLP
- Compute function  $E = \frac{1}{2m} \sum_{t=1}^m (\hat{y}_t - y_t)^2$
- Want to  $\min_{\Theta_{i,j}^{(l)}} E(\Theta_{i,j}^{(l)})$  so calculate  $\frac{\partial E(\Theta_{i,j}^{(l)})}{\partial \Theta_{i,j}^{(l)}}$
- Use  $\frac{\partial E(\Theta_{i,j}^{(l)})}{\partial \Theta_{i,j}^{(l)}}$  to update weights with learning rate
- $\Theta_{i,j}^{(l)} := \Theta_{i,j}^{(l)} - \alpha \frac{\partial E(\Theta_{i,j}^{(l)})}{\partial \Theta_{i,j}^{(l)}}$

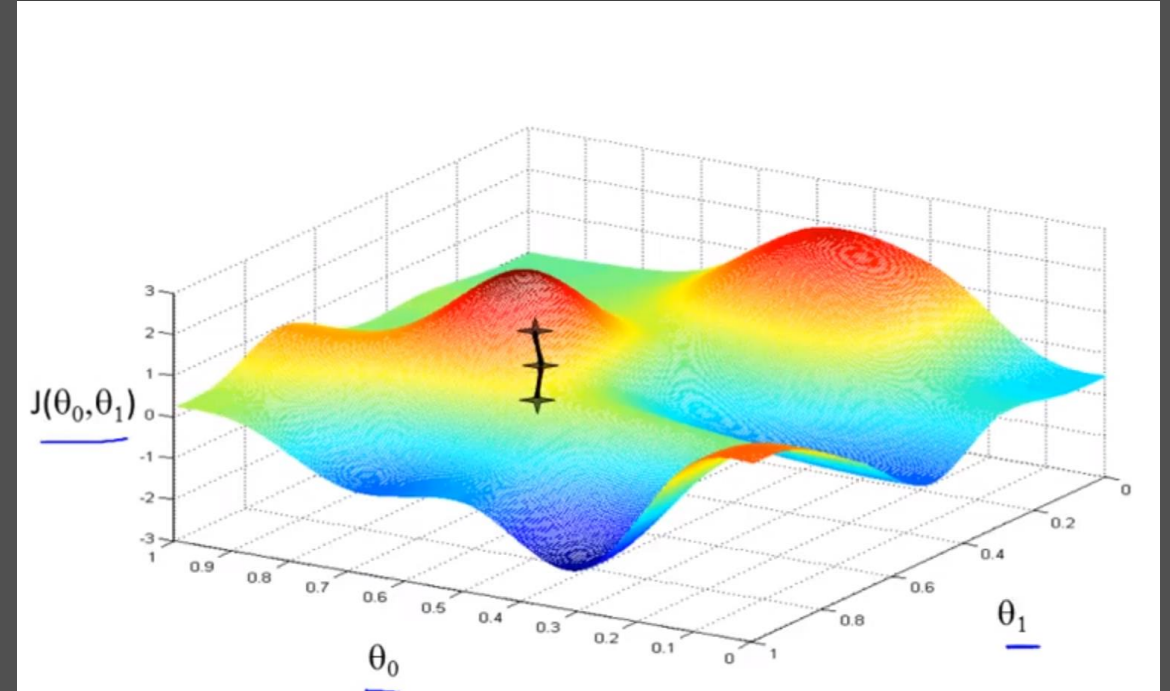
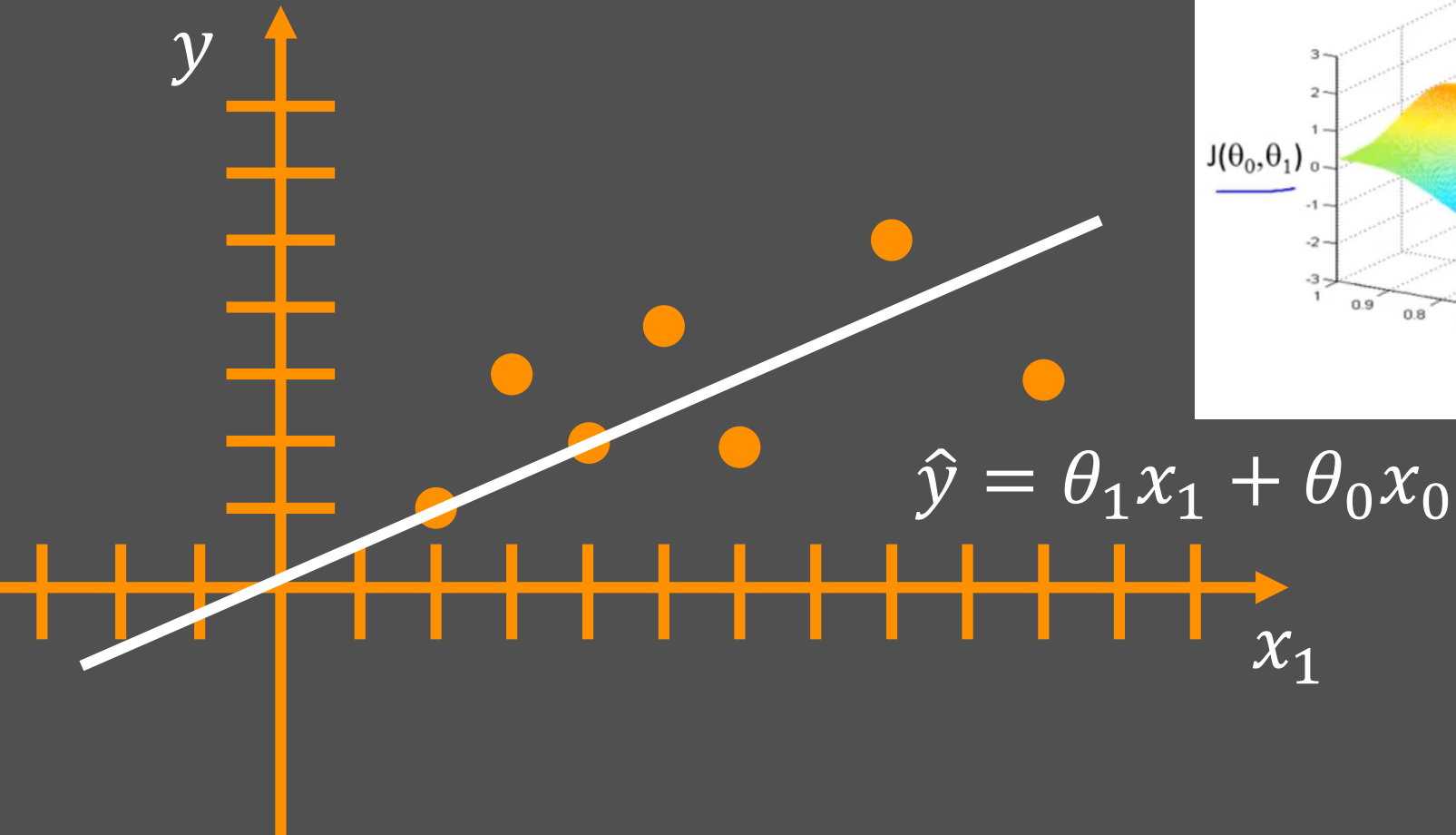
# Backward Propagation



# Backward Propagation



# Backward Propagation



# Hands-On Session



### 3 Scripts:

- **estimator.py**: Script that trains a **DNNClassifier**
- **load\_data.py**: Utility functions to load data
- **predict.py**: Real world prediction with unlabeled data

## Your task:

- Fill in the gaps which are highlighted with **TODO**
- Use internet and documentation

- **Pull code & slides from repository**
  - <https://github.com/mati3230/modalg181>
- **Download dataset from Nextcloud**
  - iris\_flower and Magic\_Gamma\_Telescope
  - <https://nextcloud.mirevi.medien.hs-duesseldorf.de/index.php/s/kPXwJiac7vTQVeu>