Ejercicio 1 ★

Evaluar en el intérprete CBN las siguientes expresiones.

i) $(\lambda x.\, x)\,\mathrm{zero}$

$$\dfrac{\dfrac{}{\vdash \lambda x.\, x \hookrightarrow \langle x, x, \emptyset\rangle}\quad \dfrac{}{\vdash \mathrm{zero} \hookrightarrow \mathrm{zero}}}{\vdash (\lambda x.\, x)\,\mathrm{zero} \hookrightarrow \mathrm{zero}}\; x = \langle \mathrm{zero}, \emptyset\rangle \vdash x \hookrightarrow \mathrm{zero}$$

ii) $(\lambda x.\, \lambda x.\, x)\,\underline{2}\,\underline{3} = ((\lambda x.\, \lambda x.\, x)\,\underline{2})\,\underline{3}$

$$\dfrac{\dfrac{\dfrac{}{\vdash \lambda y.\, \lambda x.\, x \hookrightarrow \langle y, \lambda x.\, x, \emptyset\rangle}\quad y = \langle \underline{2}, \emptyset\rangle \vdash \lambda x.\, x \hookrightarrow \langle x, x, y = \langle \underline{2}, \emptyset\rangle\rangle}{\vdash (\lambda y.\, \lambda x.\, x)\,\underline{2} \hookrightarrow \langle x, x, y = \langle \underline{2}, \emptyset\rangle\rangle}\quad y = \langle \underline{2}, \emptyset\rangle,\, x = \langle \underline{3}, \emptyset\rangle \vdash x \hookrightarrow \underline{3}}{\vdash ((\lambda y.\, \lambda x.\, x)\,\underline{2})\,\underline{3} \hookrightarrow}\qquad \dfrac{}{\vdash \underline{3} \hookrightarrow \underline{3}}$$

iii) $(\lambda x.\, \lambda y.\, (\lambda x.\, \mathrm{if\ isZero}(x)\ \mathrm{Then}\ y\ \mathrm{else}\ x)\,x)\,\underline{3}\,\underline{4}$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{x = \langle \underline{3}, \emptyset\rangle, y = \langle \underline{4}, \emptyset\rangle \vdash x \hookrightarrow \underline{3}}}{r', x = \langle x, r'\rangle \vdash x \hookrightarrow \underline{3}}\qquad x = \langle \underline{3}, \emptyset\rangle, y = \langle \underline{4}, \emptyset\rangle \vdash x \hookrightarrow \underline{3}}{r', x = \langle x, r'\rangle \vdash \mathrm{isZero}(x) \hookrightarrow \mathrm{false}}\qquad r', x = \langle x, r'\rangle \vdash x \hookrightarrow \underline{3}}{r', x = \langle x, r'\rangle \vdash \mathrm{if\ isZero}(x)\ \mathrm{Then}\ y\ \mathrm{else}\ x \hookrightarrow}$$

$$\dfrac{x = \langle \underline{3}, \emptyset\rangle, y = \langle \underline{4}, \emptyset\rangle \vdash \lambda x.\, h \hookrightarrow \langle x, h, (x = \langle \underline{3}, \emptyset\rangle, y = \langle \underline{4}, \emptyset\rangle)\rangle}{\vdash \lambda x.\, g \hookrightarrow \langle x, g, \emptyset\rangle}\quad \langle x = \underline{5}, \emptyset\rangle \vdash g \hookrightarrow \langle y, h, x = \langle \underline{3}, \emptyset\rangle\rangle \qquad r', x = \langle x, r'\rangle \vdash h \hookrightarrow$$

$$\dfrac{\vdash M_1 \hookrightarrow \langle y, h, x = \langle \underline{3}, \emptyset\rangle\rangle}{\vdash (\lambda x.\, \lambda y.\, (\lambda x.\, \mathrm{if\ isZero}(x)\ \mathrm{Then}\ y\ \mathrm{else}\ x)\,x)\,\underline{3}\,\underline{4} \hookrightarrow}\quad x = \langle \underline{5}, \emptyset\rangle, y = \langle \underline{4}, \emptyset\rangle \vdash (\lambda x.\, \mathrm{if\ isZero}(x)\ \mathrm{Then}\ y\ \mathrm{else}\ x)\,x \hookrightarrow$$

r' · h · g · M_1