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MagAnalyst: A MATLAB Toolbox for Anhysteretic Magnetization Analysis

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We introduce MagAnalyst, a user-friendly Matlab toolbox designed to streamline the fitting process of anhysteretic magnetization curves using the Langevin-Weiss equation of state. Built upon the framework proposed by Silveyra and Conde Garrido, our toolbox facilitates the description of magnetically anisotropic materials through negative molecular fields, and of magnetically inhomogeneous materials through multicomponent magnetizations. This study showcases the reliability and accuracy of MagAnalyst, enabling the direct extraction of key parameters such as the anisotropy mean field and the induced anisotropy energy constant. By making this toolbox freely available at https://github.com/matias-gonz/mag-analyst/, we aim to provide a valuable tool for comprehending the magnetic properties of soft magnets, thereby contributing to the development of cutting-edge magnetic devices.

Index Terms— **Langevin, magnetization, Matlab toolbox, modeling, soft magnetic materials.**

I. INTRODUCTION

THE ANHYSTERETIC magnetization curve serves as a valuable tool for researchers delving magnetization physics and engineers designing devices with soft magnetic cores. This curve, typically free of inflection points in most materials, proves challenging to measure directly but can be approximated as the half-sum of the left and right branches of the hysteresis loop for each M value [1]. The widely used Jiles-Atherton hysteresis model uses the anhysteretic curve as its backbone, often characterizing it through the Langevin-Weiss function [2],

$$M = M_S \mathcal{L}\left(\frac{H+\alpha M}{a}\right), \quad (1)$$

where H is the applied field and $\mathcal{L}(h) = \coth(h) - 1/h$ is the Langevin function. The three model parameters are: the saturation magnetization M_S, a, and Weiss' molecular field constant α. This analytical function offers a physically plausible representation of magnetic saturation without the ripples present in experimental curves.

Jiles and Atherton explained that this simple equation of state yields sigmoid-shaped curves for isotropic media [2] but, as Jiles et al. argued, "it should not be thought of as totally general" [3]. Ramesh, Jiles, and Roderick later proposed a more intricate equation for materials with uniaxial anisotropy, necessitating numerical integration [4].

Determining the model parameters from experimental data poses a challenge. Jiles et al. believed that it was not immediately clear which "fixed reference points" should be used [3]. Instead, they opted to treat M_S as a known value and arbitrarily set α. They used the initial susceptibility,

$$\chi_{in} = \partial M/\partial H|_{H\to 0} = M_S/(3a - \alpha M_S), \quad (2)$$

to derive parameter a and iteratively optimized a and α. They acknowledged that "the model parameters can be rather sensitive to small changes in the curve" and suggested that "further improvements in the algorithm may be possible" [3]. Stochastic optimization methods have been employed to address these issues, constraining a and α within specified limits [5] and requiring "some trial-and-error adjustments" [6].

Recent research has introduced innovative methods for extracting parameters from anhysteretic magnetization curves. Carosi et al. devised an iterative algorithm to define the seed values of a and α, assuming known M_S, showcasing robustness and efficacy in converging to accurate solutions [7]. Alternatively, Silveyra and Conde Garrido proposed using the tip of the data curve as a fixed reference to find the model parameters, transforming the search space into two intrinsically bounded parameters. This approach challenges the conventional notion that Weiss' constant must be positive, revealing the potential for negative constants to accurately characterize materials with uniaxial magnetic anisotropy [8]. The scientific community is increasingly accepting the possibility of negative α [9, 10].

Silveyra and Conde Garrido outlined two procedures for extracting the model parameters within their framework: a simple method based on an additional reference point [8] and a method relying on the critical field of the semi-log magnetization derivative curve [11]. The latter offers the advantage of extension to multicomponent magnetizations for describing inhomogeneous materials [11, 12]. However, researchers still encounter the challenge of developing custom code for data analysis.

To simplify the analysis of anhysteretic magnetization curves within the Silveyra and Conde Garrido framework, we developed MagAnalyst, a user-friendly Matlab toolbox available under the MIT License at https://github.com/matias-gonz/mag-analyst/ (the alpha version of the software has been recently presented at the 21st conference organized by the Latin American and Caribbean Consortium of Engineering Institutions [13]). MagAnalyst can handle various scenarios, including isotropic or anisotropic, and homogeneous or inhomogeneous magnetic materials. Next, we will delve into the underlying equations and algorithms of our toolbox, validate its effectiveness by fitting experimental data and comparing it with theoretical curves from other authors. Additionally, we will demonstrate how MagAnalyst streamlines the analysis of magnetization curves for alloys with varying degrees of induced uniaxial anisotropy.

II. THEORETICAL BACKGROUND

MagAnalyst employs a comprehensive strategy and equations to determine the model parameters for an anhysteretic magnetization curve.

To calculate the saturation magnetization, MagAnalyst uses the tip of the data curve (H_{TIP}, M_{TIP}) as a fixed reference point,

$$M_S = \frac{M_{TIP}}{m(H_{TIP})}, \tag{3}$$

where $m(H_{TIP})$ is called the reduced magnetization at the maximum applied field. The algorithm solves for m at H_{TIP} an implicit form of the Langevin-Weiss function

$$F(m) = \mathcal{L}\left(\frac{H+\alpha M_S m(H)}{a}\right) - m(H) = 0. \tag{4}$$

The solution, which falls within the range $0 < m < 1$, is determined by finding the point where $F(m)$ changes sign. MagAnalyst employs the Matlab built-in `fzero` function to find this solution. The algorithm, developed by Dekker, implements a combination of bisection, secant, and inverse quadratic interpolation methods [14]. When dealing with the Langevin function, $\mathcal{L}(h)$, for a reduced magnetization $|h| \le 0.001$, MagAnalyst computes the Taylor expansion about the origin to avoid catastrophic cancellation

$$\mathcal{L}(h) \cong \frac{h}{3} - \frac{1}{45}h^3 + \frac{2}{945}h^5. \tag{5}$$

To solve (4), we first need to find the a parameter and the αM_S product. To this end, we use a mathematical property of the Langevin-Weiss function: its semi-log derivative presents a maximum when plotted against the magnetic field H.

By identifying the corresponding critical field, H_{cr}, in a semi-log magnetization derivative plot of the data and estimating $m(H_{cr})$ (usually about 0.5), we can calculate the a parameter as [11]

$$a = H_{cr} P\big(m(H_{cr})\big)\left(Q\big(m(H_{cr})\big) \pm \sqrt{Q^2\big(m(H_{cr})\big) - 1}\right), \tag{6}$$

being

$$P(m) = \frac{\mathcal{L}'\big(\mathcal{L}^{-1}(m)\big)}{m - \mathcal{L}^{-1}(m)\mathcal{L}'\big(\mathcal{L}^{-1}(m)\big)}, \tag{7}$$

$$Q(m) = \left(\frac{m}{\mathcal{L}'\big(\mathcal{L}^{-1}(m)\big)}\right)^2 \frac{\Big(-\mathcal{L}''\big(\mathcal{L}^{-1}(m)\big)\Big)}{2\Big(m - \mathcal{L}^{-1}(m)\mathcal{L}'\big(\mathcal{L}^{-1}(m)\big)\Big)} - 1. \tag{8}$$

MagAnalyst utilizes Kröger's approximant to evaluate the Langevin inverse, providing a balance between simplicity and accuracy (its maximum relative error is below 0.3%) [15]

$$\mathcal{L}^{-1}(m) \cong \frac{3m - \frac{m}{5}(6m^2 + m^4 - 2m^6)}{1 - m^2}. \tag{9}$$

$\mathcal{L}'$ and $\mathcal{L}''$ are the first and second derivatives of the Langevin function with respect to the reduced magnetic field (h)

$$\mathcal{L}'(h) = -\operatorname{csch}^2(h) + 1/h^2, \tag{10}$$

$$\mathcal{L}''(h) = 2\coth(h)\operatorname{csch}^2(h) - 2/h^3, \tag{11}$$

which are also approximated by their Taylor expansion about the origin for $|h| \le 0.001$

$$\mathcal{L}'(h) \cong \frac{1}{3} - \frac{h^2}{15} + \frac{2}{189}h^4, \tag{12}$$

$$\mathcal{L}''(h) \cong -\frac{2}{15}h + \frac{8}{189}h^3 - \frac{2}{225}h^5. \tag{13}$$

The αM_S product is given by

$$\alpha M_S = \frac{\mathcal{L}^{-1}\big(m(H_{cr})\big)a - H_{cr}}{m(H_{cr})}. \tag{14}$$

Finally, the molecular field constant can be calculated as

$$\alpha = \alpha M_S / M_S. \tag{15}$$

Both H_{cr} and $m(H_{cr})$ can be iteratively optimized until the error function reaches its minimum.

III. OPTIMIZATION TECHNIQUE

MagAnalyst currently utilizes the Matlab function `minimize`, developed by Oldenhuis [16], to find the constrained minimum of the objective function based on the user's initial estimates. This function uses `fminsearch` [17] as its engine, which is a Matlab built-in function that employs the Nelder-Mead simplex method, a heuristic search method. `minimize` shares the same syntaxis of `fmincon` [18], (which offers deterministic algorithms such as the interior-point method) but has the advantage of being freely distributed (unlike `fmincon`, which requires Matlab's Optimization Toolbox).

By default, MagAnalyst minimizes the mean orthogonal distance error, also known as the diagonal distance error, between the data and theoretical magnetization curves in a normalized $(X, Y) = (\log H, M)$ plane [11]

$$\mathit{Diagonal\ error} = \frac{1}{N}\sqrt{\sum_{i=0}^{N} \Delta o_i^2}, \tag{16}$$

being

$$\Delta o_i = \Delta y_i \cos(\operatorname{atan}(\Delta y_i / \Delta x_i)), \tag{17}$$

$$\begin{cases} \Delta x_i = \frac{1}{X_N}\left|\hat{X}(Y_i) - X_i\right| \\ \Delta y_i = \frac{1}{Y_N}\left|Y\left(\hat{X}(Y_i)\right) - Y_i\right| \end{cases}. \tag{18}$$

The unhatted values represent the data curve, while the hatted values represent the theoretical curve. The *N*-th value corresponds to the curve tip with the arrays sorted in ascending order. This approach has been found to provide better fittings compared to conventional techniques that minimize the normalized root-mean-squared error of either M at constant H or H at constant M [11, 12]; vertical and horizontal errors, respectively

$$\mathit{Vertical\ error} = \frac{1}{N}\frac{1}{Y_N}\sqrt{\sum_{i=0}^{N}\left(Y_i - \hat{Y}(X_i)\right)^2}, \tag{19}$$

$$\mathit{Horizontal\ error} = \frac{1}{N}\frac{1}{X_N}\sqrt{\sum_{i=0}^{N}\left(X_i - \hat{X}(Y_i)\right)^2}. \tag{20}$$

The user has the option to optimize either of the conventional objective functions if desired, as they are faster to compute. The toolbox utilizes the Matlab built-in `interp1` function [19] for evaluating both data and theoretical curves at fields not present in the given set of values, using linear interpolation and extrapolation methods.

It is not that straightforward to compute the theoretical magnetization curve (1) for a given set of model parameters and a magnetic field range –notice that $M(H)$ appears in the argument of the Langevin function. We have thus adopted the strategy of using (4) to determine the reduced magnetizations for a range of magnetic field values up to H_{TIP}. The magnetization curve is then computed as

$$M(H) = M_S m(H). \tag{21}$$

Additionally, MagAnalyst calculates the theoretical magnetic susceptibility as [11]

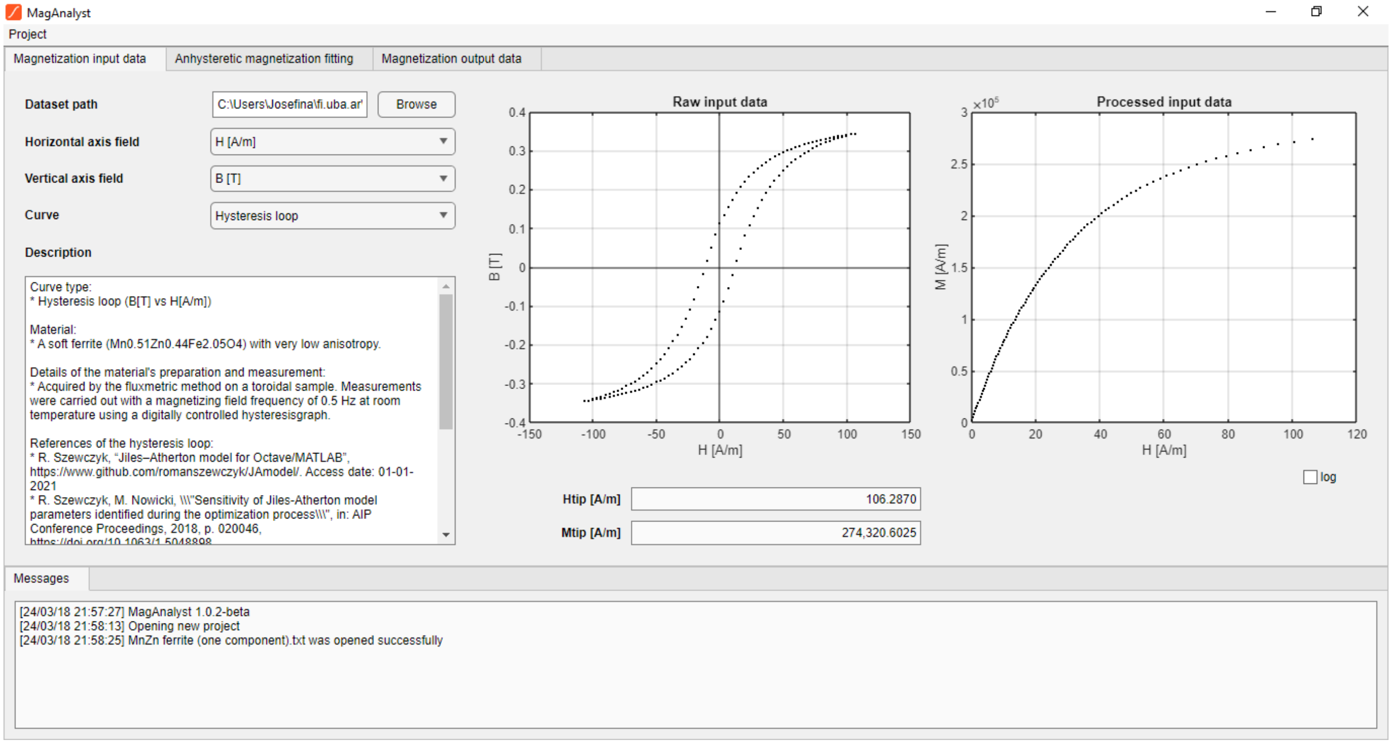

Fig. 1. "Magnetization input data" tab after importing a quasistatic hysteresis loop and calculating the anhysteretic magnetization curve.

$$\frac{\partial M}{\partial H}(H) = M_S \frac{\mathcal{L}'\left(\mathcal{L}^{-1}(m(H))\right)/a}{1-\alpha M_S \mathcal{L}'\left(\mathcal{L}^{-1}(m(H))\right)/a}, \tag{22}$$

and the theoretical semi-log M derivative as [11]

$$\frac{\partial M}{\partial \ln H}(H) = H \frac{\partial M}{\partial H}(H). \tag{23}$$

IV. Graphical User Interface

MagAnalyst offers a user-friendly graphical user interface (GUI) that simplifies the process of fitting an anhysteretic curve using the Silveyra-Conde Garrido approach. Users can easily set up a new project and save their progress for future analysis. The GUI includes tabs for setting up magnetization input data (Figure 1) and configuring the anhysteretic magnetization fitting process (Figure 2).

Upon configuring the input fields and units, MagAnalyst automatically converts the input data into $M[A/m]$ vs $H[A/m]$. The currently supported input fields for the horizontal axis include: $H[A/m]$, $H[kA/m]$, $H[Oe]$, $H[kOe]$, $B_{ext}[T]$, $B_{ext}[G]$, and $B_{ext}[kG]$, while for the vertical axis field: $M[A/m]$, $M[kA/m]$, $M[MA/m]$, $M[emu/cm^3]$, $J[T]$, $B[T]$,

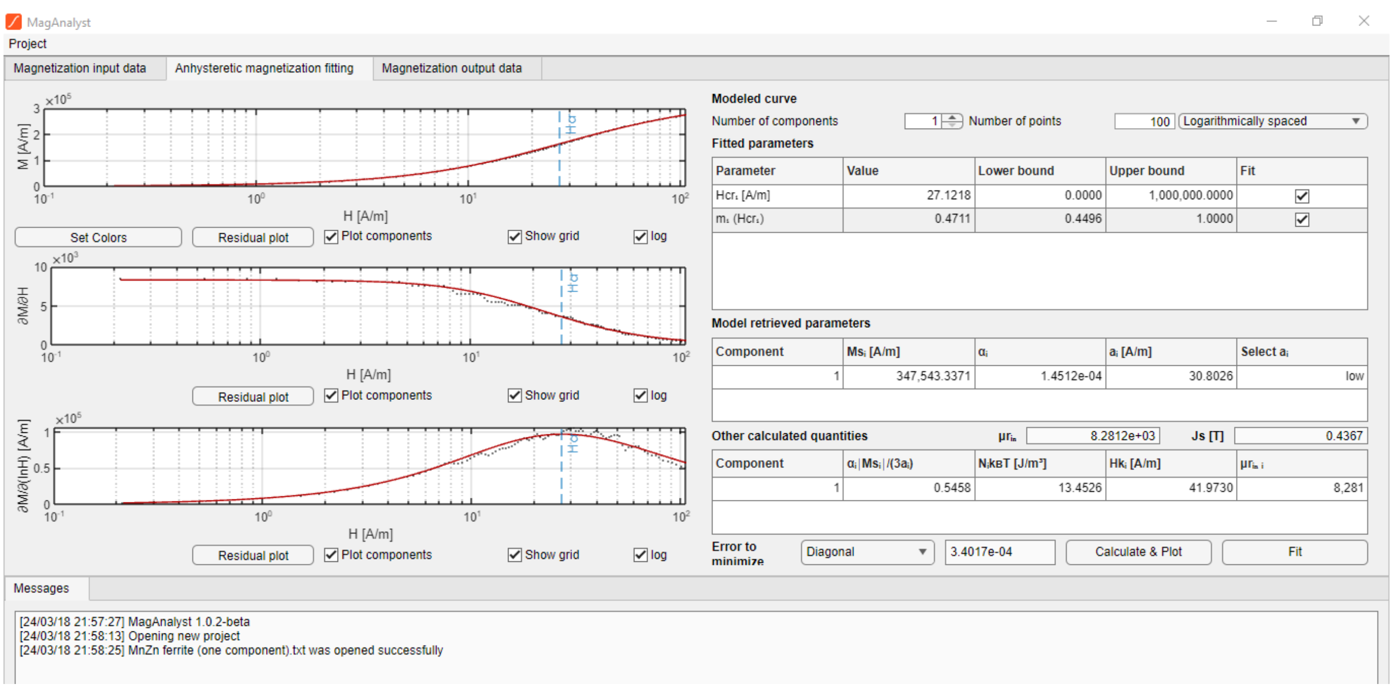

Fig. 2. "Anhysteretic magnetization fitting" tab with plots and values optimized after the completion of the fitting process.

$B[G]$, and $B[kG]$ (additional input fields available upon request). For detailed conversion formulae, the reader can refer to the toolbox's documentation. Magnetization input data can be either an anhysteretic curve or a symmetric hysteresis loop. For the latter case, the software calculates the anhysteretic curve as the mean of the left and right branches of the M vs H loop. MagAnalyst assumes that the anhysteretic curve exhibits odd symmetry and restricts the analysis to the first quadrant.

Users can analyze inhomogeneous magnetic materials by setting up models with multiple components (MagAnalyst employs an extension of the equations provided in Section II and a law of mixtures proposed by Silveyra and Conde Garrido [11]). Providing estimations for critical fields and reduced magnetizations, MagAnalyst is ready to optimize the problem after a simple click. As illustrated in Figure 2, the toolbox displays the material's initial relative permeability

$$\mu_{r\,in} = 1 + \sum_i \chi_{in\,i} = 1 + \sum_i \frac{M_{Si}}{(3a_i - \alpha_i M_{Si})}, \tag{25}$$

and saturation polarization

$$J_S = \mu_0 \sum_i M_{S\,i}, \tag{26}$$

both being significant quantities for technological applications.

Tutorial videos are available to guide users through setting up single- and double-magnetic component cases and understanding the toolbox's capabilities [20]. Also, examples of a soft ferrite, a nanocrystalline alloy, a non-oriented silicon steel, and a grain-oriented silicon steel are included in the release.

V. Evaluation of MagAnalyst's performance

The evaluation of MagAnalyst's performance involved fitting the anhysteretic curve of a carbon steel sample (0.06 wt. % C) using data from Jiles and Atherton (Fig. 8 of [2], experimental conditions are reported therein) digitized with the free online toolbox WebPlotDigitizer [21]. We compared the results obtained using MagAnalyst with the solutions originally found by Jiles and Atherton [2] and recently optimized by Carosi et al. [7]. Table I reports the values of the other studies, together with the model parameters we retrieved (by minimizing the diagonal distance error). Figure 3 illustrates the experimental and theoretical curves, as well as the corresponding residuals computed as

$$r_i(H) = M(H_i) - \widehat{M}(H_i). \tag{24}$$

We observed that the model parameters obtained using MagAnalyst closely matched those from the other studies (Table I). Yet, they reached superior accuracy. Not only is the relative error of our solution significantly lower than those from the literature, but the residual plot of our solution is more randomly scattered about zero (Fig. 3). This highlights the consistency and reliability of the toolbox to calculate optimal model parameters for a given dataset.

TABLE I
PARAMETERS RETRIEVED FOR A CARBON STEEL ANHYSTERETIC CURVE

Study	M_S (MA/m)	a (A/m)	α	Relative error*
Jiles & Atherton [2]	1.60	1100	1.60×10^{-3}	3.0×10^{-3}
Carosi et al. [7]	1.60	1304.9	2.10×10^{-3}	1.3×10^{-3}
This work	1.53	1144	1.92×10^{-3}	7.3×10^{-4}

* Diagonal distance error between the data and the theoretical curve [12].

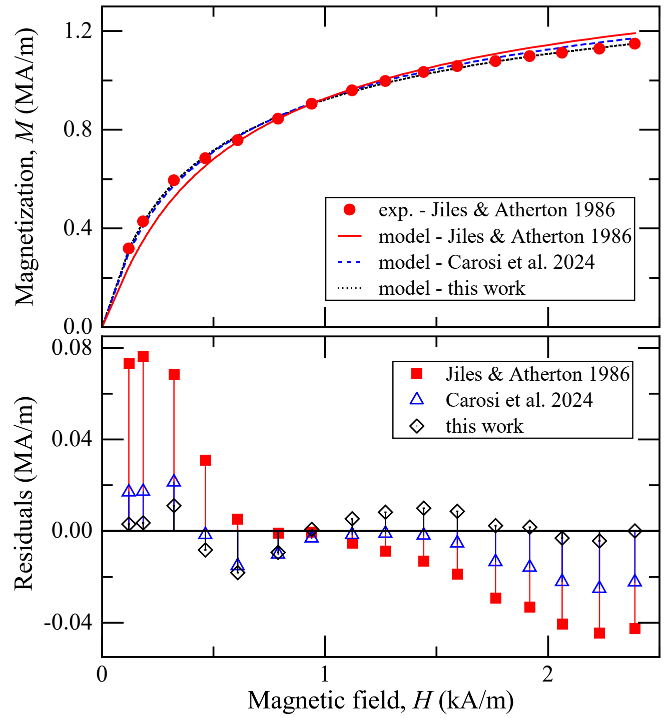

Fig. 3. Above: Anhysteretic magnetization curve of a carbon steel sample: measured data points reported by Jiles and Atherton [2], and theoretical curves calculated with model parameters reported by Jiles and Atherton [2], Carosi et al. [7] and optimized in this work (see Table I). Below: Residuals of the theoretical curves.

VI. Analysis of samples with induced anisotropy

We also utilized MagAnalyst to examine the impact of Co content in $Fe_{74-x}Co_xSi_{13}B_8Nb_2Cu_1Mn_2$ alloys annealed under tensile stress. Cai et al. recently reported their initial magnetization curve [22] (measurement details described therein), which lies below the anhysteretic magnetization curve [1]. However, due to the low hysteresis of these materials, both curves are closely aligned. Figure 4 displays the measured curves and MagAnalyst's fittings as B vs H, where the flux density field is $B = \mu_0(H + M)$ (with μ_0 denoting the vacuum permeability).

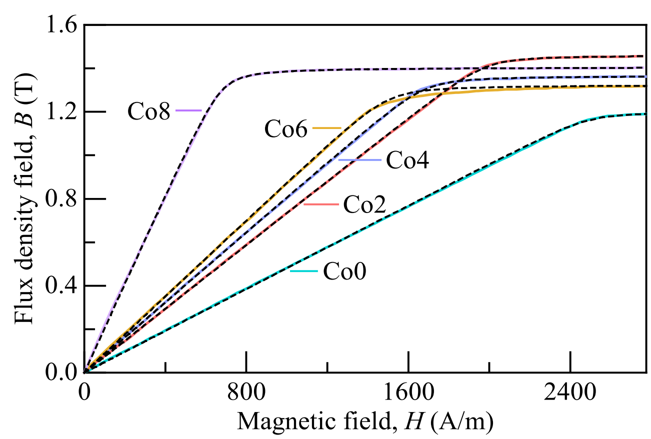

Fig. 4. Magnetization curves of strained annealed $Fe_{74-x}Co_xSi_{13}B_8Nb_2Cu_1Mn_2$ alloys (x = 0, 2, 4, 6, and 8) [22] digitized with WebPlotDigitizer [21] (continuous colored curves) together with theoretical curves fitted with MagAnalyst (dashed black curves).

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The negative molecular field constant proposed by Silveyra and Conde Garrido [8], and identified for all five theoretical magnetization curves (Table II), enabled an accurate description of the magnetization curves of these materials with strong uniaxial anisotropy. The physical reinterpretation of an internal field that can either aid or oppose the applied magnetic field has been linked to crystal electric forces aligning the elementary effective magnetic moments to minimize the crystal field and spin–orbit energies [12].

TABLE II
MODEL PARAMETERS AND OTHER QUANTITIES RETRIEVED FOR $Fe_{74-x}Co_xSi_{13}B_8Nb_2Cu_1Mn_2$ ALLOYS ANNEALED WITH TENSILE STRESS

Co content (at. %)	Modeled parameters M_S (kA/m)	a (A/m)	α	Calculated H_k (A/m)	K_u (J/m^3)	Reported [22] H_k (A/m)	K_u (J/m^3)
0	952	2.57	-2.61×10^{-3}	2494	1492	2484	1485
2	1160	2.78	-1.71×10^{-3}	1989	1449	1994	1453
4	1085	3.46	-1.55×10^{-3}	1691	1153	1694	1157
6	1051	4.07	-1.42×10^{-3}	1510	995	1507	1000
8	1116	4.03	-6.05×10^{-4}	687	482	686	482

In their study [22], Cai et al. determined the anisotropy mean field H_k by graphically identifying the intercept point between "the tangent line drawn from the gradient of the initial magnetization" and the extrapolation of "the gradient near the saturation region to $H = 0$". The induced anisotropy energy constant K_u was obtained as the area of the triangle formed by these two lines and the flux density axis.

In contrast, MagAnalyst simplifies this process since these quantities can be readily calculated from the model parameters. Assuming a constant magnetic susceptibility up to saturation at H_k, i.e., $\chi = M_S/H_k$, yields that the magnetic susceptibility is equal to the initial magnetic susceptibility $\chi_i = M_S/(3a - \alpha M_S)$ [8] and, thus,

$$H_k = 3a - \alpha M_S, \tag{27}$$

When the magnetic susceptibility remains practically constant up to saturation, the anisotropy mean field can be correlated to the induced anisotropy energy constant as

$$K_u = \frac{1}{2}\mu_0 M_S H_k, \tag{28}$$

For comparison, we present the values obtained in the literature using graphical methods as described above, alongside values analytically calculated by MagAnalyst (Table II).

VII. Conclusion

In conclusion, we offer MagAnalyst on our GitHub repository together with documentation, tutorials, and examples. This resource benefits material scientists in magnetization analysis, as well as engineers in modeling magnetically soft components by providing smooth magnetization and susceptibility curves.

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