Data-driven Newsvendor Problem and Applications in Electricity Markets

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1 Introduction

The implementation of renewable energies in the energy mix has become a global undertaking, necessary in order to combat climate change. This transition entails various technical problems, a major one being the differences in how conventional power generators produce electricity versus how renewable power generators produce electricity.
In this report, focus will be on the uncertainty present when electricity producers are bidding in the conventional northern European energy market (Nordpool). Renewable energy producers are especially subject to risk when bidding in this market, due to the stochastic nature of their production, which is largely dependant on the weather[1][2]
Thus, it is essential for a renewable energy producer, as well as a conventional electricity generator, to have some sort of methodology in place in order to minimize loss of revenue by over or under production.
A specific mathematical model for bidding in the day-ahead market is developed, which accounts for both uncertainty in power production as well as penalty prices. The final model proposed is a stochastic program with roots in the newsvendor problem. Analytical solutions and simulations of the different bidding approaches are derived and presented where applicable as an alternate approach to the stochastic program. These solutions will be compared and evaluated against each other.

2 The Electricity Market

The modern electricity market is divided into four parts. From the perspective of the market operator, there is the day-ahead market and the intra-day market. From the view of the system operator there is the ancillary services and balancing markets. Producers can bid in any of these markets, covering the necessary demand[3].
In this report, the specific focus will be on the day-ahead and balancing markets from an electricity producer viewpoint. The day-ahead market is, in the context of the Nordpool market, a marketplace where electricity providers and consumers can bid for production and consumption, this happens as seen in figure 2[4]. The balancing stage of this market is a real time marketplace where deviations from the schedule determined in the day-ahead market are balanced. The models developed will attempt to define a way to bid in the day-ahead market based on the forecasts of energy production and market realization after bidding. The final goal is to formulate a model that can be utilized in an adversarial setting, where the producers bid is expected to influence how other producers will bid in the market. In 2019, the market had 381TWh day-ahead turnover, with an average cost of 39 euro per MWh, thus the market value is around 150 billion euro. This signifies that a producer sat on a fraction of this market stand to win or lose hundreds of thousands of euros based on their bid down to a few decimals[5].
When an electricity producer places a bid in the day-ahead market they are paid a "day-ahead" price for the bid of electricity. This "day-ahead" price is calculated as the clearance of the market, which consist of electricity demand and aggregated supply across electricity

producers that wish to bid[3]. When the electricity production is realized, penalties are incurred in the case of over/under production. The penalty prices can be seen as the market clearing problem of the balancing market, with different prices for upwards and downwards balancing depending on producers, realization of electricity production and the consequent balancing necessary[3].
In the development of the models in this paper, it is assumed that the markets have been cleared already, and thus that penalty prices as well as day-ahead prices are known. Realized production for the individual producer is seen as uncertain, and therefore the penalty to be paid is only realized when clearing the balancing market.
Figure 1 shows the timeline of the electricity market the models in this paper will consider.

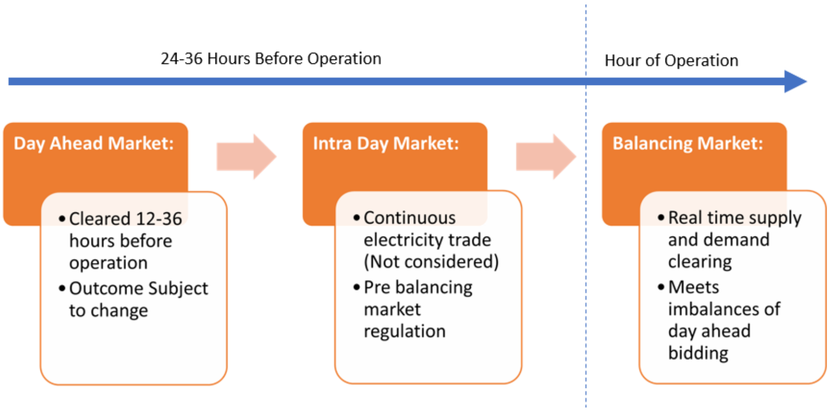

Figure 1: Timeline of the electricity market. All bids are anonymized and cleared by the market operator.

3 Newsvendor Problem Formulation

The Newsvendor Model has roots back to 1888[6], and deals historically with the problem of determining optimal inventory levels for a product, typically one with an expiration date. The problem is referred to as stochastic or uncertain when there exists imperfect information in the system, for example variable demand. In the specific case of the electricity market, and subsequently the models developed in this paper, the uncertainty stems from the fact that the forecast information used for both determining power production levels and thus the bids each producer will want to place in the market, are uncertain. These uncertain parameters will be considered by assuming an underlying predictive distribution for forecasting and power production.
In short, the true distribution functions for both power bids and forecasting of production would have to be estimated before implementation of the model in a real context - for the purposes of this report, these distributions will be represented by various β-distributions. The newsvendor problem can be treated as a Linear Program (LP) where it is desired to minimize costs associated with inventory management[7]. In the specific case of the energy market, we will instead attempt to minimize penalty payment while supplying the optimal

amount of energy. It should be noted that there is no expiration date on electricity, and further that it is supplied instantaneously, thus the desired problem to investigate forks from a classical newsvendor problem. The version proposed here thus needs to have "instantaneous" transaction from inventory to demand, as there is no storage.
An LP formulation of the newsvendor problem is defined by [7]:

$$\min_{x} \quad \sum_{k=1}^{K} p_k y_k \tag{1a}$$

$$\text{s.t.} \quad y_k \geq (c-b)x + bd_k\,, \quad \forall k \in K \tag{1b}$$

$$y_k \geq (c+h)x - hd_k\,, \quad \forall k \in K \tag{1c}$$

$$x \geq 0 \tag{1d}$$

Where y_k is the cost to order, p_k is the probability of each scenario k, x is a decision variable for the amount of items bought, d_k is demand per scenario, c is the cost per unit, b is the cost of not selling, and h is the cost of recycling (in other words selling back unsold items). This relatively simple stochastic program is expanded upon in an electricity market context as discussed earlier.

4 Basic model

The initial model for electricity bidding proposed will be based on the models proposed in Bremnes [1] and Pinsson [4].
The penalty incurred by a market participant bidding in the day-ahead market and either over or under producing can be formulated as:

$$P^{+} = (\omega - x) \cdot \pi^{+}\,, \quad \forall \omega > x \tag{2a}$$

$$P^{-} = (x - \omega) \cdot \pi^{-}\,, \quad \forall \omega < x \tag{2b}$$

Where P is the penalty, either for overproduction (+) or undeproduction (-), ω is the realization of energy production, x is the bid and π is the penalty cost for under/over production, which can be defined in different way depending on market and location[4].
In the starting formulation of the problem, it is simply assumed that the bidder is price taker and has no influence over market prices and penalty costs, but that these are known.
Now equation (2) is expanded upon to include a longer time frame t. The penalty incurred per unit of time could be seen as the expectation of each realization and bid across all bids and units of time:

$$P^{+} = \mathbb{E}_t[\omega_t - x_t) \cdot \pi^{+}]\,, \quad \forall \omega_t > x_t \tag{3a}$$

$$P^{-} = \mathbb{E}_t[(x_t - \omega_t) \cdot \pi^{-}]\,, \quad \forall \omega_t < x_t \tag{3b}$$

In this manner it is possible to find a bid value that *on average* minimizes the penalty across all times and realizations of the power production.

To expand this to encapsulate the whole day-ahead and balancing market, the function I is defined as the sum of penalties added to the sum of revenues, integrated across all possible bid values, where the realization is given as a function f representing the distribution of possible bids:

$$I(x) = \mathbb{E}[f, x, p_s, \pi^+, \pi^-)] = \int_0^x ((\omega - x)\pi^+)f(\omega)d\omega + \int_x^{w_p} ((x - \omega)\pi^- f(\omega)d\omega \tag{4}$$

Where w_p is the bid limit (or maximal capacity of the producer) and p_s is the day-ahead price. To get the optimal bid, it is necessary to find the minima of this function w.r.t x. This minima is given by equation (5), when assuming that the range from 0-w_p is normalized to be a value between 0-1.

$$\frac{\partial}{\partial x}(\mathbb{E}[f, x, p_s, \pi^+, \pi^-)]) = 0 \quad \Longrightarrow \quad x = F^{-1}(\frac{\pi^-}{\pi^+ + \pi^-}) \tag{5}$$

The full derivation is seen in appendix 11.1. F is the predictive distribution function of the function f.
This approach can be further investigated by simulation and visualisation across different bids and resulting penalties. To realize this, it is assumed that the realized power production follows a predictive distribution, here emulated by three different beta distributions:

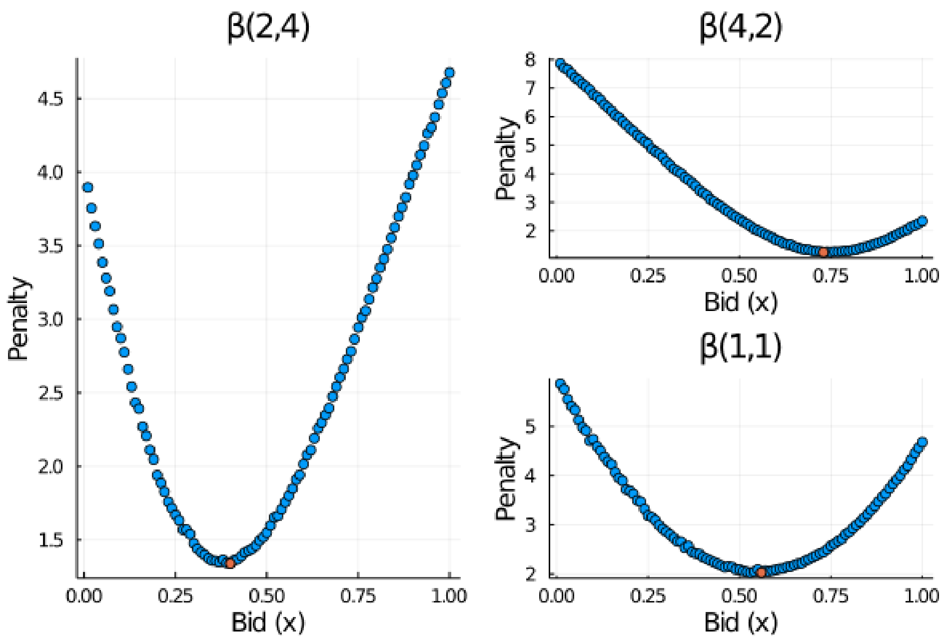

Figure 2: Minima of the problem across different distributions of ω

The runs are initialized using 10000 draws from three different beta distributions the $\beta(2,4)$, $\beta(4,2)$ and $\beta(1,1)$ distributions respectively. These distribution represent a predictive distribution function for the true distribution function f in equation (4). Here $\pi^+ = 12$ and $\pi^- = 7$, chosen arbitrarily.

Another approach to solving this problem would have been to create a Stochastic Linear Program (SLP), reminiscent of the one seen in (1) and developed in [3]. Given the previously defined details of the problem, this is defined as:

$$\begin{aligned} \min_{x} \quad & I = -p_s x + \frac{1}{N}\sum_{i=1}^{N}((p_s+\pi^+)y_{2,t} - (p_s - \pi^-)y_{3,t}) && \text{(6a)} \\ \text{s.t.} \quad & y_{1,t} + y_{3,t} = \omega_t \\ & y_{1,t} + y_{2,t} = x \\ & x, y_{1,t}, y_{2,t}, y_{3,t} \geq 0 \end{aligned}$$

It should be noted that when we are formulating the stochastic linear program for the problem, the here and now decision is the bid x, while the wait and see decisions $y_{1,t}$, $y_{2,t}$ and $y_{3,t}$ ensure that demand is fulfilled across any value of the realized electricity production ω_t for any time period t.

The solution to this initial SLP formulation becomes $I = 1.9$ and $x = 0.366$, which is seen to be slightly better than what we had from the analytical solution, when using $\beta(1,1)$ as the predictive distribution function.

5 Investigation of Price Taker Assumption.

In a desire to expand upon the previous model, the solution is saved as a constant and the problem is extended with a variable λ, that is seen as a capacity available for up/down regulation after bidding x. This variable is introduced in order to test if, when given the chance, the program would wish to change its bid by some increment if the original bid is set.

In practice, this would signify that the price taker assumption can be discarded. Thus, the model could be improved, if the penalty prices π are realized and the producer bids accordingly, thus changing the prices by a known value β. A new expression for the price is thus postulated as:

$$\alpha^+ = \pi^+ + \beta^+\lambda \tag{7a}$$

$$\alpha^- = \pi^- + \beta^-\lambda \tag{7b}$$

Where π are the balancing prices as defined before and β are the resulting changes in price after bidding x and changing with the increment λ. It is assumed that the connection between these prices can be emulated linearly. The expectation of penalty can be calculated as:

$$P^+ = \mathbb{E}_t[\omega_t - x_t + \lambda_t) \cdot \alpha^+] \,, \quad \forall \lambda_t > x_t \tag{8a}$$

$$P^- = \mathbb{E}_t[(x_t + \lambda_t - \omega_t) \cdot \alpha^-] \,, \quad \forall \lambda_t < x_t \tag{8b}$$

With lambda values ranging from -x to $(w_p - x)$ and 1000 draws from the β distributions, the penalties are simulated across all bids, as visualised in figure 3.

It is not possible to calculate an analytical solution in the same manner as in 4 given as the integral over all possible values of λ, due to complexity introduced:

$$I(x) = \mathbb{E}[f, \lambda, p_s, \alpha^+, \alpha^-, \lambda)] = \int_{-x}^{\lambda} ((\omega - x + \lambda)\alpha^+ f(\omega) d\omega + \int_{\lambda}^{(w_p - x)} ((x + \lambda - \omega)\alpha^-) f(\omega) d\omega \tag{9}$$

The derivation can be seen in appendix 11.2 The stochastic simulation used to find a numerical solution is created by taking the mean across bids from -x to (1-x) in steps of 0.01 using 1000 different realization of electricity production. The parameter values used are the same as in the previous section, and further $\beta^+ = 0.5$ and $\beta^- = 0.4$:

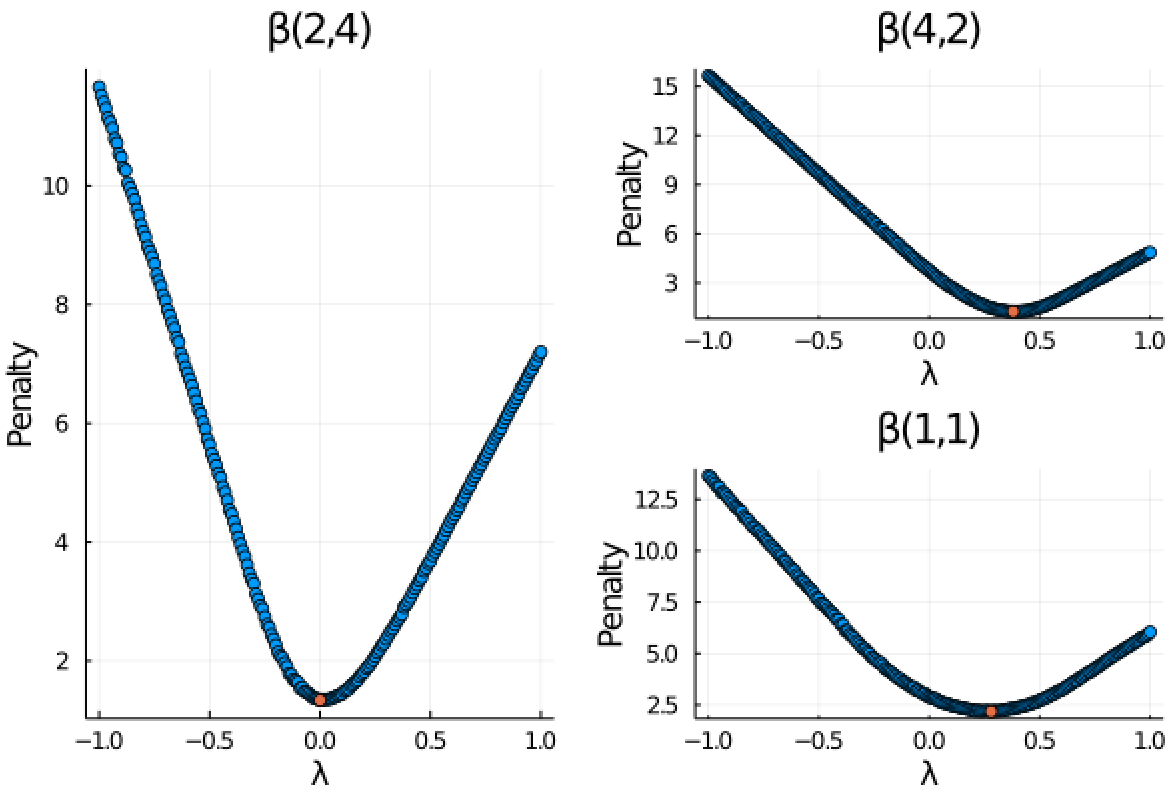

Figure 3: Visualised penalty when assuming that the bidder influences market price.

Every version of the run results in non-zero λ values, indicating that there could be a benefit to moving away from the price-taker formulation. The benefit does seem to vary depending on both distributions used and the increment of λ-values.

Distribution	$\beta(2,4)$	$\beta(4,2)$	$\beta(1,1)$
λ	0.028	0.398	0.248
Penalty	1.328	1.248	2.183

Table 1: λ values and corresponding penalties

The next step is to formulate this as an SLP. The variable β is introduced as the price of up or down regulation with λ, in in the stochastic simulation. In this program, x is the bid value found by solving equation (10) and is inserted as a constant.

$$\min_{\lambda} I = -p_s(x+\lambda)+\frac{1}{N}\sum_{i=1}^{N}((p_s+(\beta^+\lambda+\pi^+))y_{3,t}-(p_s-(\beta^-\lambda+\pi^-))y_{2,t}) \tag{10a}$$

$$\begin{aligned} \text{s.t.} \qquad & y_{1,t}+y_{3,t}=\omega_t \\ & y_{1,t}+y_{2,t}=(x+\lambda) \\ & \lambda, y_{1,t}, y_{2,t}, y_{3,t} \geq 0 \end{aligned}$$

This new formulation of the problem is harder to solve, as the multiplication of decision variables yields a non-convex problem. It is possible to solve the problem using a gurobi with significant time investment, and the found λ value closely corresponds to the value found in the stochastic simulation, corresponding to the value found in equation (10), if using the same realization values ω across the two.
As a result, it is realized that if the bidder has an impact on the balancing prices, the problem can be further optimized to account for this.

6 Initial Price Maker Formulation

In the previous section it was hinted that there may be a benefit for the bidder to move away from a price taker assumption, and in some way model that the bid they make has an influence on the market prices.
In order for the model to be able to account for the fact that its bid has an effect on the price, it is necessary to develop a function that can account for the effect our bid has across an aggregated market, with multiple players. This relationship is introduced by modeling the penalty prices as a linear function of the difference in bid and realization:

$$\alpha^+ = \pi^+ + \beta^+(\omega - x) \tag{11a}$$
$$\alpha^- = \pi^- + \beta^-(x - \omega) \tag{11b}$$

Thus the new penalties depending on bid can be seen as:

$$P^+ = \mathbb{E}_t[(\omega_t - x_t) \cdot \alpha_t^+], \quad \forall \omega_t > x_t \tag{12a}$$
$$P^- = \mathbb{E}_t[(x_t - \omega_t) \cdot \alpha_t^-], \quad \forall \omega_t < x_t \tag{12b}$$

Where α refers to equation (11). Using this, it is possible to once again simulate penalty outcomes for different bid values and distributions, using the same values for π and β as previously:

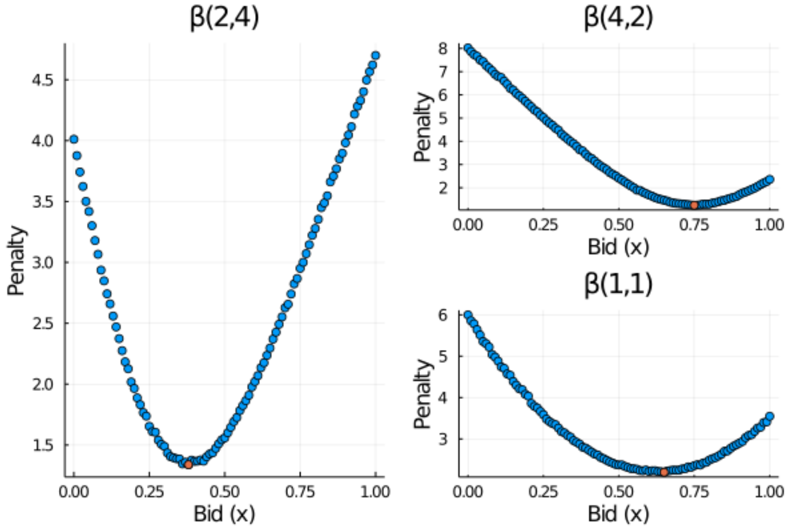

Figure 4: Third penalty Distribution

The results for the specific run are given in table 2.

	$\beta(2,4)$	$\beta(4,2)$	$\beta(1,1)$
Bid	0.38	0.75	0.65
Penalty	1.34	1.26	2.20

Table 2: Minimal penalty values and corresponding bids.

This new penalty function could also be seen as the integral across all possible bid values. The initial equation (2) is then expanded upon and now becomes:

$$\begin{aligned}\mathbb{E}[f, e_b, p_s, \pi^+, \pi^-, \beta^+, \beta^-)] = &\int_0^x (\pi^+ + \beta^+(\omega - x))(\omega - x) f(\omega) \partial\omega \\ &+ \int_x^1 (\pi^- + \beta^-(x - \omega))(x - \omega) f(\omega) \partial\omega\end{aligned} \tag{13}$$

In an attempt to find an analytical solution we once again differentiate and equate to zero, but there is no constant solution and the problem must be treated numerically. A derivation is seen in appendix 11.3. From the form of the solution, it does not seem as if there is a useful available constant minimum to the problem, meaning no analytical solution is readily available.

Since no analytical solution is available, it is of interest to expand the previous LP (10), with the newly formulated terms for price as a function of bid (11).

$$\begin{aligned}
\min_{x} \quad & I = -p_s x + \frac{1}{N}\sum_{i=1}^{N}((p_s + (\pi^+ + \beta^+ y_{2,t}))y_{2,t} - (p_s - (\pi^- + \beta^- y_{3,t}))y_{3,t}) && \text{(14a)}\\
\text{s.t.} \quad & y_{1,t} + y_{3,t} = \omega_t \\
& y_{1,t} + y_{2,t} = (x + \lambda) \\
& x, y_{1,t}, y_{2,t}, y_{3,t} \geq 0
\end{aligned}$$

In this form, the problem is quadratic and a quadratic solver is necessary. For this purpose, gurobi is used and the problem terminates with an objective value of I = 2.01, which corresponds to a bid of x = 0.399, when using $\beta^+ = 0.5$ and $\beta^- = 0.4$.

6.1 Expanding the Price Maker Model for Stochasticity

The next problem that would naturally occur when examining model (14) would be to realize that the balancing prices would be subject to uncertainty, as it is very unlikely to know exactly how the slope β would react to different bidders in the market. Thus, we introduce a final version of (14) that can account for this:

$$\begin{aligned}
\min_{x} \quad & I = -p_s x + \frac{1}{N}\sum_{i=1}^{N}((p_s + \sum_{s=1}^{S}\alpha_s^+)y_{2,t,s} - (p_s - \sum_{s=1}^{S}\alpha_s^-)y_{3,t,s}) && \text{(15a)}\\
\text{s.t.} \quad & y_{1,t} + y_{3,t} = \omega_t && \text{(15b)}\\
& y_{1,t} + y_{2,t} = x && \text{(15c)}\\
& \alpha^+ = \pi^+ + \tau_s \beta_s^+ y_{2,t} && \text{(15d)}\\
& \alpha^- = \pi^- + \tau_s \beta_s^- y_{3,t} && \text{(15e)}\\
& x, y_{1,t}, y_{2,t}, y_{3,t} \geq 0 && \text{(15f)}
\end{aligned}$$

This model can be seen as an adversarial price maker bidding model, that takes into account the total market bids (and thus clearing prices) as a function of its own bid, thus optimizing the bid across both different realizations of energy production, as well as different realizations of penalty costs that depend on what is bid and how other market participants react to that bid. The solution becomes x = 0.377 with an objective of 4.175.

7 Case Study

With the formulation of the final stochastic program (15) it is possible to compare solutions. The results of the second formulation (10) indicated that discarding the price taker assumption for the producer may add value to the bidding strategy. In order to further verify this, the solutions found in (6) and (14) are inserted into (15).

Solution	Bid value	Objective Value
Price Taker	0.362	4.180
Price maker (scenario independant)	0.399	4.184
Price maker (scenario specific)	0.377	4.175

Table 3: Overview over different solutions performance in the final problem formulation.

It is seen that the final solution does best in the context of the problem defined in (15), but that the price taker solution (6), actually outperforms the scenario in dependant price maker solution found in (14).
It is worth noting that the value of both the bid and the overall objective value are higher in the solution to (15).
The final outcomes of the different solutions are visualised in figure 5.

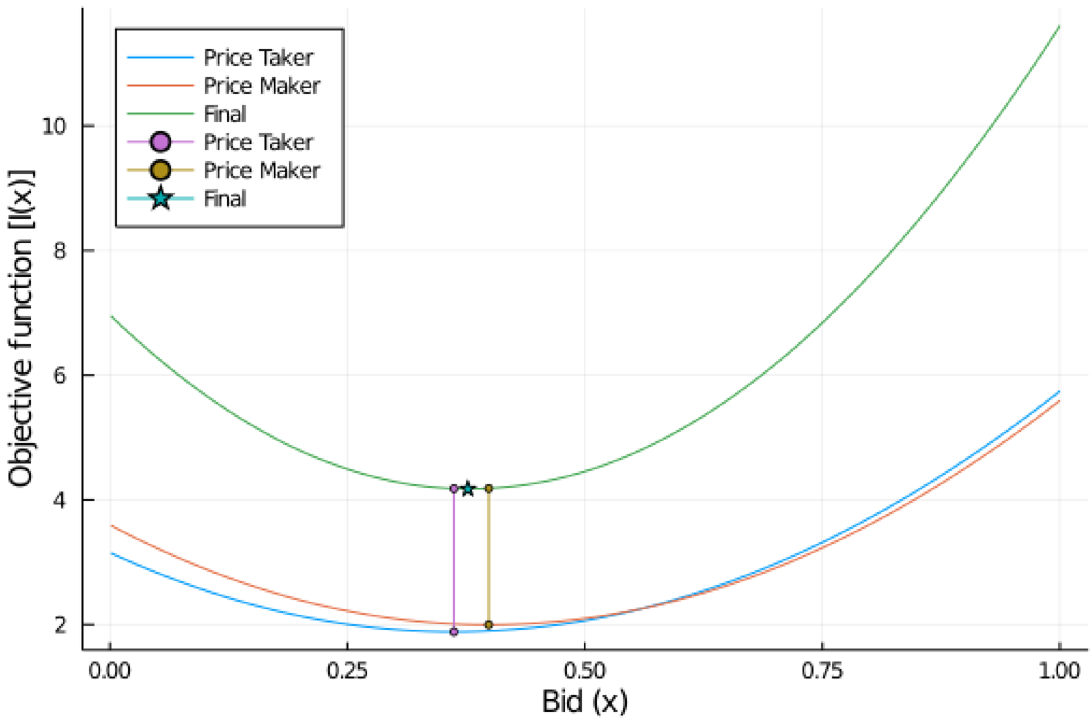

Figure 5: A view over the different solutions, across the three objective functions.

Here it is seen how the different bid values perform in the context of the their respective problem formulations, and the final formulation seen in equation (15). While the bids are close, and their respective minima do seem to lie in the area of the actual minima in the final problem formulation, the difference in a real world context might amount a large sum of money saved when bidding across the market.

8 Discussion

The proposition of a bidding strategy that moves away from price-taker assumptions will need thorough testing. Both in respect to defining the actual distributions that define the realization of electricity production, instead of the β-distributions used in this report, as well as actual values for the uncertainty in both penalty prices dependant on the bids. The $\beta^{+/-}$ and $\pi^{+/-}$ values that define the way we interpret the bid to effect the market may have very different values when estimated using real data. It could even be argued that the linear relationship introduced in equation (11) may not be representative of this correlation and a more complicated function may be necessary to accurately represent the connection between bids.
Testing all of these factors necessitates accurate data and study in an as close to real market setting as possible, which was outside the scope of this report.
When these investigations can be reported with accuracy, it will then be necessary to compare the solution offered by the last model (15) to the first (6), to ensure that the extra computational difficulty is worth the expenditure.

9 Conclusion

The initial bidding strategy for an electricity producer with an uncertain production realization was investigated. This was done both using stochastic simulation and by stochastic programming models, and further, an analytical solution to the problem was derived where applicable.
The program was then expanded upon in order to move away from the price taker assumption and to introduce further uncertainty in the penalty prices when bidding in the market. The models were in general seen to give realistic solutions and values when considering the simplified setup.
Solutions across the different versions of the problem were compared, in order to make sure that the final proposed stochastic quadratic program outperforms any previous solutions in the context of an adversarial price maker bidding in the market where both production realization and penalties are uncertain, which was seen to be the case.
It was concluded that for the models proposed here to be implemented in a real world setting, the distributions for both electricity production and pricing as a function of bid would have to be determined accurately, and vigorous testing and comparison of the programs would

have to be carried out in order to determine the value of the solutions. It can be concluded that the final model adds value to the solution with little added complexity in comparison to the initial model.

10 Future Work

As mentioned previously. the model needs testing.
If the desire is to further expand upon the models different things could be considered.
The model could be viewed as a part of a larger system where energy storage in some shape is available, thus reverting from the zero storage assumption from when the newsvendor problem was first introduced in (1).
Furthermore, the fact that there is more time in the intra day market to change the bid could be considered if the model is expanded with a time horizon beyond that of the different realizations x_t. Thus, the model could continually optimize the bid until gate closure dependant on a continuous stream of forecast information in the hours before operation.
Finally, from a practical point of view, it may be possible to expand the model to involve some kind of decomposition technique to help improve calculation times. This is especially important if the functions for adversarial bidding (11) are expanded upon and become more complex.

11 Apppendix

11.1 Derivation of First Analytical Solution

Initially we have:

$$\mathbb{E}[f, x, p_s, \pi^+, \pi^-)] = \int_0^x (\omega p_s - (\omega - x)\pi^+) f(\omega) d\omega + \int_x^{w_p} (\omega p_s - (x - \omega)\pi^- f(\omega) d\omega \quad (16)$$

In order to find the minima, the expressions derivative is found:

$$\begin{aligned} \frac{\partial}{\partial x} \mathbb{E}[f, x, p_s, \pi^+, \pi^-)] = \frac{\partial}{\partial x} \int_0^x (\omega p_s - (\omega - x)\pi^+) f(\omega) d\omega \\ + \frac{\partial}{\partial x} \int_x^{w_p} (\omega p_s - (x - \omega)\pi^- f(\omega) d\omega \end{aligned} \quad (17)$$

Using Leibnitz rule of integration [8]:

$$\frac{\partial}{\partial x}\mathbb{E}[f,x,p_s,\pi^+,\pi^-)] = \int_0^x \frac{\partial}{\partial x}(\omega p_s - (\omega - x)\pi^+)f(\omega)d\omega \\ + \int_x^{w_p} \frac{\partial}{\partial x}(\omega p_s - (x-\omega)\pi^- f(\omega)d\omega \\ = \int_0^x \pi^+ f(\omega)d\omega + \int_x^{w_p} -\pi^- f(x)d\omega \\ = \pi^+ F(x) - \pi^-(F(w_p) - F(x)) \tag{18}$$

Now setting the derivative to zero, and assuming that the power production from 0-w_p can be represented as a scalar from 0-1 (seen as a percentage of max capacity):

$$\frac{\partial}{\partial x}\mathbb{E}[f,x,p_s,\pi^+,\pi^-)] = 0 \\ \pi^+ F(x) - \pi^-(1 - F(x)) = 0 \longrightarrow \\ x = F^{-1}(\frac{\pi^-}{\pi^+ + \pi^-}) \tag{19}$$

11.2 Derivation of Second Analytical Solution

Initially we have:

$$\mathbb{E}[f,\lambda,\pi^+,\pi^-)] = \int_{-x}^{\lambda} ((\omega - x + \lambda)(\pi^+ + \beta^+\lambda)f(\omega)d\omega + \\ \int_{\lambda}^{w_p - x} ((x + \lambda - \omega)(\pi^- + \beta^-\lambda)f(\omega)d\omega \tag{20}$$

In order to find the minimum, the expressions derivative is found:

$$\frac{\partial}{\partial \lambda}\mathbb{E}[f,\lambda,\pi^+,\pi^-)] = \frac{\partial}{\partial \lambda}\int_{-x}^{\lambda} ((\omega - x + \lambda)(\pi^+ + \beta^+\lambda)f(\omega)d\omega \\ + \frac{\partial}{\partial \lambda}\int_{\lambda}^{w_p - x} ((x + \lambda - \omega)(\pi^- + \beta^-\lambda)f(\omega)d\omega \tag{21}$$

Using Leibnitz rule of integration [8]:

$$\frac{\partial}{\partial \lambda} \mathbb{E}[f, \lambda, \pi^+, \pi^-)] = \int_{-x}^{\lambda} \frac{\partial}{\partial \lambda}((\omega - x + \lambda)(\pi^+ + \beta^+ \lambda) f(\omega) d\omega$$
$$+ \int_{\lambda}^{w_p - x} \frac{\partial}{\partial \lambda}((x + \lambda - \omega)(\pi^- + \beta^- \lambda) f(\omega) d\omega$$
$$= \int_{-x}^{\lambda} (-\beta^+ \lambda - \pi^+ - (\omega - x + \lambda)\beta^+) f(\omega) d\omega$$
$$+ \int_{\lambda}^{w_p - x} (-\beta^- \lambda - \pi^- - (x + \lambda - \omega)\beta^-) d\omega \tag{22}$$

Calculating this integral results in (using integration by parts as defined in [9]:

$$\mathbb{E}[f, \lambda, \pi^+, \pi^-)] =$$
$$- 2F(\lambda)\beta^+\lambda - 2F(\lambda)\lambda + F(\lambda)\beta^+ x - F(\lambda)\pi^+ + F(-x)\beta^+\lambda$$
$$- F(-x)x - F(-x)\beta^+ x + F(-x)\pi^+ - 2F(w_p - x)\beta^-\lambda + \beta^- F(w_p - x)(w_p - x) - F(w_p$$
$$- x)\beta^- x - F(w_p - x)\pi^- + 2F(\lambda)\beta^-\lambda + F(\lambda)\beta^- x + F(\lambda)\pi^-$$
$$+ \beta^+ \int -F(-x)\mathrm{d}(-x) + \beta^- \int F(w_p - x)\mathrm{d}(w_p - x)$$

We set this equal to zero and attempt to isolate for the value of λ, but this does not yield a readily available constant due to the integrals in the expression.

11.3 Derivation of Third Analytical Solution

Initially:

$$\mathbb{E}[f, \lambda, \pi^+, \pi^-)] = \int_0^x ((\omega - x)(\pi^+ + \beta^+(\omega - x)) f(\omega) d\omega$$
$$+ \int_x^1 ((x - \omega)(\pi^- + \beta^-(x - \omega)) f(\omega) d\omega \tag{23}$$

As before we differentiate and attempt to find a minima:

$$\frac{\partial}{\partial x} \mathbb{E}[f, \lambda, \pi^+, \pi^-)] = \frac{\partial}{\partial x} \int_0^x ((\omega - x)(\pi^+ + \beta^+(\omega - x)) f(\omega) d\omega$$
$$+ \int_x^1 ((x - \omega)(\pi^- + \beta^-(x - \omega)) f(\omega) d\omega \tag{24}$$

Using Leibniz rule:

$$\begin{aligned}\frac{\partial}{\partial x}\mathbb{E}[f,\lambda,\pi^+,\pi^-)] &= \frac{\partial}{\partial x}\int_0^x ((\omega-x)(\pi^+ + \beta^+(\omega-x))f(\omega)d\omega \\ &+ \frac{\partial}{\partial x}\int_x^1 ((x-\omega)(\pi^- + \beta^-(x-\omega))f(\omega)d\omega \\ &= \int_0^x \frac{\partial}{\partial x}((\omega-x)(\pi^+ + \beta^+(\omega-x))f(\omega)d\omega \\ &+ \int_x^1 \frac{\partial}{\partial x}((x-\omega)(\pi^- + \beta^-(x-\omega))f(\omega)d\omega\end{aligned} \tag{25}$$

Which yields:

$$\begin{aligned}\frac{\partial}{\partial x}\mathbb{E}[f,\lambda,\pi^+,\pi^-)] &= \int_0^x -2\beta^+ f(\omega)\omega + 2\beta^+ f(\omega)x - f(\omega)\pi^+ d\omega \\ &+ \int_x^1 -2\beta^- f(\omega)\omega + 2\beta^- f(\omega)x + f(\omega)\pi^- d\omega\end{aligned} \tag{26}$$

These expressions are integrated and yield (using integration by parts as defined in [9]):

$$\begin{aligned}\frac{\partial}{\partial x}\mathbb{E}[f,\lambda,\pi^+,\pi^-)] =& -2\beta^+(F(x)x - \int F(x)dx) + 2\beta^+F(x) - F(x)\pi^+ + 2\beta^- \\ &+ \pi^- - 2\beta^-(F(x)x - \int F(x)dx) + 2\beta^-F(x)x + F(x)\pi^-\end{aligned} \tag{27}$$

The expression is set equal to zero and isolated for x, but it is apparent that there will be no constant solution to the equation and so the hope of finding a usable analytical minima is discarded.

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