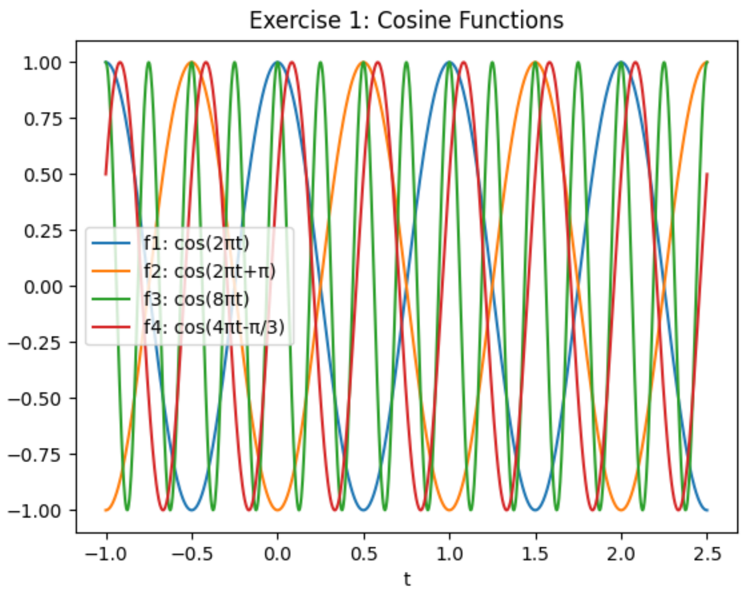
Ex01

1

a)



b)

Top left: Frequency = 2, phase = 0, amplitude = 2

$$\Rightarrow \underline{\underline{f(t) = 2\cos(2\pi \cdot 2 \cdot t + 0)}}$$

Top right: Freq = 4, phase = π, A = 1

$$\Rightarrow \underline{\underline{f(t) = \cos(2\pi \cdot 4 \cdot t + \pi)}}$$

Bottom left: Freq = 1, phase = $\frac{\pi}{2}$, A = 1

$$\Rightarrow \underline{\underline{f(t) = \cos(2\pi \cdot 1 \cdot t + \frac{\pi}{2})}}$$

Bottom right: Freq = 0.5, phase = $-\frac{\pi}{4}$, A = 1

$$\Rightarrow f(t) = \cos(2\pi \cdot 0.5 \cdot t - \pi/4)$$

(2)

a)

1. $\cos(0.5n + \pi/2)$

$\Rightarrow \cos(0.5n + \pi/2) = \cos(0.5(n+N) + \pi/2)$

$\Rightarrow \cos(0.5n + \frac{\pi}{2}) = \cos(0.5n + 0.5N + \pi/2)$

Så hvis $0.5N = 2\pi k$ for $k \in \mathbb{Z}$. Dette går ikke da $N = 4\pi k$ for at denne likningen skal gå opp. <u>Ikke periodisk</u>

2. $\cos(\pi n + \pi/2)$

$\Rightarrow \pi N = 2\pi k \Rightarrow N = 2k \Rightarrow \underline{N=2}$

<u>Periodisk</u>

3. $\cos\left(\frac{\sqrt{2}}{2}\pi n\right)$

$\Rightarrow \frac{\sqrt{2}}{2}\pi N = 2\pi k \Rightarrow N = \frac{4}{\sqrt{2}} k$

<u>Ikke periodisk</u>

(2)

b)

$$f(t) = \cos\left(2\pi \cdot \frac{1}{T} \cdot t\right)$$

$$\Rightarrow x[n] = f\left(t = \frac{nT}{10}\right) = \cos\left(2\pi \cdot \frac{1}{T} \cdot \frac{nT}{10}\right)$$

5 samples per half period equals 10 samples per period!

$$\Rightarrow \underline{\underline{x[n] = \cos\left(2\pi \frac{n}{10}\right)}}$$

c) $f(t) = \cos(t)$

$$\Rightarrow x[n] = f(t=n) = \underline{\underline{\cos(n)}}$$

(3)

1. $\underline{\underline{z^* = re^{-j\theta}}}$

2. $zz^* = re^{+j\varphi} re^{-j\varphi} = r^2 e^{j\varphi - j\varphi} = \underline{\underline{r^2}}$

$$\Rightarrow zz^* = r^2 + j \cdot 0 = \underline{\underline{r^2}}$$

3. $\underline{\underline{z^k = re^{j\varphi k}}}$

4. $z + z^* = re^{j\varphi} + re^{-j\varphi} = \underline{\underline{r(e^{j\theta} + e^{-j\theta})}}$

$$= a + jb + a - jb = \underline{\underline{2a}}$$

5. $z - z^* = r(e^{j\theta} - e^{-j\theta}) = \underline{\underline{rj\sin\theta}}$

$$z - z^* = a + jb - a + jb = \underline{\underline{2jb}}$$

6. $z^{-1} = \frac{1}{re^{j\varphi}}$, $z^{-1} = \frac{1}{a+jb}$

Ok, leste "Observe" nå, det blir jo da litt mer komplisert:

$\Rightarrow se^{j\theta} = \frac{1}{re^{j\varphi}} \Rightarrow s = \frac{1}{r}$, $\theta = -\varphi \Rightarrow \frac{1}{r} \cdot e^{-j\varphi} = z^{-1}$

$\Rightarrow \frac{1}{(a+jb)} \cdot \frac{a-jb}{(a-jb)} = \frac{a-jb}{a^2+b^2} = z^{-1}$

7.

$z + z^* = r(e^{j\varphi} + e^{-j\varphi}) = 2r\cos\varphi \Rightarrow \cos\varphi = \frac{e^{j\varphi} + e^{-j\varphi}}{2}$

$z - z^* = r(e^{j\varphi} - e^{-j\varphi}) = 2rj\sin\varphi \Rightarrow \sin\varphi = \frac{e^{j\varphi} - e^{-j\varphi}}{2}$

8. $z^{-1} = \frac{1}{z} = \frac{1}{re^{j\varphi}} \neq z^* = re^{-j\varphi}$

$z^{-1} = \frac{1}{re^{j\varphi}} = \frac{1}{r} e^{-j\varphi} = \frac{1}{r^2} z^* \Rightarrow z^{-1} = \frac{1}{r^2} z^*$

$|z|^2 = (\sqrt{zz^*})^2 = \sqrt{re^{j\varphi} re^{-j\varphi}}^2 = \sqrt{r^2}^2 = r^2 \Rightarrow z^{-1} = \frac{z^*}{|z|^2}$

b)

1. $\underline{\underline{-1 = -e^{j2\pi k}}}$

2. $(-1)^k = (-e^{j2\pi k})^k = (-1)^k e^{j2\pi k}$

Hette dette ledet ingen vei, men...

$(-1)^k = e^{j\pi k} = \cos(\pi k) + j\sin(\pi k) = \cos(\pi k) = (-1)^k$

$\Rightarrow$ Altså $\underline{\underline{(-1)^k = e^{j\pi k}}}$

3. $j^k = e^{j\frac{\pi}{2}k} = \cos(\frac{\pi}{2}k) + j\sin(\frac{\pi}{2}k) = j\sin(\frac{\pi}{2}k)$

$\Rightarrow$ $\underline{\underline{j^k = j\sin(\frac{\pi}{2}k) = e^{j\frac{\pi}{2}k}}}$

4

a) 1) $|3+j4| = \sqrt{3^2+4^2} = \underline{\underline{5}}$

2) $\frac{1}{3+j4} \cdot \frac{3-j4}{3-j4} = \frac{3-j4}{(3+j4)(3-j4)}$

$= \frac{3-j4}{25} = \underline{\underline{\frac{3}{25} - j\frac{4}{25}}}$

3) $\frac{1+j2}{1+e^{j\pi/2}} = \frac{1+j2}{1+j} \cdot \frac{1-j}{1-j} = \frac{(1+j2)(1-j)}{(1+j)(1-j)} = \frac{1-j+2j-j^2}{1+j-j-j^2} \Rightarrow$

$= \frac{1+j+1}{1+1} = \frac{2+j}{2} = \underline{\underline{1+\frac{1}{2}j}}$

a) 4

$$(-1)^n + e^{j\pi n} = (-1)^n + (-1)^n = \underline{\underline{2(-1)^n}}$$

b)

$$(\cos\theta + j\sin\theta)^n = (e^{j\theta})^n = e^{j\theta n} = \underline{\underline{\cos(\theta n) + j\sin(\theta n)}}$$