$$|6(x-5)| < \epsilon \Rightarrow |6x - 30 + 9 - 9| < \epsilon$$

$$\Rightarrow |6x + 9 - 39| < \epsilon$$

$$\therefore \lim_{x \to 5} 6x + 9 = 39$$

2) Demostrar por definición que: $\lim_{x \to 3} x^2 = 9$

S/

$$\lim_{x \to 3} x^2 = 9 \Leftrightarrow \forall \epsilon > 0, \epsilon \in R, \exists : \delta > 0, \delta \in R, \delta = \delta$$

$$\forall x: 0 < |x-3| < \delta \Rightarrow |x^2-9| < \epsilon$$

Se parte de:

$$|x^2-9| < \epsilon \Rightarrow |(x+3).(x-3)| < \epsilon$$

$$\therefore |x+3|.|x-3| < \epsilon \quad (1)$$

Se considera, por ejemplo: $\delta_1 = 1$

En consecuencia:

$$0 < |x-3| < \delta_1 = 1$$

$$|x-3| < 1$$

$$-1 < x-3 < 1$$

$$-1+6 < x-3+6 < 1+6$$

$$5 < x+3 < 7$$

Tomando: $x+3 < 7$, para relacionarlo

resulta: $|x+3| < 7$, luego:

$$|x+3||x-3| < 7|x-3| < \epsilon$$

$$\therefore |x+3||x-3| < |x-3| < \frac{\epsilon}{7}$$

Tomando: $\delta = Min\left(\delta_1, \frac{\epsilon}{7}\right) = Min\left(1, \frac{\epsilon}{7}\right) \Rightarrow$

$$0 < |x-3| < \delta \Rightarrow |x^2-9| = |x-3||x+3|$$

__Propiedades:__

① $\lim_{x \to a} f(x) = L \Leftrightarrow \lim_{x \to a} [f(x) - L] = 0$

② si $\exists$: $\lim_{x \to a} f(x) = L \Rightarrow$ Este valor es único.