29/04/21

Ej. Ⓐ:

$$\int \ln(x)\,dx \Rightarrow \text{sea: } u = \ln(x) \Rightarrow du = \frac{1}{x}\,dx$$
$$dv = dx \Rightarrow v = \int dx = x$$

Entonces:
$$\int \ln(x)\,dx = x.\ln(x) - \int \cancel{x}.\frac{1}{\cancel{x}}\,dx$$
$$= x.\ln(x) - \int dx$$
$$= x.\ln(x) - x + C$$
$$\therefore \boxed{\int \ln(x)\,dx = x\left(\ln(x) - 1\right) + C}$$

Comprobación:
$$\left(x.(\ln(x) - 1) + C\right)' = \left[x.(\ln(x) - 1)\right]' + \underbrace{C'}_{=0}$$
$$= 1(\ln(x) - 1) + x.\frac{1}{x}$$
$$= \ln(x) \cancel{- 1} \cancel{+ 1}$$

Ej. Ⓒ: Integrales cíclicas

$$\int e^x.\operatorname{sen}(x)\,dx \Rightarrow u = e^x \Rightarrow du = e^x.dx$$
$$dv = \operatorname{sen}(x)\,dx \Rightarrow v = \int \operatorname{sen}(x)\,dx$$
$$\therefore v = -\cos(x)$$

$$\therefore \int e^x.\operatorname{sen}(x)\,dx = -e^x.\cos(x) - \int (-\cos(x)).e^x\,dx$$
$$= -e^x.\cos(x) + \underbrace{\int \cos(x).e^x\,dx}_{Ⓐ}$$

En: Ⓐ $p = e^x \Rightarrow dp = e^x dx$
$$dq = \cos(x)\,dx \Rightarrow q = \int \cos(x)\,dx = \operatorname{sen}(x)$$
$$\therefore \int \cos(x)e^x\,dx = \underbrace{e^x.\operatorname{sen}(x) - \int \operatorname{sen}(x).e^x\,dx}_{Ⓐ'}$$

Reempl. Ⓐ' en Ⓐ:
$$\int e^x \operatorname{sen}(x)\,dx = -e^x.\cos(x) + e^x.\operatorname{sen}(x) - \int e^x.\operatorname{sen}(x)\,dx$$
$$\therefore 2\int e^x.\operatorname{sen}(x)\,dx = e^x.\left(\operatorname{sen}(x) - \cos(x)\right)$$
$$\Rightarrow \boxed{\int e^x.\operatorname{sen}(x)\,dx = \frac{e^x}{2}\left(\operatorname{sen}(x) - \cos(x)\right) + C}$$