# Equivalence Principle in Chameleon Models (based in arXiv: 1511.06307 to appear in PRD)

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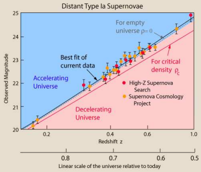
# Outline

- Motivation
- Experiments designed to test the Weak Equivalence Principle
- Introduction to the chameleon model
- The model and the one body solution
- The two body problem
- Predictions for the Eötvös parameter
- The energy criterion and comparison with other results
- Conclusions and work in progress

# Motivation





- In 1998 two independent research teams (Supernova Cosmology Project and High-Z SN search) reported strong evidence that the late universe expansion is accelerating.
- Data from the Cosmic Microwave Background and Large Scale Structure confirm the late-time acceleration of the universe.
- This resulted in a modification of the standard cosmological model: The $\Lambda_{CDM}$ model where a constant is added to Einstein's equation.

# Alternative models to explain the late-time acceleration of the universe:

- Scalar Fields minimally coupled to matter and gravity
  - quintessence
  - k-essence
- Scalar Fields with non-minimal coupling to matter:
  - dilatons, symmetrons: coupling to matter depends on the environment
  - chameleons: *thin shell* effect; coupling to matter depends on the mass $m(\phi)$.
  - galileons, beyond Horndesky: Vainstein Mechanism
- Alternative theories of gravity: f(R), massive gravity

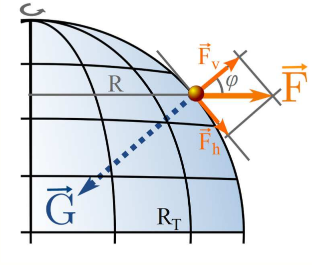
# Motivation

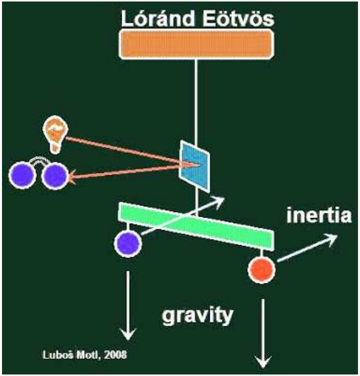
- Low energy limit of string theory
  - JHEP 1103 061 (2011)
- Conexion with $f(R)$ theories.
  - Phys.Rev. D78, 104021 (2008), Phys.Rev.D80:104002 (2009)
- Possible Variation of fundamental constants.

# The Weak Equivalence Principle (WEP)

- The principle of equivalence has historically played an important role in the development of gravitation theory, being a cornerstone of Newtonian Mechanics and Einstein's General Relativity (RG)
- One elementary equivalence principle is the kind Newton had in mind when he stated that the property of a body called "mass" is proportional to the "weight"
- An alternative statement of WEP is that the trajectory of a freely falling "test" body is independent of its internal structure and composition.
- In the simplest case of dropping two different bodies in a gravitational field, WEP states that the bodies fall with the same acceleration (this is often termed the Universality of Free Fall (UFF)) which can be tested directly.
- It can be regarded as implying the universal coupling between gravitation and matter.

# The Eötvös experiment: the torsion balance



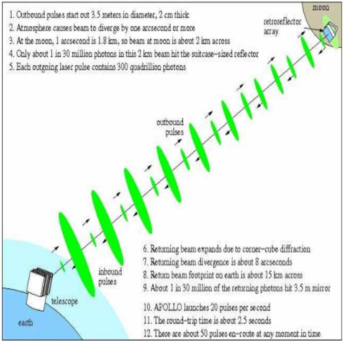




$$\tau \sim \eta = 2\frac{|a_1 - a_2|}{a_1 + a_2} = 2\frac{\frac{m_{g2}}{m_{i2}} - \frac{m_{g1}}{m_{i1}}}{\frac{m_{g2}}{m_{i2}} + \frac{m_{g1}}{m_{i1}}}$$

$$\vec{a}_i = \vec{a}_{i\varphi} + \vec{g}$$

# The Lunar Laser Ranging (LLR) experiment

- Reflectors placed on the lunar surface allow the measurement of the round-trip travel time of short pulses of laser light, and thus set limits on the differential acceleration of the Earth-Moon system in free fall towards the Sun.



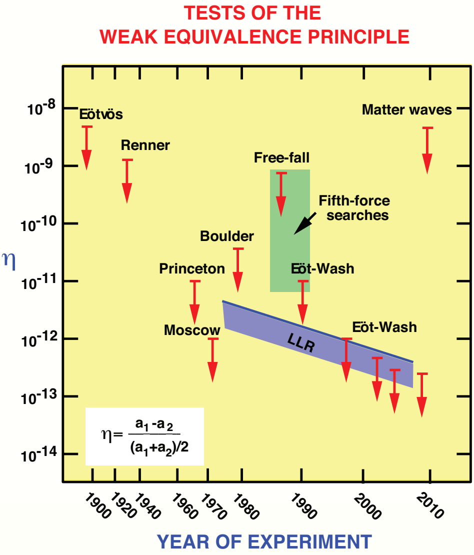

# The Lunar Laser Ranging (LLR) experiment

Nordtvedt (1968) found a radial perturbation of the Moon's orbit

$$\Delta r = S[(m_G/m_I)_{\mathrm{E}} - (m_G/m_I)_{\mathrm{M}}] \cos \Omega t, \tag{1}$$

where $\Omega = \omega_M - \omega_s$ and $S$ is a scaling factor of about $-2.9 \times 10^{13}$ mm. If the EP is violated, the lunar orbit will be displaced along the Earth-Sun line, producing a range signature having a 29.53 day synodic period.

# Experimental constraints on WEP



# Introduction to the chameleon model

- The chameleon model was proposed in 2004 by J.Khoury and A.Weltmann .
  - Phys.Rev.Lett. 93 171104 (2004), Phys.Rev. D69 044026 (2004)
- Further developments were made by Brax et al, Mota & Shaw, Hui, Nicolis and Stubbs
  - Phys.Rev.D82 083503 (2010) Phys.Rev.D75 063501,2007, Phys.Rev.D80 104002,2009

# Introduction to the chameleon model

- The key ingredient of the model is the coupling between matter and the scalar field.
- The *thin shell* effect prevents the model from violation of the WEP: the mass of the fluctuations depends on $\phi$ and thus the fifth force is screened in high density environments.

# Experiments used to constrain the chameleon model parameters

- WEP: torsion balance, torsion pendulum, Lunar Laser Ranging
  - Phys.Rev. D86, 102003 (2012)
- Atomic interferometry: cesium matter-wave interferometer near a spherical mass in a ultra-high vacuum chamber
  - Science 349, 849 (2015), Phys.Rev.D93, 104036 (2016), Phys.Rev. D94,044051 (2016)
- Ultra cold Neutron Bouncing: resonance spectroscopy measurements of quantum states of ultra-cold neutrons
  - Phys.Rev. Lett 112, 151105 (2014)
- Combinations of atomic precision measurements of electric and muonic hydrogen
  - Phys.Rev D83, 035020 (2011)

# The chameleon model

The model involves a scalar-field $\varphi$ and the action can be written as:

$$
\begin{aligned}
S[g_{\mu\nu}, \Psi_m^{(i)}, \varphi] &= M_{pl}^2 \int d^4x \sqrt{-g} \left[ \frac{R}{2} - \frac{1}{2} g^{\mu\nu} (\nabla_\mu \varphi)(\nabla_\nu \varphi) - V(\varphi) \right] \\
&\quad - \int d^4x L_m \left( \Psi_m^{(i)}, g_{\mu\nu}^{(i)} \right),
\end{aligned}
$$

where $M_{pl}$ is the reduced Planck mass, $R$ is the Ricci scalar associated with $g_{\mu\nu}$, and $\Psi_m^{(i)}$ represents the different matter fields. The scalar field $\varphi$ couples *non-minimally* to the matter through a conformal factor that relates each metric $g_{\mu\nu}^{(i)}$ with the Einstein metric $g_{\mu\nu}$:

$$g_{\mu\nu}^{(i)} = \exp\left[ \frac{2\beta_i \varphi}{M_{pl}} \right] g_{\mu\nu}.$$

# The chameleon model

We consider the potential for the chameleon field:

$$V(\varphi) = \lambda M^{4+n} \varphi^{-n},$$

where $M$ is a constant, and $n$ is a free parameter that can be taken to be either a positive integer or a negative even integer and $\lambda = 1$ for all values of $n$ except when $n = -4$ where $\lambda = \frac{1}{4!}$.
The equation of motion for the the chameleon $\varphi$ is

$$\Box\varphi = \frac{\partial V_{\text{eff}}}{\partial \varphi},$$

where $V_{\text{eff}}$ represents the effective potential defined by:

$$V_{\text{eff}} = V(\varphi) - T^m \frac{\beta \varphi}{M_{pl}},$$

For a perfect fluid model, $T^m = -\rho + 3P$.

# The one body problem

Let us consider a spherically-symmetric and *homogeneous* body of radius $R$ and density $\rho_{\rm in}$ immersed in an external medium of density $\rho_{\rm out}$.

$$\rho = \begin{cases} \rho_{\rm in} & r \leq R \\ \\ \rho_{\rm out} & r > R \end{cases} ,$$

where $R$ is the radius of the body.

# Approximations to the effective potential

There are two regimes analyzed in the literature:

- *Quadratic* approximation to the effective potential

$$V_{\rm eff}^{\rm in,out}(\varphi) \simeq V_{\rm eff}^{\rm in,out}(\varphi_{\rm min}^{\rm in,out}) + \frac{1}{2}\partial_{\varphi\varphi}V_{\rm eff}^{\rm in,out}(\varphi_{\rm min}^{\rm in,out})[\varphi - \varphi_{\rm min}^{\rm in,out}]^2,$$

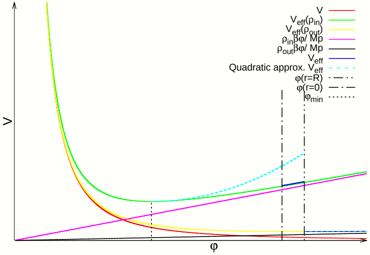
and therefore the effective mass of the chameleon is defined :

$$m_{\rm eff}^{2\rm in,out}(\varphi_{\rm min}^{\rm in,out}, \beta_i, \rho^{\rm in,out}) = n\,(n+1)\,M^2 \left(\frac{\beta\rho_{\rm in,out}}{nM_{\rm Pl}M^3}\right)^{\frac{n+2}{n+1}}$$

- *Linear* approximation to the effective potential

$$V_{\rm eff}^{\rm in,out}(\varphi) \simeq \frac{\beta\varphi\rho^{\rm in,out}}{M_{pl}}$$

In all cases, the *quadratic* approximation is used outside the body, the diference betwen the *thin shell* and the *thick shell case* is the aproximation of the potential inside the body



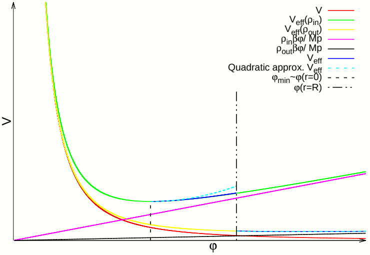




FIGURE: Effective potential inside and outside the large body. The light blue line shows the quadratic approximation Left: The *thick shell* case; Right: The *thin shell* case.

# The solution for the one body problem: *The thin shell* case

Inside the body the field is constant except for a *thin shell* near the radius where the *linear* approximation is considered:

$$\varphi = \begin{cases} \varphi_c := \varphi_{\min}^{\rm in} & 0 \leq r \leq R_{\rm S} \\ \varphi_{\rm s}(r) & R_{\rm S} \leq r \leq R_{\rm S} + \Delta R = R \\ \varphi_{\rm thin}^{\rm out}(r) & R \leq r \end{cases}$$

Here

$$\frac{\Delta R}{R} = -\frac{(\varphi_c - \varphi_\infty)}{6\beta M_{pl}\Phi_N} \ ,$$

where $\Phi_N = G\mathcal{M}/R$ stands for the Newtonian potential at the surface of the body, $\mathcal{M} = 4\pi\rho^{\rm in}R^3/3$ is the mass of the body and $\phi_\infty = \varphi_{\min}^{\rm out}$.

# The solution for the one body problem: *The thin shell* case

Imposing continuity for the field and its derivative at $r = R$, the solution reads:

$$\varphi_s(r) = \frac{\beta\rho^{\mathrm{in}}}{6M_{pl}}\left(r^2 + \frac{2R_{\mathrm{S}}^3}{r}\right) - \frac{\beta\rho^{\mathrm{in}}}{2M_{pl}}R_{\mathrm{S}}^2 + \varphi_c \qquad (R_{\mathrm{S}} \leq r \leq R)$$

$$\varphi_{\mathrm{thin}}^{\mathrm{out}}(r) \approx -\frac{\beta}{4\pi M_{pl}}\frac{3\Delta R}{R}\mathcal{M}\frac{e^{-m_{\mathrm{eff}}^{\mathrm{out}}r}}{r} + \varphi_\infty \qquad (R \leq r)$$

Since in this approximation $\frac{\Delta R}{R} << 1$, the chameleon field outside the body is strongly suppressed.This is what is usually named as the *thin shell* effect.

# The solution for the one body problem: *The thick shell* case $R_s \to 0$

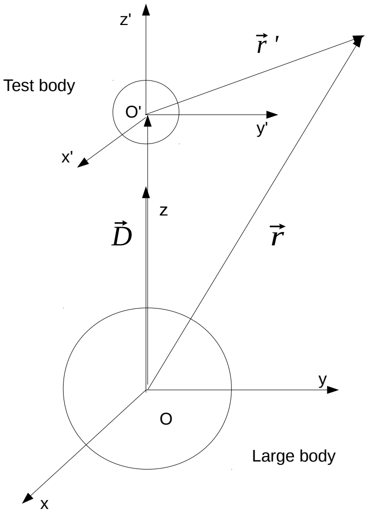
Inside the body, the *linear* approximation for the potential is considered

$$
\begin{aligned}
\varphi_{\rm in}(r) &= \frac{\beta\rho^{\rm in}}{6M_{pl}}r^2 + \varphi_0 \qquad (0 \le r \le R) \ , \\
\varphi^{\rm out}_{\rm thick}(r) &= -\frac{\beta\mathcal{M}}{4\pi M_{pl}}\frac{e^{-m^{\rm out}_{\rm eff} r}}{r} + \varphi_\infty \qquad (R \le r) \ .
\end{aligned}
$$

where $\mathcal{M}$ refers to the mass of the body.

# Differences between the standard approach and our proposal

- In the standard approach the one body problem is calculated.
- The force is computed using hand-waving arguments.
- In our proposal we compute the two body problem and calculate the force from first principles.
- In this work we use always the *quadratic* approximation to the effective potential. We develop an energy criterion to determine for which value of the model parameters this approximation holds.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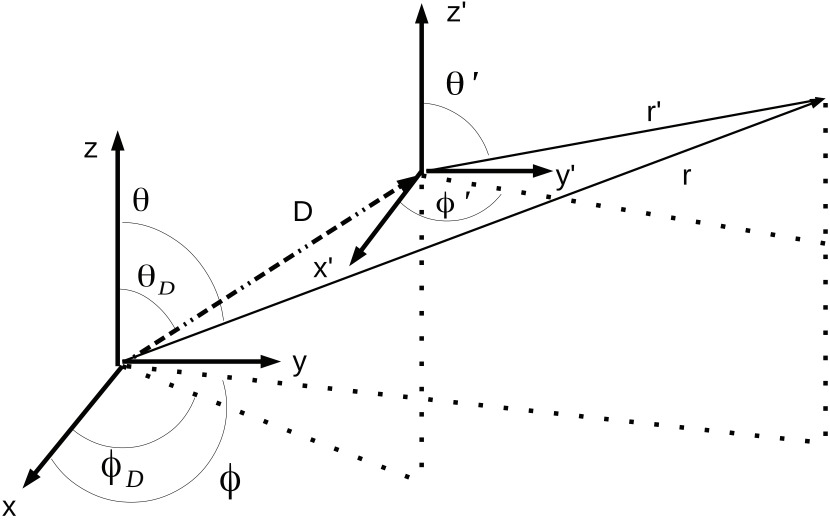
$\vec{r}\,'$
Test body
O'
y'
x'
z
$\vec{D}$
$\vec{r}$
y
O
Large body
x

# Our proposal to the two body problem arXiv:1511.06307

We expand the most general solution in complete sets of solutions of the differential equation in the inside and outside regions determined by the two bodies:

$$\varphi = \begin{cases} \varphi_{\text{in1}} = \sum_{lm} C_{lm}^{\text{in1}} i_l(\mu_1 r) Y_{lm}(\theta,\phi) + \phi_{\text{1min}}^{\text{in}} & (r \leq R_1) \\ \varphi_{\text{out}} = \sum_{lm} C_{lm}^{\text{out1}} k_l(\mu_{\text{out}} r) Y_{lm}(\theta,\phi) + C_{lm}^{\text{out2}} k_l(\mu_{\text{out}} r') Y_{lm}(\theta',\phi') \\ \qquad +\varphi_\infty \qquad (\text{exterior solution}) \\ \varphi_{\text{in2}} = \sum_{lm} C_{lm}^{\text{in2}} i_l(\mu_2 r') Y_{lm}(\theta',\phi') + \varphi_{\text{2min}}^{\text{in}} & (r' \leq R_2) \end{cases}$$

where $\mu_1 = m_{\text{eff}}^{\text{large body}}$, $\mu_{\text{out}} = m_{\text{eff}}^{\text{out}}$, $\mu_2 = m_{\text{eff}}^{\text{test body}}$ and $R_1, R_2$ are the radii of the *large* and *test* bodies, respectively, and $i_l$ and $k_l$ are Modified Spherical Bessel Functions (MSBF).


z'
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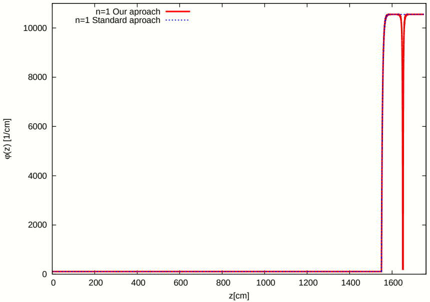
# Comparison of results between the standard approach and our solution



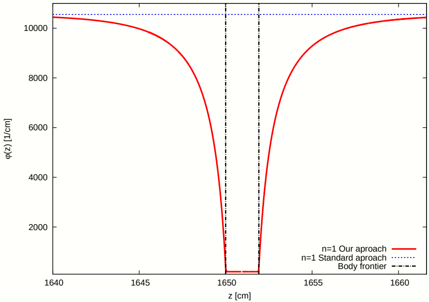
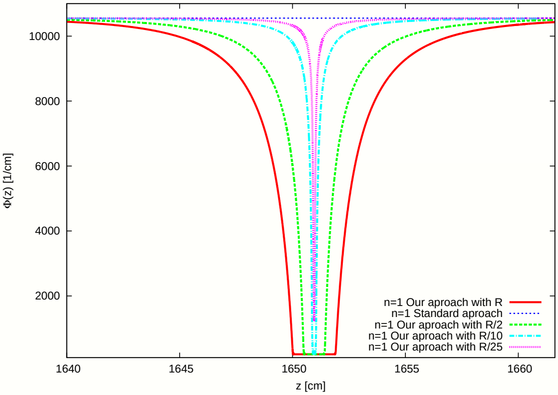




Figure: Blue: the standard approach, Red: our solution to the two body problem

# The two body problem for $R \to 0$

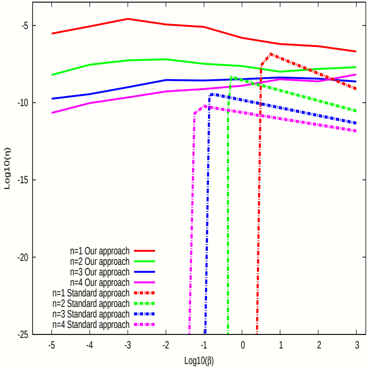


# Calculation of the Eötvös parameter

- We compute the energy using the solution for the chameleon field for the two body problem
- We compute the chameleon mediated force using $F_{z\varphi} = -\frac{\partial U}{\partial D}$, where $D$ is the distance between the center of the two bodies.
- We calculate the Eötvös parameter $\eta = 2\frac{|a_1 - a_2|}{a_1 + a_2}$

**Large body**: a hillside of 28 m located close to the laboratory
$\rho_{\rm hill} \simeq 9.27 \text{ g cm}^{-3}$
**Test bodies**: $m_{\rm Be} = m_{\rm Al} \approx 10 \text{ g}$ and $\rho_{\rm Be} \simeq 1.85 \text{ g cm}^{-3}$,
$\rho_{\rm Al} \simeq 2.70 \text{ g cm}^{-3}$

# Results for the torsion balance



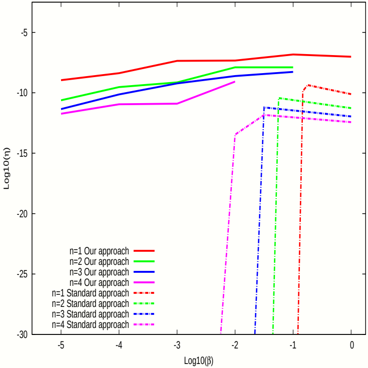
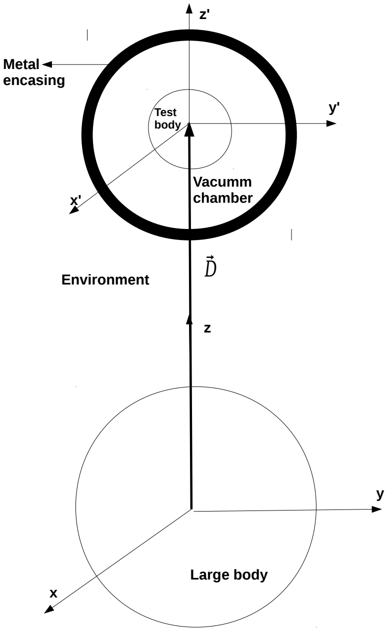




Figure: $M = 2.4 \times 10^{-3}\mathrm{eV}$. Left: $\rho_{\mathrm{out}} = 10^{-7}\mathrm{g\ cm}^{-3}$; Right: $\rho_{\mathrm{out}} = 10^{-3}\mathrm{g\ cm}^{-3}$. The predictions computed by Khoury & Weltman are also included and labeled as the *standard approach*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chamber
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Environment
z
y
Large body
x

# Results including the metal shell

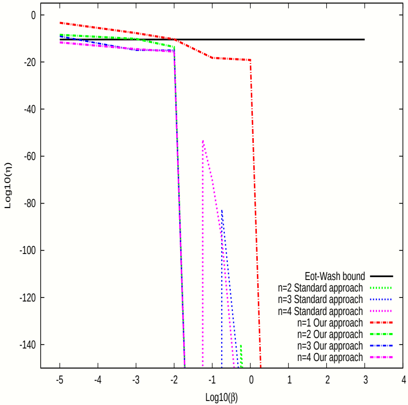



Figure: $\rho_{\rm out} = 10^{-7}{\rm g\ cm}^{-3}$. Also shown are the predictions computed by Khoury & Weltman with the effect of the metal encasing modeled by multiplying their estimates of $\eta$ by the factor $\mathrm{sech}(2\mathrm{m_{shell}d})$.

# Results including the metal shell

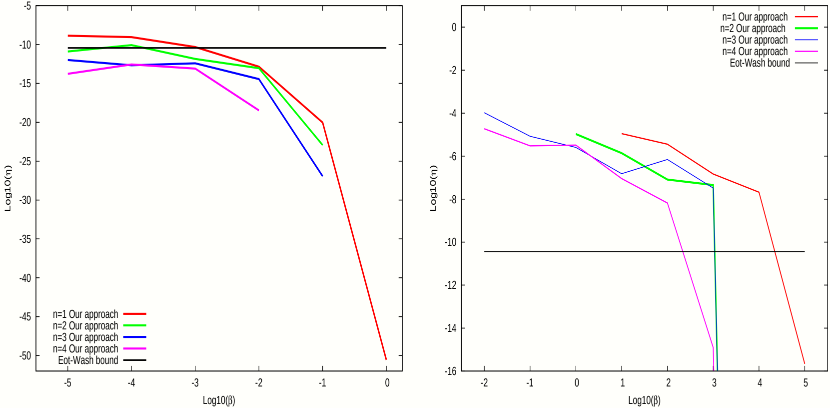



Figure: The density of the environment surrounding the Earth $\rho_{\rm out}$ is assumed to be equal the Earth's atmosphere $\rho_{\rm out} = 10^{-3}{\rm g\ cm}^{-3}$; the density of the environment inside the vacuum chamber is assumed to be $\rho_{\rm vac} = 10^{-7}{\rm g\ cm}^{-3}$; Left: $M = 2.4 \times 10^{-3}{\rm eV}$ (cosmological chameleon); Right: $M = 10\ {\rm eV}$.

# Results for the Lunar Laser Ranging Experiment

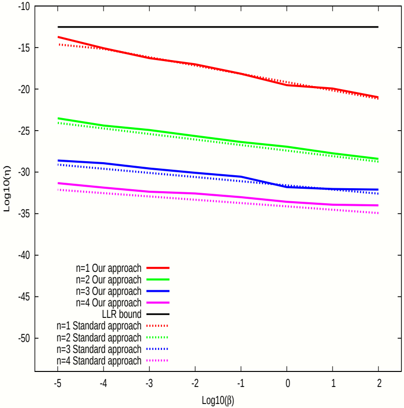



Figure: All bodies are surrounded by the interstellar medium. $M = 2.4 \times 10^{-3}\mathrm{eV}$ (cosmological chameleon).

# Results for the Lunar Laser Ranging Experiment

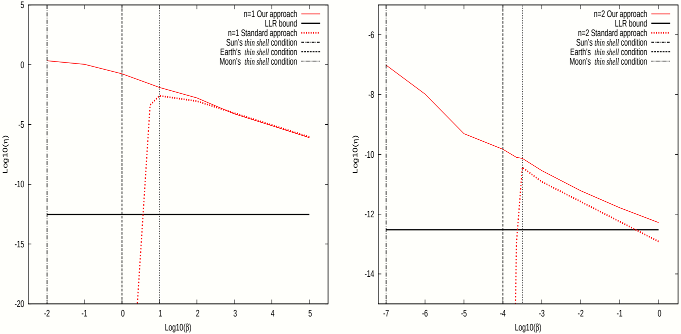



Figure: Left: $n = 1$; Right: $n = 2$. All bodies are surrounded by the interstellar medium. $M = 10$ eV.

# The energy criterion

In order to determine where our solution to the two body problem is better than the standard approach one (one body problem) we developed a single energy criterion. For this we define the energy of the system as:

$$U^*_{\text{eff}} = \int_V \left[ -\frac{1}{2}\varphi\nabla^2\varphi + V_{\text{eff}}(\varphi) - V_{\text{eff}}(\varphi^{\text{out}}_{\text{min}}) \right] dV \ \ .$$

And thus calculate the value of this functional for the different values of the chameleon space parameters $n$ and $\beta$.

# Relative difference of energy $\left(\frac{U_{\text{standard}} - U_{\text{our approach}}}{U_{\text{standard}}}\right)$ computed for the Eötvos experiment.

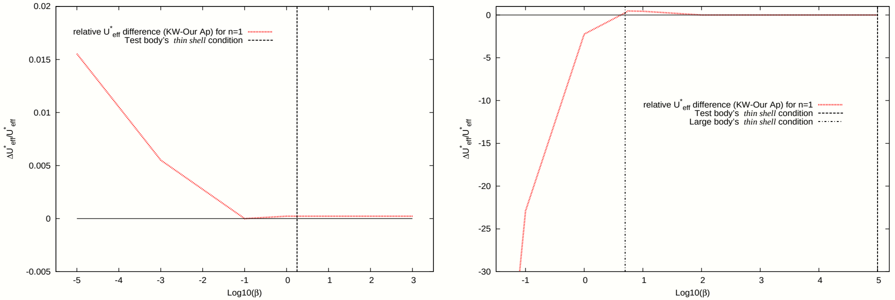
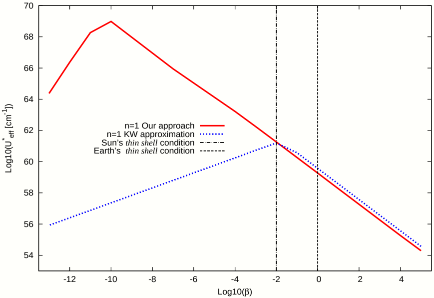



**Figure:** Left: $M = 2.4$ meV, $\rho_{\text{out}} = 10^{-7}$g cm$^{-3}$ and $n = 1$. Right: $M = 10$ eV, $\rho_{\text{out}} = 10^{-3}$g cm$^{-3}$ and $n = 1$.

# Comparison of the energy computed for the LLR experiment



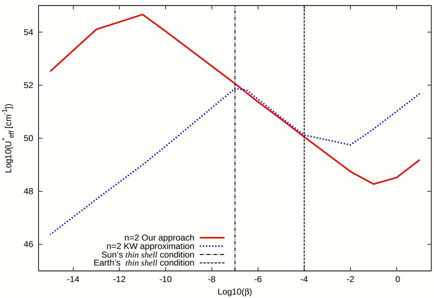
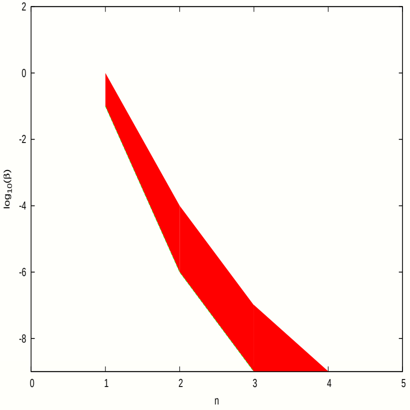
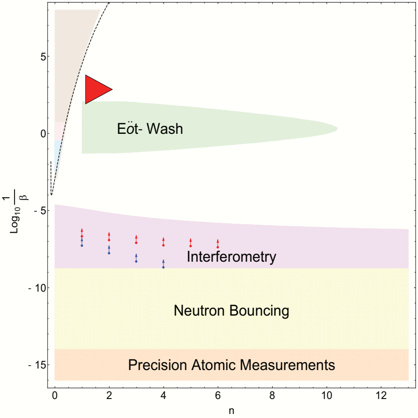




Figure: Red: our approach; Blue: the standard one taking $M = 10$ eV. The test body is assumed to be the Earth and the environment corresponds to the interstellar medium with density $\rho_{out} = 10^{-24}\mathrm{g\ cm}^{-3}$ . Left:$n = 1$; Right: $n = 2$

# Region of parameter space where our solution is different from the standard approach



# Bounds on the model parameters from different experiments



# Summary

- We have examined, in a critical manner, the degree to which the chameleon models evade existing bounds on the violation of UFF as a result of the famous *thin shell* effects.
- We have proposed a method to solve the two body problem for the chameleon field, using a quadratic approximation for the potential.
- We confirm that the force on the test body depends on the test body composition and for some values of the free parameters the estimates are varios orders of magnitude larger than predictions made in the literature.
- We have also compared our predictions with the available bounds from experiments that test violations of the WEP. We were able to restrict a new region in the plane $n - \beta$.
- The effect of the metal shell surrounding the test bodies in the experimental setup, is to reduce the predicted effect.

# Work in progress.

- We will extend the analysis for the cases where the *thick shell* approximation for the large body is appropiate.
- We will extend the analysis for $f(R)$ theories.
- We will calculate the predictions for the violations of the WEP for the future measurements of the STEP satellite.

# Muchas gracias!!!

In the Jordan frame the action of the chameleon can be written as follows:

$$\begin{aligned} S &= M_{pl}^2 \int d^4x\sqrt{-g}\left[\frac{\Omega^2(\varphi)R}{2} - \frac{1}{2}h(\varphi)(\nabla_\mu\varphi)(\nabla_\nu\varphi) - \Omega^4(\varphi)V(\varphi)\right] \\ &\quad - \int d^4x L_m\left(\Psi_m, g_{\mu\nu}\right), \end{aligned}$$

Here the scalar field $\varphi$ couples *non-minimally* to the metric. For universal $\beta$ the action is similar to a Brans-Dicke model with a potential.

# The Lunar Laser Ranging (LLR) experiment

In the solar system barycentric inertial, the quasi-Newtonian acceleration of the Moon with respect to the Earth, $\mathbf{a} = \mathbf{a}_M - \mathbf{a}_E$, is calculated to be:

$$\mathbf{a} = -\mu^* \frac{\mathbf{r}_{\mathrm{EM}}}{r_{\mathrm{EM}}^3} - \left[\frac{m_G}{m_I}\right]_{\mathrm{M}} \mu_{\mathrm{S}} \frac{\mathbf{r}_{\mathrm{SM}}}{r_{\mathrm{SM}}^3} + \left[\frac{m_G}{m_I}\right]_{\mathrm{E}} \mu_{\mathrm{S}} \frac{\mathbf{r}_{\mathrm{SE}}}{r_{\mathrm{SE}}^3}, \tag{2}$$

where $\mu^* = \mu_{\mathrm{E}}(m_G/m_I)_{\mathrm{M}} + \mu_{\mathrm{M}}(m_G/m_I)_{\mathrm{E}}$ and $\mu_k = Gm_k$. The first term on the right-hand side of the equation above, is the acceleration between the Earth and Moon with the remaining pair being the tidal acceleration expression due to the solar gravity.

# The Lunar Laser Ranging (LLR) experiment

Rearranging the above equation emphasizes the EP terms:

$$\begin{aligned}\mathbf{a} = -\mu^* \frac{\mathbf{r}_{\mathrm{EM}}}{r_{\mathrm{EM}}^3} \quad &+ \quad \mu_{\mathrm{S}} \left[ \frac{\mathbf{r}_{\mathrm{SE}}}{r_{\mathrm{SE}}^3} - \frac{\mathbf{r}_{\mathrm{SM}}}{r_{\mathrm{SM}}^3} \right] + \\ &+ \quad \mu_{\mathrm{S}} \left[ \left( \left[ \frac{m_G}{m_I} \right]_{\mathrm{E}} - 1 \right) \frac{\mathbf{r}_{\mathrm{SE}}}{r_{\mathrm{SE}}^3} - \left( \left[ \frac{m_G}{m_I} \right]_{\mathrm{M}} - 1 \right) \frac{\mathbf{r}_{\mathrm{SM}}}{r_{\mathrm{SM}}^3} \right] \end{aligned} \tag{3}$$

The second term is the Newtonian tidal acceleration. The third term gives the main sensitivity of the LLR test. Treating the EP related tidal term as a perturbation Nordtvedt (1968) found a radial perturbation of the Moon's orbit

$$\Delta r = S[(m_G/m_I)_{\mathrm{E}} - (m_G/m_I)_{\mathrm{M}}] \cos \Omega t, \tag{4}$$

where $\Omega = \omega_M - \omega_s$ and $S$ is a scaling factor of about $-2.9 \times 10^{13}$ mm. If the EP is violated, the lunar orbit will be displaced along the Earth-Sun line, producing a range signature having a 29.53 day synodic period.