$$\vec{R}_{cm} = \frac{\sum_{i=1}^{L} \vec{r}_i}{L} \sim \, ?$$

- D: Dim of the Shared Param space
- N: Num of datasets
- $L = \frac{N(N-1)}{2}$: Num of Tension Vectors

Primero: $\vec{r}_i \sim ?$

$$X_i \sim N(\vec{\mu}_i, \Sigma_i) \Rightarrow \vec{r}_i = \underbrace{A_i^{-1/2} X_i}_{} \sim N\Big(\overbrace{A_i^{-1/2}\vec{\mu}_i}^{=\vec{0}\,(H_0)}, \overbrace{A_i^{-1/2}\Sigma_i A_i^{-1/2}}^{=\mathbb{1}_{D\times D}}\Big)$$

$$\begin{cases} X_1 \simeq \text{"}\theta_A - \theta_B\text{"} \sim N(\underbrace{\vec{\theta}_A - \vec{\theta}_B}_{\vec{\mu}_1}, \underbrace{C_A + C_B}_{=\Sigma_1}) \\ A_1 = C_A + C_B \end{cases}$$

- $\vec{r}_i \sim N(\vec{0}, \mathbb{1}_{D\times D})$
- $\sum_{i=1}^{L} \vec{r}_i \sim N(\vec{0}, L\cdot\mathbb{1}_{D\times D})$
- $\vec{R}_{cm} = \frac{\sum_{i=1}^{L}\vec{r}_i}{L} \sim N\left(\vec{0}/L, \frac{L\cdot\mathbb{1}_{D\times D}}{L^2}\right) = N\Big(\vec{0}, \overbrace{\frac{\mathbb{1}_{D\times D}}{L}}^{=\tilde{\Sigma}_{D\times D}}\Big)$
- $\tilde{Q} = |\vec{R}_{cm}|^2 \sim \sum_{i=1}^{D} \underbrace{\lambda_i}_{} \chi^2_1 = \frac{1}{L}\sum_{i=1}^{D}\chi^2_1 = \frac{1}{L}\chi^2_D = \Gamma\left(\frac{D}{2}, \frac{2}{L}\right)$

  $\hookrightarrow$ AUTOVAL $(\tilde{\Sigma}_{D\times D})\big|_i = \frac{1}{L}$

(*)

(*) $\chi^2_D = \Gamma\Big(\overset{\text{Shape ("}\alpha\text{")}}{\frac{D}{2}}, \overset{\text{scale ("}\theta\text{")}}{2}\Big)$

Caracteristicas de las $\Gamma\left(\frac{D}{2}, \frac{2}{L}\right)$:

- $L = 1 \; (N=2) \rightarrow \tilde{Q} \sim \Gamma\left(\frac{D}{2}, 2\right) = \chi^2_D$
- $\frac{D}{2} = 1 \quad (D=2) \longrightarrow \tilde{Q} \sim \exp\left(\frac{2}{L}\right)$
- $\frac{D}{2} \to \infty \wedge \frac{2}{L} \to 0 \quad \left(\begin{matrix} D, L \to \infty \\ \mu \equiv \frac{D}{L} = cte \end{matrix}\right) \longrightarrow \tilde{Q} \sim N\left(\mu = \frac{D}{L}, \sigma = \frac{2D}{L^2}\right)$