

Problem 3.1

a) $f(n) = 5n$ and $g(n) = 5n^3$

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \lim_{n \rightarrow \infty} \frac{5n}{5n^3} = 0 \quad \left| \quad \lim_{n \rightarrow \infty} \frac{g(n)}{f(n)} = \lim_{n \rightarrow \infty} \frac{5n^3}{5n} = \frac{5n^2}{1} = \infty \right|$$

$$f \in o(g)$$

$$g \in \omega(f)$$

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = 0 < \infty$$

$$f \in O(g)$$

$$\lim_{n \rightarrow \infty} \frac{g(n)}{f(n)} = \infty > 0$$

$$g \in \Omega(f)$$

b) $f(n) = 9n^{0.8} + 2n^{0.3} + 14 \log n$ and $g(n) = \sqrt{n}$

$$\lim_{n \rightarrow \infty} \frac{g(n)}{f(n)} = \frac{n^{0.5}}{9n^{0.8} + 2n^{0.3} + 14 \log n} = \frac{0}{\infty} = 0 < \infty$$

$$g \in o(f)$$

$$g \in O(f)$$

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \frac{9n^{0.8} + 2n^{0.3} + 14 \log n}{n^{0.5}} = \frac{\infty}{n^{-0.5}} = 9n^{0.3} = \infty > 0$$

$$f \in \omega(g)$$

$$f \in \Omega(g)$$

c) $f(n) = n^2 / \log n$ and $g(n) = n \log n$

$$\lim_{n \rightarrow \infty} \frac{g(n)}{f(n)} = \frac{n \log n}{\frac{n^2}{\log n}} = \frac{\log^2(n)}{n} = 0 < \infty$$

$$g \in o(f)$$

$$g \in O$$

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \lim_{n \rightarrow \infty} \frac{n^2}{n \log^2(n)} = +\infty > 0$$

$$f \in w(g)$$

$$f \in \Omega(g)$$

d) $f(n) = (\log(3n))^3$ and $g(n) = \frac{1}{2} \log n$

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \frac{(\log(3n))^3}{\frac{1}{2} \log n} = +\infty > 0$$

$$f \in w(g)$$

$$f \in \Omega(g)$$

$$\lim_{n \rightarrow \infty} \frac{g(n)}{f(n)} = \frac{\frac{1}{2} \log n}{(\log(3n))^3} = 0 < \infty$$

$$g \in o(f)$$

$$g \in O(f)$$

Ex 3.2

a) for $i = 1$ to n

 min

 temp

 index

 for $j = i+1$ to n

 if $\text{min} > \text{array}[j]$

 min = array[j]

 index = j

temp = array[i]

array[i] = min

array[index] = temp

b) loop invariant: after each iteration, array $[1 \dots i-1]$ is sorted. When $i = n$, the whole array is sorted.

c) Selection Sort has the same number of swaps every time. That means that best case, worst case and average case perform the same number of element comparisons, because selection sort does not take advantage of the existing order of elements.