Ex 4.2

a) $T(n) = 36\,T(n/6) + 2n$

$a = 36 \quad b = 6 \quad f(n) = 2n$

$n^{\log_b a} = n^2 > f(n)$

$f(n) = O(n^{2-\varepsilon})$

Thus, $T(n) = \Theta(n^2)$

b) $T(n) = 5\,T(n/3) + 17n^{1,2}$

$a = 5 \quad b = 3 \quad f(n) = 17n^{1,2}$

$n^{\log_b a} = n^{1,46} > f(n)$

$f(n) = O(n^{1,46-\varepsilon})$

Thus, $T(n) = \Theta(n^{1,46})$

c) $T(n) = 12\,T(n/2) + n^2 \log n$

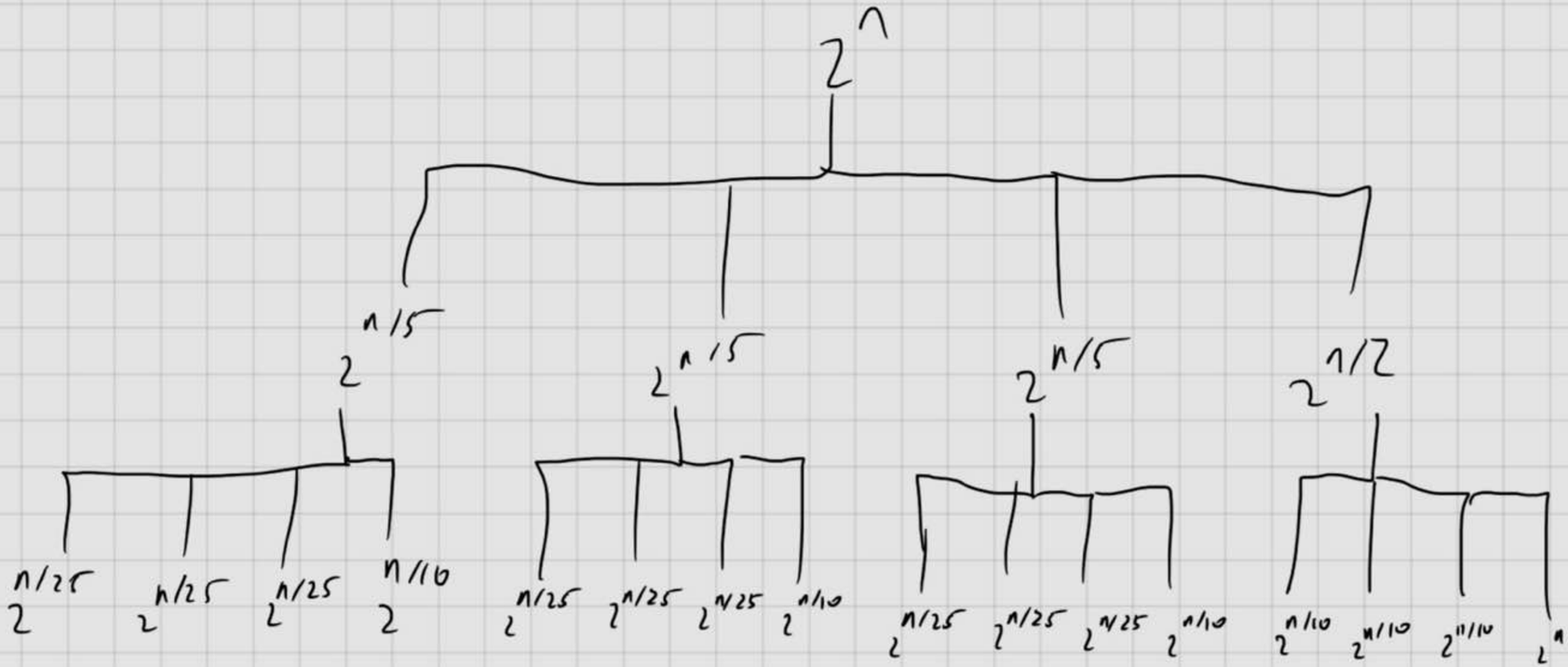
$a = 12 \quad b = 2 \quad f(n) = n^2 \log n$

$n^{\log_b a} = n^{3,6} > f(n)$

$f(n) = O(n^{3,6-\varepsilon})$

Thus, $T(n) = \Theta(n^{3,6})$

d) $T(n) = 3T(n/5) + T(n/2) + 2^n$



$$\frac{\text{Sum}}{2^n}$$

$$\left(3 \cdot 2^{1/5} + 2^{1/2}\right) \cdot 2^n$$

$$\left(3 \cdot 2^{1/25} + 6 \cdot 2^{1/10} + 2^{1/4}\right) \cdot 2^n$$

constant

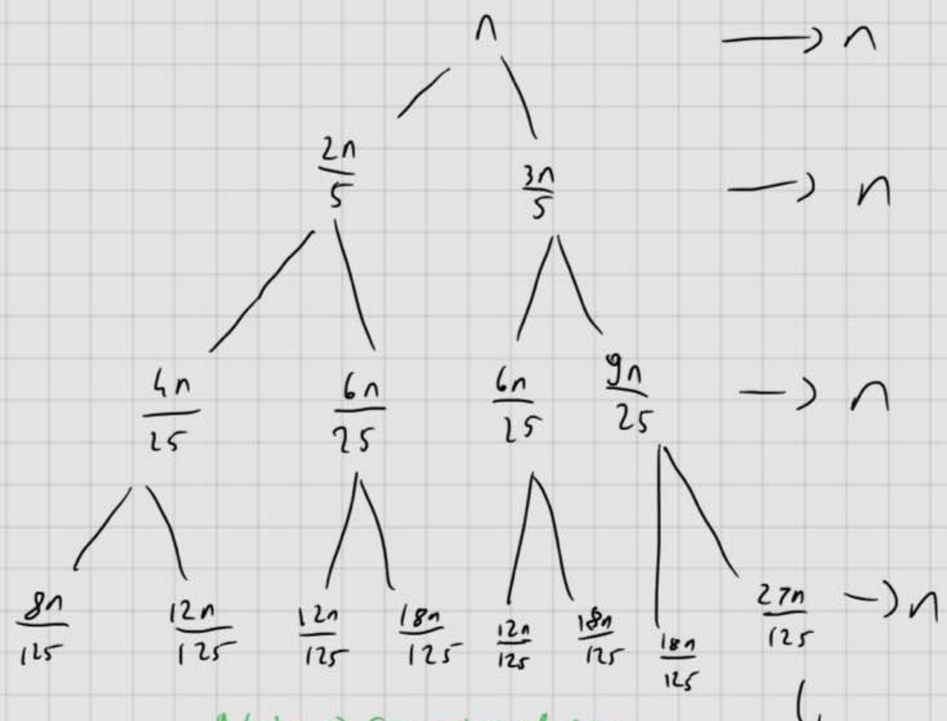
This doesn't add much to 2^n

Hence, $T(n) < 4T(n/2) + 2^n$

$T(n) = O(2^n)$

Thus, $\theta(2^n)$

eg $T(n) = T(2n/5) + T(3n/5) + \Theta(n)$



Note => r = number of row

lower bound

$\left(\frac{2}{5}\right)^r \cdot n$

$\left(\frac{2}{5}\right)^r \cdot n = 1$

$n = \left(\frac{5}{2}\right)^r$

$\log_{5/2} n = r$

upper bound

$\left(\frac{3}{5}\right)^r \cdot n$

$\left(\frac{3}{5}\right)^r \cdot n = 1$

$n = \left(\frac{5}{3}\right)^r$

$\log_{5/3} n = r$

Total = n · R

$\Theta(n) = n \cdot \log_{5/3} n \Rightarrow n \log n$ ✓ worst case