The dynamics of linear operators consists of the study of the topological properties of the orbits (i.e. sets of countable iterations) of linear operators on Banach spaces (in our case function spaces). *Hypercyclic operators*, a key concept, are operators for which the orbit of some element is dense in the space. Another relevant concept is *chaoticity*, which refers to operators that, in addition to being hypercyclic, have a dense set of periodic points.

Multiplication operators associated with certain functions φ (called *multipliers*) are of interest. These are denoted by $M\varphi$, and they assign each element f the function $M\varphi(f) = \varphi f$.

My thesis includes an in-depth study of the dynamics of multiplication operators (and their adjoints) in a Hardy space of Dirichlet series, denoted $\mathscr{H}_2$. Dirichlet series are analytic functions of the form

$$f(s) = \sum_{n=1}^{\infty} a_n n^{-s},$$

with complex coefficients a_n. The space $\mathscr{H}_2$ refers to Dirichlet series with square-summable coefficients, that is, such that

$$\|f\|^2 := \sum_{n=1}^{\infty} |a_n|^2 < \infty.$$

In this setting, we give an original characterization of the adjoint multiplication operators M_φ^* that exhibit hypercyclic and chaotic behavior in $\mathscr{H}_2$. The characterization is relevant and useful for identifying simple operators that induce relatively complex behavior, because of a sufficient condition involving only the image of the multiplier. Namely, if the image of φ intersects the complex torus $\mathbb{T}$, then the operator M_φ^* is chaotic.

A crucial tool in this work is the Bohr transform, a mapping that identifies Dirichlet series with analytic functions in infinitely many variables—that is, functions defined on spaces of infinite sequences, in our case $B_{c_0} \cap \ell_2$ (B_{c_0} denoting the unit ball of c_0). Analytic functions in in high dimensions are easier to handle than Dirichlet series, and the Bohr transform allows us to prove dynamics properties in the Hardy space $H_2(B_{c_0} \cap \ell_2)$, and then transport the results to analogous spaces of Dirichlet series.

The last chapter of the thesis consists of a thorough account of a seminal result by Gordon and Hedenmalm characterizing composition operators C_Φ in $\mathscr{H}_2$, that is, operators which, given a function f in the given space, yield the function $C_\Phi(f) = f \circ \Phi$.

Since graduating, I have continued to work on this topic with my supervisors, and we have generalized the original characterization to the general Hardy space $\mathscr{H}_p$ of Dirichlet series, $1 < p < \infty$. These spaces are defined as $\mathfrak{B}H_p(B_{c_0} \cap \ell_2)$, where $\mathfrak{B}$ is the Bohr transform and $H_p(B_{c_0} \cap \ell_2)$ is the general Hardy space of holomorphic functions in $B_{c_0} \cap \ell_2$ satisfying

$$\|f\|_{H_p(\ell_2 \cap B_{c_0})} = \sup_{n\in\mathbb{N}} \sup_{0<r<1} \left(\int_{\mathbb{T}^n} |f(rw_1, \dots, rw_n, 0, \dots)|^p d(w_1, \dots, w_n) \right)^{\frac{1}{p}} < \infty.$$