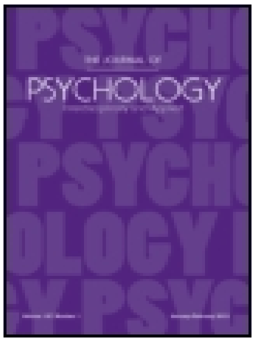

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ABSTRACT
Patterning, or the ability to understand patterns, is a skill commonly taught to young children as part of school mathematics curricula. It seems likely that some aspects of executive function, such as cognitive flexibility, inhibition, and working memory, may be expressed in the patterning abilities of children. The primary objective of the present study was to examine the relationship between patterning and executive functioning for first grade children. In addition, the relations between patterning, executive functioning, mathematics, and reading were examined. The results showed that patterning was significantly related to cognitive flexibility and working memory, but not to inhibition. Patterning, cognitive flexibility, and working memory were significantly related to mathematical skills. Only patterning and working memory were significantly related to reading. Regression analyses and structural equation modeling both showed that patterning had effects on both reading and mathematics measures, and that the effects of cognitive flexibility were entirely mediated by patterning. Working memory had independent effects on reading and mathematics, and also effects moderated by patterning. In sum, these findings suggest that cognitive flexibility and working memory are related to patterning and express their effects on reading and mathematics in whole or in part through patterning.

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Patterning; executive function; mathematics

Patterning, or the ability to identify and manipulate patterns, is a skill that has been found to be beneficial for young children both in the United States and around the world (Lee, Ng, Bull, Pe, & Ho, 2011; Mulligan & Mitchelmore, 2009; Papic, 2007; Waters, 2004). Patterning is commonly taught to young children as part of school mathematics curricula, and patterning instruction has been included as a mathematics component of the Common Core State Standards (National Governors Association Center for Best Practices, 2010) for children beginning in kindergarten. In the initial years of schooling, the first patterns taught in the classroom typically consist of five types of simple repeating patterns (Liljedahl, 2004; Rittle-Johnson, Fyfe, McLean, & McEldoon, 2013; Threlfall, 2004), such as red tan red tan red tan, or square square oval square square oval, or small big big small big big, or cat cat dog dog cat cat dog dog, or car bus truck car bus truck. Such patterns are usually taught to children

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with colors, shapes, or objects in preschool and primary grades, and manuals are available that describe how to teach such patterning skills (Burton, 1982; Ducolon, 2000; Jarboe & Sadler, 2003). This instruction of patterning relies on explanation and practice, so that children learn that the same pattern can be expressed in different elements. That is, the same abstract pattern rule can govern the relation between concrete items in any dimension, and is independent of whether the items are colors or shapes or sizes. Children are usually taught first to copy an alternating pattern with the same items in which it is presented, and then to make the same pattern but with different items. For example, instead of a pattern of alternating blue and pink beads, they may be asked to make the same pattern with large and small cubes. That is, instead of the pattern of blue bead pink bead blue bead pink bead, they are to make a pattern of large cube small cube large cube small cube. Next they may be asked to draw a pattern of beads or cubes which consists of alternating items, then to extend the pattern by adding more beads or cubes, then to draw extended patterns expressing the same rule. After they have become proficient in constructing such simple "abab" alternations, the children are helped to master "aabaab" patterns by the same method of explanation and practice. Next, come "abbabb" patterns, "aabbaabb," and finally "abcabc" patterns. Patterning is usually taught in preschools and primary grades throughout the United States and is also taught abroad (Papic, Mulligan, & Mitchelmore, 2011; Warren & Cooper, 2006).

Correlations Between Patterning and Achievement

Two very recent longitudinal studies show that there are correlations between young children's understanding of patterning and later achievement. Fyfe, Rittle-Johnson, Hofer, and Farren (2015) reported significant regression coefficients between preschoolers' mastery of alternating patterns and a composite Woodcock-Johnson scales 10 and 18 measure of mathematics achievement in fifth grade. Rittle-Johnson, Hofer, Fyfe, and Farren (2015) also reported significant correlations between second graders' patterning scores and several KEY-math measures of mathematics achievement. Hence, the extent to which children understand patterns at young ages does predict their later mathematics achievement as measured by well-regarded standardized tests.

Empirical Studies of Patterning Instruction

Herman (1973) examined the effects on mathematics achievement of teaching simple patterns to kindergarten students in her dissertation. The children were taught first to copy alternating patterns such as those described above, then to complete a pattern by filling in a missing item, then extending a pattern already started, then creating and naming patterns. Results on the Numbers Subtest of the Metropolitan Readiness Test (Hildreth, Griffiths, & McGauvran, 1969) revealed that children coming from English-speaking homes who received the patterning instruction did significantly better on measures of numeracy than did the control group. Meanwhile, children in the intervention group who came from Spanish-speaking homes did not attain higher scores than the control group. However, a major problem with this dissertation is that the control group came from kindergartens in different schools, and so may have been better or worse at the outset of the research.

Over three decades later, Hendricks, Trueblood, and Pasnak (2006) worked with first graders who were identified by their teachers as having difficulty with their schoolwork.

Children were randomly assigned to be taught either a large variety of patterns or academic material recommended by their teachers. Children in the patterning group were taught 480 patterns throughout the academic year, which ranged from simple alternating patterns to more complex multidimensional patterns. The patterning and control instruction sessions were matched in timing and duration. At the end of the year, the children who were given patterning instruction were able to identify and interpret novel patterns which followed the same rules but were made of different items significantly better than were the children who did not receive patterning instruction. In addition, after the intervention and control groups were statistically equated on IQ, children who had made gains in their understanding of patterning also made significantly greater gains in mathematics and language scales of the Diagnostic Achievement Battery-2 than did children in the control groups. Thus, the results of this study suggest that patterning instruction may not only improve mathematics skills, but also reading skills.

In 2005, Papic and Mulligan (2015) sought to answer the following two questions: is there a connection between children's patterning abilities and their development of pre-algebraic skills, and can a patterning intervention program result in long-term benefits for children's development in mathematics? The researchers conducted their investigation in two similar Sydney,

Australia preschools. The researchers did not report on what the similarities were, except that they were "typical of centres in this region." Children in one preschool received the intervention and children in the other acted as the control group. The results of the individual task-assessment interviews suggested that children in the intervention school were better at tasks for all patterning categories designed by the researchers. Observations also were reportedly made by teachers and parents in the classroom about how the patterning interventions were translating to the children's mathematical development throughout the course of the study. When a follow-up study was conducted a year later, the children in the intervention group still demonstrated a better understanding of patterning than did the control group, again as measured by the task-assessment interviews (Papic & Mulligan, 2007). However, no statistical analyses were conducted, and children in the control preschool may have been inferior to those in the intervention preschool in regards to these skills prior to the conduct of the intervention. Thus, the results of this study are open to the same criticisms as that of Herman (1973).

A study with a similar design was conducted by Papic, Mulligan, and Mitchelmore (2011). Researchers examined the impacts of a patterning intervention in two Australian preschools. The researchers reported that these preschools had equal enrollments, "comparable" staffing levels and resources, and "similar" approaches to curriculum. Children in one preschool received the patterning intervention while those in the other preschool served as the control group. In the school receiving the intervention, researchers implemented a six-month patterning intervention that emphasized repeating and spatial patterns. At the end of the intervention, as well as one year later, children from the intervention school performed better on patterning tasks than did children from the control school. Compared to children in the control group, children in the intervention group showed a greater understanding of units of repeat and spatial structure and they were also better able to explain and extend growing patterns. For example, if a pattern started with one small square, then had four small squares arranged in the configuration of a large square, and then had nine small squares arranged in the configuration of a larger square, the children could make the next item: 16 small squares arranged in the configuration of a larger square. They were also able to verbalize the rule of the patterns, that is, that a large square was made of the same number of small squares in height and width, and each had more small squares than the previous one in the pattern.

However, this also was not an adequately controlled study, as the children in the preschools may have differed in unknown ways that may have affected the results. Moreover, the parents of children in the preschool receiving the intervention participated in observations and surveys, and the teachers in both preschools participated in surveys. The fact that they were not blind to instructional conditions may have contributed to differences in the results obtained from the two preschools; therefore, the results may be invalid.

Also, in Australia, Warren and Cooper (2006) studied third grade children's understanding of visual growth patterns. In this study, patterning lessons were taught in a two-lesson sequence to children in two third grade classrooms. The two lessons included copying and continuing simple patterns, using language to describe patterns, and predicting and thinking about patterns. Pre- and posttest scores seemed to demonstrate a growth in the children's ability to understand, describe, and predict patterns. Warren and Cooper stated that they hoped the study would help to convey children's ability to understand complex patterns in the early elementary school years, and also emphasize the importance of incorporating patterning into elementary school classroom curricula. Nonetheless, this study has no control group and so must be interpreted with caution.

In 2013, Kidd, Gadzichowski, Gallington, Boyer, and Pasnak examined the effectiveness of patterning instruction with first graders. In this study, children who lagged behind their peers in patterning abilities in pretest assessments were randomly assigned to one-on-one 15 minute lessons, three times a week on patterning or reading or mathematics or social studies. Children in the patterning group were taught patterns that included single and double alternations, symmetrical patterns, progressive patterns, rotation patterns, and random repeating patterns. Children in the other groups were given lessons based on the State Standards of Learning, which parallel the Common Core State Standards (National Governors Association Center for Best Practices, 2010). Scores on the posttests given to the children after six and half months of lessons showed that the children who received patterning instruction substantially outperformed the children in other instruction groups—mathematics, reading, or social studies—on an assessment of patterning abilities and on the Woodcock-Johnson III (WJ-III): 18A—Quantitative Concepts—Concepts (WJ-18A), which measures understanding of mathematical symbols, counting, and terms like greater and lesser (Woodcock, McGrew, & Mather, 2001). Children in the patterning instruction group scored significantly higher on the WJ-III: 18B - Quantitative Concepts - Number Series (WJ-18B) than the children in any other group, and children in the mathematics instruction group scored higher than children who received reading or social studies instruction on this scale. The results of this study suggest that instructing young children in patterns may not only improve children's patterning abilities, but also their understanding of mathematical concepts. However, there were no significant differences between any of the groups on the WJ-10-Applied Problems, which measures mathematics skills, nor on the WJ scales 1, 2, or 9, which measure different aspects of reading.

The possibility of using patterning instruction to improve both reading and mathematics outcomes was also assessed by Kidd et al. (2014). As in in the Kidd et al. (2013) study, first-grade children who scored the lowest in their classroom on a pretest patterning assessment were randomly assigned to either a patterning instruction group or to groups receiving instruction in reading, or mathematics, or social studies. The patterning, reading, mathematics, and social studies instruction were the same as those used by Kidd et al. (2013), but the achievement tests administered were different. The reading achievement measures administered in this study were the Gray Oral Reading Test-4 (GORT; Wiederholt & Bryant, 2001), the Test of Word Reading Efficiency (TOWRE; Torgesen, Wagner, & Rashotte, 1999), and the Test of Early Reading Ability-3 (TERA; Reid, Hresko, & Hammill, 2001). The mathematics achievement measures administered were the WJ-18A and 18B, as well as the KEYmath 3 test (Connolly, 2007). At the end of the intervention, children in the patterning group scored significantly higher in the assessment of the understanding of patterns than did children in any other group. The children in the patterning group and reading group scored significantly higher than did the math group or social studies group on the TERA Meaning and the TOWRE Word measures. In regards to the mathematics assessments, the patterning group and math group scored significantly higher on the WJ-18A and WJ-18B than did children in the reading or social studies groups. Notably, the patterning group also scored significantly higher than all other groups on a number of the KEYmath 3 subscales (Connolly, 2007), which included numeration, addition, algebra, measurement, foundations, computation, data problems, and applied problems. Thus, the results of this study seem to provide compelling evidence to suggest that instruction in patterning can yield positive results, not only in regards to the understanding of patterns, but also mathematics and reading.

Theoretical Speculations

The causal mechanism by which improving patterning improves performance in other areas, such as mathematics, is still largely unknown. However, theorists have proposed numerous ideas just as to why and how patterning might work to help facilitate other positive outcomes. As stated by Clements and Sarama (2007c), patterns may encourage higher order thinking that can help with more advanced concepts, as "identifying patterns helps bring order, cohesion, and predictability to seemingly unorganized situations and allows one to recognize relationships and make generalizations beyond the information directly available" (p. 507). Hendricks et al. (2006) proposed "development from forming and understanding simple sequences to forming and interpreting complex sequences presumably corresponds to development from intuitive, perceptually driven thought to more logical thought" (p. 80).

The inclusion of patterning in today's classroom curricula is largely based on a consensus of educators who believe children's understanding of patterns is of great educational value in kindergarten through eighth grade, and beyond (National Council of Teachers of Mathematics, 2000). According to McGarvey (2012), "recognizing patterns is seen as critical to algebraic thinking because it develops students' ability to express generalities by recognizing commonalities, articulating rules and relationships, and eventually representing those relationships using symbols" (p. 312). The ability to detect the structure in patterns has been described by researchers as a demonstration of "pre-algebraic thinking" in children (Mulligan & Mitchelmore, 2009).

Patterning and Executive Function

Executive functions (EF), which include working memory, inhibition, and cognitive flexibility, are key characteristics of the thinking of children and adults. They have been repeatedly shown to predict achievement in reading and mathematics (Blair, 2002; Cartwright, 2012; de Beni & Palladino, 2000; Lan, Legare, Ponitz, Li, & Morrison, 2011; Sesma, Mahone, Levine, Eason, & Cutting, 2009; van der Sluis, de Jong, & van der Leij, 2007). Manipulating and using information that is immediately available in memory is termed “working” memory. Being able to control and prevent natural or automatic responses in favor of those more suitable to a task or situation is termed “inhibition.” Recognizing that different rules are appropriate for different tasks and being able to change the basis for one’s responses accordingly is termed “cognitive flexibility” (Duan, Wei, Wang, & Shi, 2010). While they are quite different and distinguishable, there are sometimes significant correlations between these components of thought (Miyake, Friedman, Emerson, Witzki, & Howerter, 2000).

These three types of executive function might each be involved in patterning. Cognitive flexibility would be needed to apply the same pattern rule to quite different items, or to switch from one pattern rule to another as a child deals with patterns that has the same kind of items but follows different rules. Working memory would be needed to compare the items from which a child was to select with the pattern rule that they were to express. Inhibition would be needed to suppress the tendency to apply a favorite pattern rule, or the rule of a pattern dealt with just previously, when asked to interpret or make a new and different pattern. Whether one, two, or all three EF are actually involved in patterning is an empirical question that has not been investigated until very recently.

Several studies have shown that patterning and EF are related, but the results are inconsistent. Both working memory and inhibition were correlated with preschool children’s understanding of alternating patterns in a study by Collins and Laski (2015). These results contrast in part with those of Miller, Rittle-Johnson, Loehr, and Fyfe (2015), who also investigated the relations between EF and preschoolers’ patterning abilities. These researchers found that working memory and cognitive flexibility correlated with the children’s understanding of alternating patterns, but inhibition did not. Hence, both studies show an effect of working memory, but only Collins and Laski (2015) found an effect for inhibition. The latter researchers did not measure cognitive flexibility.

Pasnak et al. (2015), worked with older children (first graders) and more complicated patterns: rows of pictures, numbers, or letters that either increased or decreased in size, value, or position of the alphabet, or were symmetrical. These researchers found that cognitive flexibility was related to patterning and to reading achievement as measured by the Test of Early Reading Ability (TERA). There was also a significant relation between patterning and this measure of reading. Working memory and inhibition were not assessed in this study.

Bock (2015) also studied first graders, using the same patterns as Pasnak et al. (2015), and measured all three aspects of EF, along with reading. She found that patterning correlated with cognitive flexibility, but not with the reading measure used (the Gray Oral Reading Test, or GORT) nor with working memory or inhibition.

In sum, cognitive flexibility is the EF most consistently related to patterning, although relations to working memory and inhibition have been found with preschool-aged children. Whether the differences between the outcomes of these four studies depend on the age and other characteristics of the children, or the patterns and EF measures used, is an open question at this point.

The results Bock (2015) obtained suggested that only cognitive flexibility is related to patterning skills. However, researchers have yet to try to replicate these findings. One limiting factor in Bock’s study was that the patterning task included a somewhat limited amount of patterns. The children in this study were asked to complete a set of 18 patterns. Each consisted of a line of numbers, letters, or shapes that either increased or decreased in value, position of the alphabet, or size, or were symmetrical. All patterns were presented horizontally, and the missing item was always the last in the sequence. Both Bock and Schmerold (2015) also examined the relation between patterning and reading, and obtained different results., The relation between patterning and math was not assessed in either study. Thus, in the present study, one goal was to look at the relationship between patterning and the components of executive function when more patterns expressing more variations in pattern rules were presented. The other goals included examining the relation between patterning and mathematics, as well as between patterning and reading.

Hypotheses

Patterning was expected to be significantly correlated with each of the three components of executive function examined: cognitive flexibility, working memory, and inhibition. All of the measures of executive function (cognitive flexibility, working memory, and inhibition) were also expected to be correlated with one another. Patterning was expected to be significantly related to three measures of mathematics. These measures included (a) being able to solve mathematics problems; (b) understanding mathematics concepts; and (c) understanding number series. Cognitive flexibility, working memory, and inhibition were also expected to be significantly related to these three measures of mathematics. Finally, patterning was expected to be significantly related to the reading measure. Cognitive flexibility, working memory, and inhibition were also expected to be significantly related to the reading measure.

Method

Participants

Parental consent was obtained for first-grade children in two different elementary schools in a public school system of a metropolitan area in the mid-Atlantic region. Many of the children in this system were from immigrant families, and many lived in subsidized housing. Approximately 71% of children attending these two elementary schools qualified for free or reduced lunch. All first grade classrooms in each of the two schools were included (11 classrooms total), except for two classrooms reserved for children not yet proficient in English. The children selected for testing were among those identified by their teachers, on the basis of their overall performance during the Fall semester, as about average academically as compared to their peers. Children who the teacher felt were superior or inferior to their classmates were excluded, as were those who did not receive parental permission to participate. After attrition, 45 girls and 29 boys remained. Of these 74 children, 27 (approximately 37%) were African American, 19 (26%) were Hispanic/Latino, 12 (16%) were Middle Eastern, 12 (16%) were Caucasian, and 4 (5%) were of two or more races or of an unspecified ethnicity. The mean age for these children at the beginning of testing was 6 years, 5 months, $SD = 3.42$ months.

Measures

Eight different assessments were administered to the children, which included a measure of patterning skills, three measures of mathematical skills, one measure of reading ability, and three measures of different components of executive functioning (cognitive flexibility, working memory, and inhibition).

Patterning Assessment

A researcher-designed patterning measure was administered to examine children's ability to recognize a pattern and to then fill in a missing item in the sequence (Gadzichowski, 2012). The patterning assessment had 48 pattern problems. Due to its length, the assessment was administered during two separate testing periods, with 24 pattern problems per session. The patterns included lines of numbers, letters, shapes, pictures, and clock faces that either increased or decreased in value, position of the alphabet, or size, and also included symmetrical and rotating patterns. Half of the patterns were presented horizontally, half vertically, and each pattern problem consisted of four items and one blank space, which was presented equally often in the beginning, middle, or end of the pattern. Children were asked to examine the pattern and to then choose from four possible options shown below or next to the pattern in order to fill in the blank. The patterns were presented one at a time, and each child was allowed as much time as needed to choose an answer. The total number of patterning problems correct (out of 48) for each child was used in the analyses.

Woodcock-Johnson Tests of Cognitive Abilities III

The Woodcock-Johnson III (WJ-III) is a set of tests for assessing general intellectual ability and academic achievement (Woodcock, McGrew, & Mather, 2001). The particular subtests of the WJ-III used in the present study were: 10 - Applied Problems; 18A - Quantitative Concepts - Concepts; and 18B - Quantitative Concepts - Number Series. Subtest 10 consists of oral mathematics "word problems" and measures a student's ability to analyze and solve mathematics problems. Subtests 18A and 18B assess a student's understanding of mathematical concepts, symbols and vocabulary, and the ability to understand number patterns. For all three subtests, the questions start out fairly easy and become increasingly difficult. Each subtest was discontinued when a child made six incorrect responses consecutively for WJ-10, four for WJ-18A, and three for WJ-18B, which are the cut-off points specified in the manual. The total number of correct answers for each subtest was then calculated for analyses. The WJ-III is a very highly regarded and widely used test, with a reliability coefficient of .84 for seven- year-olds (McGrew & Woodcock, 2001). The WJ-III is also reported to have convergent validity coefficients of .68–.70 with the Wechsler Individual Achievement Test (Wechsler, 1974) and.62–.66 with the Kaufman Test of Educational Achievement (Kaufman & Kaufman, 1985).

Test of Word Reading Efficiency, Second Edition

The Sight Word Efficiency (SWE) subtest of the Test of Word Reading Efficiency, Second Edition (TOWRE) assesses print-based word reading skills and provides a quick and reliable way to measure the efficiency of sight word recognition (Torgesen et al., 1999). While the majority of academic assessments in the present study examined children's mathematical skills, this subtest provided a very efficient way to also assess the relationship of reading to executive function. The SWE measures the number of words printed in vertical lists that an individual can accurately read within 45 seconds. The subtest can be completed in approximately 5 minutes, including the time for directions and practice items. The TOWRE meets high standards for both reliability and validity, with reliability coefficients ranging from .90–.99 and validity coefficients ranging from .77–.96.

Multiple Classification Card Sorting Test

The Multiple Classification Card Sorting Test (MCCST; Cartwright, 2012) was administered in order to measure the cognitive flexibility component of executive functioning. After a researcher demonstrated the sorting with an example set of cards, each child was given 12 cards and instructed to sort them based on two different dimensions concurrently (e.g., by color, such as brown and yellow, and by object type, such as tools and instruments) into four piles within a matrix (e.g., brown tools, brown instruments, yellow tools, yellow instruments). The researcher first used a set of 12 training cards to demonstrate to the child how to complete this activity. Each child was then given one of four sets of 12 cards and was asked to sort the cards by object and color on the matrix, as demonstrated. After the child sorted the cards in each set, the researcher asked for an explanation of why he or she had sorted the cards the way they did. Children were then scored on the accuracy of their sorting, the explanation they gave for their sorting (whether they justified sorting by both color and object), and the time it took them to sort the card set. If the child sorted the cards incorrectly, the researcher would demonstrate to the child again the way that the cards could be sorted by both object and color. After sorting the first pile, each child was given three more sets of cards one at a time and was asked to sort the cards using the same instructions as before. A flexibility composite score was calculated for each child by adding the sorting score and justification score together and then dividing by the sorting time. Reliability for this measure is reportedly high, with a Cronbach's alpha of .86 (Cartwright, Marshall, Dandy, & Isaac, 2010).

Wechsler Intelligence Scale for Children Revised: Memory for Digit Span

The Wechsler Intelligence Scale for Children - Revised (WISC-R) - Memory for Digit Span - was administered in order to measure the children's working memory capabilities as a component of executive functioning (Wechsler, 1974). The Memory for Digit Span component of the WISC-R is a measure of short term memory and working memory. There are two components to the Memory for Digit Span assessment: Digits Forwards and Digits Backward. Digits Forward assesses short-term auditory memory, while Digits Backwards assesses the child's working memory ability. For the Digits Forwards portion of the assessment, each child was instructed to repeat a series of numbers read aloud by the researcher. The Digit Forwards contained 14 sets of numbers, ranging from three numbers to nine numbers, with two sets of each length. A child progressed through the increasingly lengthy sets of numbers until making errors on two consecutive sets. For the Digits Backwards portion of the assessment, the child was then instructed to listen to the sequence of numbers read aloud by the researcher and to then repeat the numbers backwards. Digits Backwards contains 12 sets of two to eight numbers, also with two sets of each length. Like Digits Forwards, testing was discontinued when the child made errors on two consecutive number sets. Each correct response was worth one point, and scores were calculated for Digits Forwards and Digits Backwards each independently, as well as together, for each child. Only the Digits Backwards scores were entered in the analyses, however, as this is the component of the assessment that measures working memory.

Stroop Color-Word Test

The Stroop Color-Word Test (SCWT) test was administered in order to examine inhibition as a component of executive functioning. The SCWT uses word reading as an automatic process, where individuals are more inclined to read a word even when instructed to name the color of the ink that the word is written in instead (Stroop, 1935). The SCWT requires participants to inhibit the dominant response of reading in order to make the required color-naming response. In the present study, children were first given a list of words where the colors and words matched (e.g., the word red was written in red ink, the word blue in blue ink, etc.) and they were timed on how quickly they could read the words. Next, the children were given a list of words where the colors and words did not match (e.g., the word red was written in blue ink, the word blue in red ink, etc.). Again, the children were timed on how long it took them to read the list of words. The number of errors children made for each set was recorded, and a score was calculated by dividing the time it took a child to read the matching set of words by the time it took to read the nonmatching words.

Procedure

After obtaining approval from an internal review board and parental permission, the children included in the study were tested individually in nine separate sessions. Assessments were administered in a quiet location in the classroom or hallway. Each session lasted approximately 5–15 minutes, depending on the assessment being given and the performance of the child. Each child was given instructions at the beginning of every assessment, and a child could choose to not participate or to stop participating at any time. The order in which children were given the assessments was counterbalanced across the classrooms.

Results

Descriptive statistics were conducted on the variables of interest (see Table 1). All variables were normally distributed except for the WJ-18A, which had a borderline significant positive kurtosis value.

Bivariate Correlations

Pearson correlation coefficients were calculated between all dependent variables.

Table 1. Descriptive Statistics for Variables.

	N	Mean	*SD*	Min	Max	Skew	Kurtosis
Patterning	72	18.57	9.25	5.00	41.00	1.02	−.08
Cognitive flexibility	66	.08	.04	.00	.17	.07	−.48
Inhibition	72	.42	.20	.16	.97	1.12	.60
Working Memory	74	3.16	1.31	.00	7.00	−.31	.92
WJ-10	74	26.82	3.67	19.00	34.00	−.25	−.56
WJ-18A	74	15.28	1.88	9.00	19.00	−.98	2.06
WJ-18B	74	9.93	2.08	4.00	14.00	−.25	−.36
TOWRE	74	45.00	15.41	9.00	75.00	−.47	−.69

Table 2. Correlations Among Variables.

	P	CF	I	WM	10	18A	18B	T
Patterning	—	.41**	.17	.23*	.54**	.54**	.52**	.33*
Cognitive flexibility		—	−.03	.01	.29*	.34**	.20	.15
Inhibition			—	−.12	−.12	−.07	−.04	−.05
Working Memory				—	.37**	.40**	.27*	.40**
WJ-10					—	.67**	.46**	.59**
WJ-18A						—	.54**	.49**
WJ-18B							—	.33**
TOWRE								—

Note. $N = 68$ for correlations of cognitive flexibility with all other variables, $N = 72$ for correlations of patterning or inhibition with variables other than cognitive flexibility. $N = 74$ for all other correlations. $^{*}p < .05$. $^{**}p < .01$.

Patterning

Patterning was found to be significantly correlated to two of the executive function measures, cognitive flexibility and working memory, but not to inhibition. Patterning was also correlated to all three mathematics measures (WJ-10, WJ-18A, & WJ-18B) and the reading measure (TOWRE) (see Table 2). The correlation coefficient is the effect size, according to Cohen (1992), who defined coefficients of .10, .30, and .50 as small, medium, and large, respectively. Thus, the correlation between patterning and cognitive flexibility is considered to be a medium effect, and the correlation between patterning and working memory is considered to be a small effect size. The correlations between patterning and each of the mathematics measures are large effect sizes, and the correlation between patterning and the reading measure is a medium effect size.

Executive Function Measures

None of the executive function measures were correlated to each other (see Table 2). Cognitive flexibility was correlated with the WJ-10 and WJ-18A (effect sizes were small and medium, respectively), but not to WJ-18B or the TOWRE. Working memory correlated with all three mathematics measures and the TOWRE. These correlations for working memory were medium, except that with WJ-18B, which was small. Inhibition did not correlate with any of the achievement measures.

Achievement Measures

All of the correlations between the mathematics and reading achievement measures were significant (see Table 2). All effect sizes were large or medium.

Regression Analysis

Regression analyses were run to examine which variables predicted patterning performance and achievement measures. All assumptions for regression analyses were met: error variance was random, errors were uncorrelated, and the variance of errors was constant across observations (homoscedasticity).

Patterning

For the first analysis, regression models included the following blocks: (a) working memory, (b) inhibition, and (c) cognitive flexibility. The analysis showed that, when patterning scores

Table 3. Hierarchical Linear Regression Predicting Patterning From Working Memory, Inhibition, and Cognitive Flexibility.

Predictors	R^2	Adj R^2	*B*	*t*
Model 1	.05	.04		
Working Memory			1.65	1.90
Model 2	.08	.05		
Working Memory			1.54	1.76
Inhibition			−7.72	−1.33
Model 3	.24	.21		
Working Memory			1.55	1.94
Inhibition			−7.12	−1.34
Cognitive Flexibility			94.61**	3.63**

Note. N = 68. $^{*}p < .05$. $^{**}p < .01$.

were predicted from the complex of all three executive function measures, only cognitive flexibility was a significant predictor (see Table 3). The relationship with working memory was borderline, but not significant. This indicates that part of the bivariate relation between working memory and patterning was due to a small, but not statistically significant, relation between working memory and cognitive flexibility, probably general intelligence (*g*).

Achievement Measures

For the achievement variables, when linear regression analyses were run with the following blocks: (a) working memory, (b) inhibition, (c) cognitive flexibility, and (d) patterning as predictor variables, patterning was a significant predictor for all achievement measures (see Tables 4, 5, 6, and 7). Working memory was a significant predictor for WJ-10 and WJ-18A mathematics measures, and the TOWRE reading measure. There was no relation between cognitive flexibility and any achievement variables.

When only the following blocks (a) working memory and (b) patterning were entered as predictor variables, patterning was again a significant predictor for all achievement measures, and working memory was again predictive of all but the WJ-18B measure. Hence, this follow-up analysis suggests that the bivariate correlation between working memory and WJ-18B was due to the former's relation to patterning, but working memory predicted the other achievement measures independently.

Further exploration with regression analyses also showed that, if only the following blocks, (a) patterning and (b) cognitive flexibility, were entered as predictors, there was no

Table 4. Hierarchical Linear Regression Predicting WJ-10 Scores From Working Memory, Inhibition, Cognitive Flexibility, and Patterning.

Predictors	R^2	Adj R^2	*B*	*t*
Model 1	.23	.19		
Working Memory			1.00**	3.24**
Inhibition			−1.25	−.61
Cognitive Flexibility			25.19*	2.50*
Model 2	.40	.36		
Working Memory			.72*	2.54*
Inhibition			.06	.03
Cognitive Flexibility			7.91	.80
Patterning			.18**	4.17**

Note. N = 68. $^{*}p < .05$. $^{**}p < .01$.

Table 5. Hierarchical Linear Regression Predicting WJ-18A Scores From Working Memory, Inhibition, Cognitive Flexibility, and Patterning.

Predictors	R^2	Adj R^2	*B*	*t*
Model 1	.26	.22		
Working Memory			.49**	3.42**
Inhibition			−.26	−.27
Cognitive Flexibility			14.18**	3.03**
Model 2	.40	.36		
Working Memory			.37**	2.74**
Inhibition			.31	.35
Cognitive Flexibility			6.67	1.43
Patterning			.08**	3.81**

Note. $N = 68$. $^{*}p < .05$. $^{**}p < .01$.

relation between cognitive flexibility and any achievement measure. Only patterning remained predictive of the achievement measures. This indicates that the bivariate correlation between cognitive flexibility and the WJ-10 and WJ-18A scores was due to cognitive flexibility's relation to patterning. When the following blocks, (a) working memory and (b) cognitive flexibility, were used as predictors, both predicted WJ-10 and WJ-18A, but only working memory predicted WJ-18B and TOWRE scores. This is identical to the relationships shown by the bivariate correlations. Hence, the correlations between cognitive flexibility and WJ-10 and WJ-18A were not due to the relation between working memory and cognitive flexibility. This is a second indication that the bivariate correlation between cognitive flexibility and the WJ-10 and WJ-18A scores was due to the relation between patterning and cognitive flexibility.

As a check, four structural equation models were tested. The first model tested whether the three measures of executive control, digit span (working memory), Stroop (inhibition), and card sorting (cognitive flexibility) had direct effects on the three mathematics measures (WJ-10, WJ-18A, & WJ-18B) and the reading measure (TOWRE), and whether these effects were moderated by patterning (see Figure 1).

In Model 2, the effects of inhibition on mathematics, reading, and patterning were tested explicitly by constraining the path between Stroop and those three variables to zero. As can be seen in Table 8, removing Stroop had no effect on model fit, with Model 2 fitting no better than Model 1, which indicates that inhibition was not affecting the outcome variables.

Table 6. Hierarchical Linear Regression Predicting WJ 18B Scores From Working Memory, Inhibition, Cognitive Flexibility, and Patterning.

Predictors	R^2	Adj R^2	*B*	*t*
Model 1	.12	.06		
Working Memory			.42*	2.17*
Inhibition			−.10	−.08
Cognitive Flexibility			10.04	1.60
Model 2	.29	.24		
Working Memory			.25	1.40
Inhibition			.67	.58
Cognitive Flexibility			−.22	−.04
Patterning			.11**	3.90**

Note. $N = 68$. $^{*}p < .05$. $^{**}p < .01$.

Table 7. Hierarchical Linear Regression Predicting TOWRE Scores From Working Memory, Inhibition, Cognitive Flexibility, and Patterning.

Predictors	R^2	Adj R^2	*B*	*t*
Model 1	.20	.16		
Working Memory			4.81**	3.62**
Inhibition			.22	.02
Cognitive Flexibility			53.90	1.24
Model 2	.26	.21		
Working Memory			4.11**	3.09**
Inhibition			3.44	.40
Cognitive Flexibility			11.03	.24
Patterning			.45*	2.20*

Note. $N = 68$. $^*p < .05$. $^{**}p < .01$.

Model 3 was identical to Model 2, except that the paths from card sorting (CF) to TOWRE and mathematics were constrained to zero (see Figure 2). In effect, this provided a test of whether cognitive flexibility had a direct effect or whether patterning moderated its effects. There was no significant difference between the fits of Models 2 and 3, with Model 3 being the more parsimonious model (see Table 8).

Finally, Model 4 tested whether working memory (digit span) had a direct effect on the mathematical measures or was simply moderated by patterning. Model 4 was identical to Model 3, except that the direct path between digit span and mathematics was constrained to zero. Model 4 resulted in poorer model fit compared to Model 3, thus providing further evidence that working memory has a direct effect on mathematical ability.

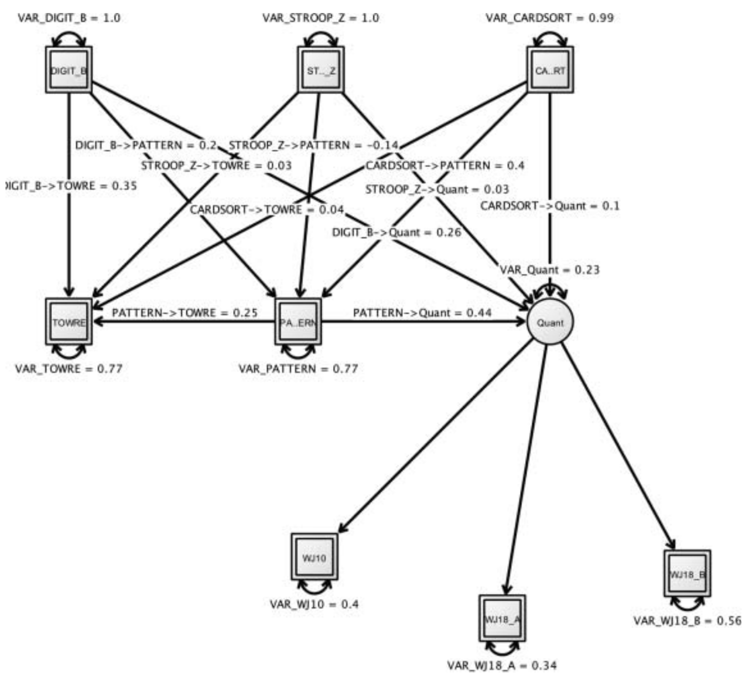

Figure 1. Model 1 is the unconstrained model testing all relations between patterning, working memory (Digit Backwards), inhibition (Stroop), cognitive flexibility (card sortng), reading (TOWRE), and mathematics (WJ10, WJ18-A, WJ18-B).

Table 8. Fit Indices for Structural Models.

Model	X^2	*df*	FP	CFI	RMSEA	AIC	ΔX^2	Δdf	*p*
Model 1	26.229	16	20	0.93	0.094	1532.10			
Model 2	28.194	19	17	0.94	0.081	1528.07			
Model 3	29.598	21	15	0.94	0.075	1525.47			
Model 4	40.573	22	14	0.88	0.108	1534.44			
Model 1 vs. Model 2							1.965	3	0.5800
Model 2 vs. Model 3							1.404	2	0.5000
Model 3 vs, Model 4							10.975	1	0.0009

Note. FP □ = number of free parameters to be estimated; CFI = comparative fit index; RMSEA = root-mean-square error of approximation; AIC = Akaike information criterion. (*N* = 68.

In summary, the structural equations models confirmed that inhibition was not related to any other variables, and that cognitive flexibility did not have an effect on the achievement variables independent of its relation to patterning. Working memory, on the other hand, had direct effects on both achievement variables and also effects moderated by patterning. The best fitting model is shown in Figure 2.

Discussion

In sum, the findings of the present study were that patterning was related to cognitive flexibility and working memory, although not to inhibition. Furthermore, patterning and

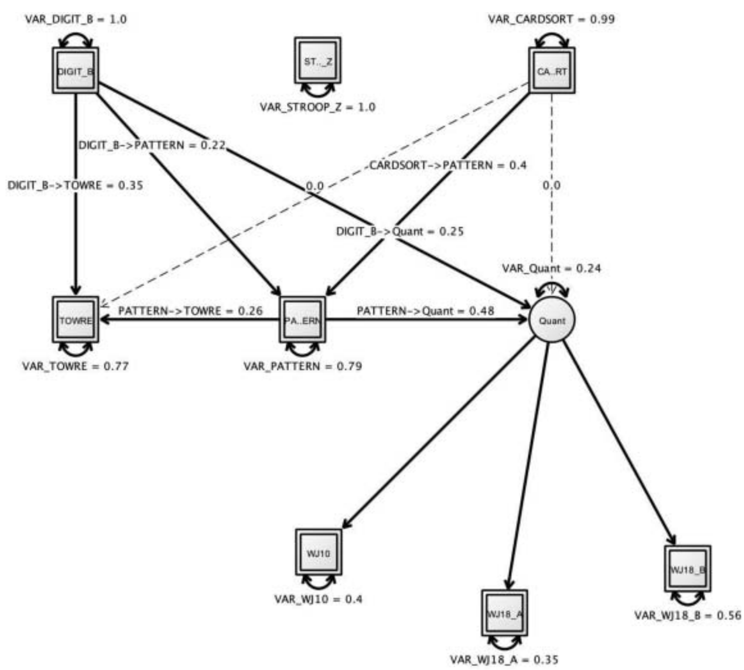

Figure 2. Model 3 confirms that inhibition has no effect on the two achievement measures, and that cognitive flexibility only has an indirect effect through patterning. Working memory has a direct effect on both achievement measures and is also moderated by patterning. Model 3 is identical to Model 2, except that the direct paths from card sorting to TOWRE and mathematics are constrained to zero.

working memory were related to mathematics and reading. The relation of cognitive flexibility to these achievement measures was entirely due to its relation to patterning. The interpretation of these findings in the context of previous research is discussed in the following section.

Executive Functions

The results obtained here contrast in part with the results of Collins and Laski (2015), who found that both working memory and inhibition correlated with preschoolers understanding of alternating patterns. Either the difference in subjects' ages—those in the present study were approximately twice as old as those of Collins and Laski—or the more complicated patterns used in the present study may account for the difference in the role of inhibition. However, neither Miller et al. (2015), who studied preschoolers' performance on alternating patterns, nor Bock (2015), who studied first graders' performance on complicated patterns, found a relation between patterning and inhibition. A more likely explanation for the difference between Collins and Laski's finding and that of the other three studies is that all four studies used different measures of inhibition. Prior researchers have found that all inhibitory tasks make demands that go beyond requiring only inhibitory ability. If a child forgets the rules of the task, for example, then he or she is likely to make an incorrect response to that picture that does not necessarily reflect a lack of inhibitory skills (Simpson & Riggs, 2005). Hence, the Head Toes Knees Shoulders task (Ponitz et al. (2008) used by Collins and Laski may reflect something other than or in addition to inhibition (or may be an especially sensitive measure of inhibition). This possibility cannot be assessed without systematic exploration of just what is measured by the different tasks which can be used to measure inhibition.

Collins and Laski (2015) did not test for a cognitive flexibility effect, but Miller et al. (2015) did assess all three measures of executive function with preschoolers, and obtained results similar to those of the present study of first graders. Both and cognitive flexibility and working memory, but not inhibition, correlated with children's initial understanding of alternating patterns. However, when Miller et al. tested the preschoolers a day later, after teacher-directed or self-directed instruction on ten patterns, only working memory was correlated with patterning. The researchers interpreted these results as indicating that working memory is an especially important factor in preschoolers' ability to solve alternating patterns, an interpretation that is congruent with our results. They offered no explanation of why the effect of cognitive flexibility found on the pretest was not found on the posttest. One possibility that should be considered is that their analysis of posttest patterning performance included pretest patterning performance as a predictor variable. Hence, what the results of Miller et al. may actually show is that the effect of cognitive flexibility on posttest scores was entirely mediated by the children's pre-existing patterning ability. This again is consistent with the results of our experiment. It is still an open question, however, whether children's ages, the type of patterns used, the amount of experience the children have with such patterns, or even the measures of executive function used, ultimately determine the results of investigations into the role of executive function.

The relation between patterning and cognitive flexibility found in the present experiment is similar to that reported by Bock et al. (2015) and by Bock (2015). Correlations range from.29 (Bock et al., 2015) to .34 (Bock, 2015) to the .41 value of r obtained here, despite differences in measures, suggesting that the size of the effect is medium (Cohen, 1992).

Working memory was found to be an important correlate of patterning ability in the present experiment and also that of Miller et al. (2015) and Collins and Laski (2015), but not in that of Bock (2015). Miller et al. and Collins and Laski studied preschoolers and alternating patterns, and the findings of both experiments agree that for such young children, working memory is a correlate of patterning. In essence, the ability of preschoolers to hold the elements of the pattern in mind while applying the appropriate alternation rule is a determinate of their success.

It is more of a problem to account for the difference in the role of working memory in the present experiment and that of Bock (2015) with first graders. Perhaps the most notable difference between the present study and that of Bock was the patterning assessment used. In the study of Bock, children were asked to complete 18 patterns consisting of lines of numbers, letters, or shapes that either increased or decreased in value, position of the alphabet, or size, or were symmetrical. All of the patterns were presented horizontally and had the last item in the pattern missing. The patterning assessment used in the present study consisted of 48 pattern problems that was administered in two separate testing periods, with 24 pattern problems per session (Gadzichowski, 2012). The different patterns consisted of numbers, letters, shapes, pictures, and clock faces that either increased or decreased in value, position of the alphabet, or size, and also included symmetrical and rotating patterns. The missing item was presented equally often in the beginning, middle, or end of the sequences, and patterns were presented either horizontally or vertically. Hence, this measure posed more difficult patterning tasks than that of Bock and made more demands on working memory in reconciling the more variable pattern rules and items. This suggests that the role of working memory in patterning depends on the difficulty of the tasks used to measure patterning and the cognitive ability or age of the children studied.

Achievement

In the present study, although there were bivariate correlations between patterning, cognitive flexibility, and achievement measures, only patterning independently predicted all achievement measures. Furthermore, while the results Bock's (2015) study did not show a significant relation between patterning and three GORT-4 reading scales, the results of the present study were that patterning was significantly related to the TOWRE reading measure. The complex patterns of the patterning assessment of the present study may be part of the reason that a relation was found between reading and patterning here, but not in the Bock study. The briefer, simpler nature of the pattern measure Bock used may have made it a less sensitive measure. Another difference is the demands of the reading measures used. For the TOWRE, the children are asked to read as fast as possible, and no concern for errors is voiced. For the GORT reading measures used by Bock, care and accuracy is required. Hence, the conflicting demands of the reading measures used in the two studies may also contribute to the difference in the relation between patterning and reading that was observed.

The greater variety of patterns and the greater number of possible positions for the missing item in the present study may also account for the disappearance of the effect of cognitive flexibility on the WJ-10 and WJ-18A. To score well on patterning, children had to consider the orientation of the pattern, whether the pattern was increasing, decreasing, symmetrical, rotating, or a clock face series when choosing from among the alternatives offered, as well as three possible positions for the missing item when considering the alternatives.

The constantly changing demands of the patterning task would subsume the effect of cognitive flexibility and, being more direct, would be primary.

The findings in the present study that working memory is related to achievement measures of math and reading are also consistent with the findings of prior studies. According to Christopher et al. (2012), working memory and general processing speed, but not inhibition, were unique predictors of both word reading and comprehension for children ages 8 through 16 years old. In addition, according to McClelland et al. (2007), behavior regulation (which includes working memory) significantly predicted pre-kindergartener's fall and spring emergent literacy, vocabulary, and math skills on the Woodcock Johnson Tests of Achievement.

The results of the present study are consistent with those of Kidd et al. (2013) in showing a significant relation between patterning and the WJ-18A, as well as between patterning and the WJ-18B. Kidd et al. (2014) also found that instruction in patterning was significantly related to higher scores on the WJ-18A and WJ-18B. The present study extends these results by showing that there was a significant relation between patterning and performance on the WJ-10 that is independent of instruction.

The results of the present study are also consistent with those of Herman (1973) and Hendricks et al. (2006), who also showed significant relations between instruction in patterning and academic achievement measures. Herman (1973), using the Numbers Subtest of the Metropolitan Readiness Test in her dissertation, found significantly higher scores for children from English-speaking homes who were instructed in patterning, as compared to the control group. In Hendricks et al. (2006), children who were taught patterning scored significantly higher than the control group on both the mathematics and language scales of the Diagnostic Achievement Battery-2, after the two groups were statistically equated on IQ. The current study is consistent with these findings, in that the results suggest not only a relationship between patterning and mathematics, but also between patterning and reading, and that this relation exists whether or not some children receive instruction in patterning.

Educational and Clinical Implications

This study confirms empirically that there is a place for patterning in early education, a proposition which rested on educational consensus rather than data for most of the past half century. It also suggests that educators might profit from extending patterning instruction to older children. Current practice here and abroad is to teach alternating patterns to preschoolers and kindergartners. The relations between understanding more complex patterns and both reading and mathematics suggests that such patterns be incorporated into the curricula for first graders (i.e., seven-year-olds). The potential advantage of such an innovation is that instruction in one subject matter could produce advances in achievement in both subject matters, a possibility enhanced by the findings of Kidd et al. (2013) and Kidd et al. (2014).

This study also revealed that fundamental aspects of intelligence are correlated with children's ability to solve patterning problems. Two of the three EFs work through patterning, in whole or in part, to predict reading and mathematics achievement. The use of intelligence tests to predict academic achievement and measure thinking ability might be enhanced by the incorporation of measures of cognitive flexibility and of working memory. We note that the Wechsler measures already include the latter. It might be an advantage to include age-appropriate measures of patterning per se in measures of thinking that are intended to be used to predict achievement, because solution of patterning problems incorporates both cognitive flexibility and relevant aspects of working memory. The multi-ethnic composition of our sample suggests that such measures might be relatively free of ethnic bias. The Pattern Reasoning scale of both the K-ABC and KABC-II, which is said to express fluid intelligence (Kaufman, Lichtenberger, Fletcher-Janzan, & Kaufman, 2005), may instead or in addition reflect working memory and cognitive complexity.

Summary and Conclusions

The correlations found in this study support the suggestions by educators that there is a relation between patterning and early mathematics skills (Clements & Sarama, 2007a, 2007b, 2007c; McGarvey, 2012; Mulligan & Mitchelmore, 2009). Children's understanding of complex patterns was, in fact, more highly related than were measures of executive function, indicating that it makes a unique contribution to progress in both mathematics concepts and mathematics achievement. Children's understanding of complex patterns was also related to reading achievement. This result confirms the prediction of Sarama and Clements (2004), and the empirical results of Kidd et al. (2014), that showed a relation between patterning and reading. It implies that better understanding of patterns provides a broad foundation for academic achievement.

The present study also shows that understanding complex patterns is related to working memory and to cognitive flexibility, but not to inhibition. However, the regression analyses show that understanding such patterns makes a contribution to mathematics and reading achievement above and beyond that made by cognitive flexibility, and, on one mathematics concepts measure, beyond that made by working memory. Hence, it appears that understanding patterns more complex than the alternations taught in preschools and kindergartens is likely to be a worthwhile enterprise, contributing to progress in both mathematics and reading.

Limitations

This prediction must be tempered, however, by the observations that different achievement and concepts measures have yielded different results even when the same procedures are followed (Kidd et al., 2013; Kidd et al., 2014) It also seems clear that the particular assessment of patterning used is likely to affect the results obtained, as appears to have been the case when the results of the present study are compared with those of Bock (2015). Sample differences, especially the age and cognitive abilities of the children studied, may also affect the generalizability of results. There is no guarantee that the same EFs are involved to the same extent in the understanding of patterns by preschoolers and older children. Other extraneous variables, unknown at the present time, may also be limitations of the present study.

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