Functional Programming for BDA 2020/2021
The Final Test

27.01.2021

Exercise 1. Let $(M, \cdot, e)$ be a monoid and (W_M, η, μ) the writer monad, i.e. $W_M : SET \to SET$ is a functor and $\eta : \mathrm{Id} \dot{\to} W_M$, $\mu : W_M^2 \dot{\to} W_M$ are natural transformations given by

$$\begin{aligned} W_M(X) &= X \times M, \\ W_M(f) &= (f, \mathrm{id}), \\ \eta_X(x) &= (x, e), \\ \mu_X((x, m_1), m_2) &= (x, m_1 \cdot m_2), \end{aligned}$$

for each set X, function f, $x \in X$ and $m_1, m_2 \in M$.

a) Show that indeed μ is a natural transformation, i.e. that the following diagram commutes for all sets X, Y and each function $f : X \to Y$:

$$\begin{array}{ccc} W_M^2(X) & \xrightarrow{W_M^2(f)} & W_M^2(Y) \\ \Big\downarrow{\scriptstyle \mu_X} & & \Big\downarrow{\scriptstyle \mu_Y} \\ W_M(X) & \xrightarrow{W_M(f)} & W_M(Y) \end{array}$$

b) Show that the following 1st "monadic diagram" commutes for every set X

$$\begin{array}{ccc} W_M(X) & \xrightarrow{\eta_{W_M(X)}} & W_M^2(X) \\ \Big\downarrow{\scriptstyle W_M(\eta_X)} & & \Big\downarrow{\scriptstyle \mu_X} \\ W_M^2(X) & \xrightarrow{\mu_X} & W_M(X) \end{array}$$

Good luck! - Marcin Michalski