

① number of atoms - $10^{98} - 10^{81}$
 length of binary string - 500
 $2^{500} = (2^{10})^{50} \approx (10^3)^{50} = 10^{150}$
 $10^{150} > 10^{82}$

② Insert Sort Quick Sort
 n^2 $2n \log n$

1 comparison - 1,2 ns

$n = 1000$

IS) $1000^2 \cdot 1.2 \text{ ns} = 1200000 \text{ ns} = 1.2 \text{ ms}$

QS) $2000 \cdot \ln(1000) \cdot 1.2 \text{ ns} \approx 16579 \text{ ns} \approx 16.6 \mu\text{s}$

$n = 1,000,000$

IS) $1.2 \cdot 10^{12} \text{ ns} = 1.2 \cdot 10^3 \text{ s} = 20 \text{ min}$

QS) $33157225 \text{ ns} \approx 33 \text{ ms}$

$n = 200,000,000$

IS) $4.8 \cdot 10^{16} \text{ ns} = 4.8 \cdot 10^7 \approx 1.5 \text{ y}$

QS) 8.8 s

1 day = 86400 s = $8.64 \cdot 10^{13} \text{ ns}$

IS) $1.2 \cdot n^2 = 8.64 \cdot 10^{13}$

$n^2 = 7.2 \cdot 10^{13}$

$n \approx 8485281$

QS) $1.2 \cdot 2n \log n = 8.64 \cdot 10^{13}$

$n \log n = 3.6 \cdot 10^{13}$

$n \approx 1.29 \cdot 10^{12}$

II

① $\vec{v}_1 = [1, 2, 3]$ $\vec{v}_2 = [5, 3, 4]$
 $d(\vec{v}_1, \vec{v}_2) = \sqrt{(1-5)^2 + (2-3)^2 + (3-4)^2} =$
 $= \sqrt{16+1+1} = \sqrt{18} = 3\sqrt{2}$

② $\vec{v}_1 = [1, 1, 1, 0, 1]$ $\vec{v}_2 = [0, 0, 1, 1, 0]$

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$\vec{v}_2 = [0, 0, 1, 1, 0]$

$H_d(\vec{v}_1, \vec{v}_2) = 4$

③ $\vec{v}_1 = [1, 2, 1]$ $\vec{v}_2 = [1, -1, 0]$

$\cos \alpha = \frac{\vec{v}_1 \cdot \vec{v}_2}{|\vec{v}_1| |\vec{v}_2|}$

$\vec{v}_1 \cdot \vec{v}_2 = 1 \cdot 1 + 2 \cdot (-1) + 1 \cdot 0 = -1$

$|\vec{v}_1| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$

$|\vec{v}_2| = \sqrt{2}$

$\cos \alpha = -\frac{1}{\sqrt{2}\sqrt{6}} = -\frac{1}{\sqrt{12}}$

④ $\ln n! = n \ln n - n$

$\binom{2n}{n} = \frac{(2n)!}{n!(2n-n)!} = \frac{(2n)!}{n!n!}$

$= \frac{2n \ln(2n) - 2n}{2(n \ln n - n)} = \frac{n \ln(2n) - n}{n \ln n - n}$

$= \frac{\ln(2n) - 1}{\ln n - 1}$

III