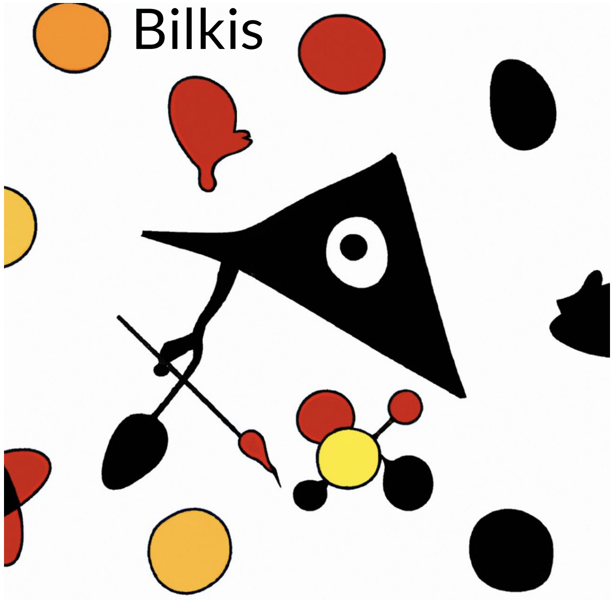
# Decision-making in quantum environments

## From model-free to model-aware learning of quantum controls

Matías
Bilkis



PhD Defense

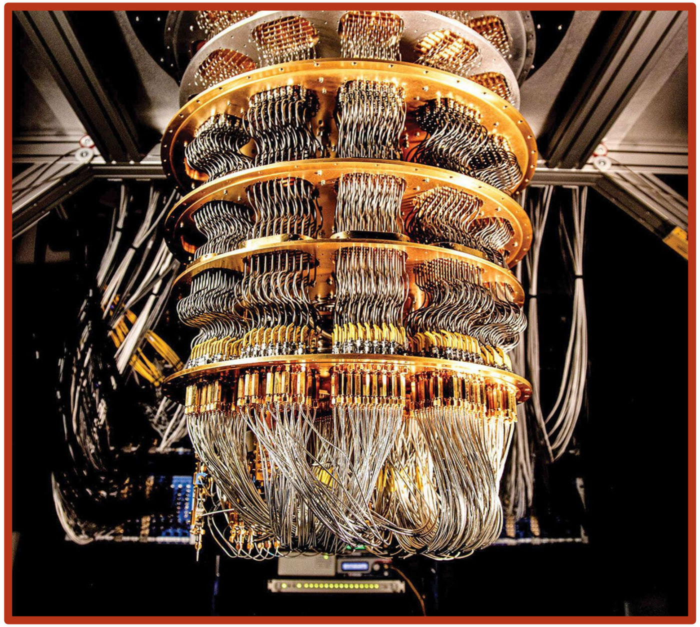
*12th June, 2023*

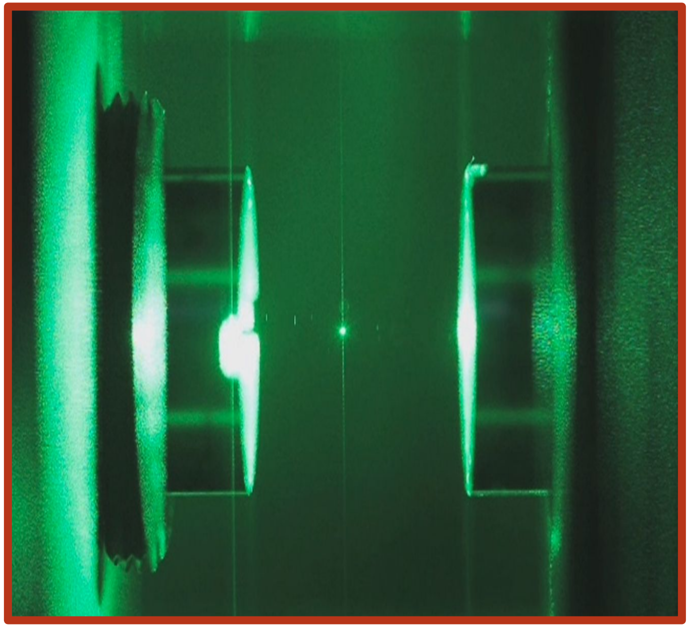
**SCAN ME**

github.com/matibilkis/PhD-thesis

*Supervisor:* Prof. John Calsamiglia

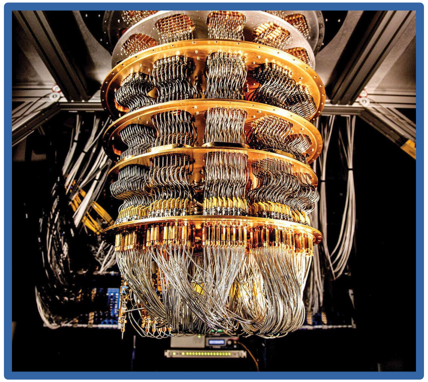
# Quantum-technology toolbox

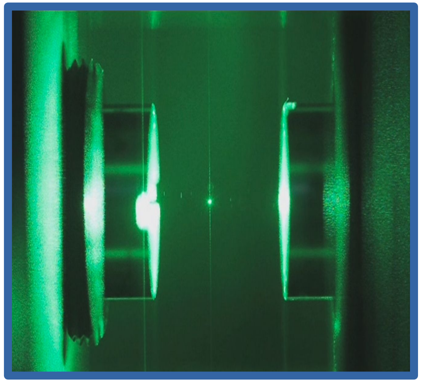




# Long-distance quantum communication

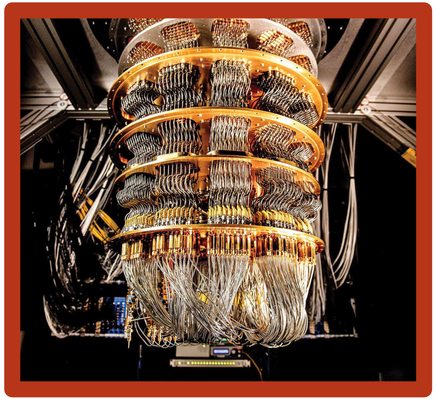
- Light as an information carrier
- Which quantum measurement?
- Calibrate measurement under unknown sources of noise.

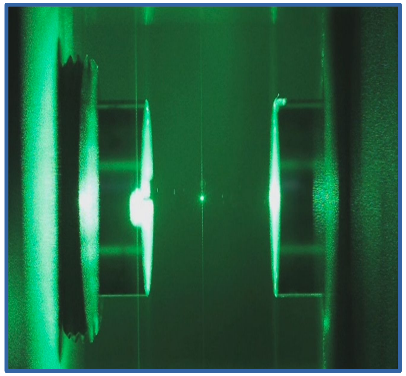




# Quantum computing

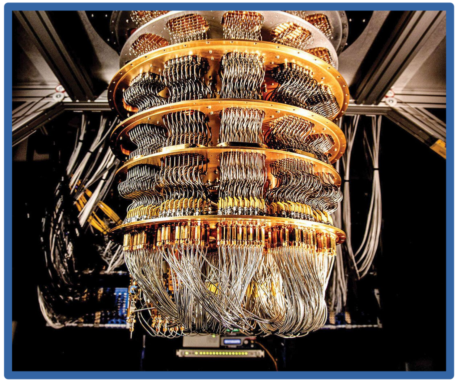
- Noisy Intermediate-Scale Quantum (NISQ) devices
- Quantum advantage experimentally observed for specific task
- How to use them for something else?

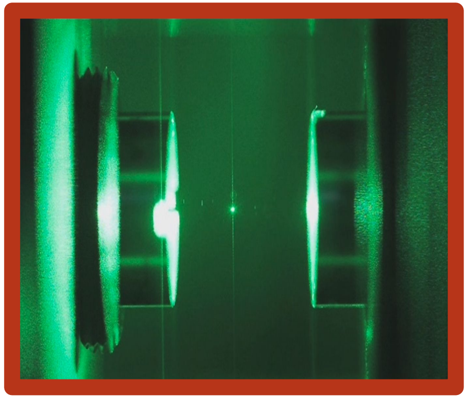




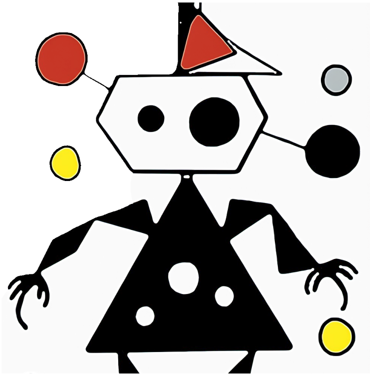
# Quantum sensing

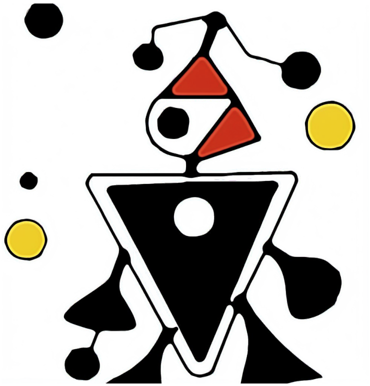
- Precise experimental control of quantum systems
- Continuous-monitoring → measurement signal
- How to best process the data acquired ?

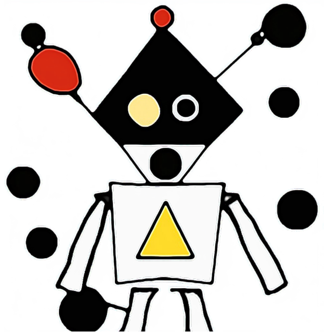




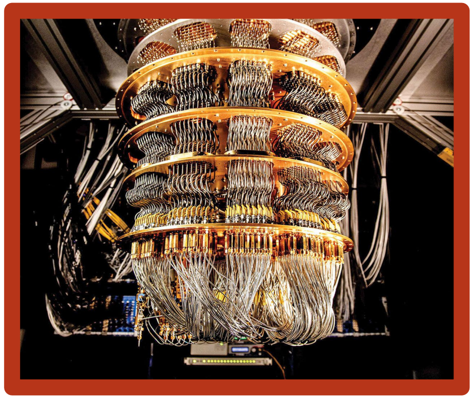
# Decision-making in quantum environments



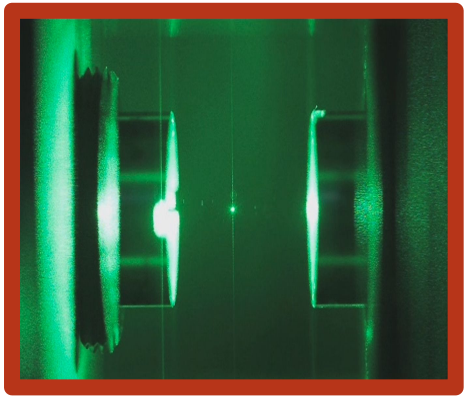




Measurement calibration

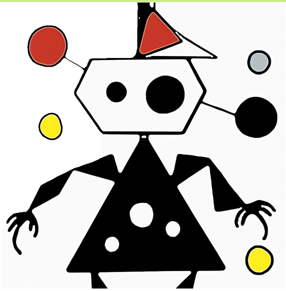


Circuit design

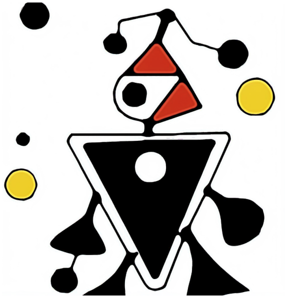


Data processing

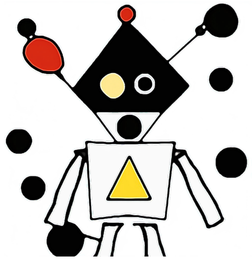
# Artificial intelligence



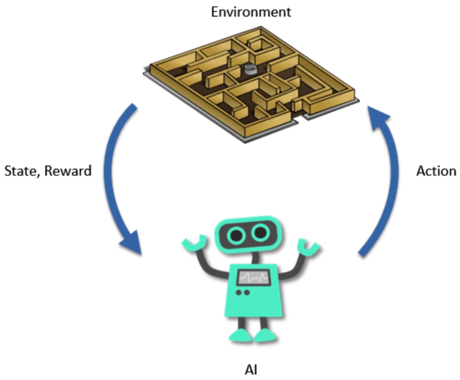
- Reinforcement-learning
  [Sutton2018Reinforcement]



- Optimization
  [Wong2010Ride]



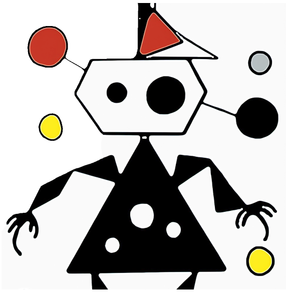
- Classification
  [Bewley2017Simple]



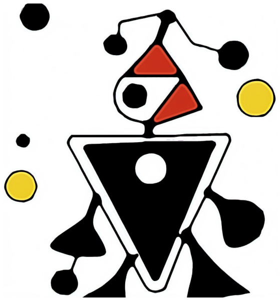


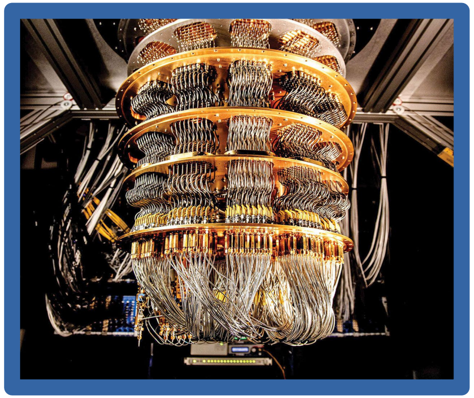
# Awareness degree

Model-free | Semi-agnostic | Model-aware

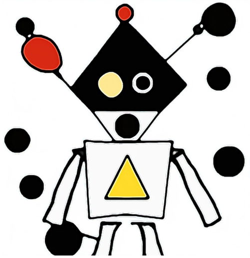


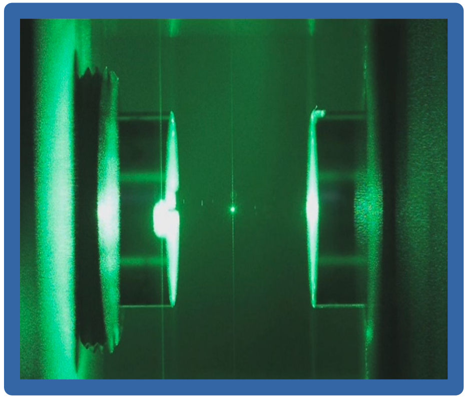
Measurement calibration





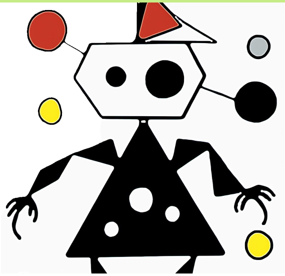
Circuit design



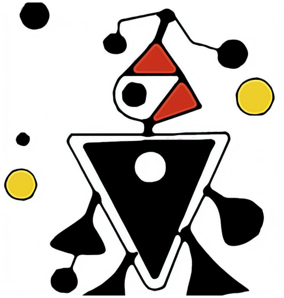


Data processing

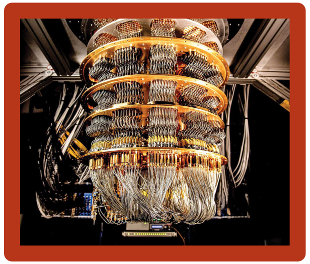
# Outline

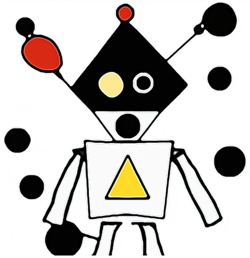


➢ Learning in the darkness

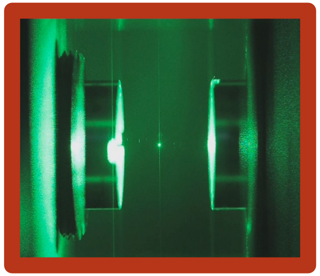


➢ Learning in the twilight





➢ Learning in the daylight

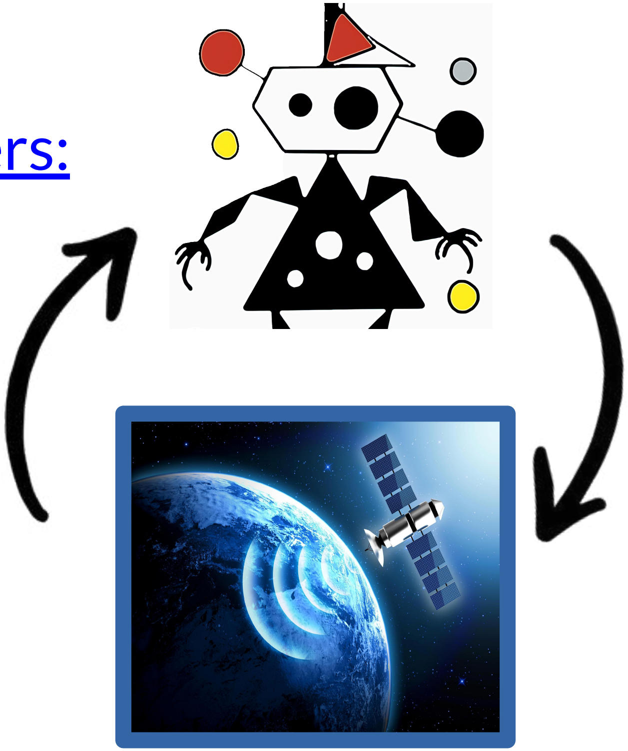


# This chapter in a nutshell

Works:

→[Real-time calibration of coherent-state receivers: Learning by trial and error]() - Bilkis et.al.

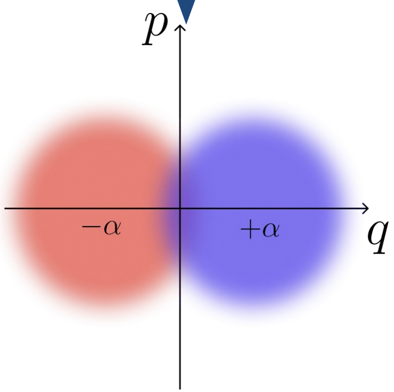
→[Reinforcement-learning calibration of coherent-state receivers on variable-loss optical channels]() - Bilkis et.al.



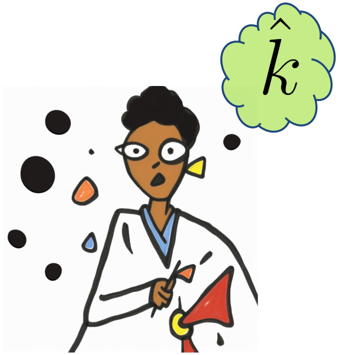
- Discrimination of coherent states
- Scenario: long-distance communication
- Model-free calibration of a measurement device

# Coherent-state discrimination

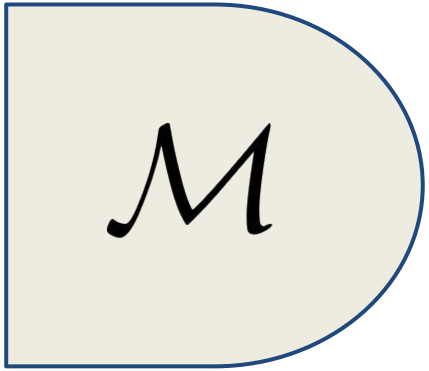

$$|\alpha_k\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{((-1)^k \alpha)^n}{\sqrt{n!}} |n\rangle$$




- Distinguish between $\{|\alpha\rangle, |-\alpha\rangle\}$
- POVM $\mathcal{M} = \{M_0, M_1\}$
- $p(n|\alpha_k) = \mathrm{Tr}\big(M_n \Phi[|\alpha_k\rangle\langle\alpha_k|]\big)$

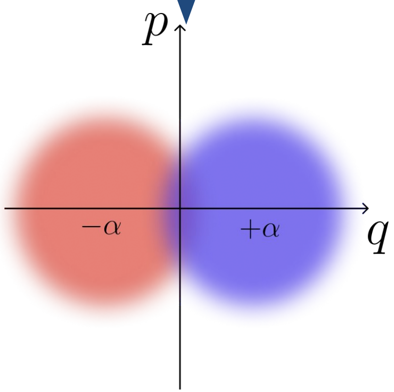



$n \sim p(n|\alpha_k)$



# Success probability & Helstrom bound


$$|\alpha_k\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{((-1)^k \alpha)^n}{\sqrt{n!}} |n\rangle$$



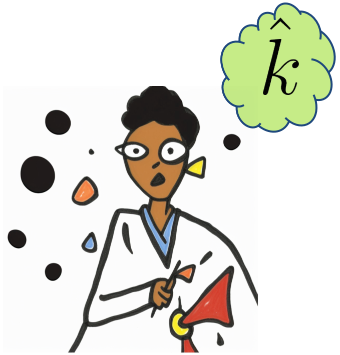

- Success probability; $g(n)$ *guessing rule*

$$P_s(\mathcal{M}, g) = \sum_n p(n|\alpha_{\hat{k}}) \mathrm{pr}(\alpha_{\hat{k}})|_{\hat{k}=g(n)}$$

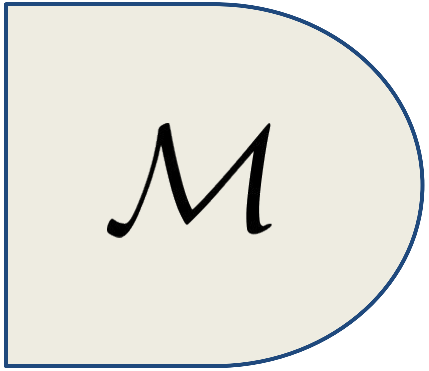
- Helstrom bound

$$\begin{aligned} P_s(\mathcal{M}, g) &\leq P_s(\mathcal{M}^*, g^*) \\ &= \frac{1}{2}\Big(1 + ||p_0\rho_0 - p_1\rho_1||_1\Big) \end{aligned}$$

- Optimal measurement projection over cat-like states $\sim |\alpha\rangle + |-\alpha\rangle$



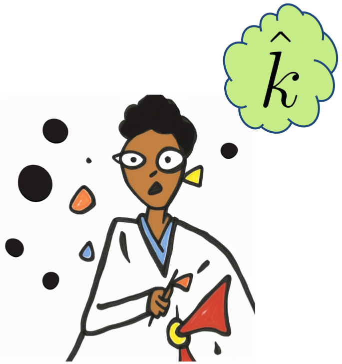

$$n \sim p(n|\alpha_k)$$



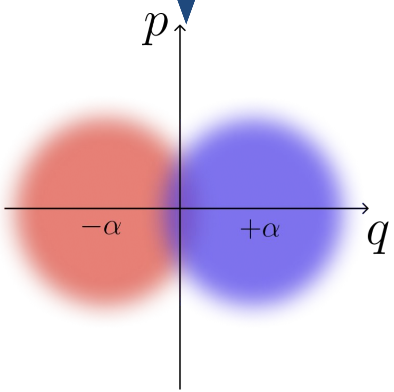
# Homodyne measurement

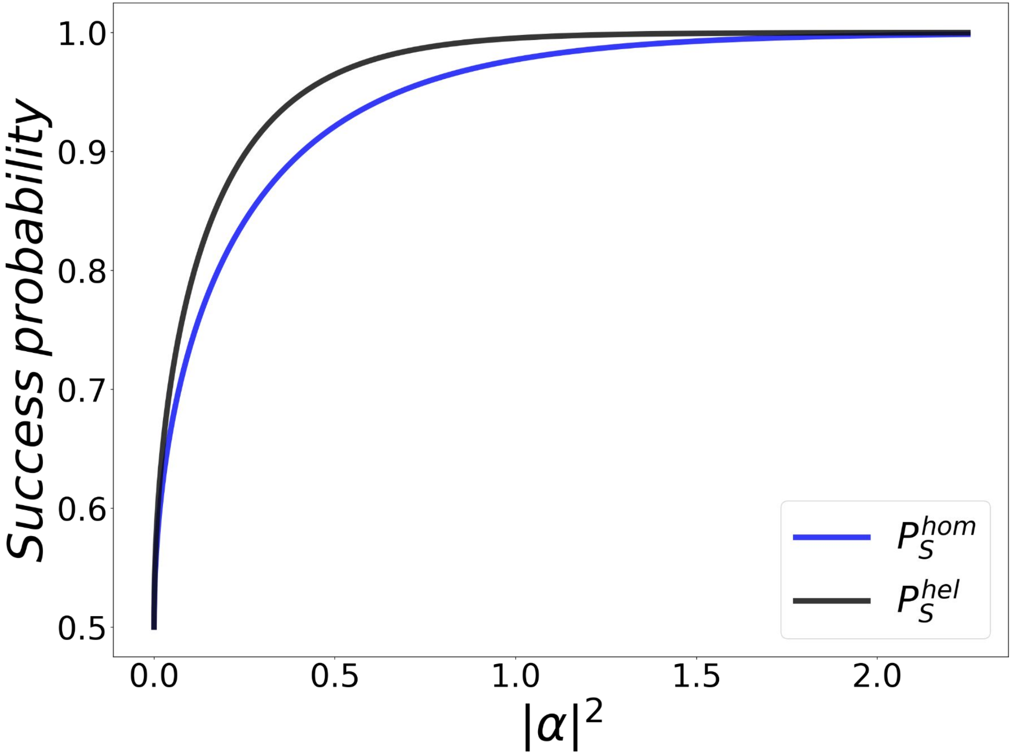

- Homodyne is optimal among all Gaussian receivers

[Takeoka2008Discrimination]
[Sabapathy2020Bosonic]

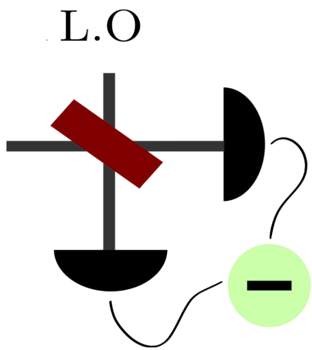



$$|\alpha_k\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{((-1)^k \alpha)^n}{\sqrt{n!}} |n\rangle$$



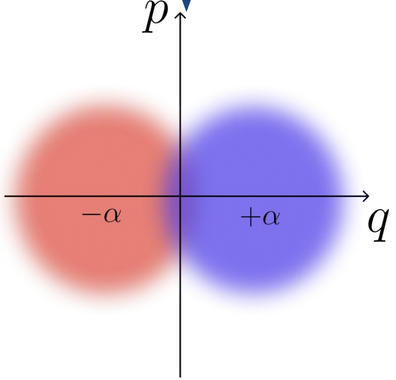

$$q \sim \sqrt{\frac{2}{\pi}} e^{-2(q-\alpha_k)^2}$$

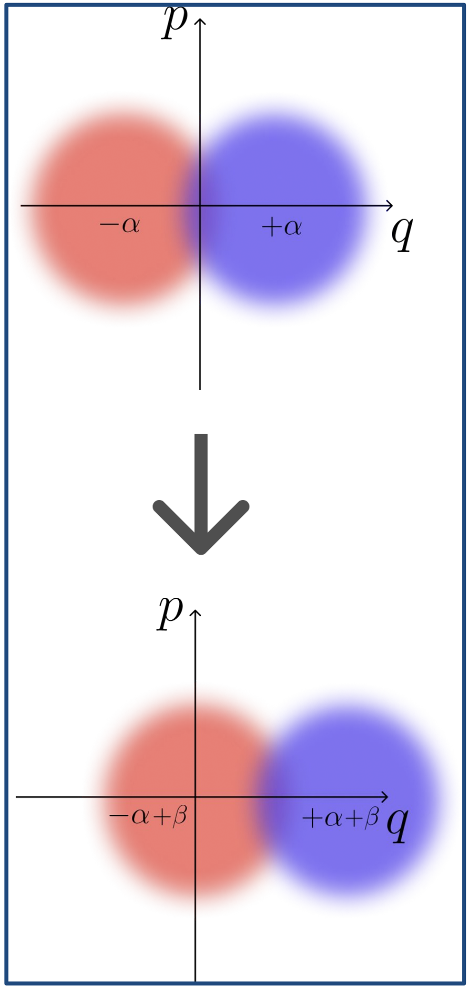


# Kennedy receiver

[Kennedy1973Near]

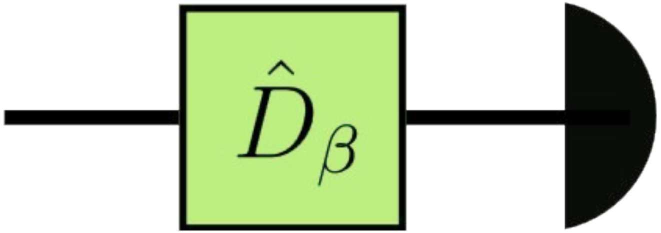

$$|\alpha_k\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{((-1)^k \alpha)^n}{\sqrt{n!}} |n\rangle$$





$$n \sim p(n|\alpha_k)$$



# Kennedy receiver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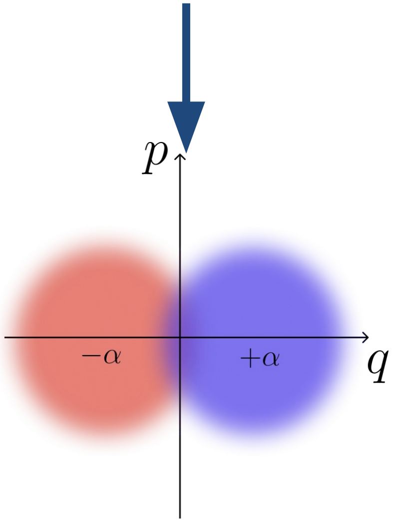
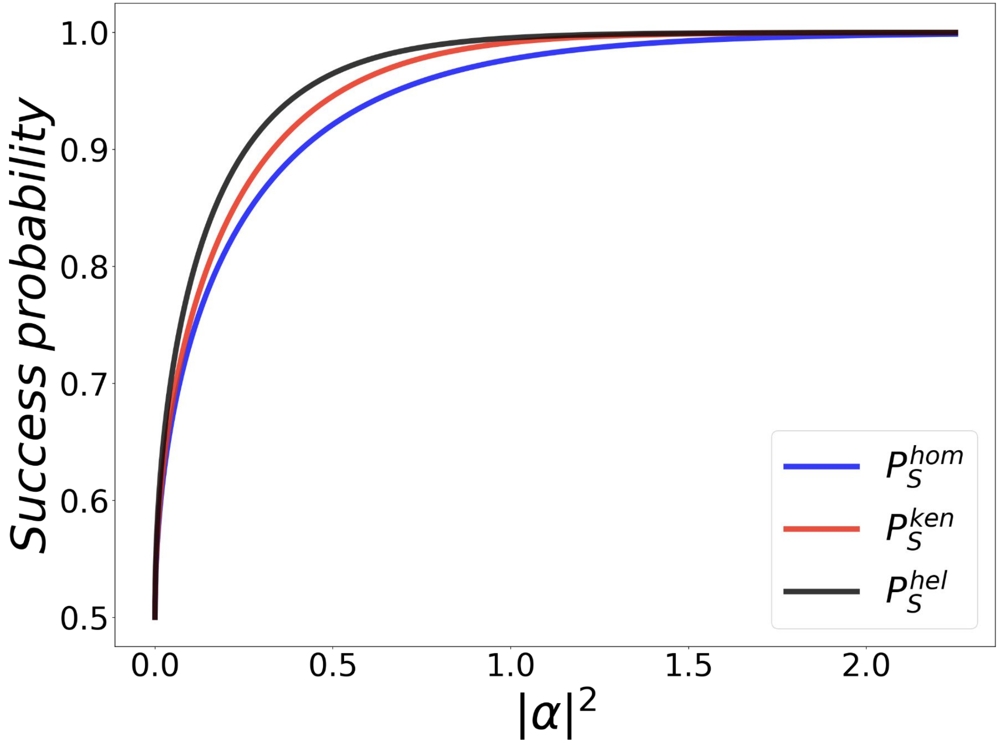
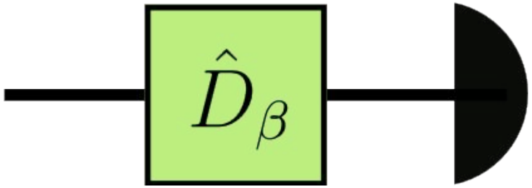



- Photon-detection measurement **(non-gaussian)** $\{|0\rangle\langle 0|, I - |0\rangle\langle 0|\}$
- $P_s^{ken} = \sum_{o=0,1} \max_k p_k p(n|\alpha^{(k)})$



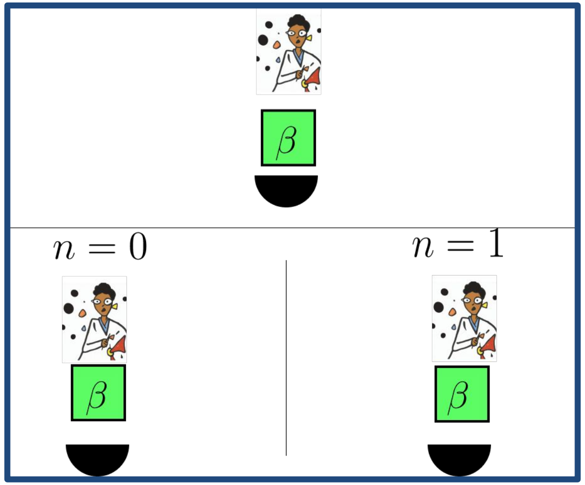


$n \sim p(n|\alpha_k)$

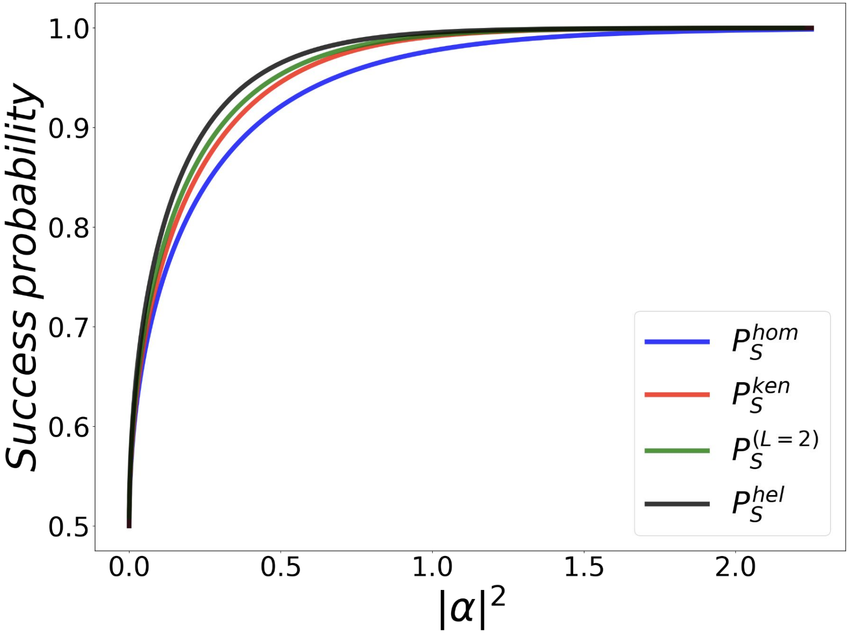


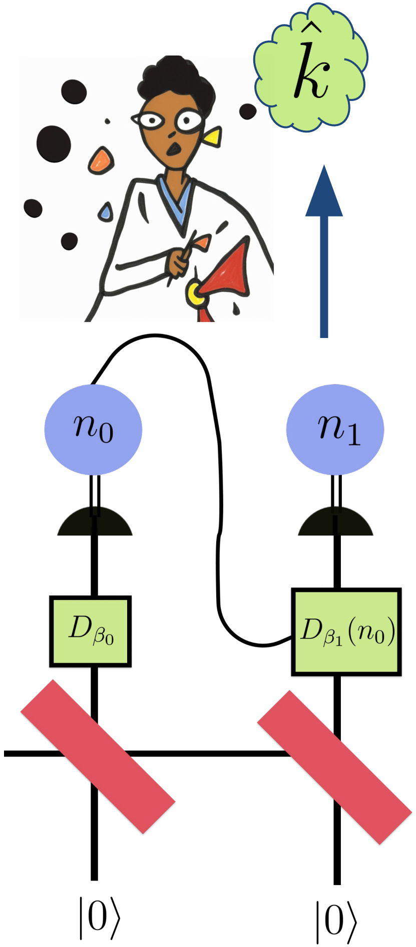
# Dolinar-like receiver

- <u>Adaptive</u>: next displacement value depends on previous outcome observed



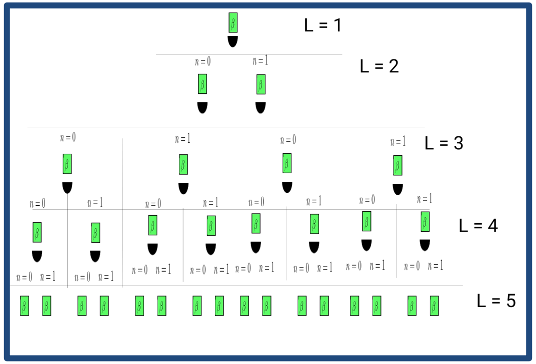

Conditional displacement tree

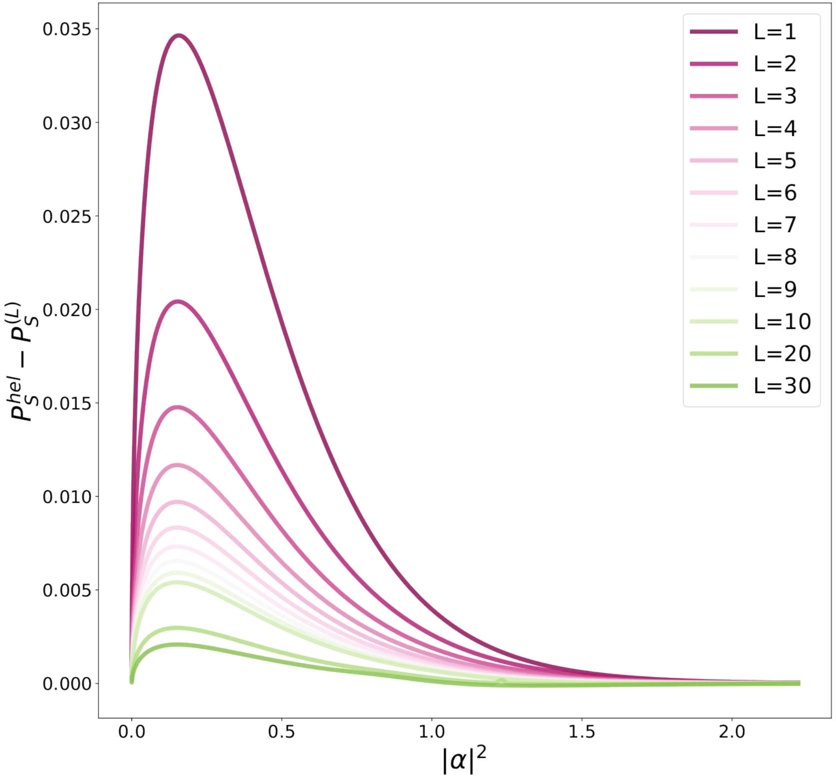


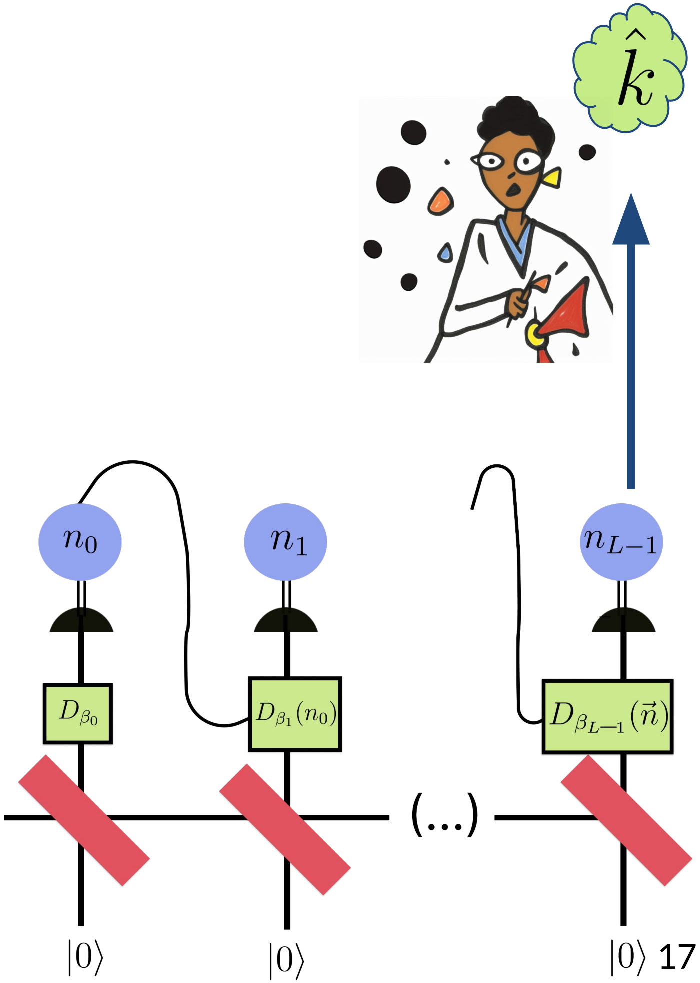



# Dynamic programming Dolinar receiver

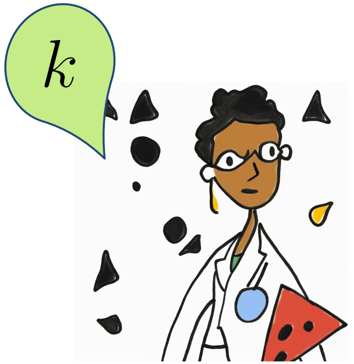
- In the limit of $L \to \infty$ is optimal [Dolinar1974Optimum]



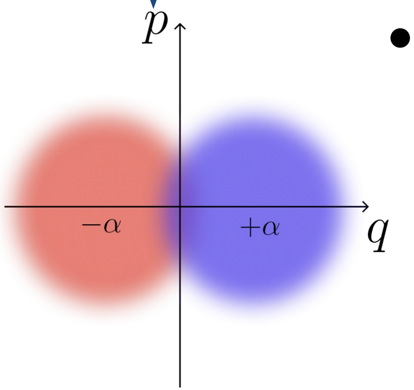



# Highlights & Issues



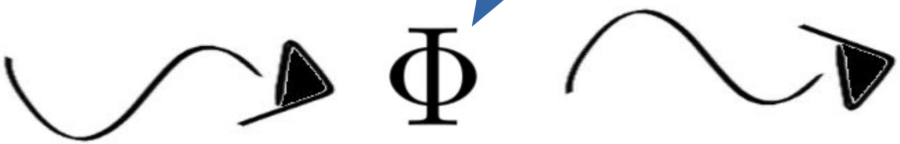

$$|\alpha_k\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{((-1)^k \alpha)^n}{\sqrt{n!}} |n\rangle$$

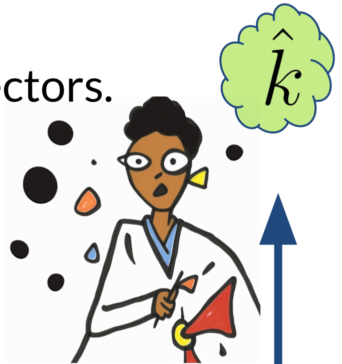


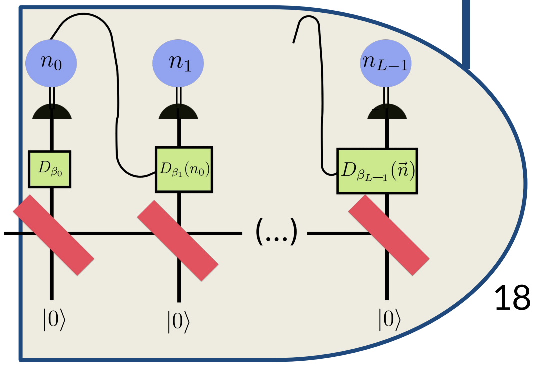

- Dolinar made out of simple linear optic elements + on/off detectors. Experimental proofs of concepts of Dolinar in labs ✓
  [Cook2007Optical]

- Performance severely limited by noise ⚠

- Communication scenarios: <u>transmission channels</u> hard-to-model ⚠

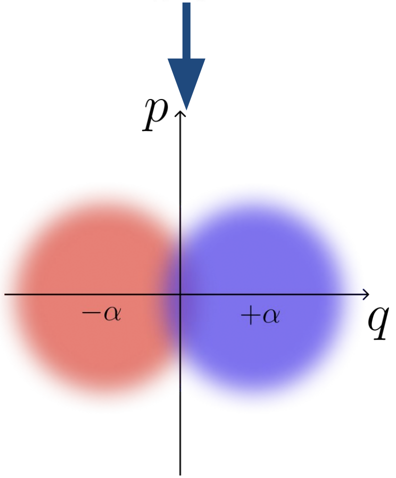


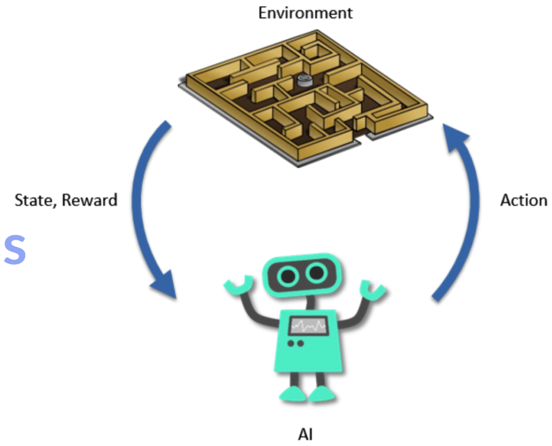
# Model-free calibration of Dolinar receiver

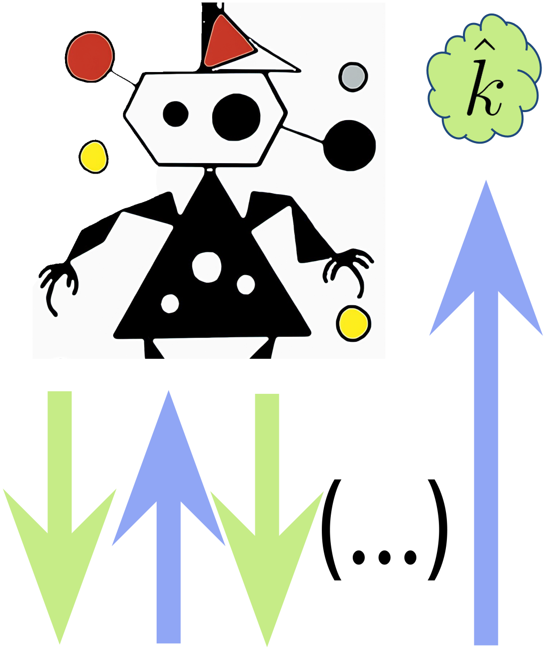

$$|\alpha_k\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{((-1)^k \alpha)^n}{\sqrt{n!}} |n\rangle$$



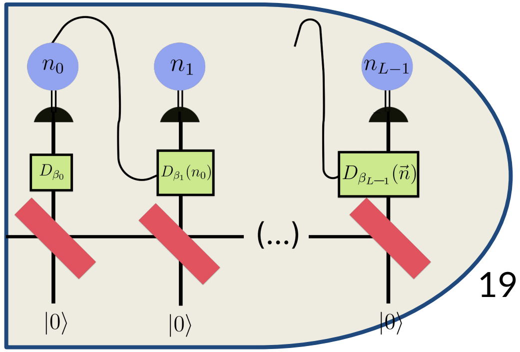

## Reinforcement learning

- Actions
- Observations
- Rewards



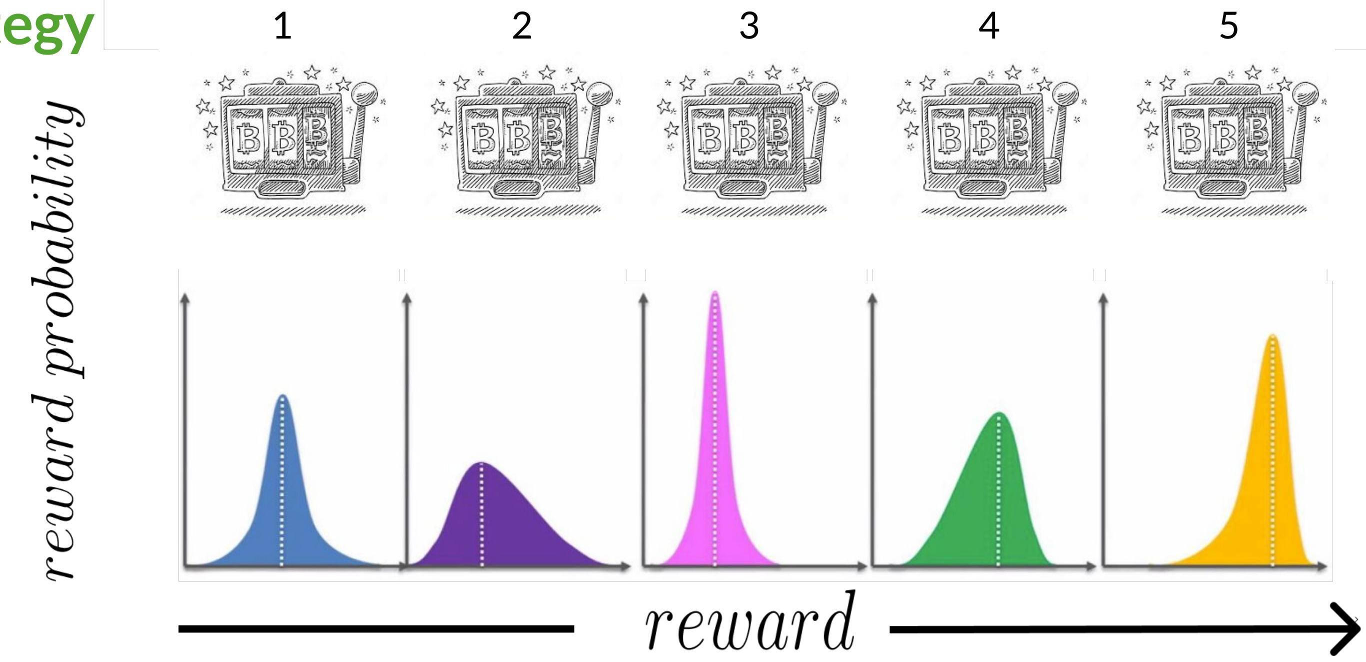




- Goal: use the receiver as best as possible
- <u>Agent is agnostic</u>: unaware of α, p(n|α)



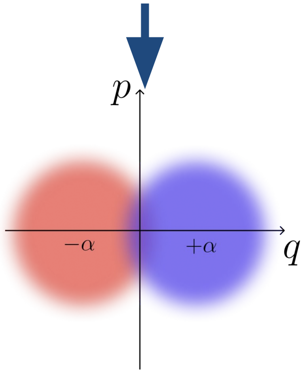
# Bandit problem

- Exploration-exploitation tradeoff: actively get as much reward as possible
- Reward obtained from black-box; sample strategies?
- Greedy strategy

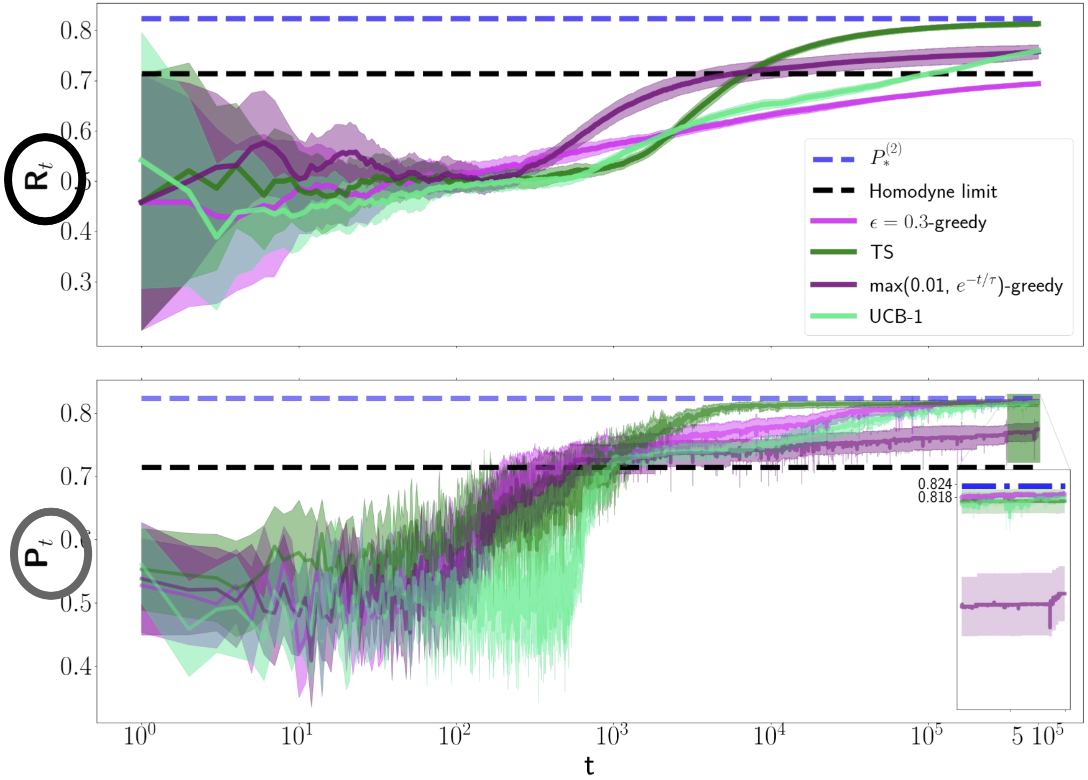



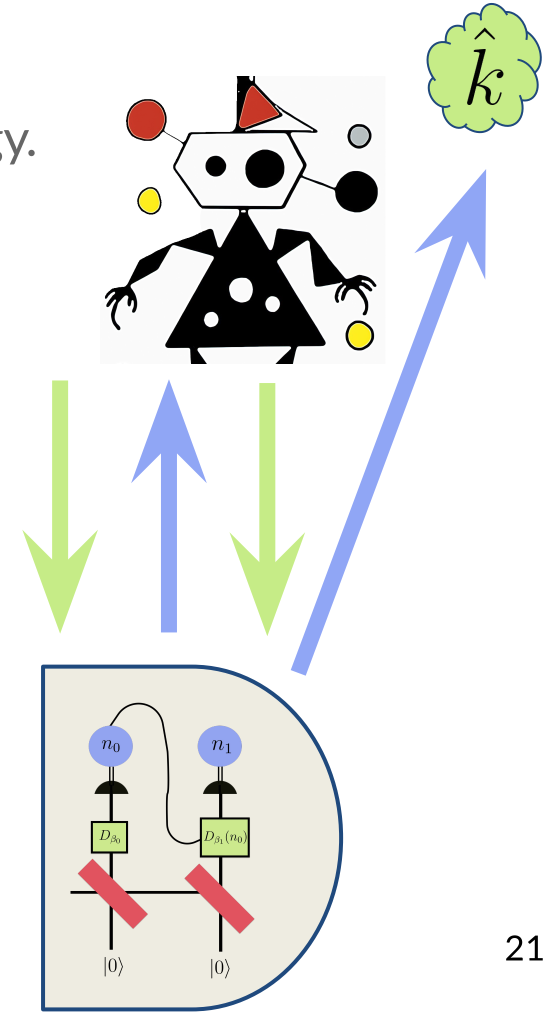
# Q-learning curves

$$|\alpha_k\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{((-1)^k\alpha)^n}{\sqrt{n}} |n\rangle$$



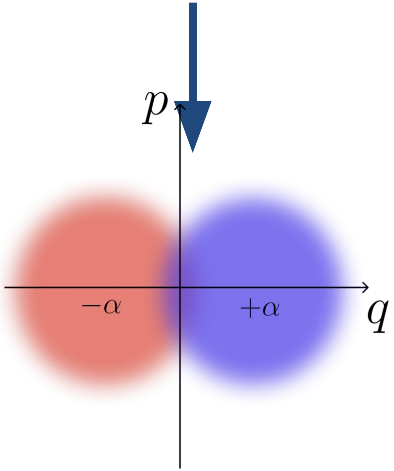

- Empirical success rate at t<u>th</u> experiment
- Success probability of agent's favorite strategy.



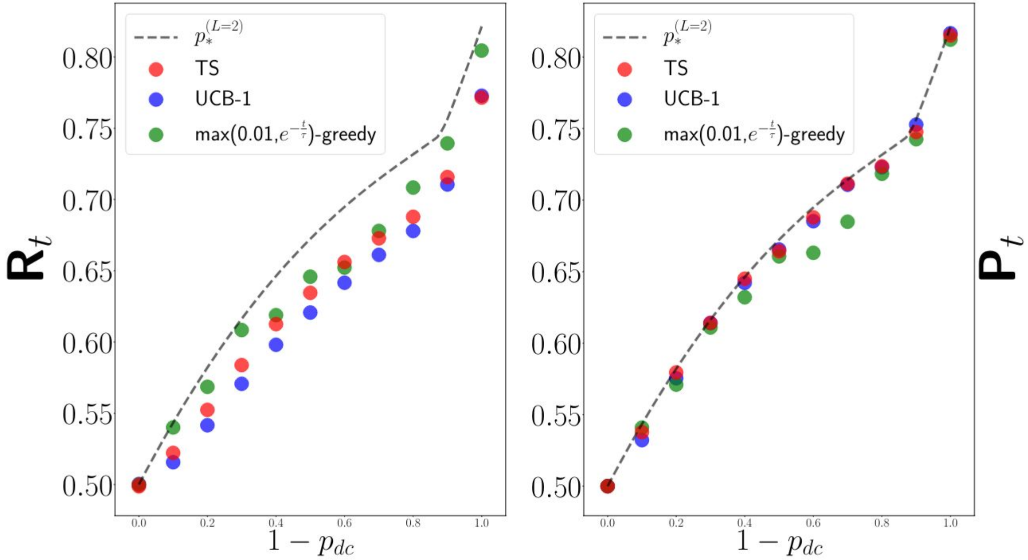
# Noise robustness

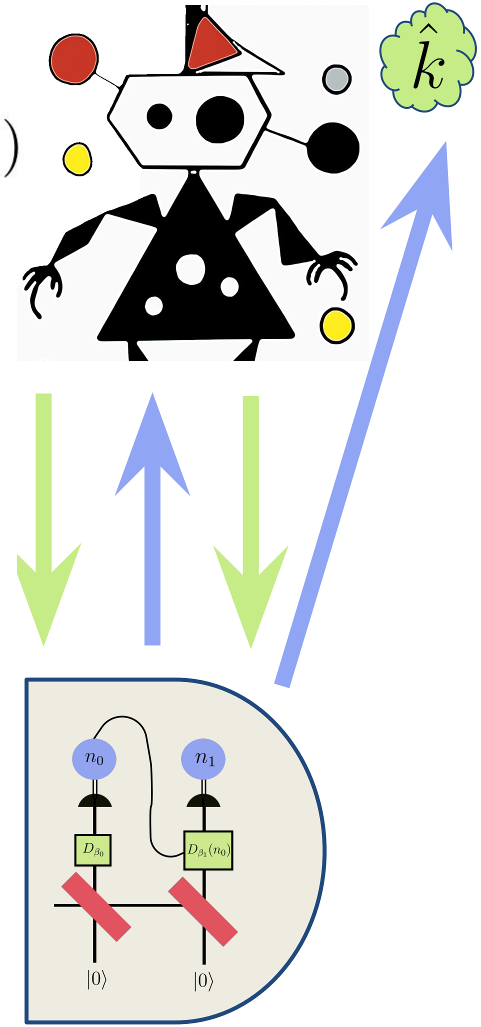

$$|\alpha_k\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{((-1)^k \alpha)^n}{\sqrt{n}} |n\rangle$$




- Dark counts

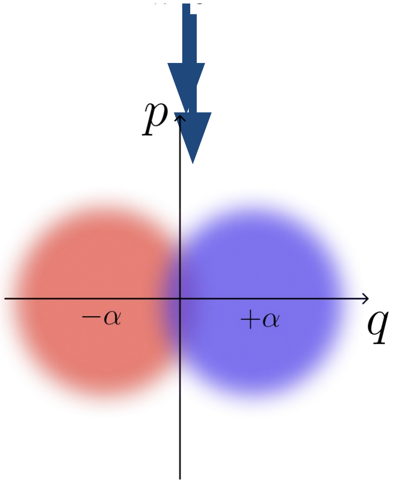
$$p(n_\ell = 0|\alpha_\pm) \rightarrow (1 - p_{\mathrm{dc}})p(n_\ell = 0|\alpha_\pm)$$



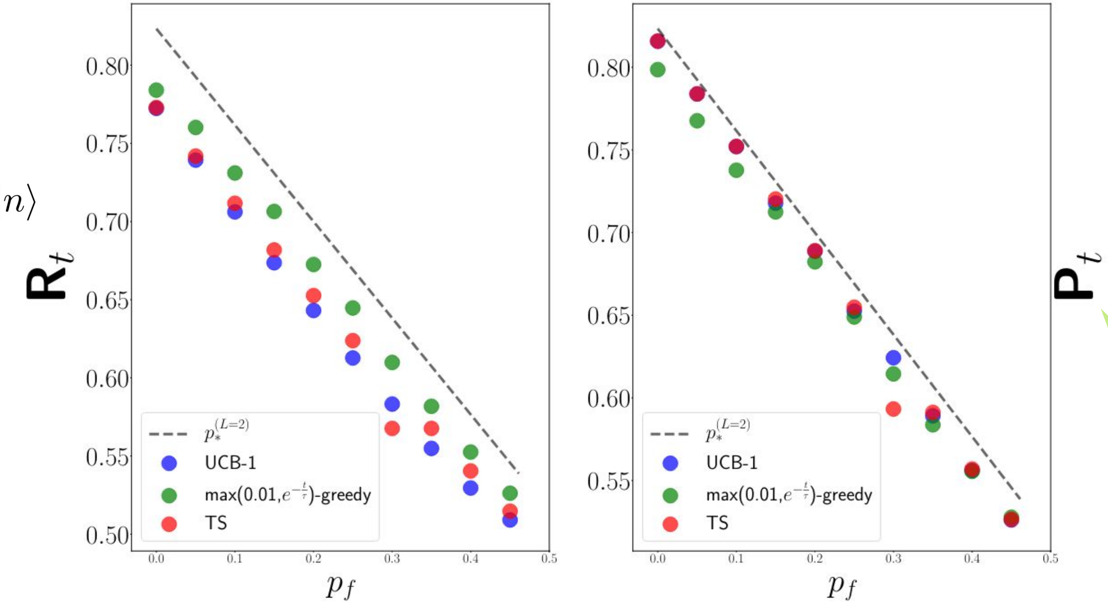
# Noise robustness

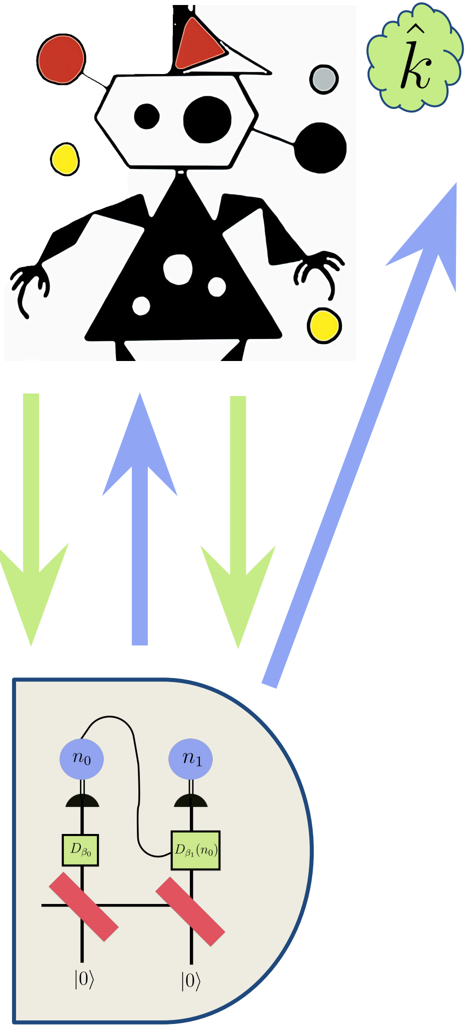

$$|c\,|\alpha_k\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{((-1)^k\alpha)^n}{\sqrt{n}} |n\rangle \quad |n\rangle$$



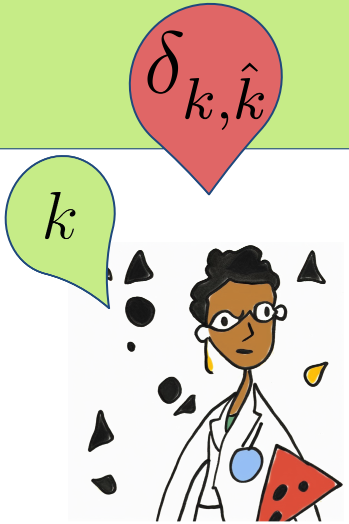

- Phase flips

$$p(\vec{n}|\alpha) \to (1-p_f)p(\vec{n}|\alpha) + p_f p(\vec{n}|-\alpha)$$

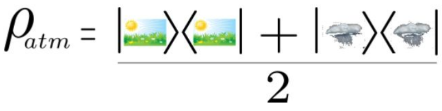


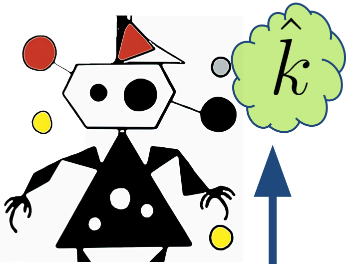



# Noise robustness

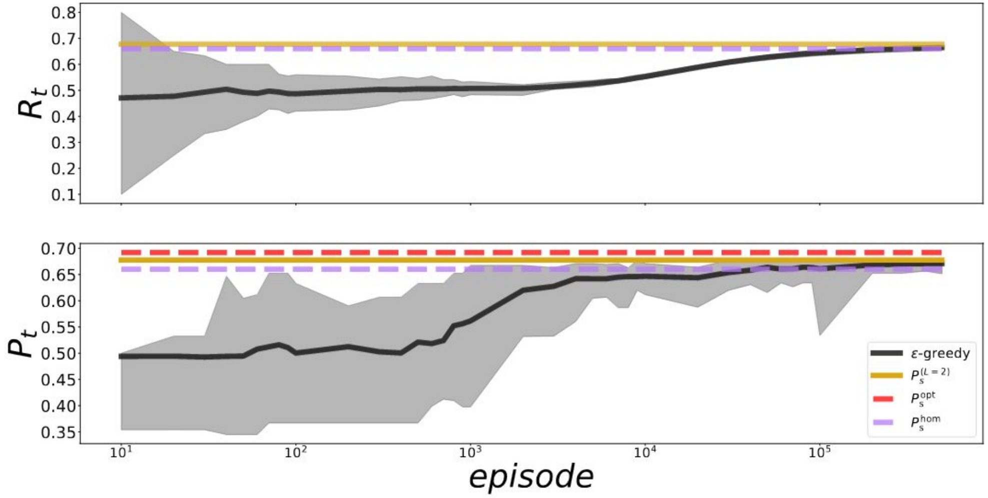


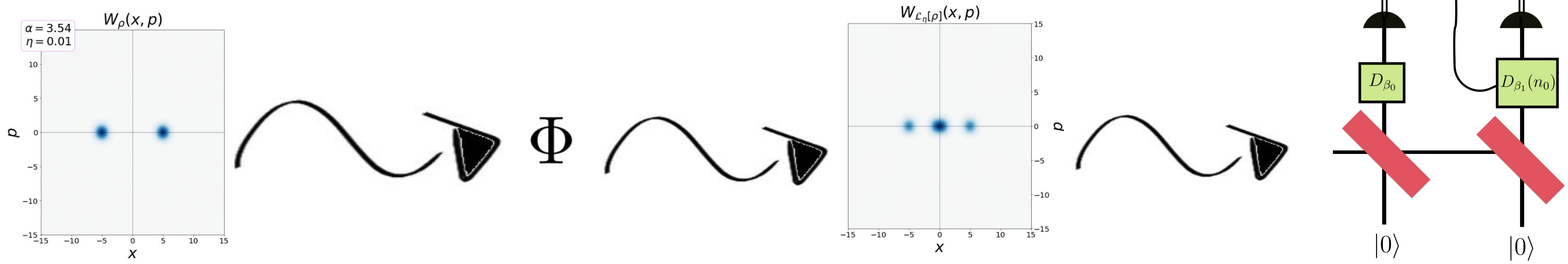

- Variable-loss lossy channel




$$|\alpha_k\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{((-1)^k \alpha)^n}{\sqrt{n!}} |n\rangle$$

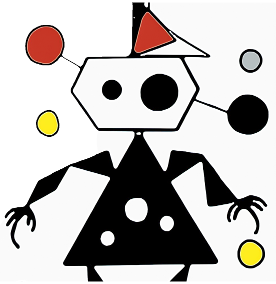


# Recap

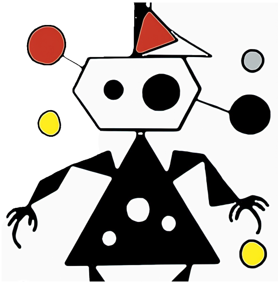
- We tackle binary coherent-state discrimination
- Optimal measurement implementable by simple optical elements

- Noise hinders a successful experimental implementation
- Model-free calibration: agent ignores everything of the experiment



- Our method can adapt to a variety of scenarios

# Highlights and next steps



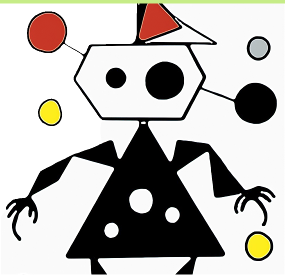
## Novelties

- First application of RL techniques to communication setting
- Agent trained solely using measurement outcomes
- Incorporation of bandit methods to Q-learning

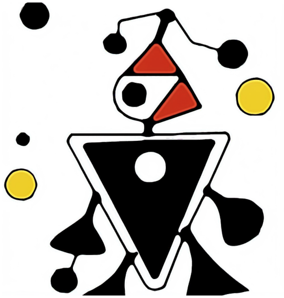
## Next steps

- Increase number of candidate states and L
- Optimization landscape (in particular for more phases and/or L>2)
- Incorporation of deep learning methods (Esté & Crosta thesis)

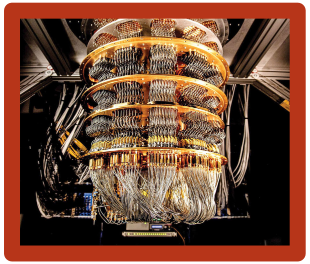
# Talk outline

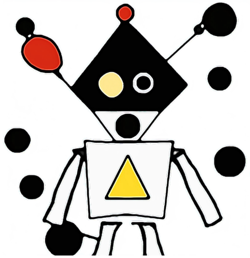


➢ Learning in the darkness

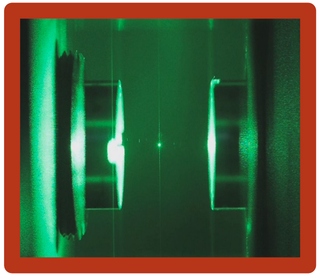


➢ Learning in the twilight





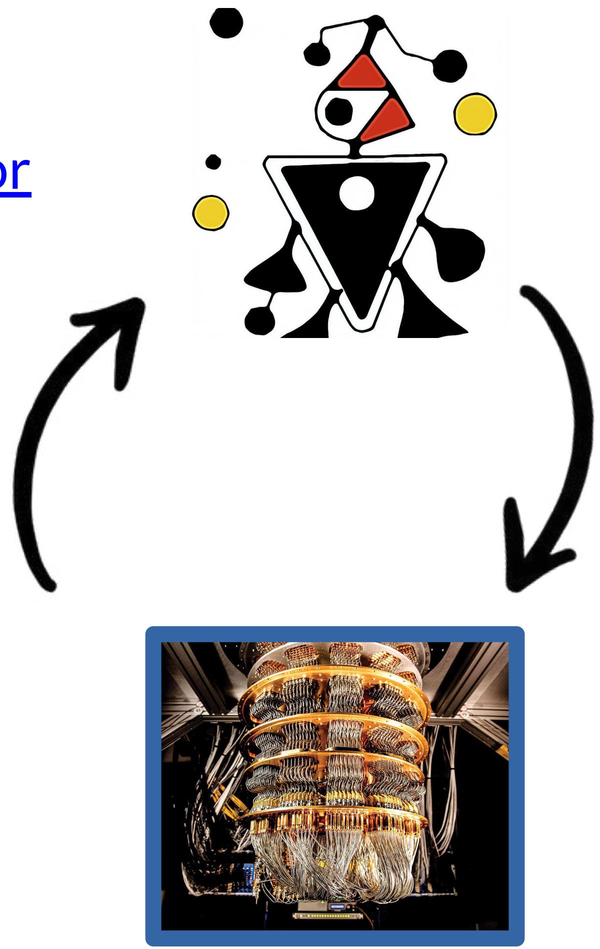
➢ Learning in the daylight



# This chapter in a nutshell

Work:

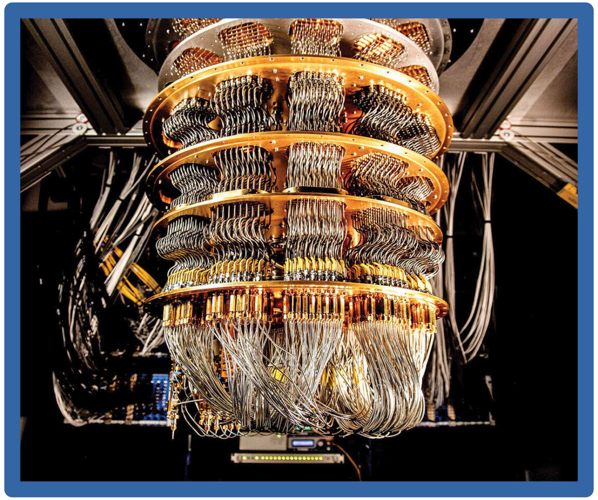
→ A semi-agnostic ansatz with variable structure for quantum machine learning - Bilkis et.al.



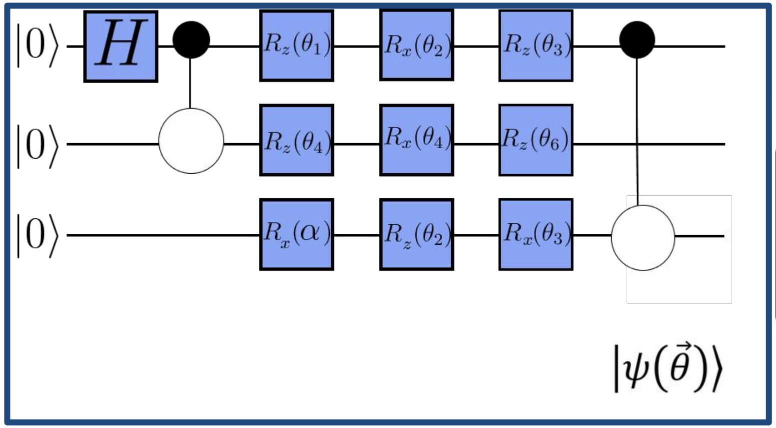
- Quantum circuit structure learning
- We propose a variable-circuit structure method

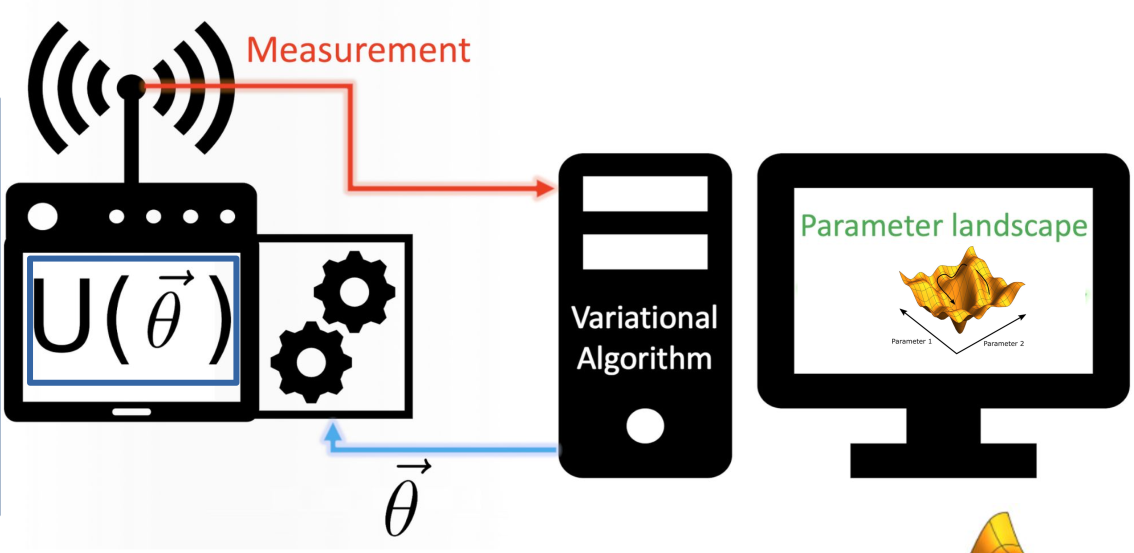
# NISQ computers

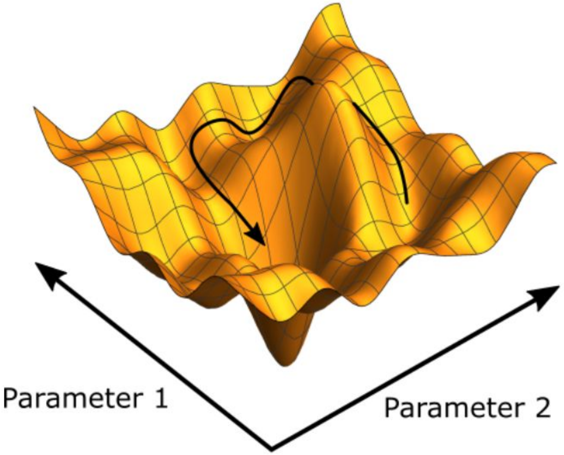
- Extremely noisy.
- However we can prepare a genuinely quantum state.
- Can we use them to accomplish some task?



# Variational Quantum Algorithms (VQAs)




<u>VQAs' *ingredients*</u>

- Quantum circuit (**ansatz)**
- Cost function
- Optimization procedure

# Gradient-shift rules

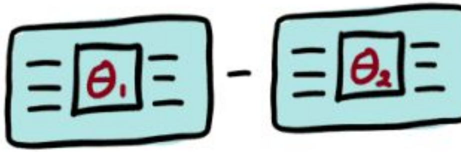
- The gradient can be estimated using the same circuit

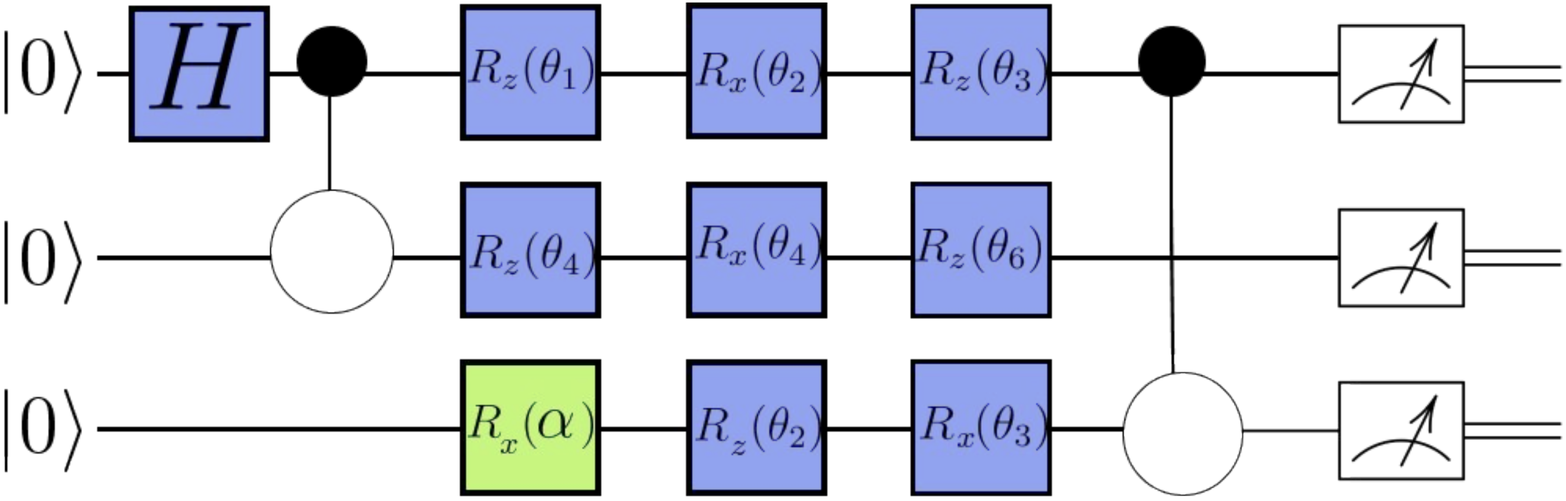
$$C_{\mathrm{VQE}}(\boldsymbol{\theta}) = \langle 0|U^{\dagger}(\boldsymbol{\theta})\hat{H}U(\boldsymbol{\theta})|0\rangle$$

- Analytical result

$$\partial_{\alpha}C(\boldsymbol{\theta}) = \frac{1}{2}\left(C(\boldsymbol{\theta})|_{\alpha\leftarrow\alpha+\frac{\pi}{2}} - C(\boldsymbol{\theta})|_{\alpha\leftarrow\alpha-\frac{\pi}{2}}\right)$$

$$\nabla_{\theta} f = f(\theta_1) - f(\theta_2)$$

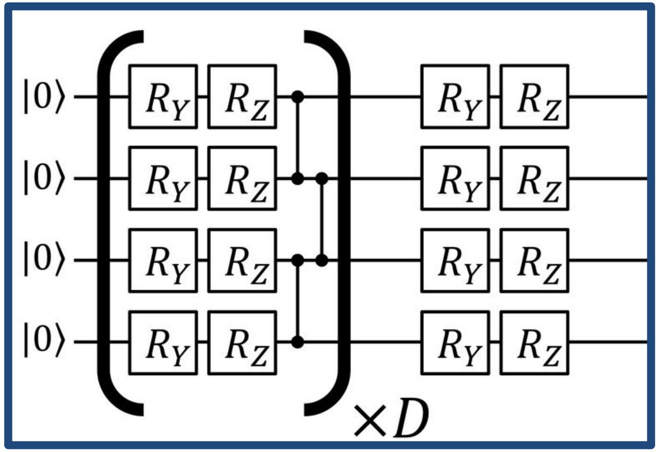


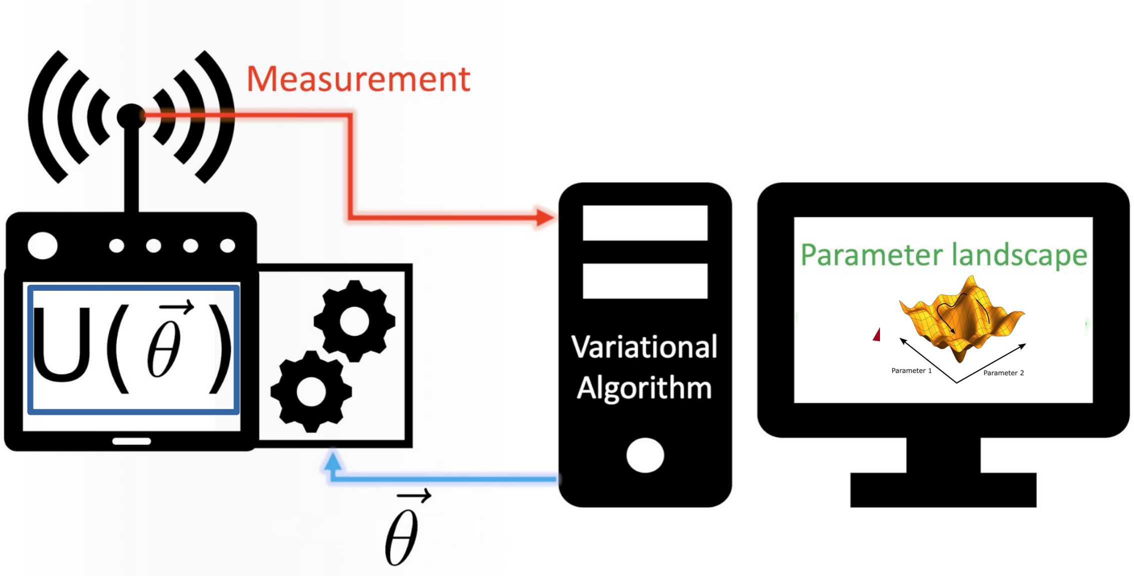





[Schuld2018Evaluating]
[Mari2021Estimating]

# Variational Quantum Algorithms (VQAs)




$|\psi(\vec{\theta})\rangle$

<u>VQAs' *ingredients*</u>

- Quantum circuit (**ansatz**)
- Cost function
- Optimization procedure

**But do they work?**

# VQAs' shortcomings

- Which layout should be used for a given task? non-trivial question
- Barren-plateaus: exponentially many measurements to estimate cost-minimizing direction (for sufficiently random quantum circuits)

[McClean2018Barren]

$$\mathrm{Var}[\partial_\alpha C_{\mathrm{VQE}}(\boldsymbol{\theta})] \leq F(n) = \mathcal{O}\left(2^{-n}\right)$$

$$\Pr[|\partial_\alpha C(\boldsymbol{\theta})| \geq c] \leq \frac{\mathrm{Var}[\partial_\alpha C(\boldsymbol{\theta})]}{c^2} = \mathcal{O}\left(2^{-n}\right)$$

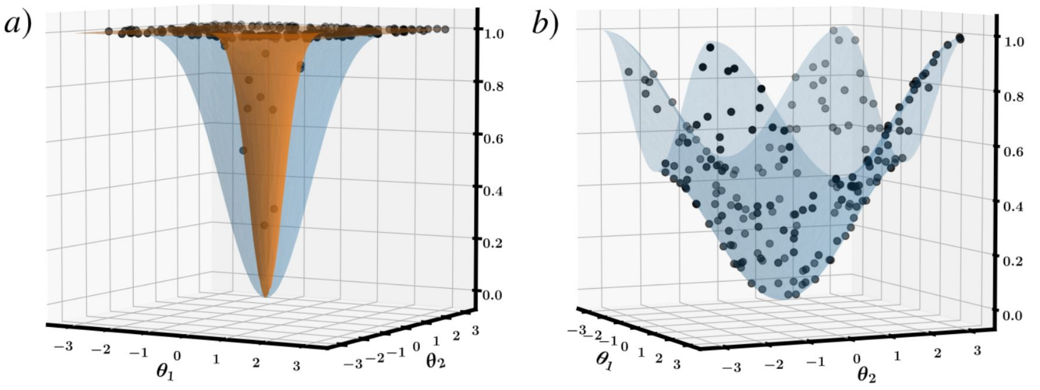
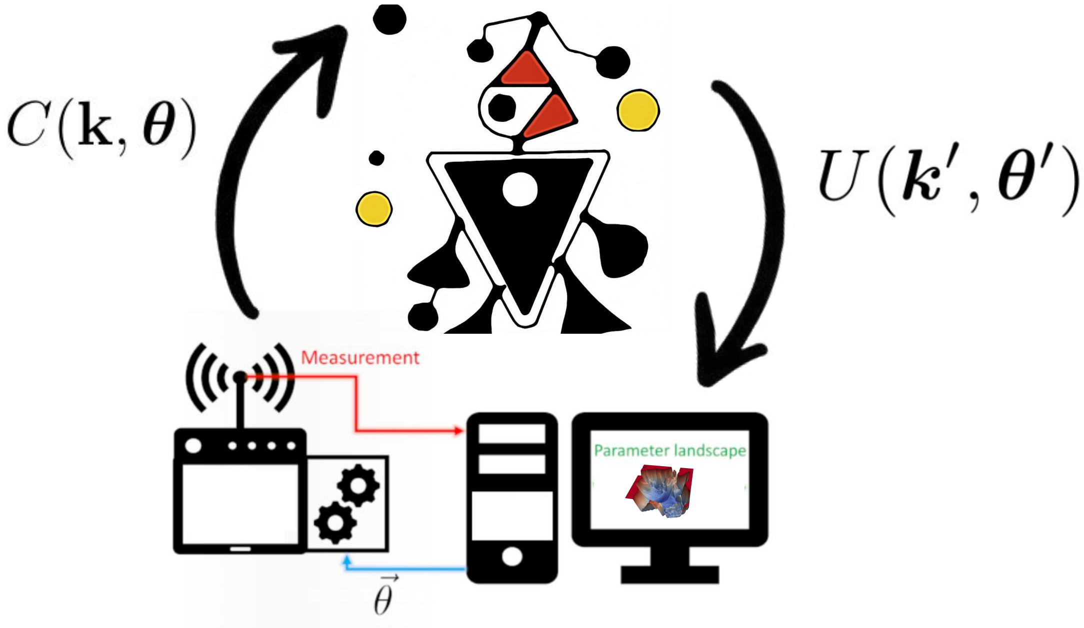



Fig. 2. from [Cerezo2021Cost]

- Noise-induced Barren Plateaus [Noise2021Wang]

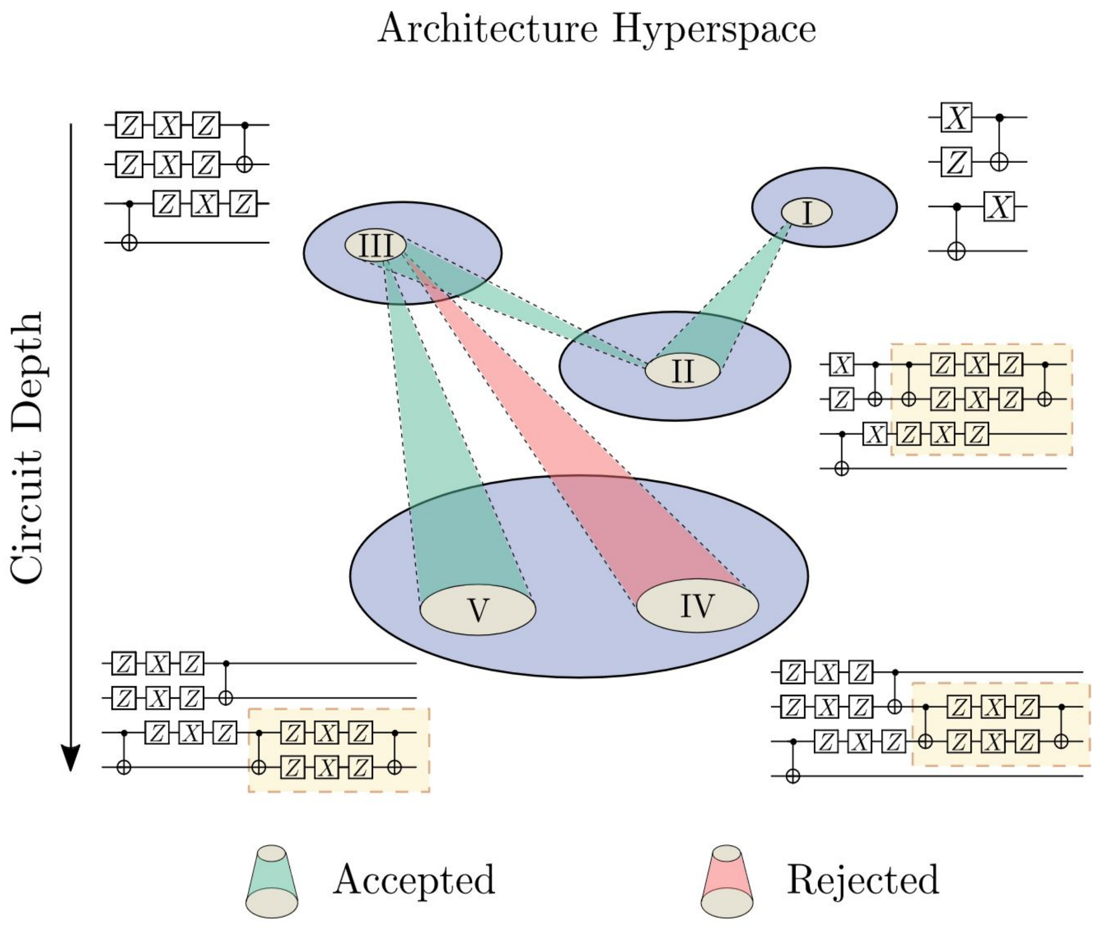
# VANs algorithm

- Exploit NISQ devices as much as possible: go beyond fixed-structured ansatz

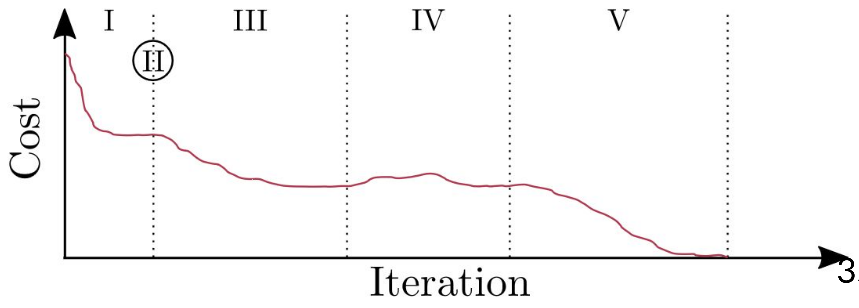



- Nested optimization loop: parameters → **ansatz** (denoted by **k)**

# VAns algorithm




- **I** optimize initial circuit + accept vs. reject
- **II** *insert* identity resolution(s)
- **III** optimize + *simplify* + accept vs. reject
- **III → IV** *insert* identity resolution(s)
- **IV** optimize + accept vs. reject
- **IV→V** *insert* identity resolution(s)
- **V** optimize

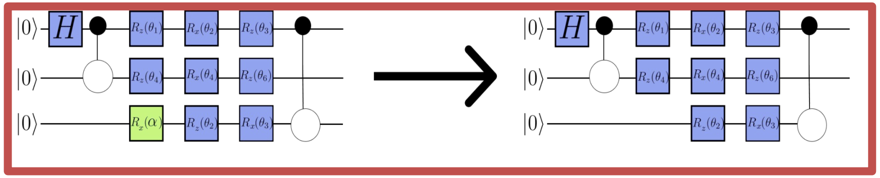


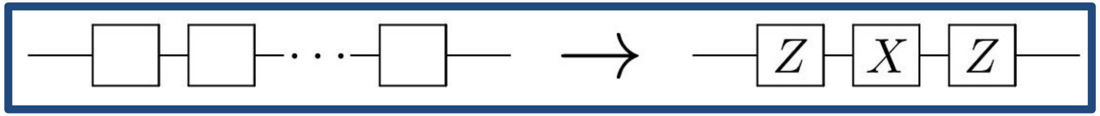

Z(X) = Parametrized rotation about *z(x) axis*

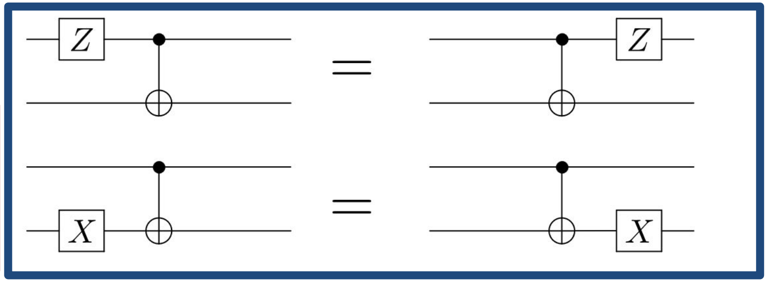
# VAns' rules in a nutshell

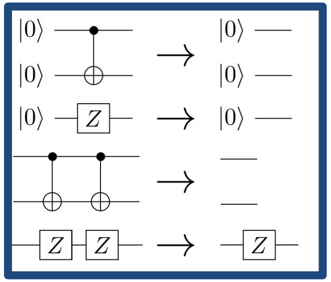
## Simplification rules

- active & passive



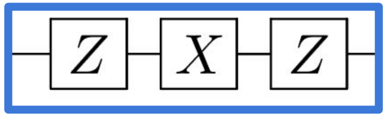


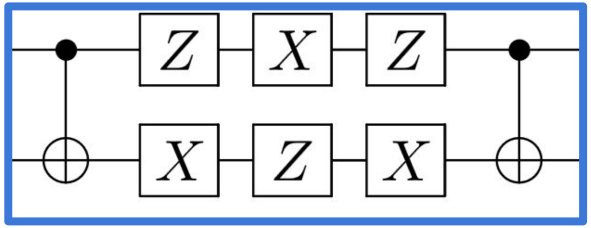




## Insertion rules

- randomly insert one/two body identity resolutions



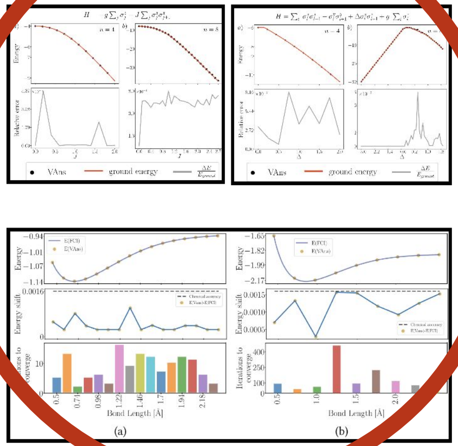




## Acceptance/rejection rules

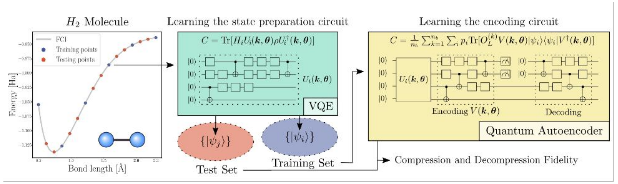
- with low probability, accept a cost-increasing circuit

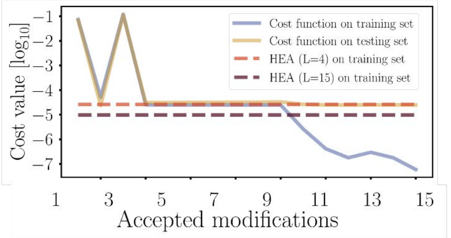
# Using VAns

## VQE

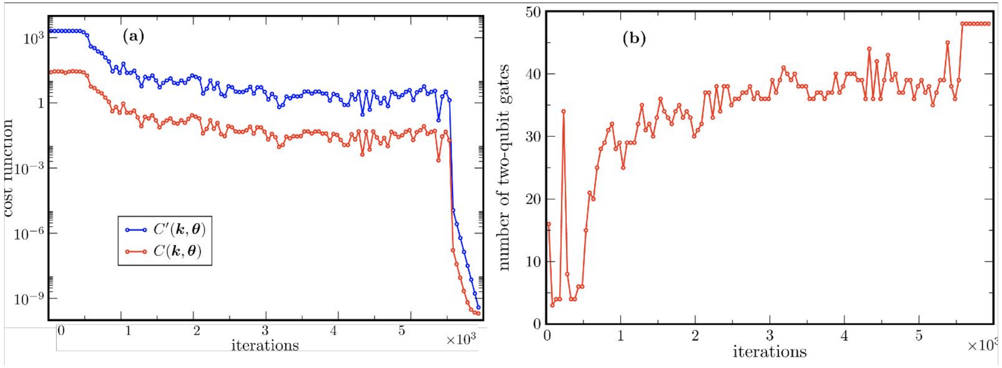


## Quantum autoencoder



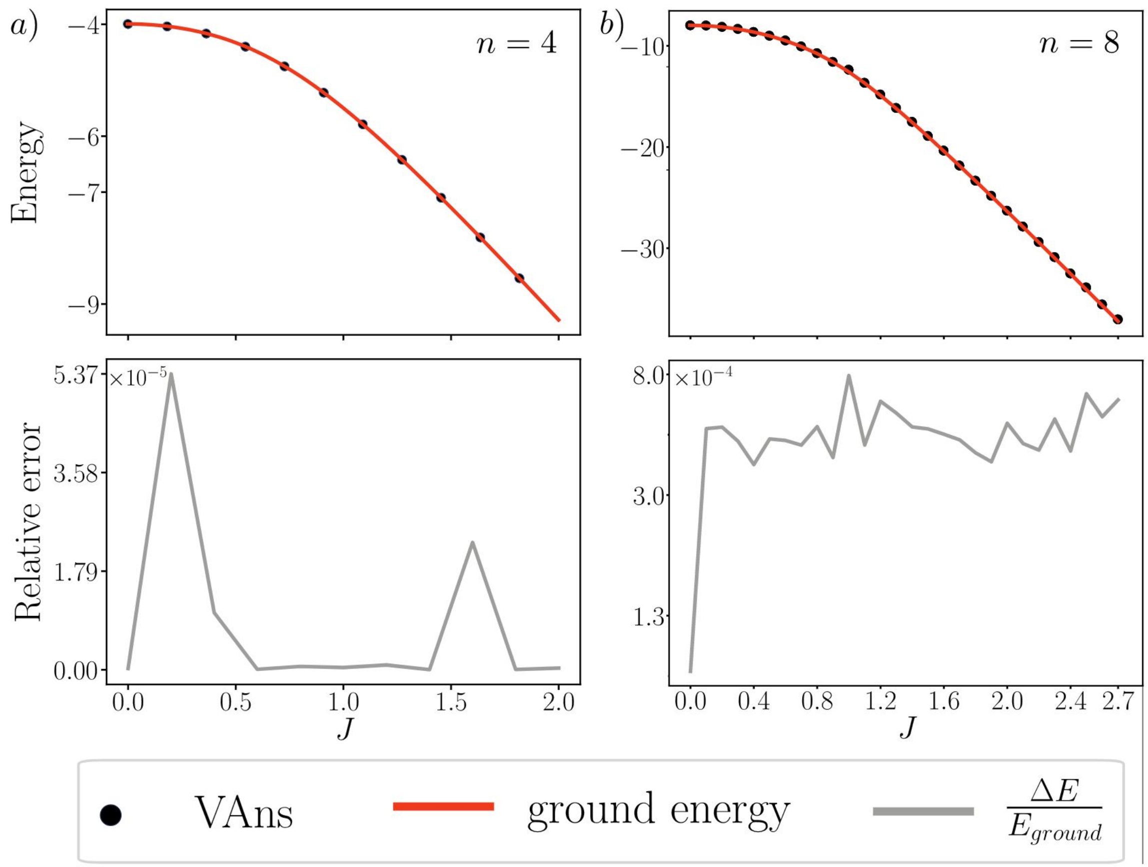

## Unitary compiling



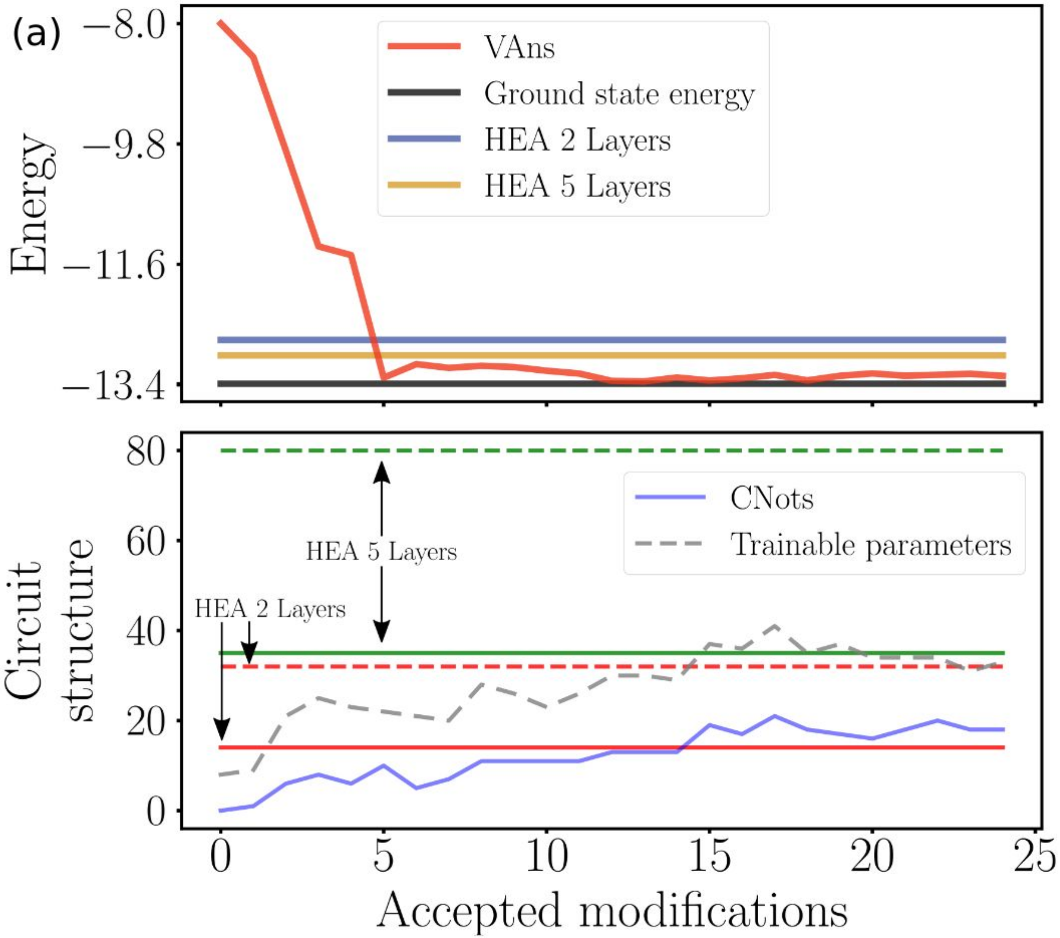

[Bilkis2021Semi]

# Using VAns: TFIM

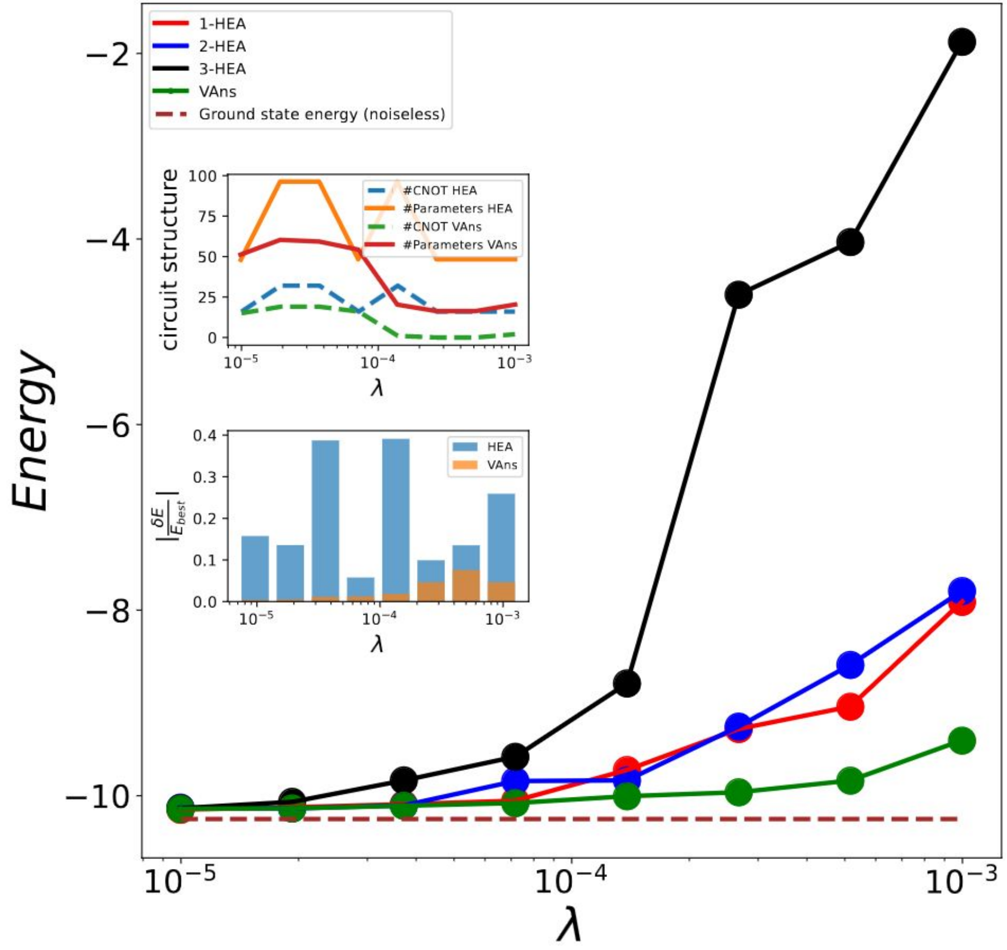
$$H = -g\sum_j \sigma_j^z - J\sum_j \sigma_j^x\sigma_{j+1}^x$$



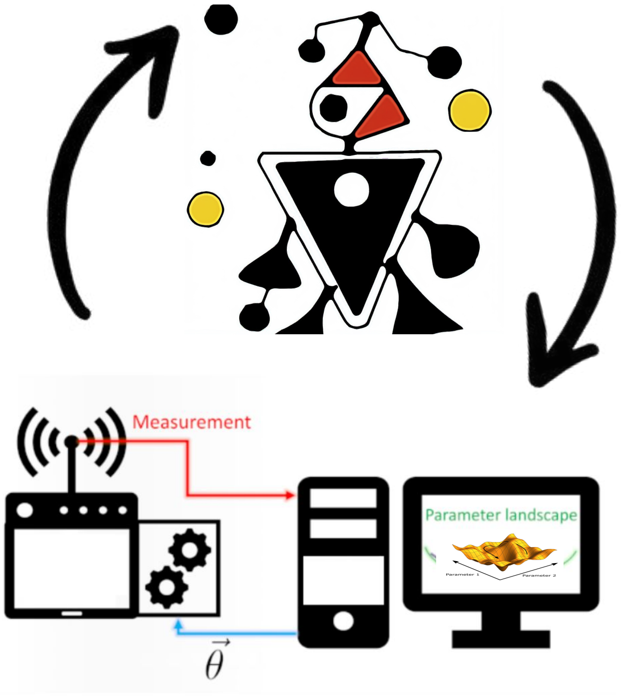
# Using VAns: TFIM



# Using VAns: TFIM under the λ-model



# Recap

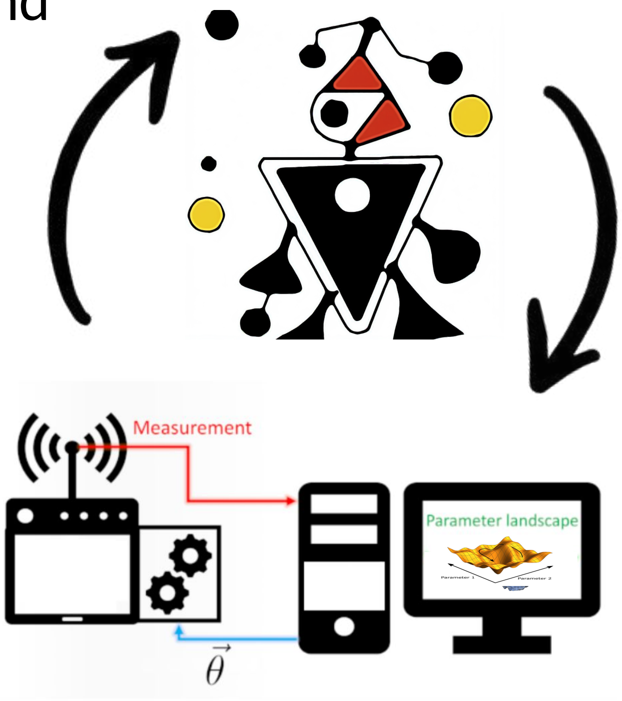



- We tackle quantum circuit structure discovery for VQAs
- We introduce a variable-ansatz approach, called VAns
- VAns sequentially grows the quantum circuit by randomly inserting identity resolutions across it
- VAns prevents the circuit to grow by usage of semi-agnostic simplification rules
- We show that VAns is able to adapt the circuit layout to hardware constraints at hand.

# Highlights and next steps

<u>Novelties</u>

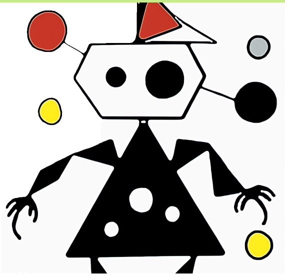
- VAns works for general cost-functions
- It adapts the quantum circuit layout to setting at hand



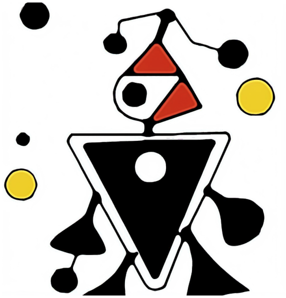

<u>Next steps</u>

- Knowledge injection for insertion rules
- Extend VAns to CV systems and use it to search for new receiver architectures

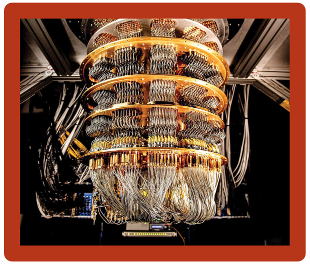
# Talk outline

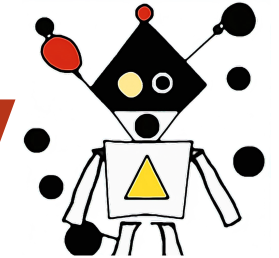


➢ Learning in the darkness

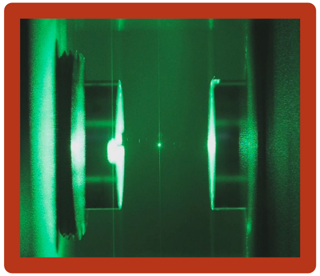


➢ Learning in the twilight





➢ Learning in the daylight

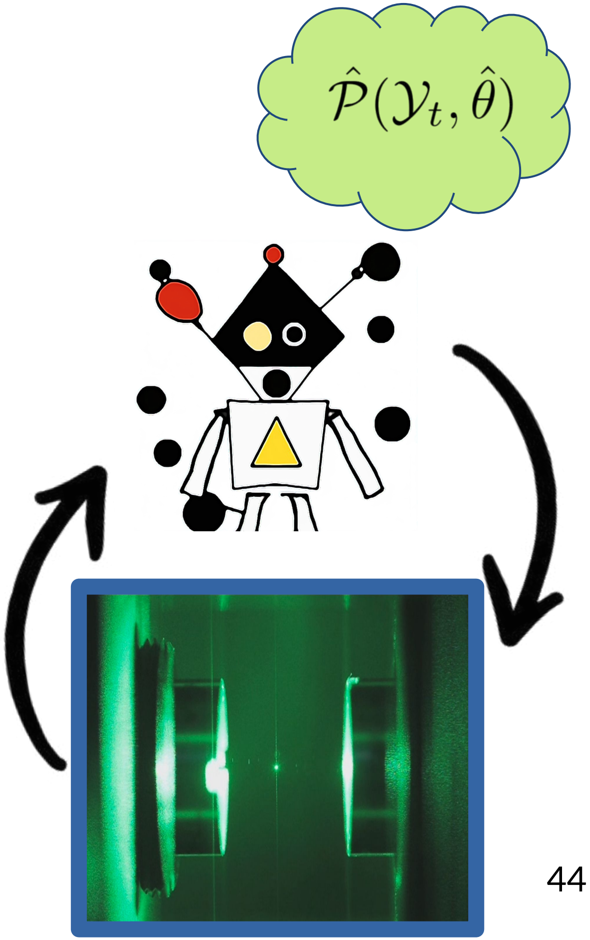


# This chapter in a nutshell

<u>Works:</u>

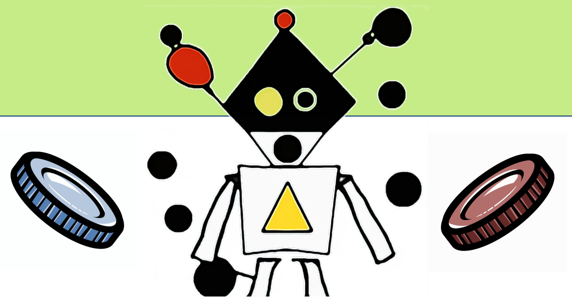
→ [Sequential testing for continuously monitored quantum systems]() - *to be submitted* - Gasbarri, Bilkis, Calsamiglia

→ [Machine-learning dynamics in continuously-monitored quantum systems -*in progress*]() - Bilkis, Gasbarri, Calsamiglia

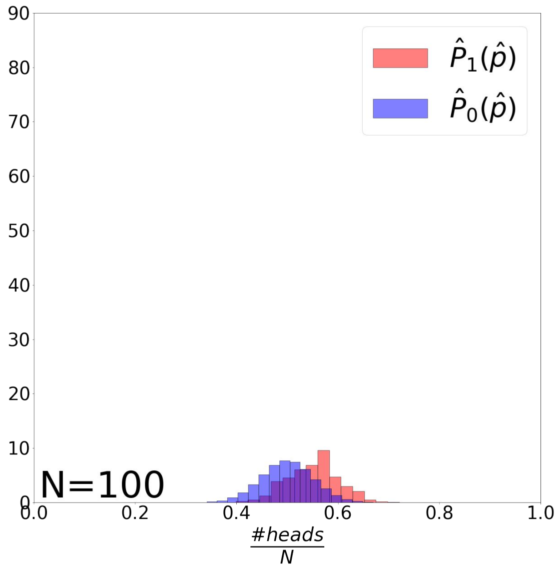


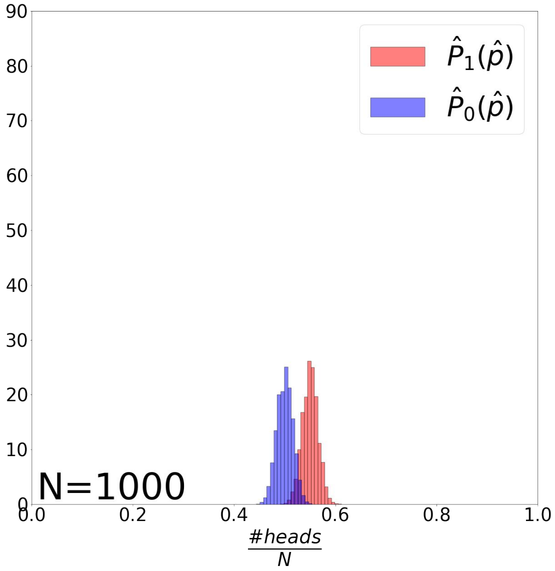

- Statistical inference in continuously-monitored systems
- Hypothesis testing: introduce sequential strategies
- Parameter estimation: machine-learning dynamics

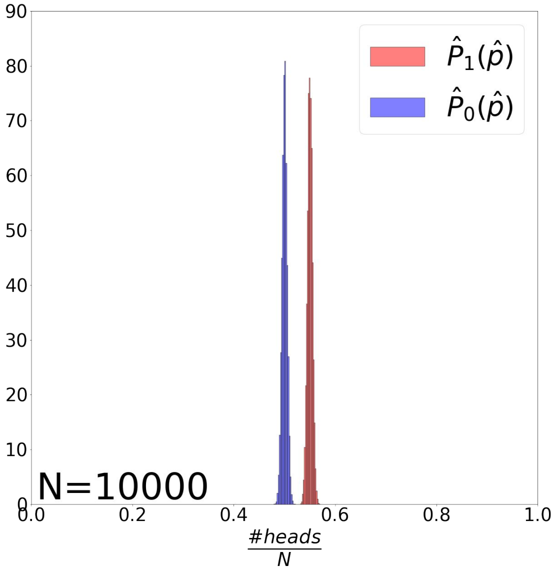
# Hypothesis testing



- Example: identify coin kind $H_0$ (bias $p_0$) vs. $H_1$ (bias $p_1$)
- After tossing the coin N times:



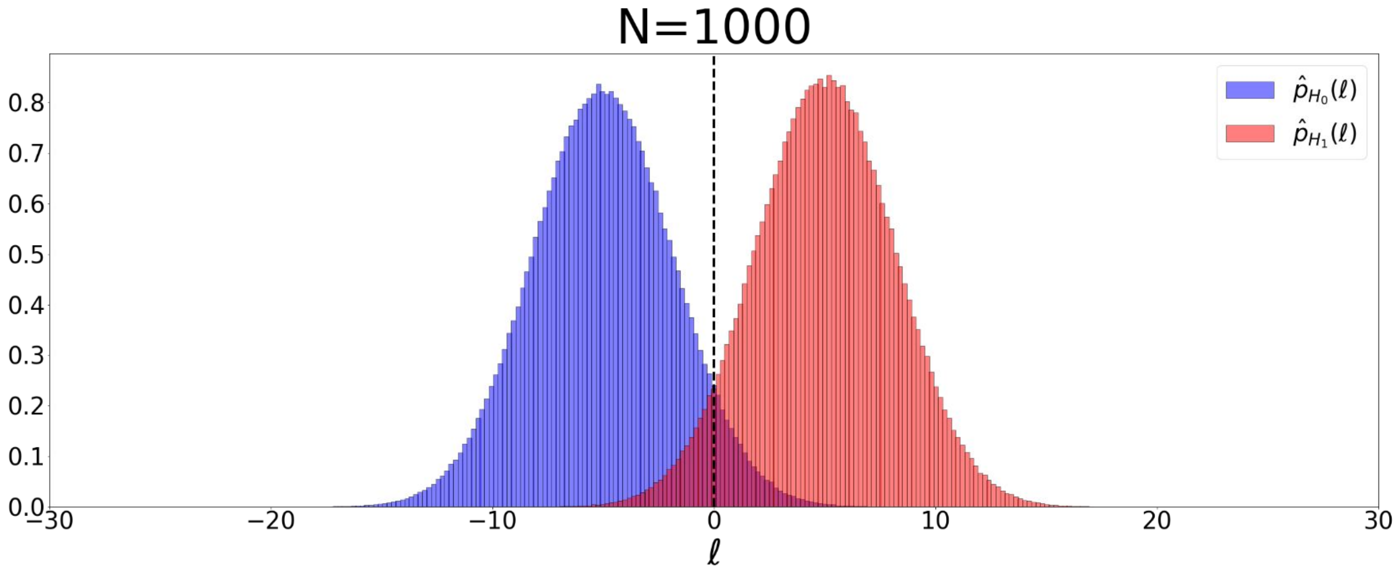




$$\mathcal{Y}_N \in \{0,1\}^{\otimes N} \quad \mathcal{P}_k(\mathcal{Y}_N) = \binom{N}{n} p_k^n (1-p_k)^{N-n}$$

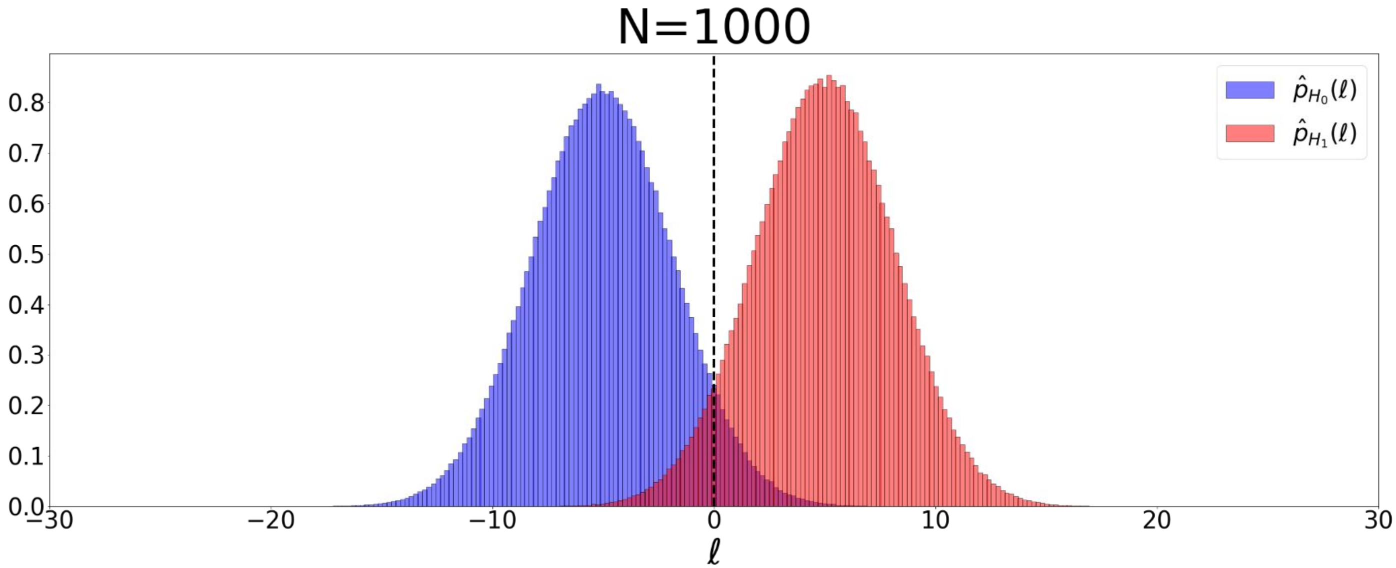
# Hypothesis testing: deterministic test

- Likelihood distribution $\ell(\mathcal{Y}_N) = \log \frac{P_1(\mathcal{Y}_N)}{P_0(\mathcal{Y}_N)}$



# Hypothesis testing: deterministic test

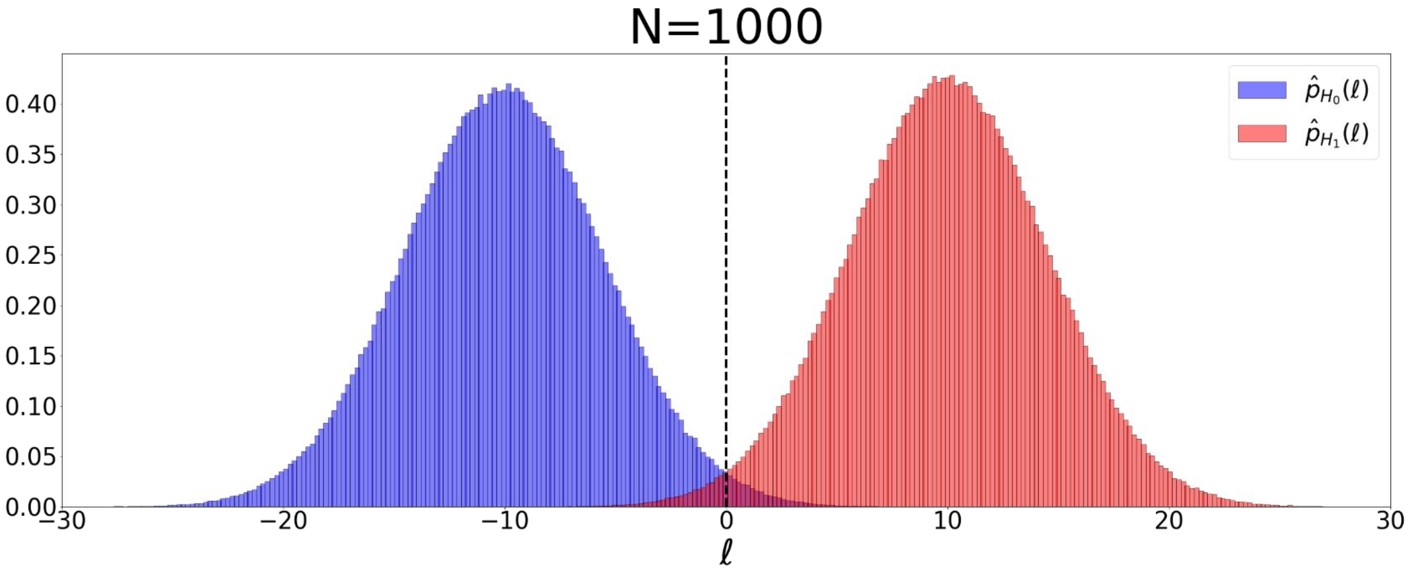
- Error probability $P_e = \frac{1}{2}p(\hat{H}_1|H_0) + \frac{1}{2}p(\hat{H}_0|H_1)$




- <u>Deterministic test</u>: acquire N samples → provide a guess

# Hypothesis testing: deterministic test

- Error probability $P_e = \frac{1}{2}p(\hat{H}_1|H_0) + \frac{1}{2}p(\hat{H}_0|H_1)$




- Exponentially decaying errors with N

# Sequential probability ratio test

- Instead of fixing N and get an error $\rightarrow$ fix error and get N
- *Strong* error condition: correct identification of $H_k$ guaranteed for each $\mathcal{Y}_N$

$$p(H_1|\mathcal{Y}_N) \geq 1-\epsilon$$
$$p(H_0|\mathcal{Y}_N) \geq 1-\epsilon$$

→ N becomes stochastic

- SPRT casts these conditions in terms of $\ell(\mathcal{Y}_N)$

$$\boxed{a := \log\left(\frac{1-\epsilon}{\epsilon}\right)}$$

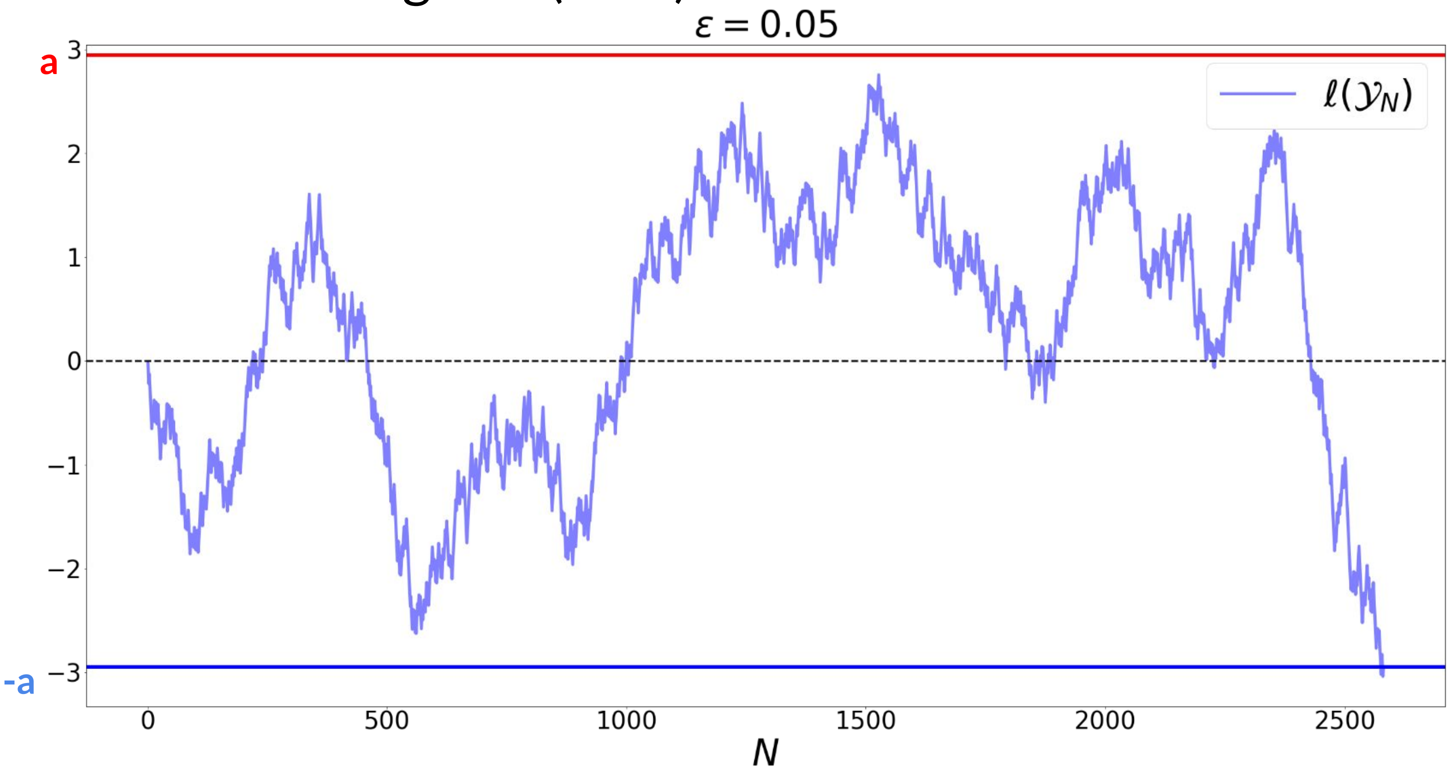
  - $\ell(\mathcal{Y}_N) \notin [-a, a]$ → *ask new sample*
  - $\ell(\mathcal{Y}_N) \geq a$ → Decide for $H_1$

    $\ell(\mathcal{Y}_N) \leq -a$ → Decide for $H_0$

[Wald1945Sequential]

# Sequential probability ratio test

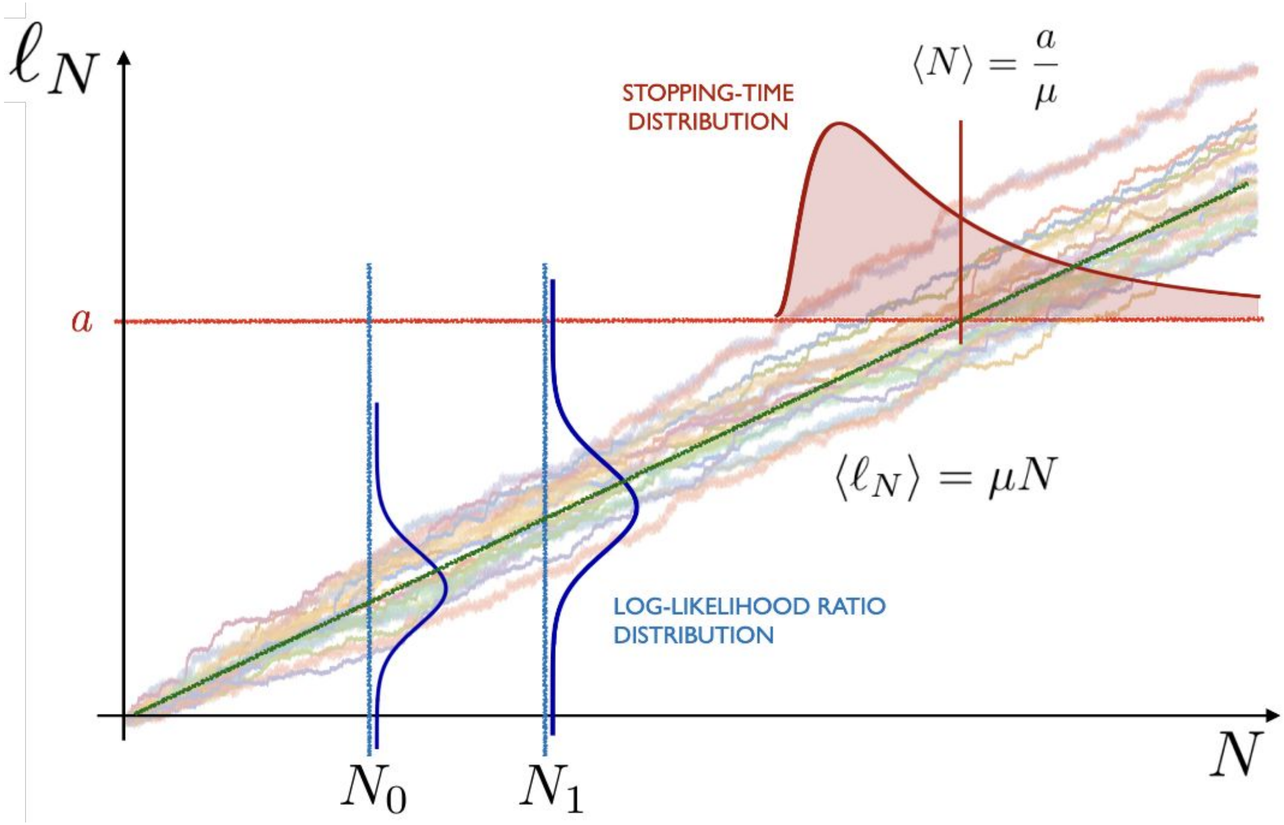
- Measurement length N (time) becomes a random variable




- In sequential analysis we aim to characterize the behaviour of N

# Sequential probability ratio test

- We aim to characterize *stopping-time* distribution



# Sequential strategies for quantum systems

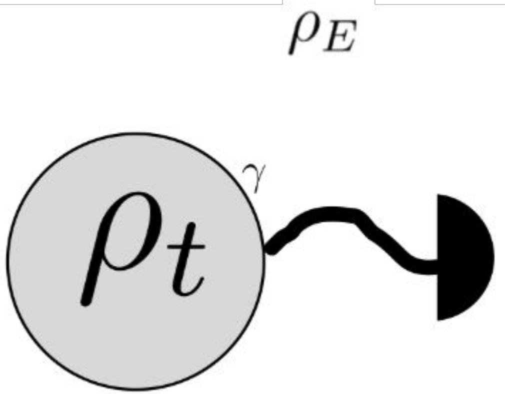
- Wald $\rightarrow$ classical probability distributions (iid) [Wald1945Sequential]

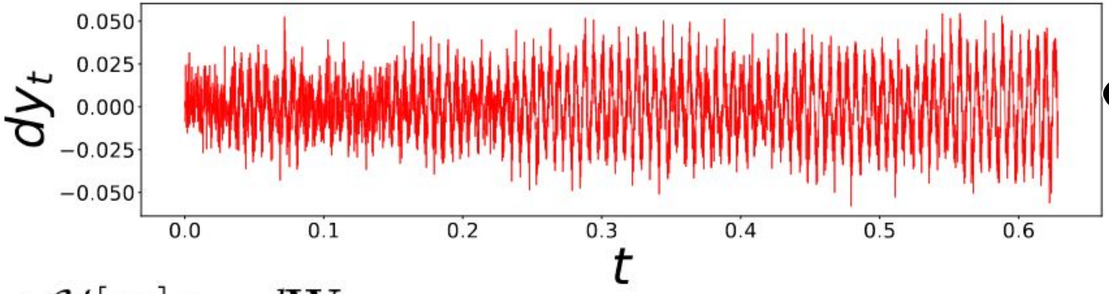
- GIQ $\rightarrow$ measure on different copies of a quantum state (iid) [Vargas2021Sequential]

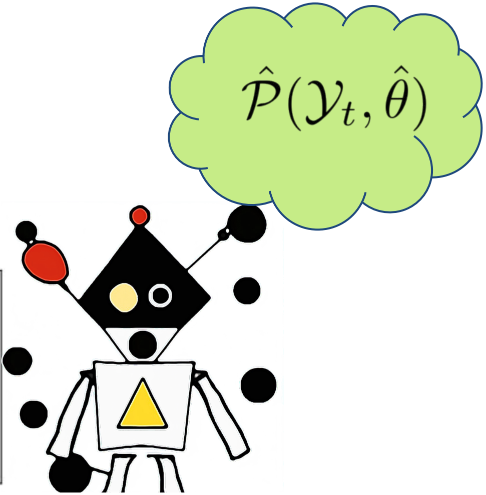
- <u>Here</u>: repeated measurements done on quantum state (non-iid)

# Continuously-monitored systems

- Continuous homodyne measurement model [[Wiseman2009Book]]
- Measurement signal $d\mathbf{y}_{t|k} = \mathrm{Tr}[\mathcal{H}_{\eta\mathbf{c}}[\rho_{t,k}]]dt + d\mathbf{W}_t$




$$d\rho_t = \mathcal{L}_\theta[\rho_t]dt + \mathcal{H}[\eta\mathbf{c}]\rho_t \cdot d\mathbf{W}_t$$

$$\mathcal{L}[\rho] = -i\,[H, \rho] + \mathcal{D}[\mathbf{c}]\rho$$

$$\mathcal{D}[\mathbf{c}]\rho = \mathbf{c}\rho\mathbf{c}^\dagger - \{\frac{\mathbf{c}^\dagger\mathbf{c}}{2}, \rho\}$$

$$\mathcal{H}[\eta\mathbf{c}]\rho = \mathcal{H}_{\eta\mathbf{c}}[\rho] - \mathrm{Tr}[\mathcal{H}_{\eta\mathbf{c}}[\rho]]$$

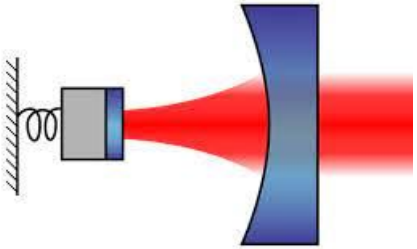
$$\mathcal{H}_{\eta\mathbf{c}}\rho = \sqrt{\eta}\mathbf{c}\rho + \rho\sqrt{\bar{\eta}}\mathbf{c}^\dagger$$

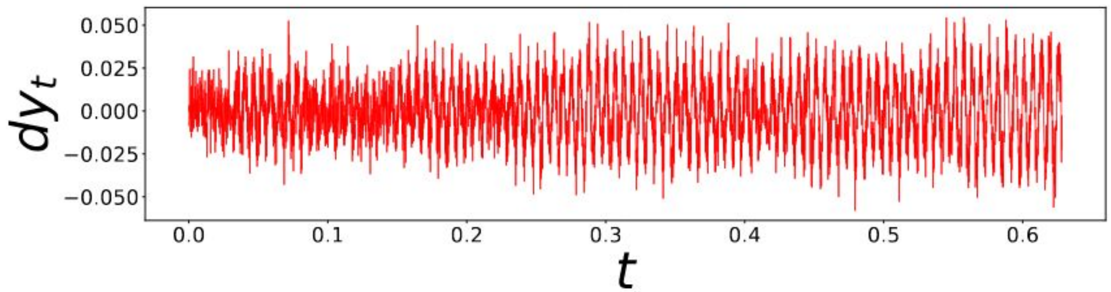
- Stochastic master equation w/ backaction term
- Physical systems:
  - atomic sensors [[Martínez2018Signal]]
  - optomechanical cavities [[Aspelmayer2010Cavity]]

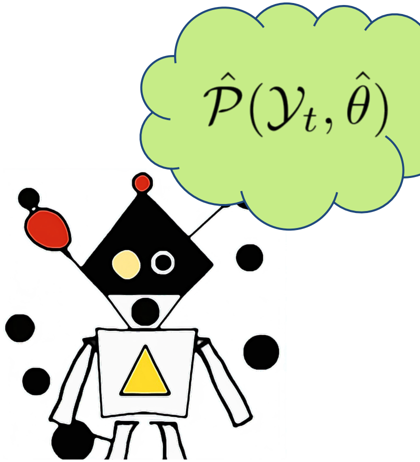
# Gaussian case

- Mechanical-mode dynamics: damped oscillator + Wiener noise

- Gaussian measurement model given $d\mathbf{y}_t = C\bar{\mathbf{r}}_t dt + d\mathbf{W}_t$






$$d\bar{\boldsymbol{r}}_t = \big(A - \chi(\boldsymbol{\Sigma}_t)C\big)\bar{\boldsymbol{r}}_t dt + \chi(\boldsymbol{\Sigma}_t)d\mathbf{y}_t$$

$$d\boldsymbol{\Sigma}_t = A\boldsymbol{\Sigma}_t + \boldsymbol{\Sigma}_t A^T + D - \chi(\boldsymbol{\Sigma}_t)^T\chi(\boldsymbol{\Sigma}_t)$$

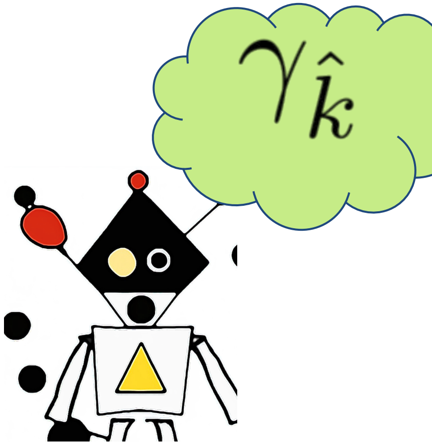
$$\chi(\boldsymbol{\Sigma}) = \boldsymbol{\Sigma}C^T$$

$$\boxed{A = \begin{pmatrix} -\frac{\gamma}{2} & -\omega \\ \omega & -\frac{\gamma}{2} \end{pmatrix}, \quad D = \left[\gamma(n + \frac{1}{2}) + \kappa\right]\mathbb{I}_2, \quad C = \sqrt{4\eta\kappa}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}}$$

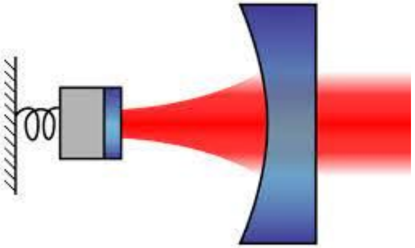
[Genoni2016Conditional]

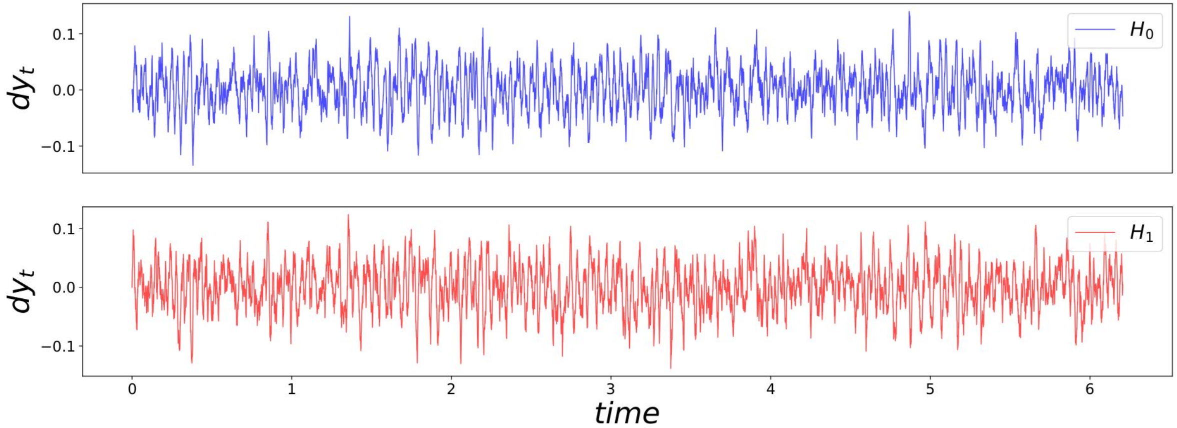
# Hypothesis testing

- Contrast two values of damping rate γ out of measurement signal
- From previous discussion → we need to study $\ell_t$




$$d\mathbf{y}_t = C\bar{\mathbf{r}}_t dt + d\mathbf{W}$$






$$d\bar{\boldsymbol{r}}_t = \big(A - \chi(\boldsymbol{\Sigma}_t)C\big)\bar{\boldsymbol{r}}_t dt + \chi(\boldsymbol{\Sigma}_t)d\mathbf{y}_t = A\bar{\boldsymbol{r}}_t dt + \chi(\boldsymbol{\Sigma}_t)dW_t$$

$$d\boldsymbol{\Sigma}_t = A\boldsymbol{\Sigma}_t + \boldsymbol{\Sigma}_t A^T + D - \chi(\boldsymbol{\Sigma}_t)^T\chi(\boldsymbol{\Sigma}_t)$$

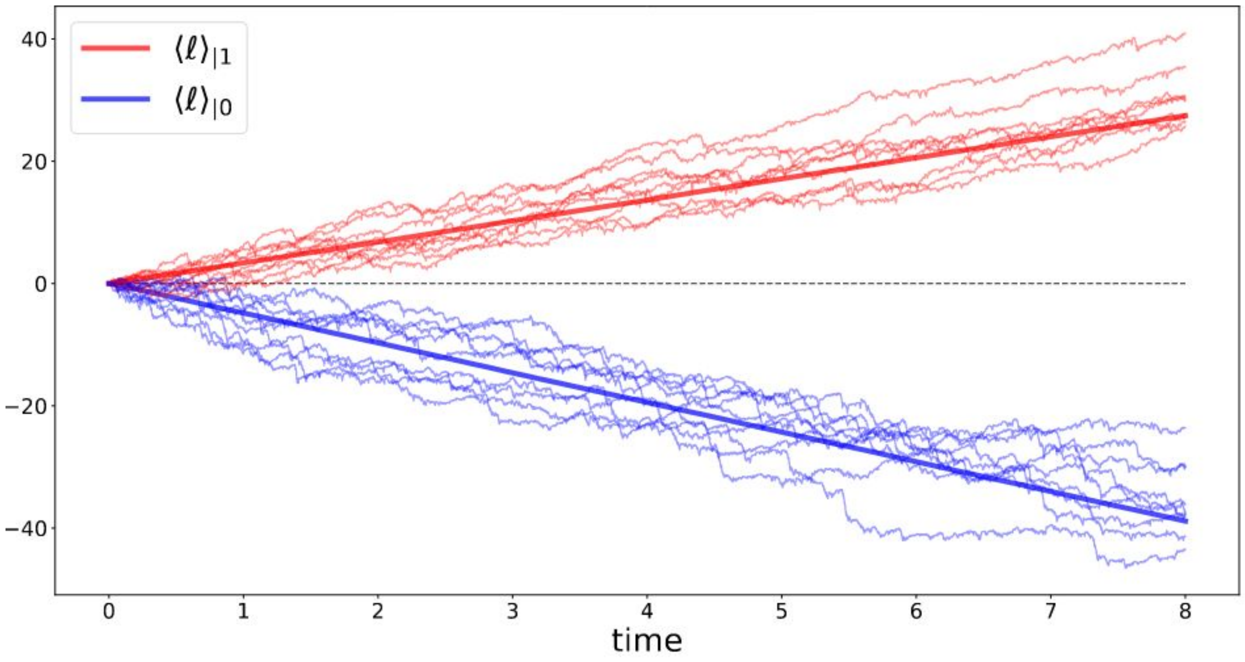
**Moving frame**

$$\boxed{C \to 2\sqrt{\eta\kappa}\,\mathbb{I}_2, \quad A \to -\frac{\gamma}{2}\mathbb{I}_2}$$

# Log-likelihood ratio evolution

- Hypothesis $H_k$ associated to a physical parameter value $\theta_k$

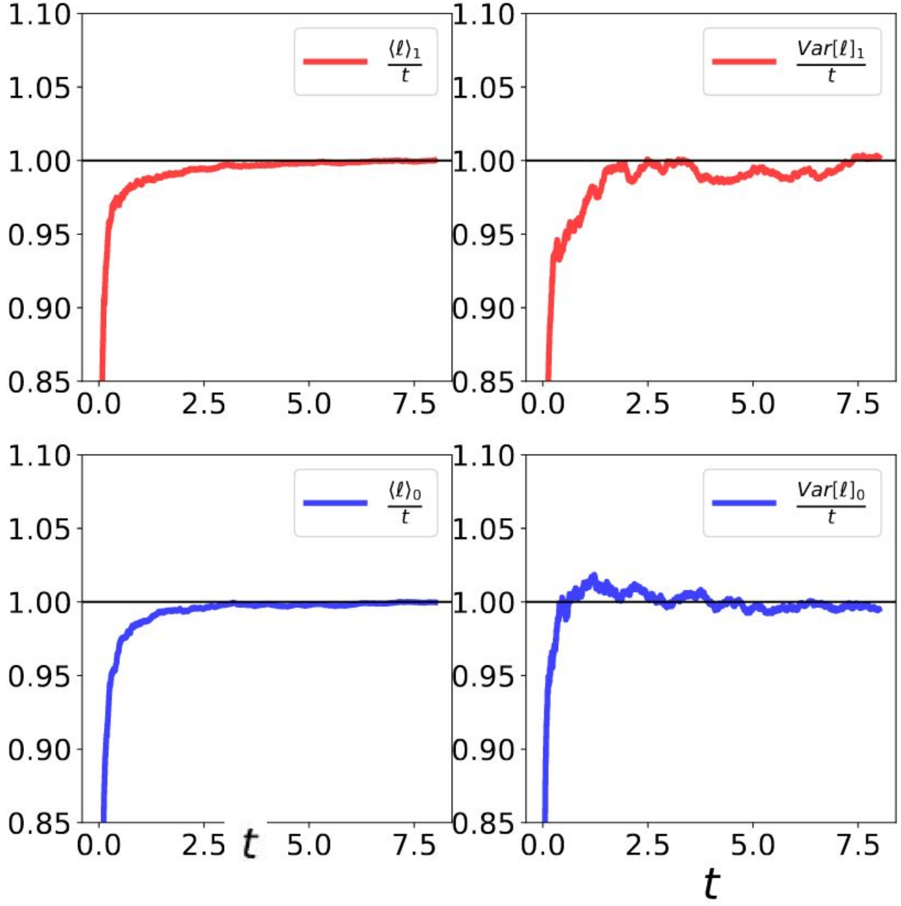
$$d\ell(\mathcal{Y}_{t|k}) = \frac{(-1)^{k+1}}{2}||C\Delta\bar{\boldsymbol{r}}_t||^2 dt + C\Delta\bar{\boldsymbol{r}}_t \cdot d\mathbf{W}_t \qquad \Delta\bar{\boldsymbol{r}}_t = \bar{\boldsymbol{r}}_1(\mathcal{Y}_{t|k}, t) - \bar{\boldsymbol{r}}_0(\mathcal{Y}_{t|k}, t)$$



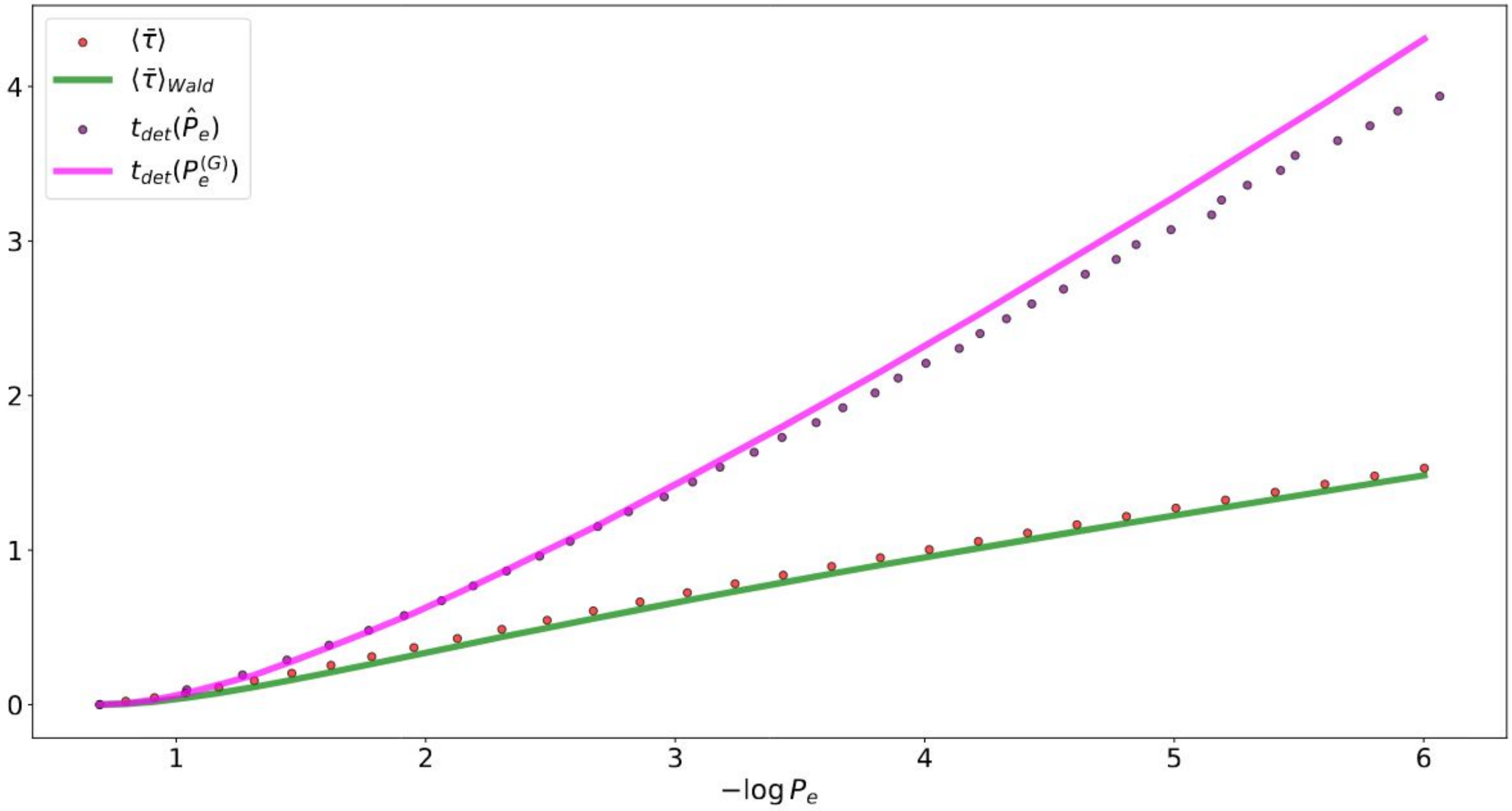
# Log-likelihood ratio evolution

$$d\ell(\mathcal{Y}_{t|k}) = \frac{(-1)^{k+1}}{2}||C\Delta\bar{\boldsymbol{r}}_t||^2 dt + C\Delta\bar{\boldsymbol{r}}_t \cdot d\mathbf{W}_t$$

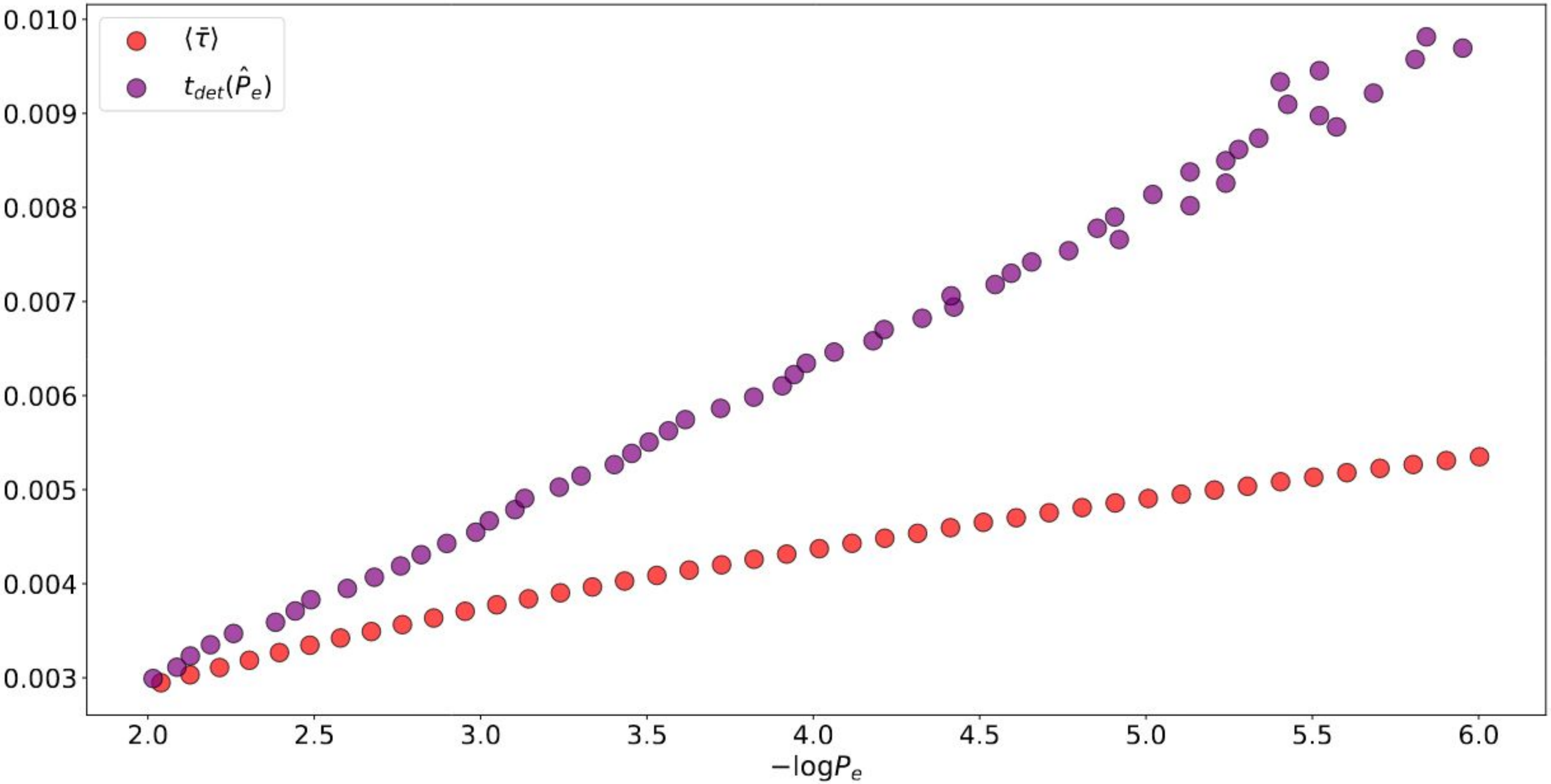
$$\Delta\bar{\boldsymbol{r}}_t = \bar{\boldsymbol{r}}_1(\mathcal{Y}_{t|k}, t) - \bar{\boldsymbol{r}}_0(\mathcal{Y}_{t|k}, t)$$



# Comparing both tests



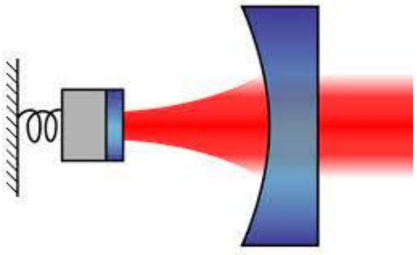
# Comparing both tests: frequency

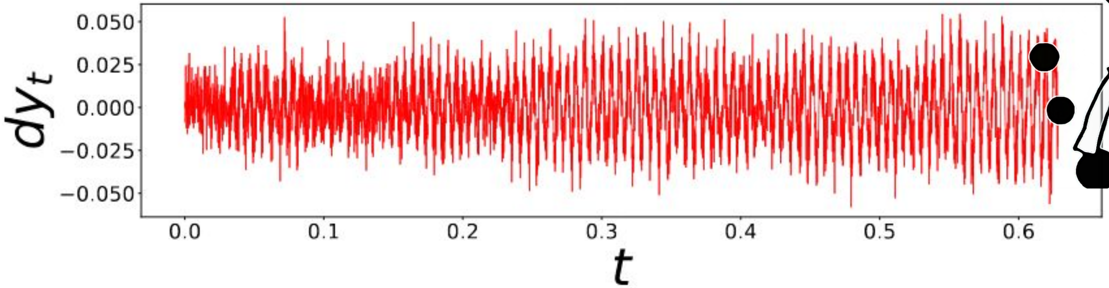


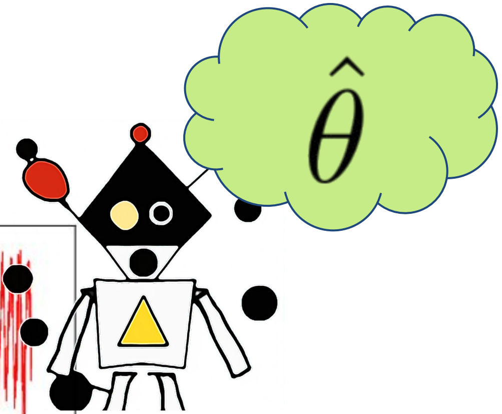
# Parameter estimation

- Given measurement samples → tell <u>parameter value</u>

$$d\mathbf{y}_t = C\bar{\mathbf{r}}_t dt + d\mathbf{W}_t$$






$$P(\mathcal{Y}_t|\theta) = e^{\lambda(\mathcal{Y}_t|\theta)} P_W(\mathcal{Y}_t)$$

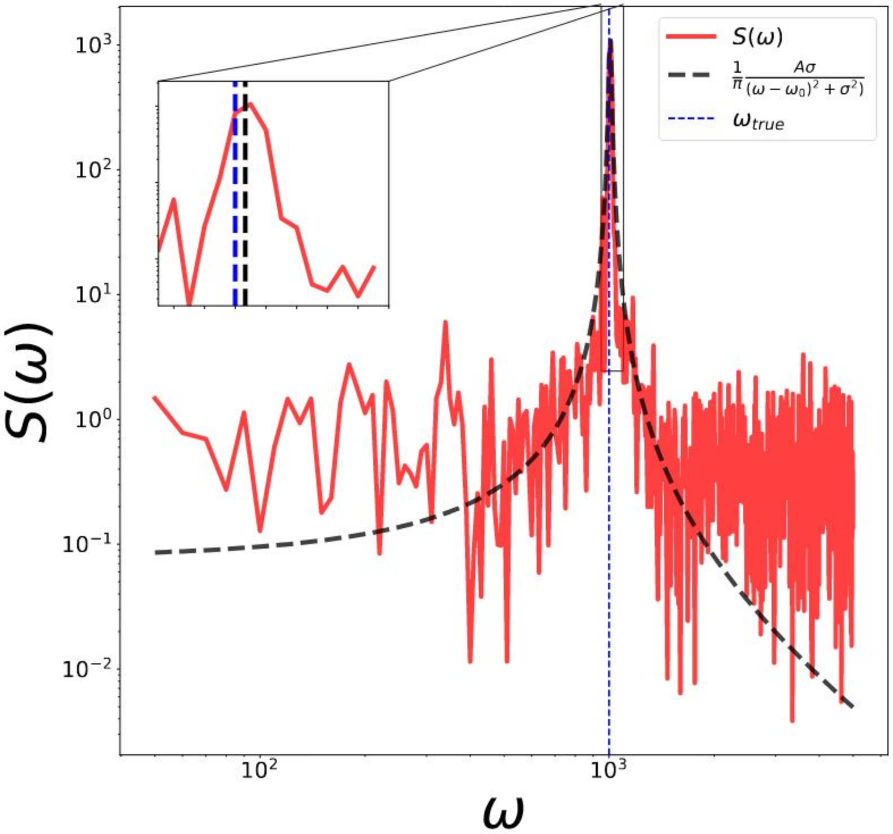
$$I_t(\theta) = \langle (\partial_\theta \lambda(\mathcal{Y}_t;\theta)^2\rangle = \langle -\partial_\theta^2 \lambda(\mathcal{Y}_t,\theta)\rangle$$

$$\mathrm{Var}[\hat{\theta}] \geq \frac{1}{NI(\theta)}$$

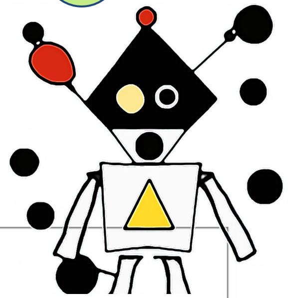
- Likelihood distribution → Fisher info
- Cramer-Rao bound
- Construct an estimator

# Parameter estimation

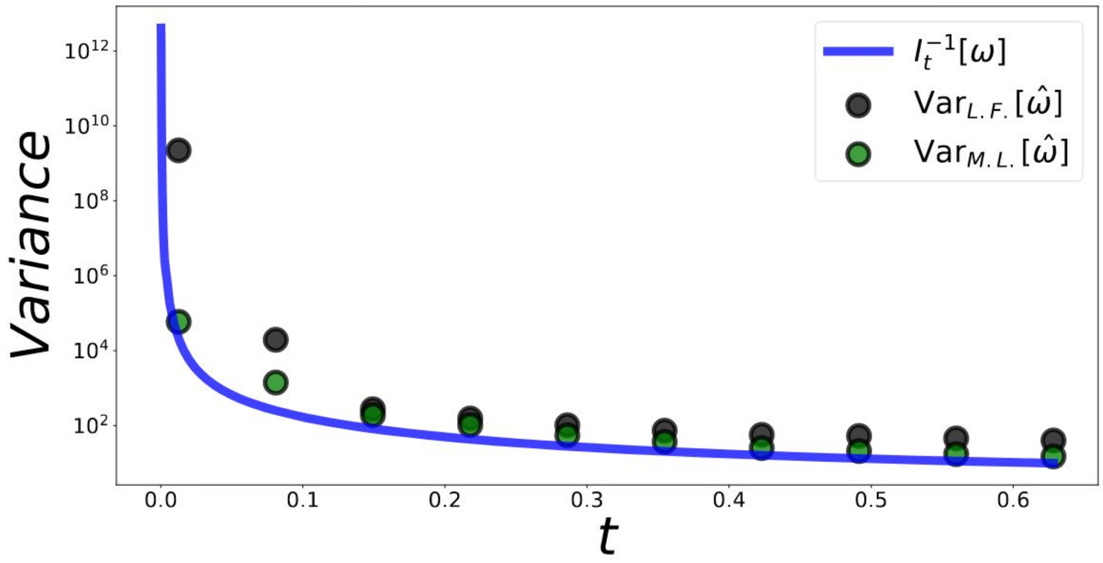
- Experimentally → Lorenztian fit on power spectrum

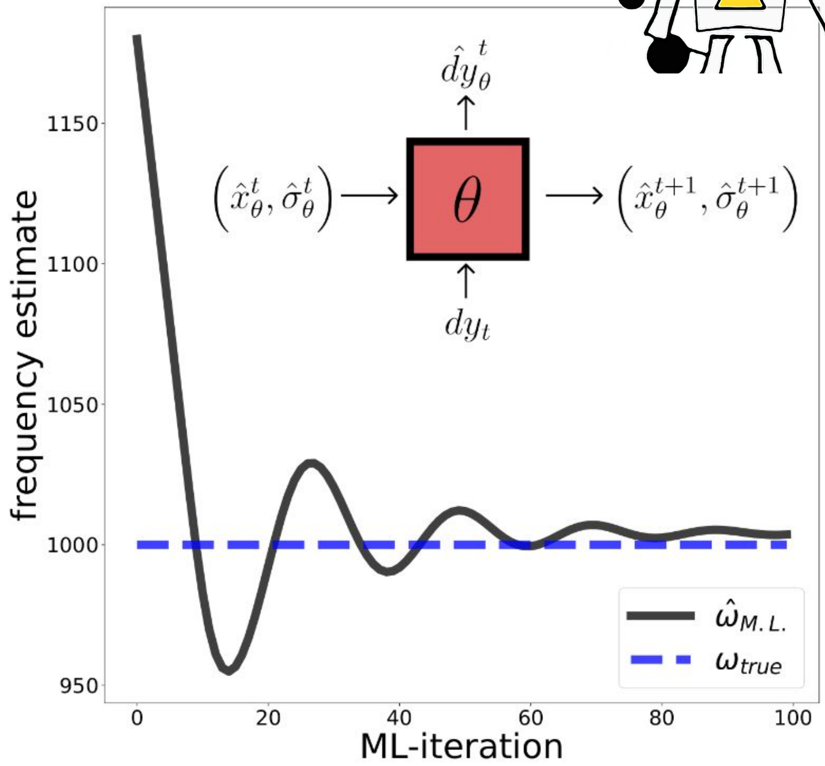



# Parameter estimation: us

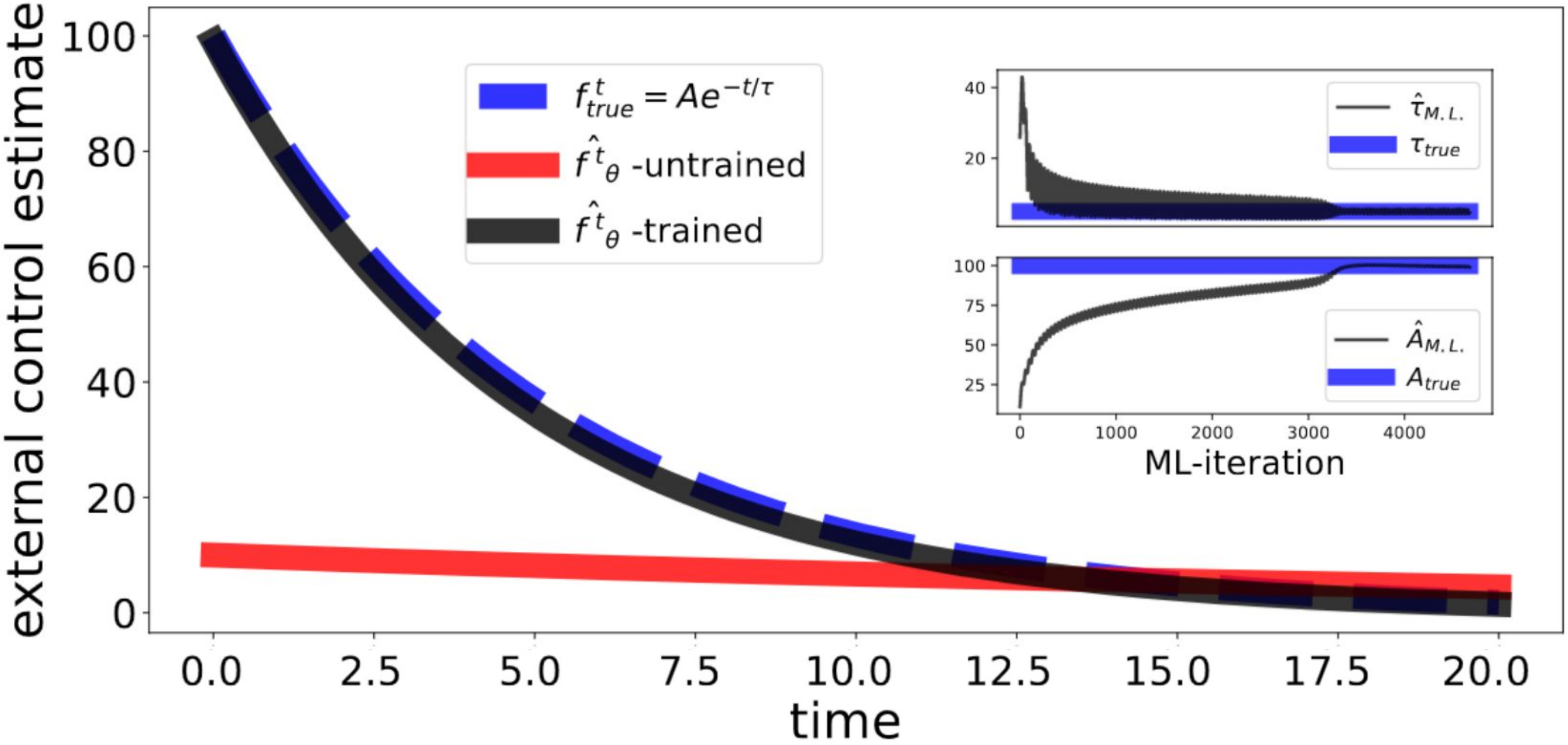
- We use recurrent structured optimized by automatic differentiation.
- Internally integrates the hidden state using current parameter guess
- Maximum likelihood $\hat{\theta} = \underset{\theta}{\mathrm{ArgMax}}\ \lambda(\mathcal{Y}_t, \theta)$

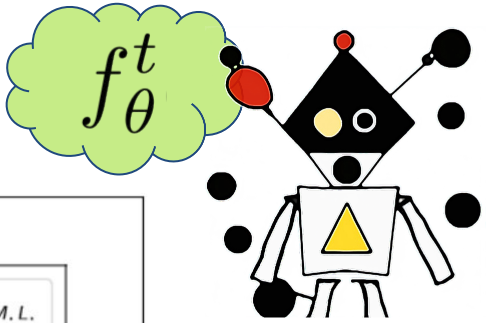


# External-force estimation

$$d\bar{\mathbf{r}}_t = \Big(A - \chi(\Sigma)\Big)\bar{\mathbf{r}}_t dt + \chi(\Sigma) d\mathbf{y}_t + f_\theta^t dt$$



# Recap

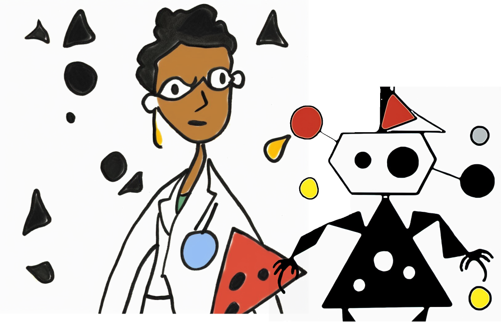
- Hypothesis testing
    - Deterministic test: pre-defined measurement time → error
    - Sequential test:
        - Is data conclusive enough? → stochastic measurement time
        - Strong error condition guarantees (for each trajectory)
    - Advantage in favour of sequential tests.
- Parameter estimation
    - Recurrent cell → maximum likelihood
    - Apply this for frequency estimation & external signal  that varies in time

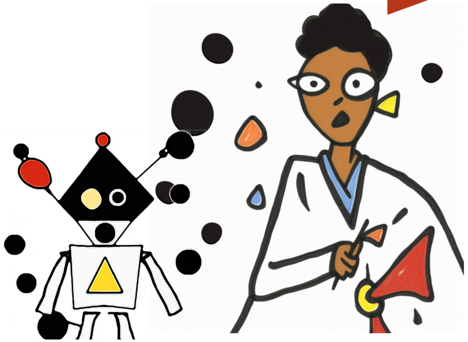
# Conclusions

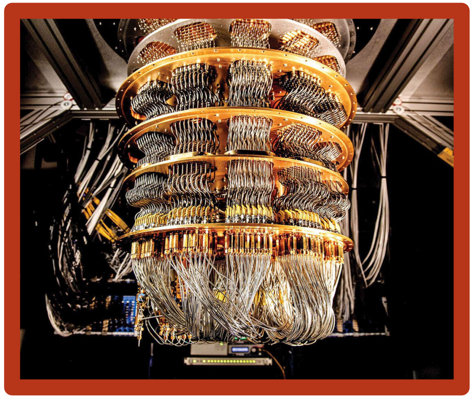
Model-free

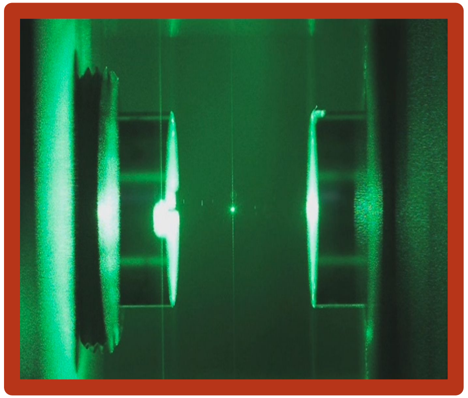
Semi-agnostic

Model-aware









# Thanks everyone!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