Echo chambers

Exploring the opinion dynamics of late-time unbounded confidence models with rewiring

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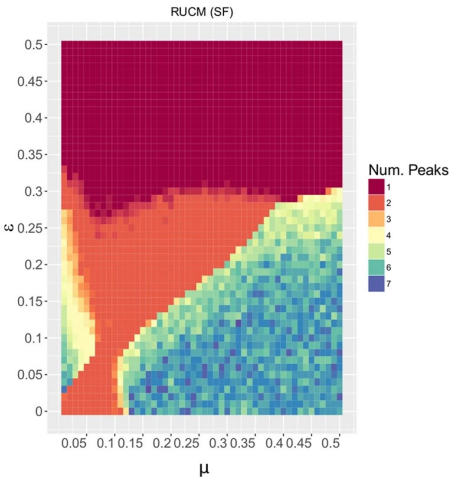
Introduction

Our work is based on the paper "Modeling confirmation bias and polarization"(2017) by Del Vicario, M., Scala, A., Caldarelli. They investigated the way opinions are distributed within social networks. They focused on the problem of polarization, a constant issue we encounter every day in our world.

Hypothesis

The result of the paper: In the paper they discovered distinct phase lines within the parameter space of their model. These are where the opinion distributions would switch modality.

Our hypothesis We will observe similar features (phase lines) within the parameter space for our different metrics: variance, similarity, modularity, and number of disconnected nodes.



Method - parameters

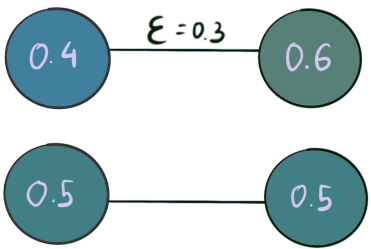
Model:
Rewire with Unbounded Confidence Model

Parameters:
- ϵ represents a tolerance threshold
- μ represents reaction strength, convergence parameter

Method - concordant opinions

If two agents have **concordant** opinions, i.e. if $|x_i - x_j|_\tau < \epsilon$, we adjust x_i and x_j as follows:

$$\begin{cases} x_i = x_i + \mu(x_j - x_i), \\ x_j = x_j + \mu(x_i - x_j), \end{cases} \tag{1}$$



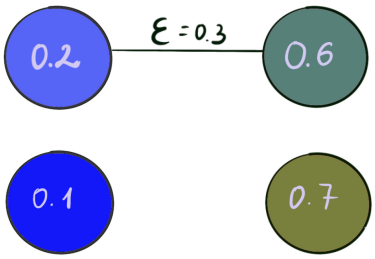
Method - disconcordant opinions

If their opinions are **discordant**, i.e. if $|x_i - x_j|_\tau \geq \epsilon$;

1. We adjust x_i and x_j as follows:

$$\begin{cases} x_i = x_i - \mu \left[x_j - x_i - \rho(x_j - x_i) \right], \\ x_j = x_j - \mu \left[x_i - x_j - \rho(x_i - x_j) \right], \end{cases} \tag{2}$$

2. The link between i and j is broken and a new link is created between i and a randomly chosen node.



Method - distance between opinions

The τ - distance is defined as:

$$|x_i - x_j|_\tau = |x_i - x_j - \rho(x_i - x_j)| \tag{3}$$

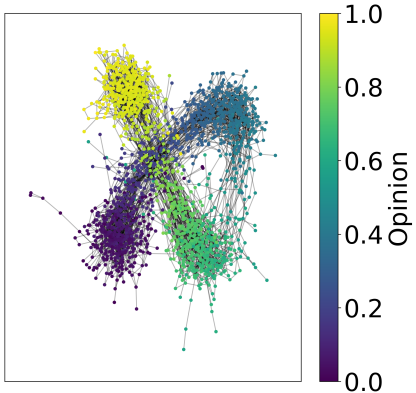
Where ρ used in the equation above is defined as:

$$\rho(x) = \begin{cases} -1, & \text{if } x \in (-1, -0.5), \\ 0, & \text{if } x \in [-0.5, 0.5], \\ 1, & \text{if } x \in (0.5, 1). \end{cases} \tag{4}$$

The Networks

Network structure: The model was applied on a scale-free network with $2 < \gamma < 3$ in $P(k) \sim k^{-\gamma}$ (typical of real-world networks).

- 2000 nodes
- All results averaged over 5 runs
- Initial opinion distribution: U[0,1]



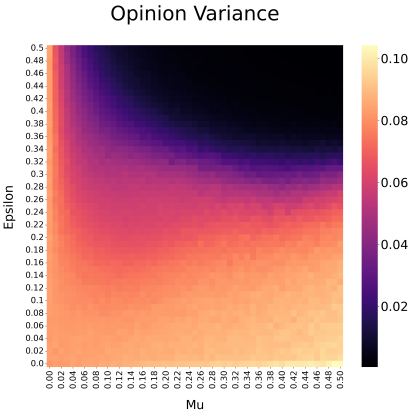
Metrics

- Variance of the opinion distribution;
- Similarity of opinions between neighbors;
- Modularity of the network;
- Number of disconnected/isolated nodes.

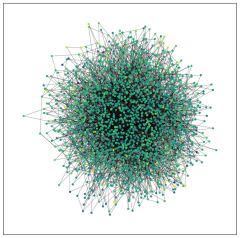
For each:

- Heatmaps covering $\epsilon = [0, 0.5], \mu = [0, 0.5]$, resolution: 51×51.
- Higher resolution "slices" of $\epsilon = [0, 0.5]$ at set μ for investigating interesting points in the parameter space.

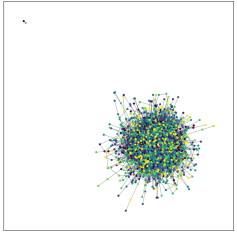
Metric 1 - Variance



$\epsilon = 0.48, \mu = 0.48$

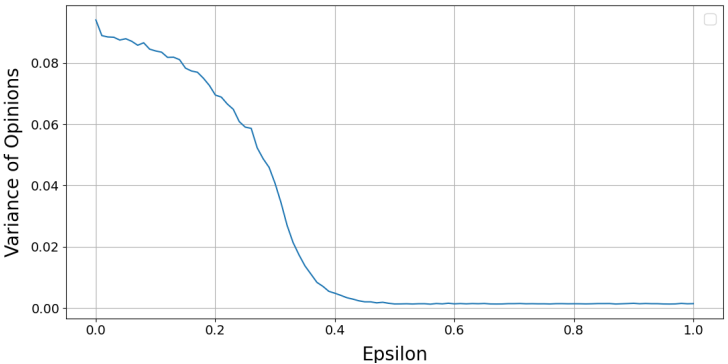


$\epsilon = 0.05, \mu = 0.48$

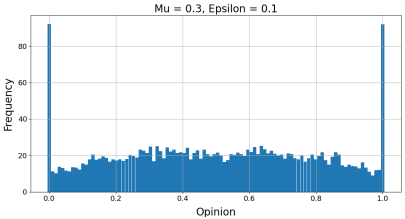


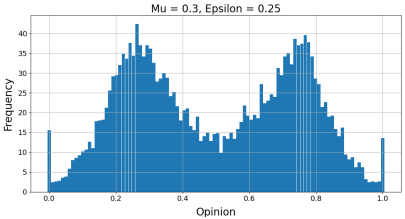
Results - Variance

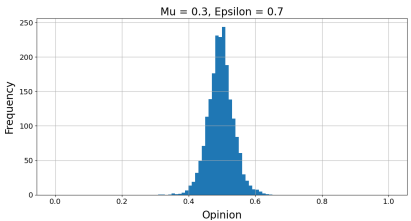
$$\mu = 0.48$$



Results - Variance







Metric 2 - Opinion Similarity

To investigate how opinions evolve within a network, we measured the **average opinion similarity between connected agents** at equilibrium.

- The similarity between two connected agents i and j was computed as:

$$S_{ij} = 1 - |x_i - x_j|$$

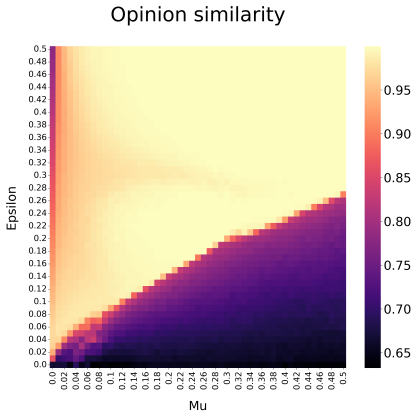
  where $S_{ij} = 1$ indicates perfect agreement and $S_{ij} = 0$ indicates maximal disagreement.

- The final metric represents the **average similarity** across all network connections.

Metric 2 - Opinion Similarity

Key Observations from the Heatmap:

- High similarity (light regions) occurs at low μ and high ϵ, indicating scenarios where opinions converge due to high tolerance and minimal resistance to change.
- Low similarity (dark regions) dominates at high μ and low ϵ, where increased resistance and limited tolerance lead to fragmented opinions.
- A sharp triangular transition boundary marks the critical balance point, where changes in μ and ϵ shift the network from cohesion to fragmentation.



Results - Opinion Similarity

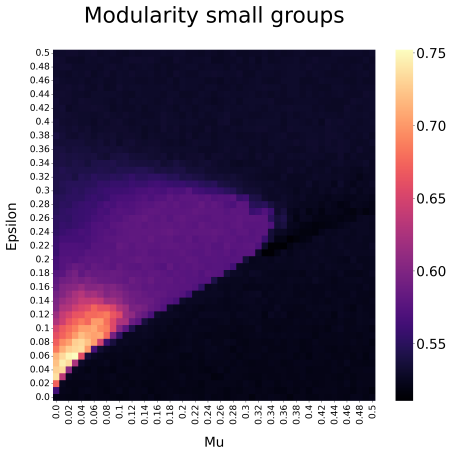
Interpretation:

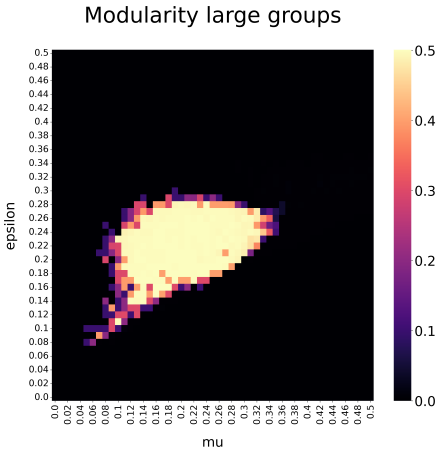
- The phase transition reflects the interplay between μ and ϵ:
  - Low ϵ: Narrow tolerance ranges enforce strong cohesion, but high μ results in fragmentation.
  - High ϵ: Tolerance allows agents to bridge differences, leading to homogeneity even with higher μ.
- The triangular boundary signifies emergent behavior where local interaction dynamics create global opinion patterns.
- These findings demonstrate the system's critical threshold, beyond which network cohesion is either maintained or lost.

Metric 3 - Modularity

- Intuitively: the 'group-ness' of the network.
- Like the paper's peak-finding, is quite subjective

Results - Modularity

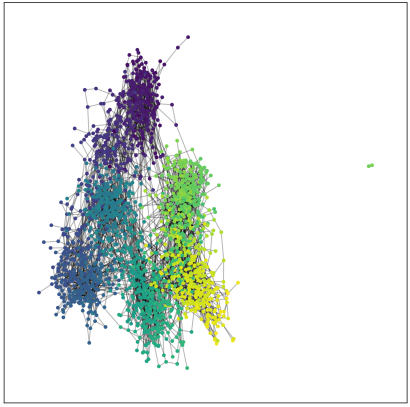




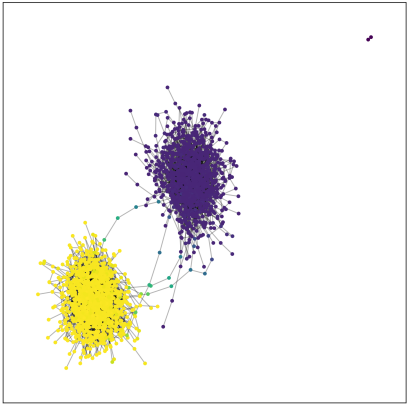
Prioritizing small communities vs prioritizing large communities

Results - Modularity

$\epsilon = 0.06, \mu = 0.03$



$\epsilon = 0.23, \mu = 0.19$

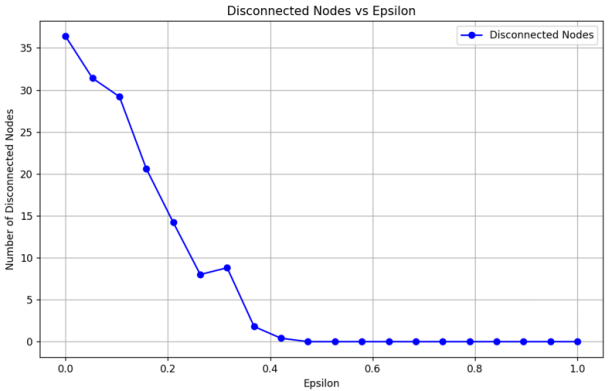


Metric 4 - Number of disconnected nodes

An isolated node means that the person in the graph does not belong to any cluster, any echo chamber.

Metric 4 - Number of disconnected nodes

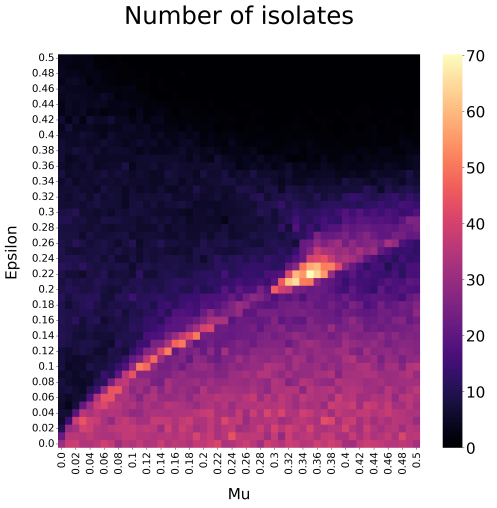
Initially, we kept a constant $\mu = 0.25$ and just varied the ε . We observe the trend is generally descending, except for the ε around 0.3, where there is an anomaly.



Results - Number of disconnected nodes

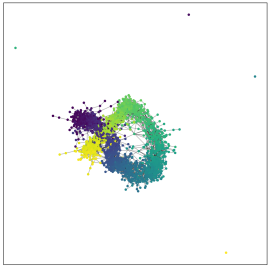
.The heatmap suggets that:

- At low ε fragmentation is low.
- At higher $\varepsilon \in (0.22 - 0.25)$, a sudden spike in fragmentation occurs.
- Diagonal pattern: where increasing both epsilon and μ together affects connectivity.
- Beyond this threshold, the system stabilizes again into a different regime.

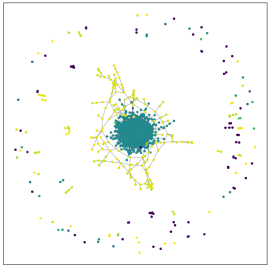


Results - Number of Disconnected Nodes

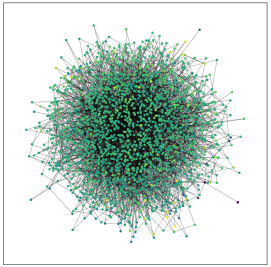
$\epsilon = 0.06, \mu = 0.1$



$\epsilon = 0.22, \mu = 0.35$



$\epsilon = 0.48, \mu = 0.40$



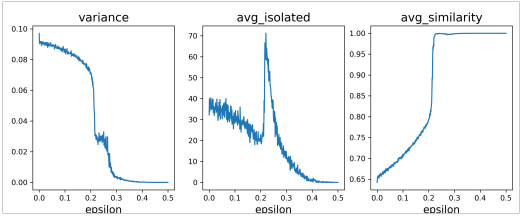
Results - Number of disconnected nodes

The phenomenon occurs at specific values of ε (0.22-0.25) and μ (0.22-0.26), where the number of disconnected nodes sharply increases.

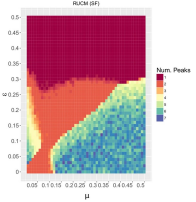
Necessary Mechanisms (ingredients required for fragmentation to emerge):
- Agent interactions based on a parameter
- Network rewiring or opinion dynamics
- Finite graph structure

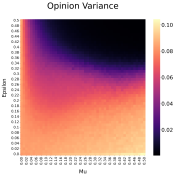
Thus, we can notice a similarity with the results from the paper, where the network transitions from one with disconnected nodes to one big cluster without isolates after a treshold.

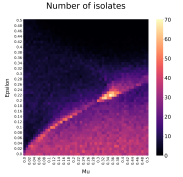
Results - unexpected behavior

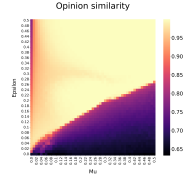


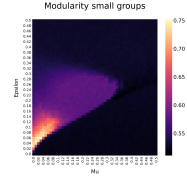
Conclusion - comparison of all heatmaps











Conclusion

• Hypothesis partially correct: we see the resemblance between our results and results from the paper.
• Three of our heatmaps (on the right) are presenting a phase line that corresponds with the multimodality (yellow/greenish) phase line in the paper.
• Our variance heatmap, only corresponds with the unimodality phase line (dark red).
• The modularity heatmap, shows portions of both these phase lines.
•We consider it logical that these different heatmaps do display different phase lines, considering these are analyzing very distinct aspects of the model.

Thank you for your attention!

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Amsterdam, February 2, 2025