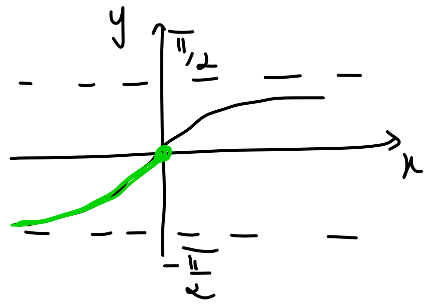
ASFORMAÇÃO
desde 1993

1 → a) $D_f = \{ x \in \mathbb{R} : (x \leq 0) \vee (x \neq 0 \wedge x > 0) \} = \mathbb{R}$

Para $x \leq 0$:



$$-\frac{\pi}{2} < \operatorname{arctg} x \leq 0$$

$$\Leftrightarrow -\frac{\pi}{2} + \frac{1}{2} < \frac{1}{2} + \operatorname{arctg} x \leq \frac{1}{2}$$

$$\Leftrightarrow \frac{-\pi + 1}{2} < f(x) \leq \frac{1}{2}$$

Para $x > 0$:

$$\frac{1}{x} > 0$$

$$\Leftrightarrow e^{\frac{1}{x}} > e^{0}$$

$$\Leftrightarrow e^{\frac{1}{x}} > 1$$

$$\Leftrightarrow f(x) > 1$$

$$CD_f = \left] \frac{-\pi+1}{2}, \frac{1}{2} \right] \cup \left] 1, +\infty \right[$$

b) Para $x < 0$ f é contínua porque é a soma de f. contínuas.

Para $x > 0$ f é contínua porque é a composta de f. contínuas

Para $x = 0$

$$\lim_{x \to 0^-} \left(\frac{1}{2} + \operatorname{arctg}(x) \right) = \frac{1}{2} + \operatorname{arctg}(0^-) = \frac{1}{2}$$

$$\rightarrow \lim_{x \to 0^+} e^{\frac{1}{x}} = e^{\frac{1}{0^+}} = e^{+\infty} = +\infty$$

Logo f não é contínua em $x = 0$

R: f é contínua em $\mathbb{R} \setminus \{0\}$

c) Definição

$x = a$ é assíntota vertical do gráfico de f se $\lim\limits_{x \to a^+} f(x) = \pm\infty$ ou $\lim\limits_{x \to a^-} f(x) = \pm\infty$

$y = mx + b$ é assíntota obliqua/horizontal do gráfico de f se $\lim\limits_{x \to \pm\infty} [f(x) - (mx+b)] = 0$

Assíntota vertical: $x = 0$ (alínea anterior)

Assíntota não vertical:

$$m = \lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \frac{e^{\frac{1}{x}}}{x} = \frac{e^{\frac{1}{+\infty}}}{+\infty} = \frac{1}{+\infty} = 0$$

$$b = \lim_{x \to +\infty} [f(x) - mx] = \lim_{x \to +\infty} \left(e^{\frac{1}{x}}\right) = e^0 = 1$$

$y = 1$ é assíntota horizontal do gráfico de f quando $x \to +\infty$

$$m = \lim_{x \to -\infty} \frac{\frac{1}{2} + \operatorname{arctg} x}{x} = \frac{\frac{1}{2} + \operatorname{arctg}(-\infty)}{-\infty} = \frac{\frac{1}{2} - \frac{\pi}{2}}{-\infty} = 0$$

$$b = \lim_{x \to -\infty} \left(\frac{1}{2} + \operatorname{arctg} x\right) = \frac{1}{2} - \frac{\pi}{2}$$

$y = \frac{1 - \pi}{2}$ é assíntota horizontal do gráfico de f quando $x \to -\infty$.

c) f não é contínua em $x=0 \Rightarrow f'(0)$ não existe

Para $x<0$: $f'(x) = \left(\frac{1}{2} + \text{arctg}\, x\right)' = \frac{1}{1+x^2}$

Para $x>0$: $f'(x) = \left(e^{\frac{1}{x}}\right)' = \left(\frac{1}{x}\right)' e^{\frac{1}{x}} = -\frac{1}{x^2} e^{\frac{1}{x}}$

$$f'(x) = \begin{cases} \frac{1}{1+x^2} & x<0 \\ -\frac{1}{x^2} e^{\frac{1}{x}}, & x>0 \end{cases} \qquad D_{f'} = \mathbb{R}\setminus\{0\}$$

e) T. Lagrange

Se $\begin{cases} f \text{ é contínua em } [a,b] \\ f \text{ é integrável em }]a,b[\end{cases}$

Então $\exists c \in]a,b[: f'(c) = \frac{f(b)-f(a)}{b-a}$

f é contínua em $[-\sqrt{3}, -1]$

f é diferenciável $]-\sqrt{3}, -1[$

Então, $\exists c \in]-\sqrt{3}, -1[: f'(c) = \frac{f(-1) - f(-\sqrt{3})}{-1-(-\sqrt{3})}$

$$= \frac{\frac{1}{2} + \text{arctg}(-1) - \left(\frac{1}{2} + \text{arctg}(-\sqrt{3})\right)}{1+\sqrt{3}}$$

$$= \frac{\cancel{\frac{1}{2}} - \frac{\pi}{4}_{\times 3} - \cancel{\frac{1}{2}} + \frac{\pi}{3}_{\times 4}}{-1+\sqrt{3}} = \frac{\frac{\pi}{12}}{-1+\sqrt{3}} = \frac{\pi}{-12+12\sqrt{3}}$$ V

f) Injetividade

$$f(a) = f(b) \Rightarrow a = b \quad \forall a, b \in D_f$$

$x \leq 0$: $f(a) = f(b) \Leftrightarrow \frac{1}{2} + \operatorname{arctg} a = \frac{1}{2} \operatorname{arctg} b$

$\Leftrightarrow \operatorname{arctg} a = \operatorname{arctg} b$

$\Leftrightarrow a = b$, arctg x é injetiva

$\forall a, b \in \mathbb{R}_0^-$

$x > 0$: $f(a) = f(b) \Leftrightarrow e^{\frac{1}{a}} = e^{\frac{1}{b}} \Leftrightarrow \frac{1}{a} = \frac{1}{b} \Leftrightarrow a = b$

e^x é inj. $\quad \forall a, b \in \mathbb{R}^+$

Logo f é injetiva.

Inversa

$x \leq 0$. $\quad y = \frac{1}{2} + \operatorname{arctg} x$

$x = \frac{1}{2} + \operatorname{arctg} y$

$\Leftrightarrow \operatorname{arctg} y = x - \frac{1}{2}$

$\Leftrightarrow \quad y = \operatorname{tg}\left(x - \frac{1}{2}\right)$

$x > 0$: $\quad y = e^{\frac{1}{x}}$

$x = e^{\frac{1}{y}}$

$\Leftrightarrow \frac{1}{y} = \ln x$

$\Leftrightarrow y = \frac{1}{\ln x}$

$$f^{-1}(x) = \begin{cases} \operatorname{tg}\left(x - \frac{1}{2}\right), & \frac{1-\pi}{2} \leq x \leq \frac{1}{2} \\ \frac{1}{\ln x}, & x > 1 \end{cases}$$

$$D_{f^{-1}} = CD_f \quad \wedge \quad CD_{f^{-1}} = D_f$$

$2 \rightarrow \displaystyle\int \frac{x^3+2}{x^2+1}\,dx$

$$\begin{array}{r|l} x^3+2 & x^2+1 \\ \cline{2-2} -x^3-x & x \\ \hline -x+2 & \end{array}$$

$$= \int \left(x + \frac{-x+2}{x^2+1}\right) dx$$

$$= \int x\,dx - \int \frac{x}{x^2+1}\,dx + 2\int \frac{1}{x^2+1}\,dx$$

$$= \frac{x^2}{2} - \frac{1}{2}\int \frac{2x}{x^2+1}\,dx + 2\operatorname{arctg} x + C$$

$$= \frac{x^2}{2} - \frac{1}{2}\ln|x^2+1| + 2\operatorname{arctg} x + C,\ C \in \mathbb{R}$$

$3 \rightarrow$ a) $\displaystyle\int_6^9 \frac{3}{x^2\sqrt{x^2-9}}\,dx$

Substituição

$x = 3\sec t$

$x' = 3\sec t \operatorname{tg} t$

x	t
6	$\pi/3$
9	$\arccos(\frac{1}{3})$

• $6 = 3\sec t$

$\Rightarrow 2 = \sec t$

$\Rightarrow \frac{1}{\cos t} = 2$

$\Rightarrow \cos t = \frac{1}{2} \Rightarrow t = \frac{\pi}{3}$

• $9 = 3\sec t$

$\Rightarrow \sec t = 3$

$\Rightarrow \cos t = \frac{1}{3} \Rightarrow t = \arccos(\frac{1}{3})$

$$= \int_{\frac{\pi}{3}}^{\arccos(\frac{1}{3})} \frac{\cancel{3} \times \cancel{3\sec t} \operatorname{tg} t}{\cancel{9\sec^2} t\sqrt{9\sec^2 t - 9}}\,dt$$

$$= \int_{\frac{\pi}{3}}^{\arccos(\frac{1}{3})} \frac{\operatorname{tg} t}{\sec t\sqrt{9(\sec^2 t - 1)}}\,dt$$

$$= \int_{\frac{\pi}{3}}^{\arccos(\frac{1}{3})} \frac{\cancel{\operatorname{tg}}\, t}{\sec t \times \sqrt{9} \times \sqrt{\cancel{\operatorname{tg}^2} t}}\,dt$$

$$= \int_{\frac{\pi}{3}}^{\arccos(\frac{1}{3})} \frac{1}{3} \times \frac{1}{\sec t}\,dt$$

$$= \int_{\frac{\pi}{3}}^{\arccos(\frac{1}{3})} \frac{1}{3} \times \frac{1}{\sec t}\,dt$$

$$= \int_{\frac{\pi}{3}}^{\arccos(\frac{1}{3})} \frac{1}{3} \times \cos t\,dt = \frac{1}{3}\left[\operatorname{sen} t\right]_{\frac{\pi}{3}}^{\arccos(\frac{1}{3})}$$

$$= \frac{1}{3}\left(\underbrace{\operatorname{sen}\left(\arccos\left(\frac{1}{3}\right)\right)} - \operatorname{sen}\frac{\pi}{3}\right)$$

$$= \frac{1}{3}\left(\sqrt{1-\left(\frac{1}{3}\right)^2} - \frac{\sqrt{3}}{2}\right)$$

$$= \frac{1}{3}\left(\sqrt{\frac{8}{9}} - \frac{\sqrt{3}}{2}\right) = \frac{1}{3}\left(\frac{\sqrt{8}}{3} - \frac{\sqrt{3}}{2}\right) = \frac{\sqrt{8}}{9} - \frac{\sqrt{3}}{6}$$

4 → $\displaystyle\int_0^2 \frac{x}{(x^2-1)^2}\,dx$

$I = [0,2] \wedge 1 \notin D_f$

$D_f = \{x \in \mathbb{R} : x^2-1 \neq 0\} = \mathbb{R}\setminus\{-1,1\}$

Integral impróprio de 2ª espécie

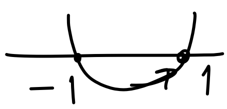
$$\int_0^2 \frac{x}{(x^2-1)^2}\,dx = \int_0^1 \frac{x}{(x^2-1)^2}\,dx + \int_1^2 \frac{x}{(x^2-1)^2}\,dx$$

- $\displaystyle\int_0^1 \frac{x}{(x^2-1)^2}\,dx = \lim_{t\to 1^-} \frac{1}{2}\int_0^t 2x(x^2-1)^{-2}\,dx$

$$= \lim_{t\to 1^-} \frac{1}{2}\left[\frac{(x^2-1)^{-1}}{-1}\right]_0^t$$

$$= \lim_{t\to 1^-} \frac{1}{2}\left[-\frac{1}{x^2-1}\right]_0^t$$

$$= \lim_{t\to 1^-} \frac{1}{2}\left(-\frac{1}{t^2-1} + \frac{1}{-1}\right)$$



$$= \frac{1}{2}\left(-\frac{1}{0^-} - 1\right) = \frac{1}{2}(+\infty - 1) = +\infty$$

Divergente.

Logo $\displaystyle\int_0^2 \frac{x}{(x^2-1)^2}\,dx$ é divergente.

5 → $H(x)=\int_{2x}^{x+x^2} f(t)\,dt$

$$\boxed{\begin{array}{l} H(x)=\int_{g_1(x)}^{g_2(x)} f(t)\,dt \\ \text{Se } g_1 \text{ e } g_2 \text{ são dif. e } f \text{ é contínua então.} \\ H'(x)=f(g_2(x))\times g_2'(x)-f(g_1(x))\times g_1'(x) \end{array}}$$

$g_1(x)=2x \qquad g_1'(x)=2$

$g_2(x)=x+x^2 \qquad g_2'(x)=1+2x$ Dif.

$f(t)$ é contínua

Então H é derivável e:

$$H'(x)=f(x+x^2)\times(1+2x)-f(2x)\times 2$$

$$\begin{aligned} H'(1) &= f(1+1^2)\times(1+2\times 1)-f(2\times 1)\times 2 \\ &= f(2)\times 3-f(2)\times 2 \\ &= f(2) \end{aligned}$$

$H(1)=\int_2^2 f(t)\,dt=0$

logo $H'(1)-H(1)=f(2)-0=f(2)$ //

7) $$\lim_{x \to 1} \frac{F(x)}{x-1} \underset{\text{Regra de Cauchy}}{\overset{\frac{0}{0}}{=}} \lim_{x \to 1} \frac{F'(x)}{(x-1)'} = \lim_{x \to 1} \frac{e^{-x^4} \times 2x - e^{-x^2}}{1}$$

$$= \frac{e^{-1} \times 2 - e^{-1}}{1} = e^{-1} \;//$$

$$F'(x) = f(x^2) \times 2x - f(x) \times 1$$
$$= e^{-x^4} \times 2x - e^{-x^2}$$

c) $f(x) = \text{arcsen}\left(e^{\frac{1}{x}}\right)$

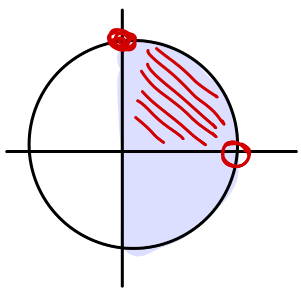
$$D_f = \{ x \in \mathbb{R} : \underbrace{-1 \leq e}_{⚠} {}^{\frac{1}{x}} \leq 1 \;\wedge\; x \neq 0 \}$$

$$0 < e^{\frac{1}{x}} \leq 1 \quad \wedge\, x \neq 0$$
$$\Rightarrow \quad e^{\frac{1}{x}} \leq 1 \quad \wedge\, x \neq 0$$
$$\Rightarrow \quad \frac{1}{x} \leq \ln 1 \quad \wedge\, x \neq 0$$
$$\Rightarrow \quad \frac{1}{x} \leq 0 \quad \wedge\, x \neq 0$$
$$\Rightarrow \quad x \leq 0 \quad \wedge\, x \neq 0$$

$$D_f = \left]-\infty, 0\right[$$

$CD_f = ?$

$$0 < e^{\frac{1}{x}} \leq 1$$
$$\Rightarrow \quad 0 < \text{arcsen}\left(e^{\frac{1}{x}}\right) \leq \frac{\pi}{2}$$



$$CD_f = \left]0, \frac{\pi}{2}\right]$$

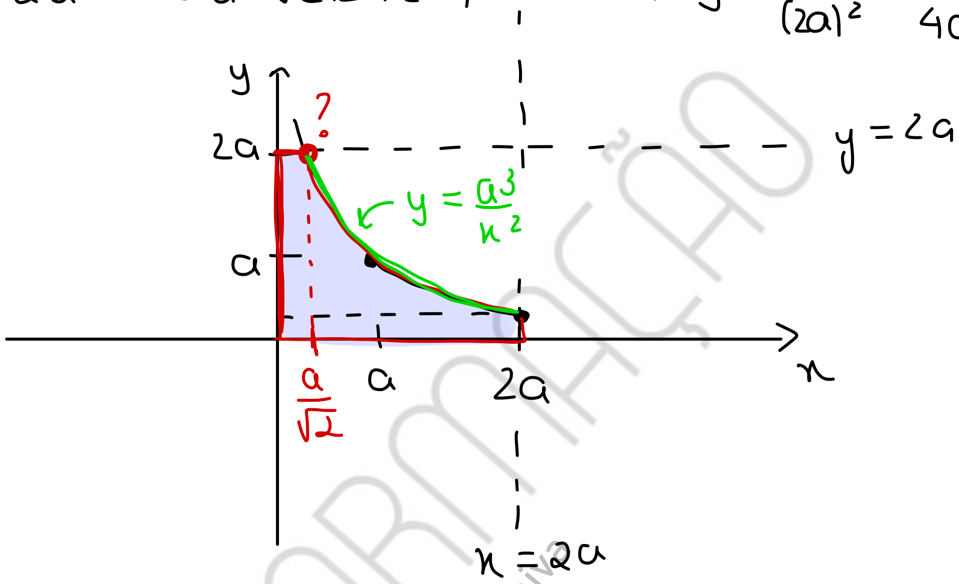
d) • $y \le 2a$ Reta Horizontal

• $x^2 y \le a^3$

$\Rightarrow \boxed{y \le \frac{a^3}{x^2}}, x \ne 0 \longrightarrow$

x	$y = \frac{a^3}{x^2}$
a	$y = \frac{a^3}{a^2} = a$
a^2	$y = \frac{a^3}{a^4} = \frac{1}{a}$
$2a$	$y = \frac{a^3}{(2a)^2} = \frac{a^3}{4a^2} = \frac{a}{4}$

• $x \le 2a$ Reta Vertical



? $2a = \frac{a^3}{x^2} \Rightarrow 2a x^2 = a^3 \Rightarrow x^2 = \frac{a^3}{2a} \Rightarrow x^2 = \frac{a^2}{2}$

$\Rightarrow x = \pm\sqrt{\frac{a^2}{2}} \Rightarrow x = \frac{a}{\sqrt{2}}$

$$A = \int_0^{\frac{a}{\sqrt{2}}} 2a \, dx + \int_{\frac{a}{\sqrt{2}}}^{2a} \frac{a^3}{x^2} \, dx$$