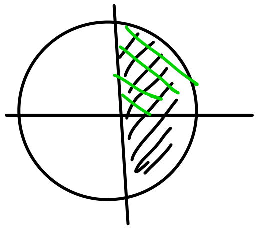
ASFORMAÇÃO

desde 1993

1 → a)

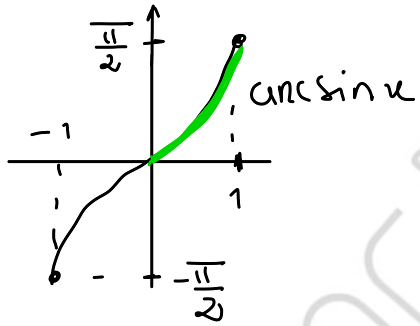
$D_f = \{x \in \mathbb{R} : \arcsin x > 0 \wedge -1 \leq x \leq 1\}$



$\arcsin x > 0$

$\Rightarrow 0 < x \leq 1$

$D_f =]0, 1]$



Domínios

$\frac{A}{B}$, $B \neq 0$

$\sqrt[n]{A}$ (n par) $A \geq 0$

$\log_a B$, $B > 0$

$\operatorname{tg} A$, $A \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$

$\left.\begin{array}{l}\arccos A \\ \arcsin A\end{array}\right\} -1 \leq A \leq 1$

$CD_f = ?$

$0 < x \leq 1 \Rightarrow 0 < \arcsin x \leq \frac{\pi}{2}$

$\Rightarrow \ln(\arcsin x) \leq \ln\frac{\pi}{2}$

$CD_f =]-\infty, \ln\frac{\pi}{2}]$

b) $D_{f^{-1}} = CD_f =]-\infty, \ln\frac{\pi}{2}]$

$CD_{f^{-1}} = D_f =]0, 1]$

Expressão: $y = \ln(\arcsin x) \Rightarrow e^y = \arcsin x$

$\Rightarrow x = \operatorname{sen}(e^y)$

$f^{-1}(x) = \operatorname{sen}(e^x)$ //

c) Se f é contínua em $[a,b]$ então f atinge em $[a,b]$ um máximo e um mínimo globais.

Não é aplicável porque o intervalo é aberto.

Adicionalmente:

$\lim_{x \to 0^+} \ln(\arcsin x) = \ln(0^+) = -\infty$

Logo f não atinge um mínimo global.

d) $\lim_{x \to 0^+} \frac{f(x)}{x} = \lim_{x \to 0^+} \frac{\ln(\arcsin x)}{x} = \frac{\ln(0^+)}{0^+} = \frac{-\infty}{0^+} = -\infty$

2 → a) $\int \frac{1}{x\sqrt{x^2-9}} dx =$

$= \int \frac{1}{\cancel{3\sec t}\sqrt{9\sec^2 t - 9}} \times \frac{\cancel{3\sec t}\,\mathrm{tg}\, t}{1} dt$

$= \int \frac{\mathrm{tg}\, t}{\sqrt{9(\sec^2 t - 1)}} dt$

$= \frac{1}{3}\int \frac{\mathrm{tg}\, t}{\sqrt{\mathrm{tg}^2 t}} dt$

$= \frac{1}{3}\int \frac{\mathrm{tg}\, t}{\mathrm{tg}\, t} dt$

$= \frac{1}{3}\int 1\, dt = \frac{1}{3} t + C, \; C \in \mathbb{R}$

$= \frac{1}{3} \mathrm{arcsec}\left(\frac{x}{3}\right) + C, \; C \in \mathbb{R}$

Funçõ	Subst.
$\sqrt{a^2-x^2}$	$x = a\,\mathrm{sen}\, t$
$\sqrt{a^2+x^2}$	$x = a\,\mathrm{tg}\, t$
$\sqrt{x^2-a^2}$	$x = a\sec t$

Substituição

$x = 3\sec t$

$\begin{cases} dx = 3\sec t\,\mathrm{tg}\, t\, dt \\ t = \mathrm{arcsec}\left(\frac{x}{3}\right) \end{cases}$

b) $\int \frac{1}{(x^2+4)(x-2)} dx = \int \left(\frac{Ax+B}{x^2+4} + \frac{C}{x-2}\right) dx = (*)$

e. Aux

$\frac{1}{(x^2+4)(x-2)} = \underset{\times(x-2)}{\frac{Ax+B}{x^2+4}} + \underset{\times(x^2+4)}{\frac{C}{x-2}}$

$\Leftrightarrow 1 = (Ax+B)(x-2) + C(x^2+4)$

$\Leftrightarrow 1 = Ax^2 - 2Ax + Bx - 2B + Cx^2 + 4C$

$\Leftrightarrow 1 = (A+C)x^2 + (B-2A)x + 4C - 2B$

$\begin{cases} A+C=0 \\ B-2A=0 \\ 4C-2B=1 \end{cases} \Rightarrow \begin{cases} C=-A \\ B=2A \\ -4A-4A=1 \end{cases} \Leftrightarrow \begin{cases} C=\frac{1}{8} \\ B=-\frac{1}{4} \\ A=-\frac{1}{8} \end{cases}$

$$\circledast = \int \frac{-\frac{1}{8}x-\frac{1}{4}}{x^2+4}\,dx + \int \frac{\frac{1}{8}}{x-2}\,dx$$

$$= -\frac{1}{8\cdot 2}\int \frac{2x}{x^2+4}\,dx - \frac{1}{4}\int \frac{1}{x^2+4} + \frac{1}{8}\int \frac{1}{x-2}\,dx$$

$$= -\frac{1}{16}\ln|x^2+4| - \frac{1}{4}\int \frac{1}{4\left(\frac{x^2}{4}+1\right)}\,dx + \frac{1}{8}\ln|x-2|$$

$$= (\dots) - \frac{1}{16}\int \frac{1}{\left(\frac{x}{2}\right)^2+1}\,dx + (\dots)$$

$$= (\dots) - \frac{1}{16}\times 2\int \frac{\frac{1}{2}}{\left(\frac{x}{2}\right)^2+1}\,dx + (\dots)$$

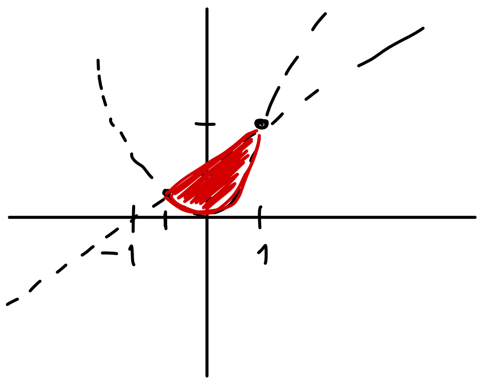
$$= -\frac{1}{16}\ln|x^2+4| - \frac{1}{8}\operatorname{arctg}\left(\frac{x}{2}\right) + \frac{1}{8}\ln|x-2| + C,\ C\in\mathbb{R}$$

3 → a) $A = \int_{-\frac{1}{2}}^{1} (x+1-2x^2)\,dx$

$$= \left[\frac{x^2}{2} + x - \frac{2x^3}{3}\right]_{-\frac{1}{2}}^{1}$$

$$= \frac{1}{2}+1-\frac{2}{3} - \left(\frac{(-\frac{1}{2})^2}{2} - \frac{1}{2} - 2\frac{(-\frac{1}{2})^3}{3}\right)$$

$(\dots)$



$y = 2x^2$

$y = x+1$

Intersecção

$2x^2 = x+1$

$\Rightarrow 2x^2 - x - 1 = 0$

$\Rightarrow x = \frac{1 \pm \sqrt{1-4\times 2(-1)}}{4}$

$\Rightarrow x = \frac{1+3}{4} \vee x = \frac{1-3}{4}$

$\Rightarrow x = 1 \quad \vee \quad x = -\frac{1}{2}$

$(1, 2) \qquad (-\frac{1}{2}, \frac{1}{2})$

b) $\int_{-\infty}^{+\infty} e^{-x}\,dx = \int_{-\infty}^{0} e^{-x}\,dx + \int_{0}^{+\infty} e^{-x}\,dx$

• $\int_{-\infty}^{0} e^{-x}\,dx = \lim_{t\to-\infty} -\int_{t}^{0} -e^{-x}\,dx =$

$= \lim_{t\to-\infty} -\left[e^{-x}\right]_t^0 = \lim_{t\to-\infty} -\left(e^{0}-e^{-t}\right)$

$= -\left(1-e^{+\infty}\right) = -\left(.-+\infty\right) = -\infty$ Divergente

Logo $\int_{-\infty}^{+\infty} e^{-x}\,dx$ é divergente.

c) $\int_{0}^{+\infty} (t^5+e^{-2t})\,e^{-2t}\,dt = \int_{0}^{+\infty} t^5 e^{-2t}\,dt + \int_{0}^{+\infty} 1\,e^{-4t}\,dt$

$\mathcal{L}\{f(t)\}(s) = \int_{0}^{+\infty} f(t)\,e^{-st}\,dt$

$= \mathcal{L}\{t^5\}(2) + \mathcal{L}\{1\}(4)$

$= \frac{5!}{2^6} + \frac{1}{4}$

$\mathcal{L}\{t^5\}(s) = \frac{5!}{s^6}$

$\mathcal{L}\{1\} = \mathcal{L}\{t^0\}$

$= \frac{0!}{s} = \frac{1}{s}$

4→ a) $f = f' \times x$ $\quad f(1)=1$

$\Rightarrow f = \frac{df}{dx} \times x$ EDO de var. sep.

$\Rightarrow \frac{1}{x}\,dx = \frac{1}{f}\,df \quad , f \neq 0$

$\int \frac{1}{x}\,dx = \int \frac{1}{f}\,df$

$\Rightarrow \ln|x| = \ln|f| + C$

$\Rightarrow f = e^{\ln|x| - C}$

$\Rightarrow \quad f(x) = |x| \times e^{-C}$

$\Rightarrow \quad f(x) = |x| \times K$

$f(1) = 1 \Rightarrow |1| \times K = 1 \Rightarrow K = 1$

$f(x) = x$, por exemplo.

b) $y' + p(x)y = q(x)$ F. Integrante: $\mu(x) = e^{\int p(x)dx}$

$\mu(x) = e^{\int \frac{1}{x}dx} = e^{\ln|x|} = x, \; x>0$

$\underbrace{x\left(y' + \frac{1}{x}y\right)} = x \times \frac{1}{x}$

$\Rightarrow (yx)' = 1$

$\Rightarrow yx = \int 1 \, dx$

$\Rightarrow yx = x + C$

$\Rightarrow y = \frac{x+C}{x}, \; C \in \mathbb{R}$

$y(1) = -1 \Rightarrow \frac{1+C}{1} = -1 \Rightarrow 1+C = -1 \Rightarrow C = -2$

$\boxed{y = \frac{x-2}{x}}$

5 + a) $y^{(iv)} + 8y'' + 16y = 0$

E.C: $R^4 + 8R^2 + 16 = 0$

$\Rightarrow (R^2+4)^2 = 0$

$\Rightarrow R^2 + 4 = 0$

$\Rightarrow R^2 = -4$

$\Rightarrow R = \pm 2i$

$\alpha = 0 \quad \beta = 2$

$SFS = \{e^0\cos(2x), e^0 \operatorname{sen}(2x)\}$

$= \{\cos(2x), \operatorname{sen}(2x)\}$

$y_h = C_1\cos(2x) + C_2 \operatorname{sen}(2x)$

$C_1, C_2 \in \mathbb{R}$

b) $b(x) = P_m(x)\, e^{\alpha x} \cos(\beta x)$ ou $b(x) = P_m(x)\, e^{\alpha x} \operatorname{sen}(\beta x)$

1º) Identificar m, $\underline{\alpha}$, $\underline{\beta}$

2º) Determinar k = multiplicidade de $\underline{\alpha} + \underline{\beta} i$ na E.C.

3º) $y_p = x^k e^{\alpha x} [P(x) \cos(\beta x) + Q(x) \operatorname{sen}(\beta x)]$

$P, Q \to$ Pol. genéricos de grau $\underline{m}$

4º) Determinar as constantes de P e Q substituindo y por y_p na EDO completa.

Neste caso $b(x) = \underbrace{\sin x}_{b_1(x)} + \underbrace{e^x}_{b_2(x)}$

Pelo princípio da sobreposição dos efeitos

$$y = y_h + y_{p_1} + y_{p_2}$$

c) $y_{p_1} = ?$ $\quad b_1(x) = \sin x$ $\quad b(x) = P_m e^{\alpha x} \operatorname{sen}(\beta x)$

1º) $\underline{m = 0}$ $\quad \alpha = 0 \quad \beta = 1$

$\quad \hookrightarrow 0 + 1i = i$ $\quad i$ não é sol da E.C.

2º) $k = 0$

3º) $y_{p_1} = x^0 e^0 [A \cos x + B \sin x]$

$y_{p_1} = A \cos x + B \sin x$

m	P	Q
0	A	B
1	$Ax+B$	$Cx+D$
2	Ax^2+Bx+C	...
⋮		

4º) $y^{(iv)} + 8y'' + 16y = \sin x$

$y'_{p_1} = -A \operatorname{sen} x + B \cos x$

$y''_{p_1} = -A \cos x - B \operatorname{sen} x$

$y'''_{p_1} = A \operatorname{sen} x - B \cos x$

$y^{iv}_{p_1} = A \cos x + B \operatorname{sen} x$

V

$$A\cos x + B\operatorname{sen} x + 8(-A\cos x - B\operatorname{sen} x) + 16(A\cos x + B\operatorname{sen} x) = \operatorname{sen} x$$

$$\Leftrightarrow 9A\cos x + 9B\operatorname{sen} x = \operatorname{sen} x \quad \begin{cases} 9A = 0 \\ 9B = 1 \end{cases} \Leftrightarrow \begin{cases} A = 0 \\ B = \frac{1}{9} \end{cases}$$

$$y_{p_1} = \frac{1}{9}\operatorname{sen} x$$

$$y_{p_2} = ? \qquad b_2(x) = e^x$$

1º) $m = 0 \quad \alpha = 1 \quad \beta = 0$

$\hookrightarrow 1 + 0i = 1$

2º) $k = 0$

3º) $y_{p_2} = x^0 e^x [A\cos 0 + B\operatorname{sen} 0]$

$\Rightarrow y_{p_2} = Ae^x$

4º) $y_{p_2}' = y''_{p_2} = y'''_{p_2} = y^{(iv)}_{p_2} = Ae^x$

$$Ae^x + 8Ae^x + 16Ae^x = e^x$$

$$\Leftrightarrow 25A = 1 \Leftrightarrow A = \frac{1}{25}$$

$$y_{p_2} = \frac{1}{25}e^x$$

$$\therefore y = C_1\cos(2x) + C_2\operatorname{sen}(2x) + \frac{1}{9}\operatorname{sen} x + \frac{1}{25}e^x, \; C_1, C_2 \in \mathbb{R}$$