Homework sheet 8 2023-07-04

Due date: 2023-07-13 17:59

Delivery format

- Create a folder called "hw08" under the "homework" folder in your git repository.
- Answer all questions to the related to the homework in a file called "README.md" in the above created folder. **The readme must be a complete and standalone solution**. You are highly encouraged to provide instructions on how to recreate your results (e.g. run this script with this dataset, or run through this Jupyter notebook).
- Put all scripts and images you've created to reproduce and comprehend your answers in the same folder. Examples for this, are scripts to create plots, visualization, certain reconstructions to compare different strengths and weaknesses and similar things.
- The concrete implementations of the homework, must be put into the "aomip" folder. You are free to choose the structure, layout and form of your implementation.
- Once you've finished your homework, commit your changes and create a git tag with `git tag -a "hw08" -m "Tag for homework 8"` and push your changes. This needs to be tagged and pushed to your "master" branch before the deadline ends. Please use this exact command.
- Finally, once you push the changes with the tag to your repository. Create a release in GitLab. You find it in your project, under the menu 'Deployments'.

Note

Generally, we expect and reward both explorativ work and good software engineering. Hence, we highly encourage you to look at the provided resources, do your own research and try different things and play around with the tools provided to you during the course.

Without explicitly permissions you are not allowed to pull any other dependencies. If you need dependencies to load or import a dataset, please get in touch with us and let us know the limitations. If you need to open `TIFF` files, use the already included dependency `tifffile`. Use it with `tifffile.imread`. See some examples at https://pypi.org/project/tifffile/#examples. This can also handle 3D TIFF-files, which makes this quite handy.

Homework 1: Subgradient Method

In the lecture, we have seen that, with some tricks, you can still use some of the tools we have covered in the course with non-differentiable functions. Of course, these are subgradients.

In this exercise, you are to implement the subgradient method and solve the generalized LASSO problem with TV regularization using the subgradient method.

$$\min_x \frac{1}{2}||Ax-b||_2^2 + \beta||\nabla x||_1 \tag{1}$$

Here ∇x is the finite difference of x in all directions. We encourage you to replace the least square term for the transmission log likelihood, if you are working with low dose data and explore how that works as well.

Subgradient Method The subgradient method seeks to find a minimizer of a (non-differentiable) function f, and its update step is defined as:

$$x_{k+1} = x_k - \alpha_k g_k, \quad g_k \in \partial f(x_k) \tag{2}$$

where $g_k \in \partial f(x_k)$ denotes a subgradient of f at the iterate x_k.

Computing the Subgradient of the ℓ^1-Norm The ℓ^1 norm is differentiable everywhere, except at $x = 0$. Hence, for $x \neq 0$ the subgradient is given as:

$$\partial f(x) = \begin{cases} -1, & x < 0 \\ 1, & x > 0 \end{cases} \tag{3}$$

Now, for $x = 0$, $\partial f(x) \in [-1, 1]$. A simple and elegant choice for the subgradient is the sign function, which can be defined as:

$$\text{sign}(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases} \tag{4}$$

Incidentally, this is exactly what `np.sign` returns (hint hint). However, as mentioned, this is not the only choice. You are encouraged to test different options and see if you can spot any differences in results or convergence.

Choice of step length A major difference compared to first-order methods is that the step lengths have to be pre-defined and known beforehand and can not be adjusted adaptively. Here, we'll focus on three different types of choices:

1. Constant step size: $\alpha_k = h$ is a constant, independent of k.
2. Square summable but not summable: the step sizes satisfy:

$$\sum_{k=1}^{\infty} \alpha_k^2 < \infty \quad \text{and} \quad \sum_{k=1}^{\infty} \alpha_k = \infty$$

3. Nonsummable diminishing: the step sizes satisfy:

$$\lim_{k \to \infty} \alpha_k = 0 \quad \text{and} \sum_{k=1}^{\infty} \alpha_k = \infty$$

for the second and third option, find at least one possible choice of sequences, show that they satisfy the conditions, and perform some tests (i.e. convergence behavior and similar). You are encouraged to try this first on a small 2D problem and present it using contour plots and similar techniques. But also try it on a larger X-ray CT problem and present your results for it.

Comparison Finally, compare it to the results from the previous homework. How does the subgradient perform compared to something like ADMM? Consider variables such as conversion speed, decrease guarantees, stability, runtime performance, and similar aspects. Please also remember that the homework submission should be standalone, and we will not remember every picture from your previous homework. So just add the necessary data, images, and analysis again to help us understand your reasoning.

Homework 2: Challenge

This is the final homework; throw everything you can at the challenge dataset and the leaderboard.

I'll give you a final trick, that might be helpful in the reconstruction. If you use a non-negativity constraint, you can make it spatially aware. You know that all the challenge data is a disc/circle. So outside the circle, there should be nothing, right? (You are not allowed to pass it the ground truth. ;-)).

In general, get nifty and play around a little!

Homework 3: Presentation

On the 11th of July, it will be the last session of the seminar, which will mostly be your presentations. The presentations should be short (around 5 minutes, no longer than 8 minutes), plus some discussion. You should show some of the things you've done during the course. The format is quite free. You can present some of your solutions to the challenge dataset that have scored particularly good results; you can present and talk about things you (dis-) liked or learned from the course; or you can show results that surprised you.

Please be aware that the projector in the lecture hall isn't all that good, so be sure to choose a color map with good contrast, and if you want to show subtleties, consider only showing a zoomed-in version (at best with some context to know what we are looking at).

Further, always mention what kind of reconstruction you have done, including setup information about how many angles, the arc, and similar information.