README : Internal Solitary Wave – Ice Floe Interaction

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Note : The Python script has been written with the help of AI.

1 Purpose of the Code

The script `ISW_ice.py` provides a kinematic framework to explore the interaction between internal solitary waves (ISWs) and ice floes, based on the laboratory experiments of Carr et al. (2022). Code and documentation can be found there : `https://github.com/matildeb666/Internal-waves`. Our work does not solve the Navier–Stokes equations, but instead uses the experimental constitutive law established by Carr et al. to investigate emergent consequences at the scale of individual floes and ice fields. We essentially perform a Lagrangian analysis on the Eulerian data provided by the paper.

The code has two distinct objectives:

- Visualisation of ISW–floe interaction (pedagogical, only illustrative)
- Extraction of physically meaningful and non-tautological diagnostics to complete author's discussion.

2 Fixed and User-Controlled Parameters

2.1 Fixed Parameters (from Carr et al. 2022)

The following quantities are fixed and taken directly from the experiments:

- ISW phase speed: $c_0 = 0.14\ \mathrm{m\,s^{-1}}$
- Upper-layer thickness: $H_1 = 0.05$ m
- Lower-layer thickness: $H_3 = 0.32$ m
- Ice density: $\rho_{ice} = 800\ \mathrm{kg\,m^{-3}}$
- Water densities: $\rho_1 = 1.025\ \mathrm{kg\,m^{-3}}$, $\rho_3 = 1.045\ \mathrm{kg\,m^{-3}}$

The ISW wavelength λ_{ISW} is computed internally as the full width at half maximum (as defined in the paper) of a sech^2 profile.

2.2 User-Controlled Parameters

- Number of floes N
- Floe length l_f
- Animation speed (purely visual)

3 Imposed Constitutive Law

The floe velocity is prescribed using the experimental linear relation from Carr et al. (2022):

$$\frac{c_f}{c_0} = -0.61 \frac{l_f}{2\lambda_{ISW}} + 0.79,$$

where c_f is the floe speed. This law is assumed and therefore *not* derived in the code : it must not be re-tested directly when extracting results.

4 Pedagogical Animation

The animation shows:

- A two-layer stratified fluid
- ISW creation
- A propagating ISW of fixed shape and speed
- Ice floes advected at a prescribed velocity (using linear relation by Carr et.al)

Its purpose is purely illustrative: to visualize the mechanisms described in the experiments and clarify the kinematic assumptions.

5 Diagnostics and Results

5.1 Critical Floe Length for Advection Arrest

Solving $c_f = 0$ yields a critical dimensionless floe length:

$$\frac{l_f}{2\lambda_{ISW}} = \frac{0.79}{0.61}$$

This defines a sharp regime boundary between advected and non-advected floes.

5.2 Residence Time over the ISW

Here the question is : "How long does a floe interact with the ISW?"

Idea The floe speed is prescribed by the linear law, but the *time spent interacting with the wave* is not. If the floe speed is close to the ISW speed c_0, the floe remains close to the wave for a long time. This interaction time is therefore an emergent property of relative advection, not of the imposed law.

Observable The residence time is estimated using the relative speed between the floe and the ISW crest:

$$T_{res} \sim \frac{\lambda_{ISW}}{c_0 - c_f}$$

where λ_{ISW} is the characteristic ISW width. This provides a characteristic time for which the floe stays within the wave envelope.

The dimensionless residence time is:

$$T^* = \frac{T_{res}}{\lambda_{ISW}/c_0}$$

Avoiding tautology

- The floe speed c_f is imposed.
- The residence time is not imposed.
- The divergence of T_{res} near $c_f \approx c_0$ emerges from relative motion.

Two floes with identical speeds may still have different interaction times depending on their phase relative to the crest, but the *scaling* of T_{res} with c_f is already nontrivial and reveals a strong nonlinear response near locking.

Comparing to the paper While Carr et al. measure velocity, damage to ice (fatigue, breaking) depends on the duration of the forcing. A floe that passes through the wave quickly experiences a transient stress. A floe that is "quasi-locked" (speed close to wave speed) experiences the wave curvature for a long time. This is critical for understanding ice breakup. Probability of a floe to transition to arrest and cumulative transport per wave are also controlled by the residency time.

This diagnostic therefore provides a measurable and physically meaningful timescale that complements the speed measurements. This quantity is particularly interesting when :

- Near the advection-arrest regime
- When c_f is close to c_0
- When floes nearly surf the crest

5.3 Phase-averaged exposure of floes to ISW curvature

Here the question is : "What part of the wave do floes actually sample?"

Idea Carr et al. measure net floe speed, but they do not address where on the ISW the floe spends its time.

The exmamined model contains:

- a finite-width ISW
- a floe moving at $c_f \neq c_0$

That automatically implies phase drift between floe and wave. So the real question is: *Does a floe mostly experience the crest, the front, or the tail of the ISW?.* we therefore define an observable that measures where on the wave the floe sits (front, crest, or rear). Since curvature (and thus bending stress) is highest at the crest/trough, knowing if a floe preferentially "hangs out" at the crest is vital for predicting fracture. This matters because curvature controls pressure gradients, and shear and bending stresses depend on where the floe sits relative to the crest. For more realistic floes (polydisperse, rough, deformable and breakable...) and waves (asymmetric, subject to attenuation...), phase-averaged investigations are particularly of interest.

Observable Define ISW phase:

$$\xi(t) = x_f(t) - c_0 t$$

Then define a phase-averaged exposure:

$$P(\xi) = \frac{1}{T} \int_0^T \delta(\xi(t) - \xi)\, dt$$

In practice: we plot a histogram of where the floe spends time relative to the crest.

Avoid tautology

- We impose only a speed
- We do not impose phase locking
- The distribution emerges from relative advection

Two floes with similar c_f can still experience very different wave regions and accumulate very different stresses.

Comparing to the paper Carr et al. implicitly assume uniform forcing over the ISW. This diagnostic shows whether that assumption is valid and whether "effective forcing" depends on floe size. This is only a qualitative result. Again, this quantity is key near the advection-arrest regime, when residence times are long and when floes nearly surf the crest.

5.4 Cumulative displacement per ISW passage

Then , one can wonder : "How effective is one wave at transporting ice?"

Idea Instead of asking "what is the instantaneous speed?", we ask: *How much net displacement does one ISW cause?*. This reframes the interaction as a transport efficiency problem and translates the micro-scale physics (wave-floe interaction) into macro-scale oceanography (mass transport). Cumulative displacemenr answers: "How effective are these waves at redistributing ice in the Arctic?"

Observable For one ISW passage:

$$\Delta X = \int_0^{T_{res}} c_f\, dt$$

Dimensionless version:

$$\frac{\Delta X}{\lambda_{ISW}}$$

We get a dimensionless measure of floe transport:

- < 1: floe barely moves
- ~ 1: floe travels one wavelength
- $\gg 1$: strong transport

Differencec with speed Even though:

$$\Delta X \sim c_f T_{res}$$

We know :

$$T_{res} \sim \frac{1}{c_0 - c_f}$$

So:

$$\Delta X \sim \frac{c_f}{c_0 - c_f}$$

This is nonlinear and diverges, even though $c_f(l_f)$ is linear.

Comparing to the paper Carr et al. do not quantify transport per wave or efficiency of ice redistribution. Our results show that small changes in floe size near the locking regime cause orders of magnitude changes in transport.

Experimental relevance This would be directly measurable on the experimental setup:

- track floes before and after a wave
- no need for instantaneous velocity measurements

5.5 Floe–wave locking and regime diagram

Our last question is : "When does a floe become dynamically trapped?". The aim here is to provide a clear visual map of the system's behavior, moving beyond simple scatter plots to a dynamical classification (Transit vs. Locking).

Idea Our model reveals three distinct regimes:

- Fast wave / slow floe: Floe slips through the ISW
- Near-locking: Floe remains near the crest for long times
- Arrest: Floe cannot be advected

The paper mainly focuses on regime (1) and hints at (3).

Locking parameter A single dimensionless number:

$$\mathcal{L} = \frac{c_f}{c_0}$$

But instead of plotting it directly, we use it to classify behavior:

Regime	Criterion
Transit	$\mathcal{L} \ll 1$
Quasi-locked	$0.7 \lesssim \mathcal{L} < 1$
Arrest	$\mathcal{L} = 0$

Plot We plot a regime diagram :

- x-axis: $l_f/(2\lambda)$
- y-axis: qualitative regime
- color: residence time or displacement

This avoids tautology because we don't plot the linear $c_f(l_f)$ law itself but rather plot behavioral outcomes.

Comparing to the paper Carr et al. show data points, and we try to add a global summarize of the structures, transitions and thresholds in floes behaviors to it.

6 Interpretations and conclusions

This script demonstrates that using the imposed experimental law that has been established by Carr et.al, we managed to extract:

- nonlinear transport effects
- regime transitions
- time-integrated interaction physics

6.1 Figure Analysis

6.1.1 Residency Time vs. Floe Length

Figure Description: The plot shows the dimensionless residence time $T^* = T_{res} \cdot c_0/\lambda_{ISW}$ as a function of dimensionless floe length. The residence time is defined as the duration a floe remains within the characteristic width of the ISW.

Interpretation: The curve exhibits a vertical asymptote (singularity) as the floe length decreases towards a critical value (approx. $l_f/(2\lambda) \approx 0.65$).

- **Large Floes (Right):** For large floes, the floe speed c_f is significantly slower than the wave speed c_0. The relative velocity is large, meaning the wave passes underneath the floe quickly. The interaction is transient.
- **Small Floes (Left):** As floe length decreases, the empirical law predicts the floe speed increases. As $c_f \to c_0$, the relative velocity $c_{rel} = c_0 - c_f$ approaches zero. Consequently, the time required for the wave to pass the floe ($T \sim \lambda/c_{rel}$) diverges toward infinity.
- **Physical Implication:** Floes near this critical size enter a "quasi-locked" state where they surf the wave for extended periods, accumulating significant bending stress cycles.

6.1.2 Ice Transport Efficiency (Cumulative Displacement)

Figure Description: This figure illustrates the cumulative distance ΔX a floe travels during a single interaction with an ISW, normalized by the ISW wavelength.

Interpretation: Similar to the residence time, the transport efficiency increases non-linearly for smaller floes.

- **Transit Regime:** Large floes correspond to $\Delta X/\lambda_{ISW} < 1$. They are barely nudged by the wave; the wave passes them before they can be transported significantly.
- **Transport Regime:** As the floe size decreases and approaches the locking limit, the displacement grows exponentially. A single wave can transport a specific size of ice floe over distances many times larger than the wave itself. This suggests that ISWs act as a selective filter, stripping away smaller floes from the marginal ice zone while leaving larger floes behind.

6.1.3 Phase-Averaged Exposure

Figure Description: This histogram displays the probability density of the floe's position relative to the wave crest phase $\xi = (x_f - x_c)/\lambda_{ISW}$ over a long interaction window.

Interpretation: The distribution is effectively uniform (flat).

- **Meaning:** This indicates that for the chosen representative floe length ($l_f = 0.6\lambda$), the floe is in the *transit regime*. It does not lock to the wave. Because there is a constant non-zero relative velocity between the floe and the wave, the floe "drifts" continuously through the wave's phase.
- **Consequence:** The floe experiences all parts of the wave (trough, crest, zero-crossing) with equal probability. It is not preferentially exposed to the maximum curvature at the crest, unlike a locked floe which would show a delta-function-like peak at $\xi \approx 0$.

6.1.4 Floe-ISW Interaction Regimes

Figure Description: A regime diagram classifying the interaction based on the locking parameter $\mathcal{L} = c_f/c_0$. The color scale indicates the interaction intensity (e.g., residence time).

Interpretation: This synthesis identifies three distinct physical behaviors derived from the linear trend:

1. **Transit (Blue):** Floes are too large and slow. The wave passes through them. Transport is inefficient.
2. **Quasi-locked (Red/Purple):** The "sweet spot" where floe speed matches wave speed. Interaction times diverge, and transport is maximal. This is the most dangerous regime for ice integrity due to sustained forcing.
3. **Arrested (Yellow/Limit):** Predicted for very small l_f (or extrapolation outside the validity of the linear fit), where the floe might theoretically move faster than the wave or be pushed continuously, though physically this likely represents the transition to surfing.